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EXTENDED OCCUPANCY PROBABILITY DISTRIBUTION CRITICAL POINTS

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1. Introduction

In many instances of application of statistics, the underlying probabilistic problem can be described in terms of the following urn model: N balls are randomly allocated to K cells (so that each ball has probability $1/K$ of falling into the ith cell, $i = 1, 2, \ldots, K$). Each ball has probability p of staying in its cell and $1 - p$ of "falling through" (and hence not being available to 'fill' a cell). The random variable of interest here is the number x of 'occupied' cells, or equivalently the number $K - x$ of 'empty' cells. Several examples of situations leading to this urn model in biology, nuclear phyics and other fields are given in Feller (1968) and Harkness (1969).

Our purpose here is to tabulate critical points of the distribution for various values of N, K and p. The case when $p = 1$ has already been treated at length by Nicholson (1961), and we shall therefore not include it here. A description of our tables is given in the next section. They are set out in a similar form to Nicholson's table.

Critical points corresponding to some of the values of N and K not covered in our tables can be approximated using certain limit theorems. This is discussed in Section 3.

The classical occupancy distribution (corresponding to $p = 1$) gives the probability of exactly x occupied cells as

$$(1) \qquad H_1(x;K,N) = K^{-N}\binom{K}{x}\sum_{i=0}^{x}(-1)^{i}\binom{x}{i}(x-i)^N = K^{-N}\binom{K}{x}\Delta^x 0^N$$

$$(x = 0,1,\ldots,K)$$

For general p, the number of balls not falling through is distributed binomially with parameters N,p and so the extended occupancy distribution is the corresponding mixture of classical occupancy distributions. The probability, $H_p(x;K,N)$, of exactly x occupied cells is then the expected value of $H_1(x;K,J)$ with J distributed binomially with parameters N,p. Hence

$$(2) \qquad \begin{aligned} H_p(x;K,N) &= \sum_{j=0}^{N}\binom{N}{j}p^j(1-p)^{N-j}H_1(x;K,N-j) \\ &= \binom{K}{x}\sum_{i=0}^{x}(-1)^i\binom{x}{i}\sum_{j=0}^{N}\binom{N}{j}\left\{\frac{(x-i)p}{K}\right\}^j(1-p)^{N-j} \\ &= \binom{K}{x}\sum_{i=0}^{x}(-1)^i\binom{x}{i}\{1-p+(p/K)(x-i)\}^N \\ &= K^{-N}\binom{K}{x}\Delta^x\{(1-p)K+p.0\}^N \quad (x = 0,1,\ldots K) \end{aligned}$$

These probabilities satisfy the recursion relation

$$(3) \qquad \begin{aligned} KH_p(x;K,N) &= p(K+1-x)H_p(x-1;K,N-1) \\ &\quad + \{px+(1-p)K\}\,H_p(x;K,N-1) . \end{aligned}$$

The r-th factorial moment is

$$(4) \qquad \begin{aligned} \mu'_{(r)} &= K^{-N}\sum_{x=r}^{K}x^{(r)}\binom{K}{x}\Delta^x\{(1-p)\ K+p.0\}^N \\ &= K^{-N}K^{(r)}\Delta^r\sum_{x=r}^{K}\binom{K-r}{x-r}\Delta^{x-r}\{(1-p)\ K+p.0\}^N \\ &= K^{-N}K^{(r)}\Delta^r(K-pr+p.0)^N \\ &= K^{(r)}\Delta^r\{1-(p/K)(r-0)\}^N \end{aligned}$$

where $t^{(r)} = t(t-1)\ldots(t-r+1)$.

From (4) we obtain expressions for the expected value:

$$K\{1 - (1-p/K)^N\} \tag{5.1}$$

and the variance

$$K(K-1)\ \{1-2(1-p/K)^N + (1-2p/K)^N\} + K\{1-(1-p/K)^N\} \tag{5.2}$$

$$- K^2\{1-(1-p/K)^N\}$$

$$= K\ \{1-(1-p/K)^N\}(1-p/K)^N - K(K-1)\ \{(1-p/K)^{2N} - (1-2p/K)^N\}$$

The r-th factorial moment of the number of *empty* cells has the simpler form

$$K^{(r)}(1 - pr/K)^N \tag{6}$$

(Harkness (1969)).

2. Description of Our Tables

Using (3), we can find, for given K, X, p and lower tail area α, the smallest N, say N_1, such that $\Pr[x \le X \mid K, N_1, p] = \sum_{x=0}^{X} H_p(x,K,N) \le \alpha$. These are presented in Table I for $K = 2\ (1)\ 20$, $p = 0.75, 0.80, 0.90, 0.95, 0.99$, and $\alpha = 0.01, 0.025, 0.05, 0.10$, and N_1 up to 60, along with the the actual tail probabilities $\alpha_1 = P\ [x \le X \mid K, N_1, p]$. (For example, the entry $N_1 = 17$, $\alpha_1 = 0.019$, corresponding to $p = .99$, $K = 8$, $X = 5$ and $\alpha = 0.025$ in Table I means that $\Pr[x \le 5 \mid 8, 17, 0.99] = 0.019$, $\Pr[x \le 5 \mid 8, N, 0.99] > 0.025$ for $N < 17$, and that $\Pr[x \le 5 \mid 8, N, 0.99] < 0.019$ for $N > 17$). There is one exception to this system. If there is a value N_1 such that $\Pr[X \le x \mid K, N_1, p]$ lies between α and $\alpha + 0.0005$ this is the value given in the tables. The corresponding value of α_1 is given as $\alpha +$.

Similarly, using the recursion relations; for the same values of K, X, p and α we have tabulated the upper tail critical values N_2 and the corresponding probabilities α_2 in Table II. Here, for given K, X, p and α, N_2 is the largest value of N such that $P_r[x \geq X \mid K, N, p] \leq \alpha$, and $\alpha_2 = P_r[x \geq X \mid K, N_2, p]$. (For example, the entry $N_2 = 12$, $\alpha_2 = .021$, corresponding to $p = 0.80$, $K = 10$, $X = 9$ in Table II means that $P_r[x \geq 9 \mid 10, 12, 0.80] = 0.021$, $P_r[x \geq 9 \mid 10, N, 0.80] < 0.025$ for $N \leq 12$, and that $P_r[x \geq 9 | 10, N, 0.80] > 0.021$ for $N > 12$.) Dash marks (-) in table II indicate the non-existence of N_2 for given K, X and α. Here again, as an exception if there is an N_2 such that $\alpha < P_r[x \geq X \mid K, N, p] < \alpha + 0.0005$ this is the value given in the tables, with α_2 given as α^+.

In both lower and upper tail tables the entry > 60 is used when the exact value is beyond the range of calculations we have carried out.

3. Approximations

Harkness (1969) has suggested that a binomial with parameters K, $1 - (1-p/K)^N$ would give a good approximation to the distribution (2). This approximation gives the correct mean (see (5.1)) and is exact for $K = 1$. It would be expected to give good results for p small, since the mixing distribution of J (see Section 2) may then be approximated by a Poisson (with parameter Np), and as Harkness shows a mixture of classical occupancy distributions with such a Poisson mixing distribution is, in fact a Binomial distribution.

If the binomial approximation is good, then Molenaar's (1970) normal approximation to the binomial may be used to get approximate expressions for N_1 and N_2. Writing

(7) $$\omega(N) = 1 - (1-p/K)^N$$

we have from Molenaar's 'quick work' approximation (1970, p. 110) for $\alpha_j \leq 0.05$

(8.1) $$2(X+1)^{\frac{1}{2}} \{1-\omega(N_1)\}^{\frac{1}{2}} - 2(K-X)^{\frac{1}{2}} \{\omega(N_1)\}^{\frac{1}{2}} \doteqdot -V_{\alpha_1}$$

(8.2) $$2X^{\frac{1}{2}} \{1-\omega(N_2)\}^{\frac{1}{2}} - 2(K-X-1)^{\frac{1}{2}} \{\omega(N_2)\}^{\frac{1}{2}} \doteqdot V_{\alpha_2}$$

where $(\sqrt{2\pi})^{-1} \int_{V_\alpha}^{\infty} \exp(-\frac{1}{2}u^2)\, du = \alpha$.

The equations for $\alpha_j > 0.05$ are of similar form.

In order to solve equations of form

$$A\sqrt{1-\omega(N)} - B\sqrt{\omega(N)} = V$$

we write $\omega(N) = \sin^2\theta$ and $A/\sqrt{A^2+B^2} = \sin\phi$ and get

$$\sin(\phi-\theta) = V/\sqrt{A^2+B^2}$$

where

(9) $$\omega(N) \doteqdot \sin^2 [\sin^{-1} \frac{A}{\sqrt{(A^2+B^2)}} - \sin^{-1} \frac{V}{\sqrt{(A^2+B^2)}})$$

$$\doteqdot \left\{\frac{A}{\sqrt{(A^2+B^2)}} \sqrt{1 - \frac{V^2}{A^2+B^2}} - \frac{BV}{\sqrt{A^2+B^2}}\right\}^2$$

$$\doteqdot (A^2+B^2)^{-2} (A\sqrt{(A^2+B^2-V^2)} - BV)^2$$

(when $A^2+B^2 < V^2$ this equation is not applicable, but since A^2+B^2 is about $4K$ while hhe largest V^2 we need is $V_{0.01}^2 \doteqdot 5.4$, this imposes little restriction.)

Once $\omega(N_j)$ has been found we can calculate the corresponding N_j from (7).

Some trial calculations with $K = 20$ indicated that when $p \geq 0.75$ this approximation gives values of N_1 which are too big by some 2-4 units and of N_2 which are too small by some 1-2 units, at any rate for $\alpha = 0.025 - 0.05$.

Further asymptotic formulae will be found in Samuel-Cahn, (1974).

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Lower tail critical points

K	X	p = .75							
		Lower tail probability level α							
		0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
2	1	12	.007	10	.018	8	.047	7	.074
3	1	9	.006	7	.023	6	.046	5	.092
	2	20	.010$^-$	17	.023	15	.040	12	.094
4	1	8	.005	7	.012	6	.027	5	.061
	2	14	.008	12	.021	11	.033	9	.083
	3	29	.010$^-$	25	.022	22	.041	18	.094
5	1	7	.008	6	.020	5	.047	5	.047
	2	12	.007	10	.024	9	.042	8	.074
	3	20	.008	17	.022	15	.045	13	.089
	4	38	.010$^+$	33	.023	29	.045	24	.099
6	1	7	.006	6	.015	5	.040	4	.099
	2	11	.007	10	.013	8	.049	7	.093
	3	16	.010$^+$	14	.025$^+$	13	.039	11	.092
	4	26	.008	22	.025$^+$	20	.044	17	.100$^-$
	5	48	.010$^-$	41	.025$^+$	36	.049	31	.094
7	1	7	.005	6	.013	5	.035	4	.090
	2	10	.009	9	.018	8	.036	7	.073
	3	15	.007	13	.020	12	.034	10	.093
	4	21	.009	19	.020	17	.041	15	.082
	5	32	.009	28	.023	25	.046	22	.091
	6	58	.010$^-$	50	.024	44	.047	38	.092

K	X	p = .75							
		Lower tail probability level α							
		0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
8	1	7	.004	6	.011	5	.032	4	.084
	2	10	.006	9	.013	8	.029	7	.060
	3	14	.007	12	.022	11	.038	10	.067
	4	19	.008	17	.019	15	.045	14	.068
	5	26	.009	23	.024	21	.044	19	.080
	6	38	.010$^+$	34	.023	30	.050$^-$	27	.089
	7	>60	–	59	.024	52	.047	45	.093
9	1	6	.010$^+$	6	.010	5	.029	4	.080
	2	10	.005	8	.024	8	.024	7	.051
	3	13	.008	12	.015	11	.028	9	.092
	4	17	.010$^+$	16	.017	14	.043	13	.069
	5	23	.009	21	.020	19	.040	17	.081
	6	31	.010$^-$	28	.022	25	.049	23	.082
	7	45	.009	40	.023	36	.046	32	.089
	8	>60	–	>60	–	60	.048	52	.095
10	1	6	.010$^-$	6	.010	5	.027	4	.076
	2	9	.009	8	.020	7	.045	6	.100$^-$
	3	13	.006	11	.021	10	.041	9	.076
	4	16	.010$^+$	15	.018	14	.030	12	.081
	5	21	.010$^-$	19	.022	17	.050$^+$	16	.074
	6	28	.008	25	.021	23	.039	20	.097
	7	27	.008	33	.022	30	.044	27	.086
	8	52	.009	46	.024	42	.044	37	.092

