Phenomenological theories of creep and long-term strength of structural materials subjected to irradiation under independent or combined quasi-static and cyclic loading

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1 INTRODUCTION

In-service operation of nuclear reactors, while consumption of power is under control, necessitates to take into account the effect of variations in loading and temperature on the lifetime of structural components. Of particular importance here is the problem of developing a fatigue life criterion which would enable establishing the allowable number of power drops and introducing the operating conditions with stress and strain values being known.

During constant-power operation periods the structural material is subjected to creep, and damage is accumulated in it, the one which we conventionally will call "static" damage.

When entering the operating conditions and during power drops the material is loaded or unloaded alongside with negative stresses occurring in it. The resulting damage may cause, regarding stress (σ) and temperature, ductile or brittle fracture. This type of damage will be further called "fatigue" damage.

Our main concern here is to find the critical value for total damage at which failure occurs.

For the quantitative evaluation of damage, the formal parameter ω was introduced which varies from the moment of loading to the unloading of the material within $0 \leqslant \omega \leqslant 1$. As indicated above, it is assumed to incorporate both "static" and "fatigue" damage ω_s and ω_f .

2 EVALUATION OF DAMAGE DURING STATIC-LOADING PERIODS

For the cases of isothermal loading at σ = const

(1) $$\omega_s = f_1(\sigma) \cdot f_2(1-\omega_s)$$

Formally, relying upon physical and logic considerations we assume

(2) $$f_1(\sigma) = B\sigma^n$$

(3) $$f_2(1-\omega) = (1-\omega)^{-s}$$

where B, n and s are coefficients. Upon substituting (2) and (3) into Eq.(1), dividing the variables and integrating at σ = const, while the initial conditions are $\omega = 0$, $t = 0$, we obtain

$$\omega_s = 1 - \left[1 - (s+1)B\sigma^n t\right]^{\frac{1}{s+1}} \tag{4}$$

Bearing in mind that at fracture $t = t_R$ and $\omega_s = 1$, the following equation is obtained

$$t_R = N\sigma^{-n}, \quad \left(N = \frac{1}{B(s+1)}\right), \tag{5}$$

which is well supported experimentally.

Now, Eq.(4) for the static-loading portion lasting may be rewritten as

$$\omega_s = 1 - \left(1 - \frac{t_B}{t_R}\right)^{\frac{1}{s+1}} = 1 - (1-K)^{\frac{1}{s+1}} \tag{6}$$

where $K = (s+1)B\sigma^n t_B$. The coefficient s may be found from the experimental dependence $t_R = f(\sigma)$.

To develop the material's creep equation we shall employ the following hardening hypothesis substituting initial stress σ for the "true" stress $S = \frac{\sigma}{1-\omega_s}$.

$$\dot{p} = p^{\alpha} \cdot f(S), \tag{7}$$

where p and $\dot{p}$ are creep strain and creep rate, respectively. Now, let us take the simplest assumption concerning function $f(S)$

$$f(S) = A\left(\frac{\sigma}{1-\omega_s}\right)^k \tag{8}$$

where A, k are coefficients. Substituting ω_s from (6) into (8) and then the result obtained into Eq.(7), upon its integration for σ = const, when the initial conditions are $p = 0$ and $t = 0$, we obtain

$$p = \left\{\frac{C}{F}\left[1 - \left(1 - \frac{t}{t_R}\right)^D\right]^F\right\} \tag{9}$$

where the following notation is introduced

$$C = \frac{A}{B(s+1-n)}, \quad D = \frac{s+1-n}{s+1}, \quad F = \frac{1}{1+\alpha}, \quad k = n \tag{10}$$

When during the static loading period relaxation occurs at a small rate according to $\sigma(t)$, then

$$K = (s+1)\int_0^{t_B} f_1[\sigma(t)]\,dt \tag{11}$$

Approximating the relaxation curve by the straight-line equation

$$\sigma(t) = \sigma - \dot{\sigma}t \tag{12}$$

and assuming by analogy with (2)

(13) $$f_1[\sigma(t)] = B(\sigma_0 - \dot{\sigma}t)^n$$

we obtain

(14) $$K = L[\sigma_0^{n+1} - (\sigma_0 - \dot{\sigma}t)^{n+1}]$$

where

(15) $$L = \frac{B(s+1)}{\dot{\sigma}\, s}$$

The value of B = const for the given type of a material and temperature is estimated using the expression

(16) $$B = \frac{\dot{\sigma}\, s}{(s+1)(\sigma_R^s - \sigma_0^s)}$$

basing on the data of tests in tension at a random rate $\dot{\sigma}$ from the initial stress σ_0 up to fracture stress σ_R. Eq.(16) is obtained assuming $\omega_s = 0$ at $t = 0$. When static loads occur periodically, the initial condition for each i-th cycle in integrating (1) will be $\omega_s = \omega_s^{(i-1)}$ at $t = 0$, and Eq.(6) will have the following form

(17) $$\omega_s^{(i)} = 1 - [(1 - \omega_s^{(i-1)})^{s+1} - K]^{\frac{1}{s+1}}.$$

3 EVALUATION OF DAMAGE FOR THE PORTIONS OF ACTIVE LOADING

The equation for the increments of the damage parameter during initial loading has the following form

(18) $$\frac{d\omega_f}{d\sigma} = \varphi_1(\sigma) \cdot \varphi_2(\omega_f).$$

The variations of parameter ω_f due to σ and the area of undamaged section will be expressed by the power function

(19) $$\varphi_1(\sigma) = M\sigma^m,$$

(20) $$\varphi_2(\omega_f) = (1 - \omega_f)^{-r},$$

where M, m, r are coefficients. Solution of eq.(18) upon substituting (19) and (20) for the initial condition $\omega_f = 0$ at $\sigma = 0$ as applied to the first loading takes the following form

(21) $$\omega_f = 1 - [1 - \frac{M(r+1)}{m+1}\sigma^{m+1}]^{\frac{1}{r+1}}.$$

Provided initial loading lasts to fracture, then $\omega_f = 1$ and $\sigma = \sigma_B$. Hence, from Eq.(21) it follows

$$\frac{M(r+1)}{m+1} = \frac{1}{\sigma_B^{m+1}}$$

and it may be rewritten as

(22) $$\omega_f = 1 - (1-R)^{\frac{1}{r+1}}, \quad R = \left(\frac{\sigma}{\sigma_B}\right)^{m+1}.$$

If Eq.(18) is being solved for the beginning of any i-th half-cycle within $1 \leqslant i \leqslant N_R^0$, where N_R^0 is the number of cycles to failure under cyclic loading (without hold-times), i.e. with the initial condition being $\omega_f = \omega_f^{(i-1)}$ at $\sigma = 0$, then we obtain

(23) $$\omega_f^{(i)} = 1 - \left[\left(1 - \omega_f^{(i-1)}\right)^{r+1} - R\right]^{\frac{1}{r+1}}$$

It is easy to show that when experimental stress dependence is approximated by the power function

(24) $$N_R^0 = P\sigma^{-l}$$

where P, l are coefficients, then $l = m + 1$ and

(25) $$R = \frac{1}{N_R^0}$$

The first approximation for r may be found assuming in analogy with the dependencies that $\omega_f = \omega_f(\sigma)$ and $\hat{\varepsilon} = \hat{\varepsilon}(\sigma)$, where $\hat{\varepsilon} = \frac{\varepsilon}