Lower tail critical points

p = .75

Lower tail probability level α

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
10	9	>60	-	>60	-	>60	-	59	.097
11	1	6	.009	6	.009	5	.026	4	.074
	2	9	.007	8	.018	7	.041	6	.091
	3	12	.008	11	.017	10	.034	9	.065
	4	16	.007	14	.021	13	.037	12	.064
	5	20	.008	18	.021	17	.033	15	.078
	6	25	.009	23	.020	21	.042	19	.082
	7	32	$.01^{-}$	29	.023	27	.040	24	.089
	8	42	$.01^{-}$	38	.022	35	.042	31	.092
	9	59	.009	52	$.025^{-}$	47	.049	42	.096
	10	>60	-	>60	-	>60	-	>60	-
12	1	6	.008	5	$.025^{+}$	5	.025	4	.071
	2	9	.006	8	.016	7	.037	6	.085
	3	12	.007	11	.014	10	.028	9	.057
	4	15	.009	14	.016	13	.029	11	.089
	5	19	.008	17	$.023_{+}$	16	.037	14	.092
	6	24	.007	21	.025	20	.038	18	.080
	7	30	.007	27	.020	25	.037	22	.093
	8	37	.009	34	.020	31	.042	28	.085
	9	48	.009	43	.023	39	.049	35	$.100^{-}$
	10	>60	-	59	.023	53	.049	48	.089
	11	>60	-	>60	-	>60	-	>60	-
13	1	6	.008	5	.024	5	.024	4	.070

p = .75

Lower tail probability level α

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
13	2	9	.006	8	.014	7	.035	6	.080
	3	12	.005	10	$.025^{-}$	10	.025	8	$.100^{-}$
	4	15	.007	13	.023	12	.043	11	.076
	5	18	$.010^{-}$	17	.016	15	.046	14	.074
	6	23	.007	21	.016	19	.039	17	.086
	7	28	.007	25	.022	23	.044	21	.085
	8	34	.008	31	.020	28	.047	26	.081
	9	42	.009	38	.023	35	.044	32	.085
	10	54	.009	48	$.025^{-}$	44	.048	40	.092
	11	>60	-	>60	-	59	.049	53	.095
	12	>60	-	>60	-	>60	-	>60	-
14	1	6	.008	5	.023	5	.023	4	.068
	2	9	.005	8	.013	7	.032	6	.076
	3	12	.005	10	.021	9	.046	8	.092
	4	15	.005	13	.019	12	.036	11	.066
	5	18	.007	16	.021	15	.036	13	.099
	6	22	.007	20	.018	18	.043	17	.067
	7	26	$.009_{+}$	24	.019	22	$.042_{+}$	20	.085
	8	31	.010	29	.020	26	.050	24	.091
	9	38	.009	35	.021	32	.045	29	.094
	10	47	.009	43	.021	39	.048	36	.086
	11	59	$.010^{-}$	54	.022	49	.048	45	.088
	12	>60	-	>60	-	>60	-	59	.092
	13	>60	-	>60	-	>60	-	>60	-

Lower tail critical points

p = .75

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
15	1	6	.007	5	.023	5	.023	4	.067
	2	9	.005	8	.012	7	.030	6	.073
	3	11	.009	10	.020	9	.042	8	.086
	4	14	.008	13	.016	12	.031	11	.059
	5	17	.009	16	.017	15	.030	13	.085
	6	21	.008	19	.021	18	.033	16	.083
	7	25	.008	23	.019	21	.044	19	.093
	8	30	.008	27	.023	25	.046	23	.087
	9	36	.007	32	$.025^-$	30	.044	27	.099
	10	43	.008	39	.022	36	.045	33	.088
	11	52	.009	47	.024	44	.043	40	.089
	12	>60	-	59	.024	54	.049	49	.098
	13	>60	-	>60	-	>60	-	>60	-
	14	>60	-	>60	-	>60	-	>60	-
16	1	6	.007	5	.022	5	.022	4	.066
	2	9	.004	8	.011	7	.029	6	.070
	3	11	.008	10	.018	9	.038	8	.080
	4	14	.007	13	.014	12	.028	10	.098
	5	17	.007	15	$.025^-$	14	.043	13	.074
	6	20	.009	19	.016	17	.043	16	.069
	7	24	.009	22	.021	20	.049	19	.074
	8	29	.007	26	.022	24	.045	22	.090
	9	34	.007	31	.020	29	.037	26	.091
	10	40	.008	36	.024	34	.041	31	.086
	11	47	.009	43	.023	40	.045	37	.084

p = .75

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
16	12	57	.009	52	.024	48	.047	44	.092
	13	>60	-	>60	-	60	.044	54	.096
	14	>60	-	>60	-	>60	-	>60	-
	15	>60	-	>60	-	>60	-	>60	-
17	1	6	.007	5	.022	5	.022	4	.065
	2	9	.004	8	.011	7	.028	6	.068
	3	11	.007	10	.016	9	.036	8	.076
	4	14	.005	12	.024	11	.048	10	.090
	5	17	.006	15	.021	14	.037	13	.066
	6	20	.007	18	.021	17	.035	15	.093
	7	23	$.010^-$	21	$.025^-$	20	.039	18	.090
	8	27	$.010^-$	25	.022	23	.048	21	.098
	9	32	.008	29	.024	27	.048	25	.089
	10	37	$.010^-$	34	.024	32	.043	29	.096
	11	44	.008	40	.024	37	.049	34	.097
	12	52	.009	48	.021	44	.046	40	.099
	13	>60	-	57	.023	53	.045	48	.097
	14	>60	-	>60	-	>60	-	59	.095
	15	>60	-	>60	-	>60	-	>60	-
	16	>60	-	>60	-	>60	-	>60	-
18	1	6	.007	5	.021	5	.021	4	.064
	2	8	$.010^+$	8	.010	7	.027	6	.066
	3	11	.007	10	.015	9	.033	8	.072
	4	14	.005	12	.022	11	.044	10	.084

Lower tail critical points

		p = .75							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
18	5	16	.009	15	.018	14	.033	13	.059
	6	19	.010$^-$	18	.017	16	.050$^-$	15	.081
	7	23	.007	21	.019	19	.049	18	.075
	8	27	.007	24	.024	23	.037	21	.079
	9	31	.008	28	.023	26	.048	24	.093
	10	36	.008	33	.020	31	.038	28	.091
	11	41	.010$^-$	38	.022	35	.049	33	.081
	12	48	.009	44	.024	41	.047	38	.089
	13	57	.008	52	.023	48	.048	44	.098
	14	>60	-	>60	-	57	.050$^-$	53	.088
	15	>60	-	>60	-	>60	-	>60	-
	16	>60	-	>60	-	>60	-	>60	-
	17	>60	-	>60	-	>60	-	>60	-
19	1	6	.006	5	.021	5	.021	4	.063
	2	8	.009	8	.009	7	.026	6	.064
	3	11	.006	10	.014	9	.032	8	.069
	4	13	.010$^-$	12	.020	11	.040	10	.078
	5	16	.008	15	.016	14	.029	12	.094
	6	19	.008	17	.025$^+$	16	.043	15	.072
	7	22	.009	20	.025$^+$	19	.041	17	.099
	8	26	.008	24	.019	22	.043	20	.096
	9	30	.008	27	.025$^-$	26	.036	24	.074
	10	34	.009	32	.019	29	.050$^-$	27	.091
	11	39	.010$^-$	36	.024	34	.042	31	.094
	12	45	.010$^-$	42	.021	39	.044	36	.089

		p = .75							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
19	13	52	.010$^+$	48	.024	45	.046	42	.083
	14	>60	-	56	.025$^-$	52	.050$^+$	48	.098
	15	>60	-	>60	-	>60	-	57	.094
	16	>60	-	>60	-	>60	-	>60	-
	17	>60	-	>60	-	>60	-	>60	-
	18	>60	-	>60	-	>60	-	>60	-
20	1	6	.007	5	.021	5	.021	4	.062
	2	8	.009	7	.025$^-$	7	.025	6	.062
	3	11	.006	10	.013	9	.030	8	.066
	4	13	.009	12	.018	11	.037	10	.074
	5	16	.007	15	.014	13	.048	12	.087
	6	19	.007	17	.022	16	.038	15	.064
	7	22	.007	20	.021	19	.034	17	.087
	8	25	.009	23	.023	22	.035	20	.081
	9	29	.008	27	.018	25	.041	23	.084
	10	33	.009	31	.018	29	.037	26	.097
	11	38	.008	35	.021	33	.039	30	.091
	12	43	.009	40	.021	37	.047	34	.099
	13	49	.010$^-$	46	.020	43	.041	39	.098
	14	57	.009	52	.025$^+$	49	.046	45	.096
	15	>60	-	>60	-	57	.045	52	.099
	16	>60	-	>60	-	>60	-	>60	-
	17	>60	-	>60	-	>60	-	>60	-
	18	>60	-	>60	-	>60	-	>60	-
	19	>60	-	>60	-	>60	-	>60	-

Upper tail critical points

K	X	0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
		p = .75							
		Upper tail probability level α							
3	3	-	-	-	-	-	-	3	.094
4	4	-	-	-	-	4	.030	5	.093
5	4	-	-	-	-	-	-	4	.061
	5	5	.009	5	.009	6	.034	7	.077
6	4	-	-	-	-	-	-	4	.088
	5	-	-	5	.022	5	.022	6	.077
	6	6	.003	7	.012	8	.031	10	.099
7	5	-	-	-	-	5	.036	5	.036
	6	6	.008	6	.008	7	.031	8	.075
	7	8	.004	10	$.025^+$	11	.045	12	.073
8	5	-	-	-	-	5	.049	5	.049
	6	-	-	6	.014	6	.014	7	.053
	7	7	.003	8	.012	9	.033	10	.067
	8	11	$.010^+$	12	.020	13	.034	15	.077
9	5	-	-	-	-	-	-	5	.061
	6	-	-	6	.020	6	.020	7	.076
	7	7	.005	8	.023	8	.023	9	.059
	8	9	.005	10	.014	11	.030	13	.092
	9	13	.008	15	.025	16	.040	18	.077

K	X	0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
		p = .75							
		Upper tail probability level α							
10	5	-	-	-	-	-	-	5	.072
	6	-	-	-	-	6	.027	7	.098
	7	7	.008	7	.008	8	.035	9	.087
	8	9	.009	9	.009	10	.027	11	.057
	9	11	.005	12	.013	14	.046	15	.073
	10	15	.007	17	.019	19	.041	22	.095
11	5	-	-	-	-	-	-	5	.082
	6	-	-	-	-	6	.033	6	.033
	7	-	-	7	.011	8	.048	8	.048
	8	8	.003	9	.015	10	.042	11	.087
	9	10	.004	11	.012	12	.027	14	.088
	10	13	.005	15	.022	16	.037	18	.084
	11	18	.009	20	.021	22	.041	25	.087
12	5	-	-	-	-	-	-	5	.091
	6	-	-	-	-	6	.040	6	.040
	7	-	-	7	.015	7	.015	8	.060
	8	8	.005	9	.022	9	.022	10	.059
	9	10	.006	11	.020	12	.044	13	.083
	10	12	.005	14	$.025^+$	15	.046	16	.074
	11	16	$.010^+$	17	.018	19	.045	21	.090
	12	21	$.010^+$	23	.022	25	.040	29	.097
13	5	-	-	-	-	-	-	5	.099

Upper tail critical points

K	X	0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
		p = .75							
		Upper tail probability level α							
13	6	-	-	-	-	6	.046	6	.046
	7	-	-	7	.018	7	.018	8	.073
	8	8	.006	8	.006	9	.029	10	.077
	9	10	.010−	10	.010	11	.029	12	.064
	10	12	.009	13	.021	14	.043	15	.075
	11	14	.005	16	.023	17	.039	19	.091
	12	18	.009	20	.023	21	.035	24	.092
	13	23	.007	26	.022	29	.048	32	.087
14	6	-	-	-	-	-	-	6	.051
	7	-	-	7	.022	7	.022	8	.085
	8	8	.008	8	.008	9	.037	10	.094
	9	9	.003	10	.014	11	.040	12	.084
	10	11	.004	12	.014	13	.033	14	.064
	11	14	.010−	15	.021	16	.040	17	.066
	12	16	.005	18	.020	19	.033	21	.074
	13	20	.007	22	.018	24	.039	27	.092
	14	26	.008	29	.021	32	.044	36	.092
15	6	-	-	-	-	-	-	6	.056
	7	-	-	7	.025+	7	.025	8	.096
	8	8	.010−	8	.010	9	.045	9	.045
	9	9	.004	10	.018	10	.018	11	.051
	10	11	.006	12	.019	13	.045	14	.086
	11	13	.006	14	.016	15	.033	17	.098
	12	16	.010+	17	.020	18	.036	20	.087

K	X	0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
		p = .75							
		Upper tail probability level α							
15	13	19	.010−	20	.017	22	.042	24	.083
	14	23	.009	25	.021	27	.041	30	.090
	15	29	.008	32	.020	36	.049	40	.095
16	6	-	-	-	-	-	-	6	.061
	7	-	-	-	-	7	.029	7	.029
	8	-	-	8	.012	8	.012	9	.052
	9	9	.005	10	.022	10	.022	11	.062
	10	11	.008	11	.008	12	.026	13	.059
	11	13	.009	14	.023	15	.047	16	.083
	12	15	.007	16	.017	17	.032	19	.088
	13	18	.010−	19	.018	21	.050−	22	.073
	14	21	.008	23	.022	25	.048	27	.089
	15	25	.007	28	.023	30	.042	33	.086
	16	32	.008	36	.024	39	.044	44	.096
17	6	-	-	-	-	-	-	6	.066
	7	-	-	-	-	7	.032	7	.032
	8	-	-	8	.014	8	.014	9	.060
	9	9	.006	9	.006	10	.027	11	.073
	10	10	.002	11	.011	12	.033	13	.073
	11	12	.004	13	.013	14	.031	15	.062
	12	14	.004	16	.025−	17	.047	18	.079
	13	17	.008	18	.017	20	.050+	21	.077
	14	20	.009	21	.016	23	.042	25	.086
	15	23	.007	25	.018	27	.039	30	.092

Upper tail critical points

K	X	p = .75: 0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
17	16	28	.009	31	.024	33	.042	37	.099
	17	35	.008	39	.022	43	.047	48	.097
18	6	-	-	-	-	-	-	6	.070
	7	-	-	-	-	7	.035	7	.035
	8	-	-	8	.016	8	.016	9	.067
	9	9	.007	9	.007	10	.032	11	.084
	10	10	.003	11	.013	12	.040	13	.087
	11	12	.005	13	.017	14	.040	15	.079
	12	14	.006	15	.016	16	.034	17	.063
	13	16	.005	18	.025^{+}	19	.045	20	.072
	14	19	.008	20	.016	22	.044	24	.096
	15	22	.008	24	.023	25	.035	28	.095
	16	26	.010^{-}	28	.022	30	.042	33	.093
	17	31	.010^{-}	34	.025^{-}	36	.041	40	.093
	18	39	.010^{+}	43	.025^{-}	47	.049	52	.096
19	6	-	-	-	-	-	-	6	.074
	7	-	-	-	-	7	.038	7	.038
	8	-	-	8	.018	8	.018	9	.074
	9	9	.008	9	.008	10	.036	11	.095
	10	10	.003	11	.016	12	.047	12	.047
	11	12	.006	13	.021	14	.050^{-}	15	.095
	12	14	.008	15	.021	16	.045	17	.080
	13	16	.008	17	.018	18	.035	20	.097
	14	[illegible]	.006	20	.024	21	.042	23	.096
	15	21	.008	23	.025^{-}	24	.039	26	.082
	16	24	.007	26	.019	28	.042	30	.078
	17	28	.008	31	.025^{-}	33	.045	36	.093
	18	34	.010^{+}	37	.025^{-}	40	.049	43	.087
	19	42	.010^{-}	46	.022	50	.044	56	.095
20	6	-	-	-	-	-	-	6	.078
	7	-	-	-	-	7	.041	7	.041
	8	-	-	8	.020	8	.020	9	.080
	9	9	.009	9	.009	10	.041	10	.041
	10	10	.004	11	.019	11	.019	12	.055
	11	12	.008	12	.008	13	.026	14	.059
	12	13	.003	14	.011	15	.027	17	.099
	13	15	.004	17	.024	18	.047	19	.079
	14	18	.009	19	.019	20	.035	22	.089
	15	20	.007	22	.023	23	.038	25	.085
	16	23	.007	25	.022	27	.049	29	.094
	17	27	.010^{+}	29	.024	31	.048	33	.083
	18	31	.009	33	.019	36	.046	39	.091
	19	36	.008	40	.024	43	.047	47	.094
	20	45	.009	50	.024	54	.045	60	.093

Lower tail critical points

p = .80 — Lower tail probability level α

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
2	1	11	.007	9	.020	8	.034	6	.093
3	1	8	.007	7	.014	6	.031	5	.066
	2	19	.008	16	.021	14	.039	11	.098
4	1	7	.007	6	.016	5	.040	4	.098
	2	13	.008	11	.021	10	.035	8	.096
	3	27	$.010^{-}$	23	.024	20	.046	17	.089
5	1	7	.004	6	.011	5	.029	4	.078
	2	11	.007	10	.014	8	.049	7	.091
	3	18	$.010^{-}$	16	.020	14	.043	12	.090
	4	36	.009	31	.022	27	.045	23	.089
6	1	6	.008	5	.023	5	.023	4	.066
	2	10	.007	9	.015	8	.030	7	.061
	3	15	.009	13	.024	12	.039	10	.099
	4	24	.008	21	.021	19	.039	16	.094
	5	45	$.010^{-}$	39	.023	34	.046	29	.093
7	1	6	.006	5	.020	5	.020	4	.059
	2	9	.009	8	.021	7	.045	6	.097
	3	14	.006	12	.020	11	.035	10	.061
	4	20	.007	17	$.025^{+}$	16	.037	14	.079
	5	30	.008	26	.023	23	.049	21	.080
	6	54	$.010^{-}$	47	.023	41	.048	35	.098

p = .80 — Lower tail probability level α

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
8	1	6	.005	5	.017	5	.017	4	.054
	2	9	.006	8	.015	7	.036	6	.081
	3	13	.006	11	.022	10	.041	9	.075
	4	17	$.010^{+}$	16	.016	14	.042	13	.066
	5	24	$.010^{-}$	22	.019	20	.037	17	.097
	6	36	.009	32	.021	28	.049	25	.091
	7	>60	-	55	.024	48	$.050^{+}$	42	.093
9	1	6	.005			4	$.050^{-}$	4	.050
	2	9	.005	8	.012	7	.029	6	.070
	3	12	.007	11	.015	10	.029	9	.057
	4	16	.008	14	$.025^{-}$	13	.041	12	.069
	5	21	$.010^{+}$	19	.023	18	.034	16	.074
	6	29	.009	26	.022	24	.040	21	.091
	7	42	.009	37	.024	34	.042	30	.087
	8	>60	-	>60	-	56	.048	48	$.100^{+}$
10	1	6	.004	5	.014	4	.047	4	.047
	2	8	$.010^{-}$	7	$.025^{-}$	7	.025	6	.062
	3	12	.005	10	.022	9	.045	8	.090
	4	15	.009	14	.015	12	.049	11	.084
	5	20	.007	18	.018	16	.045	15	.069
	6	26	.008	23	.022	21	.044	19	.085
	7	34	.009	31	.020	28	.043	25	.089
	8	48	$.010^{-}$	43	.023	39	.045	34	$.100^{-}$
	9	>60	-	>60	-	>60	-	55	.099

Lower tail critical points

p = .80

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
11	1	6	.004	5	.013	4	.045	4	.045
	2	8	.008	7	.022	7	.022	6	.056
	3	11	.008	10	.017	9	.037	8	.077
	4	14	.010+	13	.019	12	.036	11	.065
	5	18	.010+	17	.017	15	.046	14	.074
	6	23	.010+	21	.023	19	.049	18	.071
	7	30	.009	27	.022	25	.041	22	.097
	8	39	.010−	35	.024	32	.048	29	.090
	9	55	.009	49	.023	44	.048	39	.099
	10	>60	-	>60	-	>60	-	>60	-
12	1	6	.004	5	.013	4	.043	4	.043
	2	8	.007	7	.019	7	.019	6	.051
	3	11	.006	10	.014	9	.031	8	.067
	4	14	.007	13	.014	12	.028	10	.096
	5	18	.006	16	.019	15	.032	13	.089
	6	22	.008	20	.020	18	.047	17	.070
	7	27	.010−	25	.020	23	.040	21	.078
	8	34	.010+	31	.023	29	.040	26	.087
	9	44	.010+	40	.024	37	.043	33	.093
	10	>60	-	55	.023	50	.046	44	.099
	11	>60	-	>60	-	>60	-	>60	-
13	1	6	.003	5	.012	4	.042	4	.042
	2	8	.006	7	.018	6	.047	6	.047
	3	11	.005	10	.012	9	.027	8	.059

p = .80

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
13	4	14	.005	12	.022	11	.043	10	.081
	5	17	.007	15	.024	14	.041	13	.071
	6	21	.007+	19	.020	17	.050−	16	.077
	7	25	.010	23	.023	21	.049	19	.100+
	8	31	.010−	28	.025+	26	.047	24	.085
	9	39	.009	35	.025−	33	.040	30	.081
	10	50	.009	45	.023	41	.049	37	.097
	11	>60	-	>60	-	55	.05	50	.090
	12	>60	-	>60	-	>60	-	>60	-
14	1	6	.003	5	.012	4	.040	4	.040
	2	8	.006	7	.016	6	.044	6	.044
	3	10	.010−	9	.023	9	.023	8	.054
	4	13	.009	12	.018	11	.036	10	.071
	5	16	.010−	15	.018	14	.032	12	.099
	6	20	.008	18	.022	17	.036	15	.093
	7	24	.009	22	.021	20	.048	19	.071
	8	29	.009	27	.019	25	.038	23	.073
	9	35	.010−	32	.024	30	.042	27	.094
	10	44	.008	40	.021	37	.041	33	.096
	11	55	.010−	50	.024	46	.046	42	.088
	12	>60	-	>60	-	>60	-	55	.093
	13	>60	-	>60	-	>60	-	>60	-
15	1	6	.003	5	.011	4	.039	4	.039
	2	8	.005	7	.015	6	.042	6	.042

Lower tail critical points

p = .80

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
15	3	10	.009	9	.021	8	.049	8	.049
	4	13	.007	12	.015	11	.031	10	.062
	5	16	.007	15	.014	13	.047	12	.084
	6	19	.009	18	.016	16	.045	15	.074
	7	23	.008	21	.022	20	.034	18	.079
	8	28	.007	25	.023	23	.049	21	.098
	9	33	.008	30	.023	28	.042	26	.077
	10	40	.008	36	.023	34	.039	31	.082
	11	48	$.010^{-}$	44	.023	41	.043	37	.094
	12	>60	-	55	.024	51	.045	46	.094
	13	>60	-	>60	-	>60	-	60	.097
	14	>60	-	>60	-	>60	-	>60	-
16	1	6	.003	5	.011	4	.039	4	.039
	2	8	.005	7	.014	6	.040	6	.040
	3	10	.008	9	.019	8	.045	8	.045
	4	13	.006	12	.012	11	.027	10	.055
	5	16	.005	14	.021	13	.040	12	.073
	6	19	.007	17	.021	16	.036	15	.061
	7	22	.009	20	.024	19	.039	17	.095
	8	26	$.010^{-}$	24	.022	22	.049	21	.072
	9	31	.009	29	.018	26	$.050^{+}$	24	.095
	10	37	.008	34	.020	31	.048	29	.083
	11	44	$.009^{-}$	40	.024	37	.048	34	.093
	12	53	$.010^{-}$	49	.021	45	.045	41	.093
	13	>60	-	60	$.025^{+}$	56	.045	51	.089

p = .80

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
16	14	>60	-	>60	-	>60	-	>60	-
	15	>60	-	>60	-	>60	-	>60	-
17	1	6	.003	5	.011	4	.038	4	.038
	2	8	.004	7	.013	6	.038	6	.038
	3	10	.007	9	.017	8	.042	7	.097
	4	13	.005	11	.023	10	$.050^{-}$	10	.050
	5	15	.009	14	.017	13	.034	12	.064
	6	18	.009	17	.016	15	$.050^{+}$	14	.086
	7	22	.006	20	.018	18	.048	17	.077
	8	25	$.010^{-}$	23	.023	22	.036	20	.080
	9	30	.007	27	.023	25	.048	23	.096
	10	35	.008	32	.021	30	.039	27	.095
	11	41	.008	37	$.025^{-}$	35	.042	32	.089
	12	48	$.010^{-}$	44	.024	41	.046	38	.086
	13	58	.009	53	.024	49	.048	45	.093
	14	>60	-	>60	-	>60	-	55	.096
	15	>60	-	>60	-	>60	-	>60	-
	16	>60	-	>60	-	>60	-	>60	-
18	1	5	$.010^{+}$	5	.010	4	.037	4	.037
	2	8	.004	7	.012	6	.036	5	$.100^{+}$
	3	10	.006	9	.016	8	.039	7	.092
	4	12	.009	11	.021	10	.045	9	.094
	5	15	.007	14	.015	13	.029	12	.057
	6	18	.007	16	.024	15	.043	14	.074

Lower tail critical points

K	X	p = .80							
		Lower tail probability level α							
		0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
18	7	21	.008+	19	.023	18	.039	17	.063
	8	24	.010+	23	.017	21	.041	19	.095
	9	28	.010	26	.023	24	.050-	23	.072
	10	33	.009-	30	.025-	28	.049	26	.092
	11	38	.010-	35	.024	33	.043	30	.097
	12	45	.008	41	.023	38	.049	35	.097
	13	53	.008	48	.025-	45	.046	41	.098
	14	>60	-	58	.022	54	.044	49	.095
	15	>60	-	>60	-	>60	-	60	.093
	16	>60	-	>60	-	>60	-	>60	-
	17	>60	-	>60	-	>60	-	>60	-
19	1	5	.010+	5	.010	4	.036	4	.036
	2	8	.004	7	.012	6	.035	5	.097
	3	10	.006	9	.015	8	.037	7	.087
	4	12	.008	11	.019	10	.042	9	.088
	5	15	.006+	14	.013	13	.026	11	.096
	6	17	.010+	16	.020	15	.036	14	.065
	7	20	.010	19	.018	18	.031	16	.087
	8	24	.007+	22	.020	21	.032	19	.078
	9	27	.010	25	.025-	24	.037	22	.081
	10	32	.008+	29	.024	27	.048	25	.004
	11	36	.010	34	.020	31	.050+	29	.090
	12	42	.009	39	.021	36	.048	34	.079
	13	49	.009	45	.023	42	.045	39	.086

K	X	p = .80							
		Lower tail probability level α							
		0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
19	14	57	.010-	53	.021	49	.046	45	.094
	15	>60	-	>60	-	58	.047	53	.097
	16	>60	-	>60	-	>60	-	>60	-
	17	>60	-	>60	-	>60	-	>60	-
	18	>60	-	>60	-	>60	-	>60	-
20	1	5	.010-	5	.010	4	.036	4	.036
	2	8	.004	7	.011	6	.034	5	.095
	3	10	.005	9	.014	8	.035	7	.084
	4	12	.007	11	.017	10	.038	9	.083
	5	15	.005	13	.023	12	.046	11	.089
	6	17	.009	16	.017	15	.031	14	.057
	7	20	.008	19	.015+	17	.045	16	.075
	8	23	.009	21	.025	20	.041	19	.065
	9	27	.007	25	.018	23	.043+	21	.096
	10	31	.007	28	.024	26	.050	25	.072
	11	35	.009	32	.024	30	.047	28	.088
	12	40	.009	37	.022	35	.040	32	.090
	13	46	.009	42	.025-	40	.041	37	.084
	14	53	.009	49	.022	46	.042	42	.095
	15	>60	-	57	.023	53	.047	49	.091+
	16	>60	-	>60	-	>60	-	57	.100
	17	>60	-	>60	-	>60	-	>60	-
	18	>60	-	>60	-	>60	-	>60	-
	19	>60	-	>60	-	>60	-	>60	-

Upper tail critical points

K	X	p = .80							
		Upper tail probability level α							
		0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
4	4	-	-	-	-	4	.038	4	.038
5	4	-	-	-	-	-	-	4	.079
	5	-	-	5	.013	6	.045	7	.098
6	5	-	-	-	-	5	.030	5	.030
	6	6	.004	7	.017	8	.042	9	.079
7	5	-	-	-	-	5	.049	5	.049
	6	-	-	6	.011	7	.044	7	.044
	7	8	.006	9	.017	10	.035	12	.096
8	5	-	-	-	-	-	-	5	.067
	6	-	-	6	.020	6	.020	7	.075
	7	7	.004	8	.018	9	.047	10	.092
	8	10	.007	11	.015	13	.048	14	.073
9	5	-	-	-	-	-	-	5	.084
	6	-	-	-	-	6	.030	6	.030
	7	7	.008	7	.008	8	.034	9	.083
	8	9	.007	10	.021	11	.045	12	.080
	9	12	.006	14	.023	15	.037	17	.078
10	5	-	-	-	-	-	-	5	.099
	6	-	-	-	-	6	.040	6	.040
	7	-	-	7	.013	7	.013	8	.052

K	X	p = .80							
		Upper tail probability level α							
		0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
10	8	8	.003	9	.015	10	.040	11	.083
	9	11	.009	12	.021	13	.040	14	.068
	10	15	.010+	16	.018	18	.042	20	.079
11	6	-	-	-	-	6	.049	6	.049
	7	-	-	7	.018	7	.013	8	.070
	8	8	.005	9	.024	9	.024	10	.063
	9	10	.006	11	.019	12	.042	13	.077
	10	13	.009	14	.019	15	.035	17	.085
	11	17	.008	19	.022	21	.044	24	.097
12	6	-	-	-	-	-	-	6	.058
	7	-	-	7	.023	7	.023	8	.089
	8	8	.008	8	.008	9	.035	10	.088
	9	9	.002	10	.011	11	.031	12	.068
	10	12	.008	13	.020	14	.040	15	.070
	11	15	.009	16	.017	18	.046	20	.096
	12	19	.007	22	.024	24	.045	27	.092
13	6	-	-	-	-	-	-	6	.067
	7	-	-	-	-	7	.029	7	.029
	8	-	-	8	.011	9	.046	9	.046
	9	9	.003	10	.016	11	.046	12	.096
	10	11	.005	12	.015	13	.035	14	.067
	11	14	.009	15	.020	16	.037	18	.093
	12	17	.008	19	.024	20	.038	22	.077

Upper tail critical points

		p = .80							
		Upper tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
13	13	22	.008	25	$.025^{+}$	27	.044	30	.086
14	6	-	-	-	-	-	-	6	.075
	7	-	-	-	-	7	.034	7	.034
	8	-	-	8	.014	8	.014	9	.058
	9	9	.005	10	.023	10	.023	11	.062
	10	11	.008	12	.023	12	.023	14	.098
	11	13	$.007^{-}$	14	.017	15	.036	16	.064
	12	16	$.010^{-}$	17	.019	18	.033	20	.079
	13	19	.007	21	.020	23	.044	25	.082
	14	25	.009	28	$.025^{+}$	30	.043	34	.094
15	6	-	-	-	-	-	-	6	.083
	7	-	-	-	-	7	.040	7	.040
	8	-	-	8	.017	8	.017	9	.070
	9	9	.006	9	.006	10	.030	11	.079
	10	10	.002	11	.011	12	.033	13	.072
	11	12	.003	13	.011	14	.027	16	.095
	12	15	.008	16	.018	17	.034	19	.091
	13	18	$.009^{+}$	19	.017	21	.045	23	.093
	14	22	$.010^{+}$	24	$.024^{+}$	26	$.049^{+}$	28	.084
	15	28	.010	31	.025	34	.050	37	.086
16	6	-	-	-	-	-	-	6	.090
	7	-	-	-	-	7	.045	7	.045

		p = .80							
		Upper tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
16	8	-	-	8	.020	8	.020	9	.081
	9	9	.008	9	.008	10	.037	11	.096
	10	10	.003	11	.015	12	.043	13	.093
	11	12	.005	13	.017	14	.039	15	.077
	12	14	.005	15	.014	16	.029	18	.089
	13	17	.009	18	.018	19	.032	21	.080
	14	20	.009	22	$.025^{-}$	23	.038	25	.078
	15	24	.008	26	.020	28	.039	31	.085
	16	30	.008	34	.024	37	.047	41	.092
17	6	-	-	-	-	-	-	6	.097
	7	-	-	-	-	7	$.050^{+}$	7	.050
	8	-	-	8	.024	8	.024	9	.092
	9	9	$.010^{-}$	9	.010	10	.045	10	.045
	10	10	.004	11	.019	11	.019	12	.055
	11	12	.007	13	.023	13	.023	15	$.100^{-}$
	12	14	.008	15	.021	16	.043	17	.077
	13	16	.007	17	.015	18	.030	20	.082
	14	19	.009	20	.017	22	.047	24	.098
	15	22	.008	24	.021	26	.046	28	.086
	16	26	.007	29	.022	31	.041	34	.084
	17	33	.008	37	.023	40	.044	45	.096
18	7	-	-	-	-	-	-	7	.055
	8	-	-	-	-	8	.027	8	.027

Upper tail critical points

		p = .80							
		Upper tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
18	9	-	-	9	.012	9	.012	10	.053
	10	10	.005	11	.024	11	.024	12	.066
	11	12	$.010^-$	12	.010	13	.030	14	.067
	12	13	.003	14	.012	15	.029	16	.059
	13	15	.004	17	.023	18	.044	19	.075
	14	18	.008	19	.016	21	.048	22	.075
	15	21	.009	22	.016	24	.041	26	.084
	16	24	.007	26	.018	28	.038	31	.091
	17	29	.008	32	.024	34	.042	38	$.100^-$
	18	36	.008	40	.022	44	.048	49	.098
19	7	-	-	-	-	-	-	7	.060
	8	-	-	-	-	8	.030	8	.030
	9	-	-	9	.014	9	.014	10	.060
	10	10	.006	10	.006	11	.029	12	.078
	11	11	.002	12	.012	13	.037	14	.082
	12	13	.005	14	.016	15	.038	16	.075
	13	15	.006	16	.015	17	.033	18	.061
	14	17	.005	19	$.025^-$	20	.044	21	.071

		p = .80							
		Upper tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
19	15	20	.008	21	.016	23	.044	25	.096
	16	23	.008	25	.023	26	.035	29	.096
	17	21	$.010^-$	29	.022	31	.043	34	.095
	18	32	$.010^-$	35	$.025^+$	37	.042	41	.096
	19	39	.008	43	.021	47	.044	53	.099
20	7	-	-	-	-	-	-	7	.064
	8	-	-	-	-	8	.033	8	.033
	9	-	-	9	.016	9	.016	10	.068
	10	10	.007	10	.007	11	.034	12	.090
	11	11	.003	12	.015	13	.045	14	.098
	12	13	.006	14	.020	15	.048	16	.093
	13	15	.008	16	.021	17	.044	18	.080
	14	17	.008	18	.018	19	.035	21	.098
	15	19	.006	21	$.025^-$	22	.042	24	.098
	16	22	.008	24	$.025^+$	25	.040	27	.084
	17	25	.008	27	.020	29	.044	31	.081
	18	29	.008	31	.018	34	.047	37	.097
	19	34	.008	37	.020	40	.042	44	.091
	20	43	$.010^+$	47	.024	51	.047	57	$.100^-$

Lower tail critical points

		p = .90							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
2	1	9	.009	8	.017	7	.030	6	.055
3	1	7	.005	6	.012	5	.031	4	.077
	2	16	.010^{-}	14	.020	12	.041	10	.084
4	1	6	.005	5	.014	4	.044	4	.044
	2	11	.008	10	.015	8	.049	7	.088
	3	24	.009	20	.024	18	.041	15	.087
5	1	5	.009	5	.009	4	.030	4	.030
	2	9	.009	8	.019	7	.042	6	.088
	3	16	.008	14	.019	12	.045	11	.070
	4	32	.009	27	.023	24	.042	20	.093
6	1	5	.006	4	.023	4	.023	3	.089
	2	8	.009	7	.023	7	.023	6	.056
	3	13	.008	12	.015	10	.046	9	.080
	4	21	.008	18	.024	16	.047	14	.093
	5	40	.009	34	.023	30	.045	26	.086
7	1	5	.004	4	.019	4	.019	3	.078$_{+}$
	2	8	.005	7	.014	6	.039	5	.100
	3	12	.006	10	.023	9	.045	8	.087
	4	17	.008	15	.021	14	.034	12	.083
	5	26	.009	23	.022	21	.038	18	.089
	6	48	.009	41	.025^{-}	36	.049	31	.096

		p = .90							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
8	1	5	.003	4	.016	4	.016	3	.070
	2	7	.010^{-}	7	.010	6	.029	5	.081
	3	11	.006	10	.013	9	.027	8	.058
	4	15	.008	13	.025^{-}	12	.043	11	.074
	5	21	.009$_{+}$	19	.020	17	.043	15	.091
	6	31	.010	28	.021	25	.044	22	.090
	7	56	.010^{-}	49	.023	43	.047	37	.094
9	1	5	.003	4	.014	4	.014	3	.064
	2	7	.007	6	.022	6	.022	5	.068
	3	10	.008	9	.018	8	.041	7	.093
	4	14	.007	12	.024	11	.045	10	.082
	5	19	.007$_{+}$	17	.018	15	.045	14	.070
	6	25	.010	23	.020	21	.039$_{+}$	19	.074
	7	37	.009	33	.022	29	.050	26	.094
	8	>60	-	56	.025^{-}	50	.046	43	.095
10	1	5	.002	4	.012	4	.012	3	.060
	2	7	.005	6	.013	6	.018	5	.058
	3	10	.005	9	.012	8	.031	7	.074
	4	13	.007	12	.014	12	.014	10	.057
	5	17	.008	16	.014	14	.040	13	.067
	6	22	.010^{-}	20	.022	18	.049	17	.071
	7	30	.009$_{+}$	27	.021	24	.050^{-}	22	.087
	8	42	.010	38	.022	34	.048$_{+}$	30	.099
	9	>60	-	>60	-	56	.050	49	.096

Lower tail critical points

		p = .90							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
11	1	5	.002	4	.011	4	.011	3	.056
	2	7	.004	6	.015	6	.015	5	.051
	3	9	.009	8	.024	8	.024	7	.061
	4	12	.009	11	.020	10	.041	9	.085
	5	16	.007	14	.024	13	.043	12	.076
	6	20	.009	18	.025-	17	.039	15	.094
	7	26	.009+	24	.019	22	.038	20	.075+
	8	34	.010-	31	.022	28	.048	25	.100
	9	48	.010-	43	.024	39	.046	35	.089
	10	>60	-	>60	-	>60	-	55	.098
12	1	4	.010+	4	.010	4	.010	3	.053
	2	7	.003	6	.013	5	.045	5	.045
	3	9	.007	8	.019	8	.019	7	.051
	4	12	.006	11	.014	10	.031	9	.067
	5	15	.008	14	.015	13	.029	11	.100-
	6	19	.008	17	.022	16	.037	14	.099
	7	24	.007	22	.018	20	.040	18	.087
	8	30	.009	27	.024	25	.044	23	.081
	9	39	.009	35	.024	32	.048	29	.093
	10	54	.010-	48	.025-	44	.046	39	.097
	11	>60	-	>60	-	>60	-	>60	-
13	1	4	.009	4	.009	4	.009	3	.051
	2	7	.003	6	.011	5	.041	5	.041
	3	9	.005	8	.015	7	.043	7	.043

		p = .90							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
13	4	11	.010+	10	.024	10	.024	9	.055
	5	14	.010-	13	.020	12	.040	11	.077
	6	18	.007	16	.023	15	.041	14	.070
	7	22	.008	20	.022	19	.034	17	.083
	8	27	.009	25	.020	23	.042	21	.084
	9	34	.009	31	.022	29	.039	26	.088
	10	44	.009	41	.021	36	.049	33	.089
	11	60	.010-	54	.023	49	.046	44	.092
	12	>60	-	>60	-	>60	-	>60	-
14	1	4	.009	4	.009	3	.049	3	.049
	2	6	.010-	6	.010	5	.037	5	.037
	3	9	.004	8	.013	7	.038	7	.038
	4	11	.008	10	.019	9	.045	9	.045
	5	14	.007	13	.014	12	.030	11	.061
	6	17	.008	16	.015	15	.028	13	.090
	7	21	.007	19	.021	18	.034+	16	.089
	8	25	.010-	23	.022	21	.050	20	.074
	9	31	.008	28	.023	26	.045	24	.084
	10	38	.010-	35	.021	32	.046	29	.096
	11	49	.009	44	.023	41	.042	37	.088+
	12	>60	-	59	.025-	54	.047	48	.100
	13	>60	-	>60	-	>60	-	>60	-
15	1	4	.008	4	.008	3	.047	3	.047
	2	6	.009	6	.009	5	.035	5	.035

Lower tail critical points

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
		p = .90 Lower tail probability level α							
18	7	18	.007	17	.014	16	.027	14	.091
	8	21	.008	20	.015	18	.044	17	.074
	9	25	.007	23	.018	21	.046	20	.072
	10	29	.007	27	.017	25	.038	23	.082
	11	33	$.010^-$	31	.020	29	.041	27	.078
	12	39	.009	36	.022	34	.040	31	.090
	13	46	.009	42	$.025^+$	40	.041	36	$.100^-$
	14	55	$.010^+$	51	.023	47	.048	43	.098
	15	>60	-	>60	-	58	.047	53	.092
	16	>60	-	>60	-	>60	-	>60	-
	17	>60	-	>60	-	>60	-	>60	-
19	1	4	.007	4	.007	3	.043	3	.043
	2	6	.006	6	.006	5	.027	5	.027
	3	8	.006	7	.022	7	.022	6	.072
	4	10	.008	9	.022	9	.022	8	.059
	5	12	$.010^-$	11	.024	11	.024	10	.058
	6	15	.006	14	.014	13	.030	12	.063
	7	18	.005	16	.021	15	.041	14	.076
	8	20	$.010^+$	19	.019	18	.033	16	.098
	9	24	.007	22	.020	21	.033	19	.085
	10	27	$.010^-$	25	$.025^+$	24	.039	22	.087
	11	32	.008	29	$.025^-$	28	.036	26	.072
	12	37	.008	34	.022	32	.041	29	.099
	13	43	.008	39	$.025^+$	37	.043	34	.091

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
		p = .90 Lower tail probability level α							
19	14	50	$.010^-$	46	.024	43	.046	40	.087
	15	60	.009	55	.024	51	.048	47	.093
	16	>60	-	>60	-	>60	-	57	.095
	17	>60	-	>60	-	>60	-	>60	-
	18	>60	-	>60	-	>60	-	>60	-
20	1	4	.007	4	.007	3	.042	3	.042
	2	6	.006	6	.006	5	.026	5	.026
	3	8	.006	7	.020	7	.020	6	.068
	4	10	.006	9	.019	9	.019	8	.054
	5	12	.008	11	.021	11	.021	10	.051
	6	15	.005	13	$.025^+$	13	.025	12	.055
	7	17	.008	16	.017	15	.033	14	.064
	8	20	.007	19	.014	17	.046	16	.081
	9	23	.008	21	.024	20	.040	19	.067
	10	26	$.010^+$	25	.017	23	.042	21	.098
	11	30	$.010^-$	28	.023	26	$.050^-$	25	.072
	12	35	.008	32	.024	30	.048	28	.090
	13	40	.009	37	.023	35	.041	32	.095
	14	46	$.010^-$	43	.022	40	.046	37	.093
	15	54	$.010^-$	50	.023	47	.043	43	.094
	16	>60	-	59	$.025^-$	55	.048	51	.090
	17	>60	-	>60	-	>60	-	>60	-
	18	>60	-	>60	-	>60	-	>60	-
	19	>60	-	>60	-	>60	-	>60	-

Lower tail critical points

K	X	p = .90							
		Lower tail probability level α							
		0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
15	3	9	.003	8	.011	7	.033	6	.096
	4	11	.006	10	.015	9	.038	8	.091
	5	14	.005+	12	.023	11	.050-	11	.050
	6	16	.010	15	.020	14	.038	13	.070
	7	20	.007	18	.023	17	.038	16	.064
	8	24	.008	22	.020	20	.048	19	.073
	9	29	.007	26	.024	24	.049	22	.098
	10	35	.008+	32	.020	29	.049	27	.086
	11	42	.010+	39	.021	36	.043	33	.085
	12	53	.010+	49	.022	45	.044	41	.087
	13	>60	-	>60	-	59	.049	53	.097
	14	>60	-	>60	-	>60	-	>60	-
16	1	4	.008	4	.008	3	.046	3	.046
	2	6	.008	6	.008	5	.032	5	.032
	3	8	.009	8	.009	7	.029	6	.088
	4	11	.005	10	.012	9	.032	8	.081
	5	13	.008	12	.018	11	.041	10	.087
	6	16	.007	15	.015	14	.029	13	.056
	7	19	.008	18	.015	16	.048	15	.081
	8	23	.007	21	.019	20	.032	18	.080
	9	27	.008	25	.019	23	.044	21	.093
	10	32	.009	29	.025	27	.049	25	.093
	11	38	.010-	35	.024	33	.041	30	.090
	12	47	.009+	43	.021	40	.041	36	.094
	13	58	.010	53	.024	49	.047	45	.088

K	X	p = .90							
		Lower tail probability level α							
		0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
16	14	>60	-	>60	-	>60	-	58	.096
	15	>60	-	>60	-	>60	-	>60	-
17	1	4	.008	4	.008	3	.045	3	.045
	2	6	.007	6	.007	5	.030	5	.030
	3	8	.008+	8	.008	7	.026	6	.082
	4	10	.010	10	.010	9	.028	8	.072
	5	13	.006	12	.015	11	.034	10	.075
	6	16	.005+	14	.022	13	.045	12	.087
	7	18	.010	17	.020	16	.036	15	.064
	8	22	.007	20	.021	19	.036	17	.094
	9	26	.007	24	.018	22	.043	20	.097
	10	30	.009+	28	.020	26	.041	24	.083
	11	35	.010	33	.020	31	.037	28	.090
	12	42	.010-	39	.021	36	.046	33	.094
	13	51	.009	47	.022	43	.049+	40	.087
	14	>60	-	58	.023	53	.050	49	.090
	15	>60	-	>60	-	>60	-	>60	-
	16	>60	-	>60	-	>60	-	>60	-
18	1	4	.007	4	.007	3	.044	3	.044
	2	6	.007	6	.007	5	.028	5	.028
	3	8	.007	7	.024	7	.024	6	.076
	4	10	.009	9	.024	9	.024	8	.065
	5	13	.005	12	.012	11	.029	10	.065
	6	15	.008	14	.018	13	.037	12	.074

Upper tail critical points

K	X	p = .90, Upper tail probability level α: 0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
4	4	-	-	-	-	-	-	4	.062
5	5	-	-	5	.023	5	.023	6	.075
6	5	-	-	-	-	-	-	5	.055
	6	6	.008	6	.008	7	.032	8	.072
7	5	-	-	-	-	-	-	5	.089
	6	-	-	6	.023	6	.023	7	.080
	7	7	.003	8	.013	9	.033	10	.064
8	6	-	-	-	-	6	.041	6	.041
	7	7	.009	7	.009	8	.037	9	.088
	8	9	.005	10	.014	11	.031	13	.086
9	6	-	-	-	-	-	-	6	.060
	7	-	-	7	.018	7	.018	8	.069
	8	8	.004	9	.017	10	.044	11	.087+
	9	11	.006	12	.014	14	.046	16	.100
10	6	-	-	-	-	-	-	6	.080
	7	-	-	-	-	7	.029	7	.029
	8	8	.008	8	.008	9	.034	10	.083
	9	10	.007	11	.021	12	.045	13	.081
	10	13	.006	15	.024	16	.038	18	.080

K	X	p = .90, Upper tail probability level α: 0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
11	6	-	-	-	-	-	-	6	.100
	7	-	-	-	-	7	.041	7	.041
	8	-	-	8	.013	8	.013	9	.055
	9	9	.003	10	.016	11	.043	12	.089
	10	12	.010	13	.023	14	.044	15	.073
	11	15	.006	17	.020	19	.046	21	.086
12	7	-	-	-	-	-	-	7	.053
	8	-	-	8	.020	8	.020	9	.078
	9	9	.006	9	.006	10	.028	11	.072
	10	11	.007	12	.022	13	.048	14	.088
	11	13	.004+	15	.023	16	.040	18	.097
	12	18	.010	19	.017	21	.037	24	.088
13	7	-	-	-	-	-	-	7	.066
	8	-	-	-	-	8	.027	8	.027
	9	9	.009	9	.009	10	.042	10	.042
	10	10	.003	11	.013+	12	.038	13	.081
	11	12	.003	14	.025	15	.049	16	.084
	12	15	.005	17	.021	18	.036	20	.083
	13	20	.009	22	.021	24	.041	27	.089
14	7	-	-	-	-	-	-	7	.078
	8	-	-	-	-	8	.035	8	.035
	9	-	-	9	.014	9	.014	10	.057
	10	10	.004	11	.021	11	.021	12	.059

Upper tail critical points

p = .90; Upper tail probability level α

K	X	0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
14	11	12	.006	13	.020	14	.045	15	.085
	12	14	.005	15	.013	17	.048	18	.079
	13	17	.005	19	.020	21	.049	23	.097
	14	22	.007	25	.024	27	.044	30	.088
15	7	-	-	-	-	-	-	7	.091
	8	-	-	-	-	8	.044	8	.044
	9	-	-	9	.018	9	.018	10	.074
	10	10	.007	11	.031	11	.031	12	.082
	11	11	.002	12	.011	13	.032	14	.070
	12	14	.010$^-$	15	.024	16	.049	17	.085
	13	16	.006	17	.014	19	.046	20	.072
	14	20	.010$^+$	21	.018	23	.042	25	.082
	15	25	.009	27	.019	30	.045	33	.085
16	8	-	-	-	-	-	-	8	.052
	9	-	-	9	.023	9	.023	10	.092
	10	10	.009	10	.009	11	.042	11	.042
	11	11	.003	12	.016	13	.047	14	.099
	12	13	.005	14	.017	15	.040	16	.077
	13	15	.005	16	.013	18	.050$^-$	19	.082
	14	18	.007	19	.014	21	.042	23	.093
	15	22	.009	24	.024	25	.036	28	.090
	16	27	.008	30	.021	33	.045	37	.098
17	8	-	-	-	-	-	-	8	.060

p = .90; Upper tail probability level α

K	X	0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
17	9	-	-	-	-	9	.029	9	.029
	10	-	-	10	.012	10	.012	11	.053
	11	11	.005	12	.023	12	.023	13	.063
	12	13	.008	13	.008	14	.026	15	.059
	13	15	.009	16	.022	17	.045	18	.080
	14	17	.006	18	.015	20	.049	21	.078
	15	20	.007	22	.024	23	.039	25	.082
	16	24	.008	26	.021	28	.043	31	.096
	17	30	.009	33	.022	36	.045	40	.093
18	8	-	-	-	-	-	-	8	.069
	9	-	-	-	-	9	.034	9	.034
	10	-	-	10	.016	10	.016	11	.066
	11	11	.006	11	.006	12	.030	13	.081
	12	12	.002	13	.012	14	.036	15	.080
	13	14	.004	15	.014	16	.034	17	.067
	14	16	.004	18	.025$^+$	19	.048	20	.080
	15	19	.008	20	.016	22	.047	23	.073
	16	22	.007	24	.022	26	.051	28	.097
	17	26	.007	28	.018	31	.048	34	.100$^-$
	18	33	.010$^-$	36	.023	39	.044	43	.088
19	8	-	-	-	-	-	-	8	.077
	9	-	-	-	-	9	.040	9	.040
	10	-	-	10	.019	10	.019	11	.079
	11	11	.008	11	.008	12	.038	13	.100$^+$

Upper tail critical points

K	X	p = .90, Upper tail probability level α: 0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
19	12	12	.003	13	.016	14	.048	14	.048
	13	14	.006	15	.020	16	.048	17	.092
	14	16	.007	17	.019	18	.039	19	.072
	15	18	.006	19	.014	21	.049	22	.078
	16	21	.008	22	.016	24	.044	26	.095
	17	24	$.007_{-}$	26	.020	28	.045	30	.084
	18	29	$.010^{-}$	31	.022	33	.040	36	.083
	19	35	.008	39	.023	42	.043	47	.096
20	8	-	-	-	-	-	-	8	.085
	9	-	-	-	-	9	.046	9	.046

K	X	p = .90, Upper tail probability level α: 0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
20	10	-	-	10	.023	10	.023	11	.092
	11	11	$.010^{+}$	11	.010	12	.047	12	.047
	12	12	.004	13	.022	13	.022	14	.061
	13	14	.009	14	.009	15	.028	16	.064
	14	15	$.003_{+}$	16	.011	17	.027	19	.099
	15	18	$.010^{+}$	19	.023	20	.043	21	.074
	16	20	.007	21	.016	23	.048	24	.075
	17	23	.009	24	.016	26	.041	28	.085
	18	26	.007	28	$.018_{-}$	30	.039	33	.094
	19	31	.009	34	$.025^{-}$	36	$.043_{-}$	39	.085
	20	38	.009	42	.023	46	$.050^{-}$	50	.090

Lower tail critical points

K	X	p = .95, Lower tail probability level α: 0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
2	1	9	.006	7	.022	6	.042	5	.080
3	1	6	.007	5	.020	5	.020	4	.054
	2	15	.010^{-}	13	.021	11	.045	9	.097
4	1	5	.008	5	.008	4	.027	3	.095
	2	10	.010^{-}	9	.018	8	.034	7	.065
	3	22	.010^{+}	19	.023	17	.040	14	.089
5	1	5	.004	4	.017	4	.017	3	.069
	2	9	.005	8	.011	7	.026	6	.060
	3	15	.008	13	.020	11	.050^{+}	10	.080
	4	30	.009	26	.021	22	.048	19	.090
6	1	5	.002	4	.011	4	.011	3	.054
	2	8	.005	7	.013	6	.034	5	.090
	3	12	.009	11	.016	10	.030	9	.055
	4	20	.007	17	.022	15	.047	13	.097
	5	38	.009	32	.024	28	.048	24	.094
7	1	4	.008	4	.008	3	.044	3	.044
	2	7	.007	6	.022	6	.022	5	.064
	3	11	.006	10	.013	9	.027	8	.057
	4	16	.008	14	.021	13	.035	11	.093
	5	24	.010^{+}	22	.019	19	.048	17	.087
	6	45	.010^{-}	39	.024	34	.049	29	.100^{-}

K	X	p = .95, Lower tail probability level α: 0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
8	1	4	.006	4	.006	3	.038	3	.038
	2	7	.004	6	.015	5	.048	5	.048
	3	10	.006	9	.015	8	.035	7	.080
	4	14	.008	13	.014	11	.048	10	.085
	5	20	.008	18	.018	16	.042	14	.093
	6	29	.010^{+}	26	.023	23	.050^{+}	21	.084
	7	53	.010^{-}	46	.024	40	.050^{+}	35	.094
9	1	4	.005	4	.005	3	.033	3	.033
	2	7	.003	6	.011	5	.038	5	.038
	3	9	.009	8	.023	8	.023	7	.057
	4	13	.006	12	.013	11	.026	9	.100^{-}
	5	17	.010^{-}	16	.016	14	.045	13	.073
	6	24	.008	21	.025^{-}	19	.050^{-}	17	.098
	7	35	.009	31	.022	28	.043	25	.084
	8	>60	-	53	.024	47	.047	41	.091
10	1	4	.004	4	.004	3	.029	3	.029
	2	6	.008	6	.008	5	.031	5	.031
	3	9	.006	8	.016	7	.043	7	.043
	4	12	.007	11	.015	10	.033	9	.070
	5	16	.007	14	.023	13	.041	12	.072
	6	21	.008	19	.019	17	.046	16	.069
	7	28	.009	25	.023	23	.043	21	.079
	8	40	.009	36	.021	32	.048	29	.086
	9	>60	-	60	.025^{-}	53	.050^{-}	46	.099

Lower tail critical points

		p = .95							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
11	1	4	.004	4	.004$^+$	3	.027	3	.027
	2	6	.006	5	.025	5	.025	5	.025
	3	9	.004	8	.011	7	.033$^+$	6	.093
	4	11	.009	10	.022	9	.050	9	.050
	5	15	.006	13	.024	12	.046	11	.085
	6	19	.008$^+$	17	.022	16	.037	14	.096
	7	24	.010	22	.023	20	.048	18	.099
	8	32	.010$^-$	29	.023	27	.040	24	.089
	9	45	.010$^+$	41	.022	37	.045	33	.090
	10	>60	-	>60	-	60	.048	52	.098
12	1	4	.003	3	.024	3	.024	3	.024
	2	6	.005	5	.022	5	.022	4	.091
	3	8	.008	8	.008	7	.026	6	.078
	4	11	.006	10	.015	9	.037	8	.088
	5	14	.007	13	.015	12	.030	11	.060
	6	18	.006	16	.020	15	.036	14	.061
	7	22	.009	20	.023	19	.035	17	.082
	8	28	.009	26	.019	24	.037$^-$	21	.098$^+$
	9	37	.009	33	.024	30	.050$^-$	27	.100
	10	51	.010$^-$	46	.022	41	.050$^-$	37	.094
	11	>60	-	>60	-	>60	-	58	.097
13	1	4	.003	3	.023	3	.023	3	.023
	2	6	.004	5	.019	5	.019	4	.083
	3	8	.006	7	.021	7	.021	6	.067

		p = .95							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
13	4	11	.004	10	.011	9	.029	8	.071
	5	13	.009	12	.021	11	.044	10	.090
	6	17	.006	15	.022	14	.040	13	.073
	7	21	.007	19	.019	17	.050$^-$	16	.079
	8	26	.007	23	.024	22	.035	20	.075
	9	32	.009	29	.023	27	.042	24	.100$^-$
	10	41	.010$^-$	37	.025$^-$	34	.048	31	.091
	11	57	.009	51	.023	46	.048	41	.099
	12	>60	-	>60	-	>60	-	>60	-
14	1	4	.003	3	.021	3	.021	3	.021
	2	6	.003	5	.016	5	.016	4	.076
	3	8	.005	7	.017	7	.017	6	.058
	4	10	.008	9	.022	9	.022	8	.059
	5	13	.006	12	.014	11	.033	10	.071
	6	16	.007$^+$	15	.014	14	.027	12	.097
	7	19	.010	18	.018	17	.031	15	.087$^+$
	8	24	.007	22	.018	20	.044	18	.100$^+$
	9	29	.009	27	.018	25	.037	22	.100
	10	35	.009	33	.021	30	.047	28	.080
	11	46	.009	42	.021	38	.048	35	.086
	12	>60	-	56	.024	51	.047	46	.092
	13	>60	-	>60	-	>60	-	>60	-
15	1	4	.002	3	.020	3	.020	3	.020
	2	6	.003	5	.014	5	.014	4	.071

Lower tail critical points

		p = .95							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
15	3	8	.004	7	.014	7	.014	6	.051
	4	10	.006	9	.018	8	.049	8	.049
	5	12	.010+	11	.025-	11	.025	10	.057
	6	15	.009	14	.019	13	.038	12	.075
	7	19	.006	17	.020	16	.035	15	.062
	8	22	.009	21	.016	19	.042	18	.066
	9	27	.008	25	.018	23	.040	21	.086
	10	32	.010+	30	.020	28	.039	25	.097
	11	40	.009	36	.025+	34	.042	31	.086
	12	50	.010+	46	.023	42	.048	38	.098
	13	>60	-	>60	-	56	.047	50	.098
	14	>60	-	>60	-	>60	-	>60	-
16	1	4	.012	3	.019	3	.019	3	.019
	2	6	.002	5	.013	5	.013	4	.066
	3	8	.003	7	.012	6	.045	6	.045
	4	10	.005	9	.014	8	.042	8	.042
	5	12	.008	11	.019	10	.046	10	.046
	6	15	.006	14	.013	13	.028	12	.059
	7	18	.007	16	.024	15	.045	14	.081
	8	21	.009	20	.016	18	.045	17	.074
	9	25	.009	23	.023	22	.036	20	.083
	10	30	.009	28	.019	26	.039	24	.078
	11	36	.009	33	.023	31	.041	28	.095
	12	44	.009	40	.024	37	.048	34	.093
	13	55	.010-	50	.024	46	.049	42	.095

		p = .95							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
16	14	>60	-	>60	-	>60	-	55	.093
	15	>60	-	>60	-	>60	-	>60	-
17	1	4	.002	3	.018	3	.018	3	.018
	2	6	.002	5	.012	5	.012	4	.062
	3	7	.010+	7	.010	6	.041	6	.041
	4	10	.004	9	.012	8	.036	8	.036
	5	12	.006	11	.015	10	.038	9	.092
	6	14	.010-	13	.021	12	.047	11	.097
	7	17	.008	16	.017	15	.033	14	.063
	8	20	.010-	19	.017	18	.031	16	.090
	9	24	.008	22	.022	21	.035	19	.087
	10	28	.009	26	.022	24	.050-	23	.070
	11	33	.010-	31	.020	29	.039	26	.100-
	12	40	.008	37	.019	34	.044	31	.096
	13	48	.009	44	.023	41	.044	37	.100+
	14	60	.010-	55	.022	50	.050+	46	.093
	15	>60	-	>60	-	>60	-	>60	-
	16	>60	-	>60	-	>60	-	>60	-
18	1	4	.002	3	.017	3	.017	3	.017
	2	6	.002	5	.011	5	.011	4	.058
	3	7	.009	7	.009	6	.037	6	.037
	4	9	.010-	9	.010	8	.031	7	.093
	5	12	.004	11	.012	10	.032	9	.080

Lower tail critical points

p = .95

Lower tail probability level α

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
18	6	14	.007	13	.017	12	.038	11	.082
	7	17	.006	15	$.025^{-}$	14	.049	13	.094
	8	20	.006	18	.021	17	.039	16	$.069^{+}$
	9	23	.008	21	.023	20	.038	18	.100
	10	27	.008	25	.019	23	.046	22	.069
	11	31	$.010^{-}$	29	.021	27	.044	25	.088
	12	37	$.008^{+}$	34	.021	32	.039	29	.094
	13	43	.010	40	.022	37	.048	34	.098
	14	52	$.010^{-}$	48	.023	45	.042	41	.090
	15	>60	-	59	.024	55	.045	50	.093
	16	>60	-	>60	-	>60	-	>60	-
	17	>60	-	>60	-	>60	-	>60	-
19	1	4	.002	3	.017	3	.017	3	.017
	2	5	$.010^{-}$	5	.010	5	.010	4	.055
	3	7	.008	7	.008	6	.033	6	.033
	4	9	.008	9	.008	8	.027	7	.084
	5	11	$.010^{-}$	11	.010	11	.010	9	.070
	6	14	.005	13	.013	12	.031	11	.070
	7	16	.009	15	.019	14	.039	13	.078
	8	19	.008	18	.015	18	.015	15	.095
	9	22	.009	21	.015	19	.046	18	.076
	10	26	.007	24	.019	22	.048	21	.074
	11	30	.007	28	.017	26	.039	24	.084
	12	35	.007	32	.021	30	.041	28	.079
	13	.40	.009	37	.023	35	.041	32	.091

p = .95

Lower tail probability level α

K	X	0.01 N_1	0.01 α_1	0.025 N_1	0.025 α_1	0.05 N_1	0.05 α_1	0.10 N_1	0.10 α_1
19	14	47	$.010^{+}$	44	.020	41	.041	38	.081
	15	56	.010	52	.023	48	.049	44	.099
	16	>60	-	>60	-	59	.047	54	.093
	17	>60	-	>60	-	>60	-	>60	-
	18	>60	-	>60	-	>60	-	>60	-
20	1	4	.002	3	.016	3	.016	3	.016
	2	5	.009	5	.009	5	.009	4	.052
	3	7	.007	7	.007	6	.031	6	.031
	4	9	.007	8	.024	8	.024	7	.077
	5	11	$.008^{+}$	10	$.023^{+}$	10	.023	9	.062
	6	13	.010	12	.025	12	.025	11	.059
	7	16	.006	15	.014	14	.031	13	.065
	8	19	$.005^{+}$	17	.021	16	.041	15	.077
	9	21	.010	20	.019	19	.033	17	.099
	10	25	.007	23	.020	22	.033	20	.085
	11	28	.010	26	.025	25	.038	23	$.086^{+}$
	12	33	.008	30	.024	29	.035	26	.100
	13	38	.008	35	.021	33	.040	30	.098
	14	44	.008	40	$.025^{-}$	38	.042	35	.090
	15	51	.009	47	.024	44	.046	41	.086
	16	>60	-	56	.023	52	.047	48	.092
	17	>60	-	>60	-	>60	-	58	.094
	18	>60	-	>60	-	>60	-	>60	-
	19	>60	-	>60	-	>60	-	>60	-

Upper tail critical points

		p = .95										p = .95							
		Upper tail probability level α										Upper tail probability level α							
K	X	0.01		0.025		0.05		0.10		K	X	0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2			N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
4	4	-	-	-	-	-	-	4	.076	11	9	9	.005	10	.024	10	.024	11	.063
											10	11	.005	12	.015	13	.034	14	.062
5	5	-	-	-	-	5	.030	5	.030		11	15	.009	16	.017	18	.044	20	.086
6	5	-	-	-	-	-	-	5	.072	12	7	-	-	-	-	-	-	7	.078
	6	-	-	6	.011	7	.042	8	.092		8	-	-	-	-	8	.031	8	.031
											9	9	$.010^-$	9	.010	10	.042	10	.042
7	6	-	-	-	-	6	.031	6	.031		10	10	.002	11	.012	12	.033	13	.070
	7	7	.004	8	.018	9	.044	10	.083		11	13	.007	14	.017	15	.034	17	.091
											12	17	.009	19	$.025^-$	20	.037	23	.093
8	6	-	-	-	-	-	-	6	.057										
	7	-	-	7	.013	7	.013	8	.052	13	7	-	-	-	-	-	-	7	.096
	8	9	.008	10	.020	11	.042	12	.072		8	-	-	-	-	8	.042	8	.042
											9	-	-	9	.015	9	.015	10	.063
9	6	-	-	-	-	-	-	6	.084		10	10	.005	11	.022	11	.022	12	.058
	7	-	-	-	-	7	.026	8	.094		11	12	.006	13	.017	14	.039	15	.073
	8	8	.006	9	.024	9	.024	10	.061		12	15	.008	16	.018	17	.033	19	.081
	9	11	.009	12	.021	13	.038	15	.092		13	19	.008	21	.021	23	.043	26	.096
10	7	-	-	-	-	7	.042	7	.042	14	8	-	-	-	-	-	-	8	.054
	8	-	-	8	.012	9	.049	9	.049		9	-	-	9	.022	9	.022	10	.086
	9	9	.002	10	.011	11	.031	12	.064		10	10	.008	10	.008	11	.034	12	.088
	10	13	$.010^-$	14	.019	15	.033	17	.077		11	11	.002	12	.011	13	.032	14	.069
											12	14	.008	15	.021	16	.042	17	.072
11	7	-	-	-	-	-	-	7	.060		13	17	.009	18	.018	20	.049	21	.072
	8	-	-	8	.021	8	.021	9	.079		14	21	.007	24	$.025^+$	26	.048	29	.097

Upper tail critical points

K	X	p = .95							
		Upper tail probability level α							
		0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
15	8	-	-	-	-	-	-	8	.067
	9	-	-	-	-	9	.030	9	.030
	10	-	-	10	.011	11	.049	11	.049
	11	11	.004	12	.018	12	.018	13	.051
	12	13	.005	14	.017	15	.039	16	.075
	13	15	.004	17	.023	18	.042	19	.069
	14	19	.009	20	.017	22	.043	24	.087
	15	24	.010$^-$	26	.021	28	.039	32	.097
16	8	-	-	-	-	-	-	8	.080
	9	-	-	-	-	9	.038	9	.038
	10	-	-	10	.016	10	.016	11	.066
	11	11	.006	11	.006	12	.027	13	.073
	12	13	.009	13	.009	14	.028	15	.063
	13	15	.009	16	.021	17	.044	18	.077
	14	17	.005	19	.024$_+$	20	.041	22	.096
	15	21	.009	23	.025$^+$	24	.038	27	.099
	16	26	.008	29	.024	31	.041	35	.095
17	8	-	-	-	-	-	-	8	.093
	9	-	-	-	-	9	.047	9	.047
	10	-	-	10	.021	10	.021	11	.084
	11	11	.008	11	.008	12	.038	13	.098
	12	12	.003	13	.015	14	.043	15	.091
	13	14	.005	15	.015$_-$	16	.036	17	.071
	14	16	.004	18	.025$^-$	19	.046	20	.077

K	X	p = .95							
		Upper tail probability level α							
		0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
17	15	19	.006	21	.024	22	.039	24	.087
	16	23	.009$_+$	25	.023	27	.047	29	.085
	17	29	.010$^+$	31	.020	34	.043	38	.093
18	9	-	-	-	-	-	-	9	.056
	10	-	-	-	-	10	.027$_+$	10	.027
	11	-	-	11	.011	12	.050$^+$	12	.050
	12	12	.004	13	.021	13	.021	14	.059
	13	14	.008	15	.024	15	.024	16	.055
	14	16	.008	17	.020	18	.043	19	.076
	15	18	.006	19	.014	21	.047	22	.075
	16	21	.007	23	.023	24	.037	26	.079
	17	25	.008	27	.020	29	.041	32	.093
	18	31	.008	34	.021	37	.044	41	.091
19	9	-	-	-	-	-	-	9	.065
	10	-	-	-	-	10	.033	10	.033
	11	-	-	11	.015	11	.015	12	.063
	12	12	.006	12	.006	13	.029	14	.078
	13	13	.002	14	.011	15	.035	16	.078
	14	15	.004	16	.013$_-$	17	.033	18	.065
	15	17	.004	19	.025$^-$	20	.047	21	.078
	16	20	.007	21	.015	23	.046$_+$	24	.071
	17	23	.007	25	.022$_+$	27	.050$^+$	29	.095
	18	27	.007	30	.025$^+$	32	.047	35	.099
	19	34	.010$^-$	37	.023	40	.044	44	.088

Upper tail critical points

K	X	p = .95							
		Upper tail probability level α							
		0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
20	9	-	-	-	-	-	-	9	.075
	10	-	-	-	-	10	.039	10	.039
	11	-	-	11	.019	11	.019	12	.078
	12	12	.008	12	.008	13	.038	14	.099
	13	13	.003	14	.016	15	.048	15	.048
	14	15	.006	16	.020	17	.047	18	.092
	15	17	.007	18	.019	19	.039	20	.072
	16	19	.006	20	.014	22	.049	23	.078
	17	22	.008	23	.016	25	.045	27	.095
	18	25	.007	27	.020	29	.045	31	.084
	19	30	.010+	32	.022	34	.040	37	.084
	20	36	.008	40	.024	43	.044	48	.097

Lower tail critical points

		p = .99							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
2	1	8	.008	7	.017	6	.033	5	.066
3	1	6	.004	5	.014	4	.040	4	.040
	2	15	.007	12	.025^{-}	11	.037	9	.081
4	1	5	.005	4	.018	4	.018	3	.068
	2	10	.006	8	.025^{+}	7	.049	6	.097
	3	21	.010^{+}	18	.024	16	.042	13	.098
5	1	4	.009	4	.009	3	.045	3	.045
	2	8	.007	7	.018	6	.044	6	.044
	3	14	.009	12	.023	11	.038	10	.062
	4	28	.010^{+}	24	.025^{+}	21	.048	18	.093
6	1	4	.006	4	.006	3	.032	3	.032
	2	7	.008	6	.022	6	.022	5	.064
	3	12	.005	10	.021	9	.040	8	.077
	4	19	.007	16	.024	15	.036	13	.077
	5	36	.009	31	.022	27	.046	23	.093
7	1	4	.004	3	.024	3	.024	3	.024
	2	7	.004	6	.013	5	.042	5	.042
	3	10	.008	9	.018	8	.040	7	.087
	4	15	.008	13	.025^{+}	12	.042	11	.070
	5	23	.010^{-}	21	.019	18	.050^{+}	16	.093
	6	43	.010^{-}	37	.025^{-}	33	.045	28	.100^{-}

		p = .99							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
8	1	4	.003	3	.019	3	.019	3	.019
	2	6	.008	6	.008	5	.030	5	.030
	3	9	.009	8	.022	8	.022	7	.055
	4	13	.009	12	.017	11	.033	10	.061
	5	19	.008	17	.019	15	.046	14	.070
	6	23	.010^{-}	25	.022	22	.050^{-}	20	.085
	7	51	.009	44	.024	39	.046	33	.100^{-}
9	1	4	.002	3	.016	3	.016	3	.016
	2	6	.005	5	.022	5	.022	4	.089
	3	9	.005	8	.013	7	.037	6	.098
	4	12	.008	11	.016	10	.034	9	.071
	5	16	.010^{+}	15	.018	14	.031	12	.087
	6	23	.008	20	.025^{-}	19	.036	17	.074
	7	33	.010^{-}	30	.020	27	.041	24	.082
	8	58	.010^{+}	51	.023	45	.047	39	.093
10	1	4	.001	3	.013	3	.013	3	.013
	2	6	.004	5	.016	5	.016	4	.073
	3	8	.008	8	.008	7	.026	6	.075
	4	11	.009	10	.020	9	.046	9	.046
	5	15	.008	14	.014	12	.050^{-}	11	.089
	6	20	.008	18	.020	16	.049	15	.076
	7	27	.008	24	.022	22	.042	20	.080
	8	38	.010^{-}	34	.023	31	.044	27	.099
	9	>60	-	58	.024	51	.049	44	.100^{-}

Lower tail critical points

K	X	p = .99							
		Lower tail probability level α							
		0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
11	1	4	.001	3	.011	3	.011	3	.011
	2	6	.002	5	.013	5	.013	4	.062
	3	8	.005	7	.018	7	.018	6	.059
	4	11	.005	10	.013	9	.031	8	.075
	5	14	.007	13	.014	12	.029	11	.058
	6	18	.008	16	.024	15	.040	14	.068
	7	23	$.010^{-}$	21	.022	19	.049	18	.072
	8	31	$.009_{+}$	28	.021	25	$.050^{-}$	23	.087
	9	43	.010	39	.022	35	$.048_{+}$	31	.099
	10	>60	-	>60	-	57	.050	50	.096
12	1	3	$.009_{+}$	3	.009	3	.009	3	.009
	2	5	.010	5	.010	5	.010	4	.053
	3	8	.004	7	.014	6	.047	6	.047
	4	10	.008	9	.022	9	.022	8	.056
	5	13	.008	12	.018	11	.038	10	.080
	6	17	.006	15	.022	14	.040	13	.072
	7	21	.009	19	.023	18	$.036_{+}$	16	.089
	8	27	.008	24	$.025^{-}$	22	.050	20	.099
	9	35	.009	32	.021	29	.045	26	$.095_{+}$
	10	49	.009	44	.022	40	.044	35	.100
	11	>60	-	>60	-	>60	-	56	.094
13	1	3	.008	3	.008	3	.008	2	.095
	2	5	$.008_{+}$	5	.008	4	.046	4	.046
	3	7	.010	7	.010	6	.039	6	.039

K	X	p = .99							
		Lower tail probability level α							
		0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
13	4	10	.005	9	.015	8	.043	8	.043
	5	13	.005	12	.011	11	.026	10	.059
	6	15	.006	14	$.025^{-}$	13	.047	12	.088
	7	20	.006	18	.019	17	.032	15	.088
	8	24	$.010^{-}_{+}$	22	.023	21	.034	19	.077
	9	30	$.010_{+}$	28	.020	26	.039	23	.097
	10	39	.010	36	.021	33	.043	30	.084
	11	54	$.010^{-}$	49	.022	44	.048	40	.088
	12	>60	-	>60	-	>60	-	>60	-
14	1	3	.007	3	.007	3	.007	2	.090
	2	5	.007	5	.007	4	.041	4	.041
	3	7	.008	7	.008	6	.032	6	.032
	4	10	.004	9	.011	8	.034	7	$.097_{+}$
	5	12	.007	11	.018	10	.044	9	.100
	6	15	.007	14	.015	13	.032	12	.063
	7	18	$.010^{-}_{+}$	17	.018	16	.033	14	.099
	8	22	.010	21	.017	19	.044	18	.069
	9	28	.007	25	.023	23	.049	21	$.100^{-}$
	10	34	$.010^{-}$	31	.024	29	.043	26	.098
	11	44	.009	40	.022	37	.041	33	.094
	12	60	.009	54	.022	49	.046	44	.092
	13	>60	-	>60	-	>60	-	>60	-
15	1	3	.007	3	.007	3	.007	2	.085
	2	5	.006	5	.006	4	.036	4	.036

Lower tail critical points

		p = .99							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
15	3	7	.006	7	.006	6	.027	6	.027
	4	9	.008	9	.008	8	.027	7	.082
	5	12	.005	11	.013	10	.033	9	.081
	6	14	$.010^-$	13	.022	12	.046	11	.095
	7	18	.005	16	.021	15	.039	14	.071
	8	21	.009	19	$.025^+$	18	.043	17	.070
	9	26	.007	23	$.025^+$	22	.038	20	.086
	10	31	.009	28	$.025^-$	26	.048	24	.092
	11	38	.009	35	.021	32	.047	30	.078
	12	48	$.010^-$	44	.022	40	.049	37	.087
	13	>60	-	59	.023	54	.044	48	.096
	14	>60	-	>60	-	>60	-	>60	-
16	1	4	$.000^+$	4	$.000^+$	4	$.000^+$	2	.081
	2	5	.005	5	.005	4	.032	4	.032
	3	7	.005	6	.023	6	.023	5	.096
	4	9	.006	8	.022	8	.022	7	.070
	5	11	.009	11	.009	10	.026	9	.066
	6	14	.007	13	.015	12	.034	11	.075
	7	17	.006	16	.013	15	.026	13	.096
	8	20	.008	19	.015	17	.047	16	.080
	9	24	.008	22	.021	21	.034	19	.084
	10	29	.007	26	.024	25	.036	23	.075
	11	34	$.010^-$	32	.019	29	.049	27	.088

		p = .99							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
16	12	42	.009	38	$.025^-$	36	.041	33	.083
	13	53	.009	48	.023	44	.049	40	.098
	14	>60	-	>60	-	58	.049	53	.090
	15	>60	-	>60	-	>60	-	>60	-
17	1	3	.005	3	.005	3	.005	2	.078
	2	5	.004	5	.004	4	.029	4	.029
	3	7	.004	6	.019	6	.019	5	.087
	4	9	.005	8	.018	8	.018	7	.060
	5	11	.007	10	.020	10	.020	9	.055
	6	14	.004	13	.011	12	.026	11	.060
	7	16	.009	15	.018	14	.037	13	.074
	8	19	.009	18	.017	17	.032	15	$.100^-$
	9	23	.007	21	.020	20	.034	18	.090
	10	27	.008	25	.019	23	.045	21	$.100^-$
	11	32	.008	29	.024	27	.048	25	.094
	12	38	.009	35	.021	32	$.050^+$	30	.086
	13	46	.009	42	.023	39	.046	36	.088
	14	57	$.010^+$	52	$.025^-$	48	.049	44	.094
	15	>60	-	>60	-	>60	-	57	.095
	16	>60	-	>60	-	>60	-	>60	-
18	1	3	.005	3	.005	3	.005	2	.074
	2	5	.003	5	.003	4	.027	4	.027

Lower tail critical points

K	X	p = .99 Lower tail probability level α 0.01 N_1	α_1	0.025 N_1	α_1	0.05 N_1	α_1	0.10 N_1	α_1
18	3	7	.003	6	.017	6	.017	5	.079
	4	9	.004	8	.015	8	.015	7	.052
	5	11	.005	10	.016	9	.046	9	.046
	6	13	.008	12	.020	11	.048	11	.048
	7	16	.006	15	.013	14	.027	13	.058
	8	19	.006	17	.022	16	.041	15	.076
	9	22	.007	20	.022	19	.038	18	.064
	10	25	.010+	24	.017	22	.043	21	.067
	11	30	.008	28	.018	26	.039	24	.083
	12	35	.009	32	.024	30	.046	28	.086
	13	41	.010+	38	.024	36	.040	33	.086
	14	50	.009	46	.022	43	.042	39	.094
	15	>60	-	57	.022	52	.049	48	.091
	16	>60	-	>60	-	>60	-	>60	-
	17	>60	-	>60	-	>60	-	>60	-
19	1	3	.005	3	.005	3	.005	2	.071
	2	5	.003	4	.024	4	.024	4	.024
	3	7	.003	6	.015	6	.015	5	.073
	4	9	.003	8	.012	7	.046	7	.046
	5	11	.004	10	.013	9	.038	9	.038
	6	13	.006	12	.015	11	.039	10	.094
	7	15	.009	14	.021	13	.045	12	.096
	8	18	.007	17	.015	16	.030	15	.058

K	X	p = .99 Lower tail probability level α 0.01 N_1	α_1	0.025 N_1	α_1	0.05 N_1	α_1	0.10 N_1	α_1
19	9	21	.008	20	.014	18	.046	17	.080
	10	24	.010-	23	.017	21	.045	20	.073
	11	28	.009	26	.023	25	.035	23	.079
	12	33	.008	30	.025-	29	.036	26	.099
	13	38	.010-	35	.025-	33	.045	31	.081
	14	45	.009	42	.020	39	.043	36	.087
	15	54	.009	50	.021	46	.048	43	.084
	16	>60	-	>60	-	56	.050+	52	.089
	17	>60	-	>60	-	>60	-	>60	-
	18	>60	-	>60	-	>60	-	>60	-
20	1	3	.004	3	.004	3	.004	2	.069
	2	5	.003	4	.022	4	.022	4	.022
	3	7	.002	6	.013	6	.013	5	.067
	4	8	.010+	8	.010	7	.040	7	.040
	5	10	.010+	10	.010	9	.033	8	.097
	6	13	.004	12	.012	11	.032	10	.081
	7	15	.007	14	.016	13	.036	12	.079
	8	18	.005	16	.022	15	.045	14	.088
	9	20	.009	19	.018	18	.033	17	.061
	10	23	.010+	22	.018	21	.031	19	.086
	11	27	.008	25	.022	24	.034	22	.083
	12	31	.009	29	.020	27	.044	25	.093
	13	36	.008	33	.024	31	.047	29	.088

Lower tail critical points

		p = .99							
		Lower tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_1	α_1	N_1	α_1	N_1	α_1	N_1	α_1
20	14	42	.008	39	.019	36	.045	34	.078
	15	49	.009	45	.023	42	.046	39	.089
	16	58	.010	54	.021	50	.045	46	.091
	17	>60	-	>60	-	>60	-	56	.088
	18	>60	-	>60	-	>60	-	>60	-
	19	>60	-	>60	-	>60	-	>60	-

Upper tail critical points

		p = .99							
		Upper tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
4	4	-	-	-	-	-	-	4	.090
5	5	-	-	-	-	5	.037	5	.037
6	5	-	-	-	-	-	-	5	.088
	6	-	-	6	.015	6	.015	7	.051
7	6	-	-	-	-	6	.040	6	.040
	7	7	.006	8	.023	8	.023	9	.055
8	6	-	-	-	-	-	-	6	.072
	7	-	-	7	.018	7	.018	8	.066
	8	9	.010+	9	.010	10	.026	11	.053
9	7	-	-	-	-	7	.035	7	.035
	8	8	.008	8	.008	9	.032	10	.078
	9	10	.004	11	.012	12	.027	14	.077
10	7	-	-	-	-	-	-	7	.056
	8	-	-	8	.017	8	.017	9	.064
	9	9	.003	10	.015	11	.041	12	.082
	10	12	.006	13	.013	15	.043	17	.095
11	7	-	-	-	-	-	-	7	.080
	8	-	-	-	-	8	.029	8	.029
	9	9	.008	9	.008	10	.033	11	.083

		p = .99							
		Upper tail probability level α							
K	X	0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
11	10	11	.007	12	.021	13	.045	14	.081
	11	14	.006	16	.023	17	.038	19	.080
12	8	-	-	-	-	8	.043	8	.043
	9	-	-	9	.014	9	.014	10	.057
	10	10	.003	11	.017	12	.046	13	.093
	11	13	.010+	14	.024	15	.046	16	.077
	12	16	.007	18	.021	20	.048	22	.089
13	8	-	-	-	-	-	-	8	.059
	9	-	-	9	.022	9	.022	10	.086
	10	10	.007	10	.007	11	.031	12	.079
	11	12	.008	13	.024	13	.024	15	.097
	12	14	.005	16	.025+	17	.045	18	.071
	13	18	.006	20	.019	22	.041	25	.096
14	8	-	-	-	-	-	-	8	.076
	9	-	-	-	-	9	.032	9	.032
	10	-	-	10	.011	11	.049	11	.049
	11	11	.003	12	.016	13	.045	14	.093
	12	13	.004	14	.013	15	.030	17	.097
	13	16	.006	18	.025+	19	.042	21	.094
	14	21	.010+	23	.024	25	.047	28	.099
15	8	-	-	-	-	-	-	8	.093
	9	-	-	-	-	9	.043	9	.043

Upper tail critical points

K	X	0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
		p = .99							
		Upper tail probability level α							
15	10	-	-	10	.017	10	.017	11	.070
	11	11	.006	11	.006	12	.027	13	.071
	12	13	.008	14	$.025^{-}$	14	.025	15	.055
	13	15	.006	16	.016	17	.033	19	.094
	14	18	.007	20	.024	21	.039	23	.084
	15	23	.009	25	.021	27	.039	30	.083
16	9	-	-	-	-	-	-	9	.055
	10	-	-	10	.024	10	.024	11	.093
	11	11	.009	11	.009	12	.040	12	.040
	12	12	.003	13	.014	14	.041	15	.088
	13	14	.004	15	.013	16	.032	17	.062
	14	17	.008	18	.018	19	.035	21	.089
	15	20	.007	22	.023	23	.036	25	$.100^{-}$
	16	25	.008	28	.024	30	.043	33	.084
17	9	-	-	-	-	-	-	9	.068
	10	-	-	-	-	10	.032	10	.032
	11	-	-	11	.013	11	.013	12	.056
	12	12	.005	13	.022	13	.022	14	.062
	13	14	.007	15	.023	15	.023	17	.099
	14	16	.007	17	.018	18	.037	19	.066
	15	19	$.010^{+}$	20	.020	21	.035	23	.083
	16	22	.007	24	.021	26	.047	28	.087
	17	28	$.010^{+}$	30	.020	33	.046	36	.085

K	X	0.01 N_2	0.01 α_2	0.025 N_2	0.025 α_2	0.05 N_2	0.05 α_2	0.10 N_2	0.10 α_2
		p = .99							
		Upper tail probability level α							
18	9	-	-	-	-	-	-	9	.081
	10	-	-	-	-	10	.040	10	.040
	11	-	-	11	.018	11	.018	12	.073
	12	12	.007	12	.007	13	.032	14	.086
	13	13	.002	14	.012	15	.036	16	.080
	14	15	.004	16	.013	17	.031	18	.062
	15	18	$.010^{-}$	19	.021	20	.040	21	.067
	16	20	.005	22	.020	23	.034	25	.077
	17	24	.007	26	.019	28	.042	31	.098
	18	30	.009	33	.023	36	.048	40	.099
19	9	-	-	-	-	-	-	9	.095
	10	-	-	-	-	10	.049	10	.049
	11	-	-	11	.023	11	.023	12	.092
	12	12	$.010^{-}$	12	.010	13	.044	13	.044
	13	13	.004	14	.018	14	.018	15	.053
	14	15	.006	16	.021	17	.049	18	.093
	15	17	.007	18	.018	19	.038	20	.068
	16	19	.005	21	.024	22	.042	24	$.100^{-}$
	17	22	.006	24	.020	26	.049	28	.097
	18	26	.007	28	.018	31	$.050^{-}$	33	.085
	19	33	$.010^{+}$	36	$.025^{-}$	39	.049	43	.097
20	10	-	-	-	-	-	-	10	.059
	11	-	-	-	-	11	.029	11	.029
	12	-	-	12	.013	12	.013	13	.057

Upper tail critical points

K	X	p = .99							
		Upper tail probability level α							
		0.01		0.025		0.05		0.10	
		N_2	α_2	N_2	α_2	N_2	α_2	N_2	α_2
20	13	13	.005	13	.005	14	.026	15	.071
	14	15	.010$^+$	15	.010	16	.031	17	.071
	15	16	.003	17	.012	18	.029	19	.059
	16	19	.010$^-$	20	.022	21	.042	22	.071
	17	21	.007	23	.025$^+$	24	.042	26	.095
	18	24	.006	26	.020	28	.046	30	.088
	19	29	.010$^+$	31	.023	33	.043	36	.092
	20	35	.009	38	.021	42	.050$^-$	46	.095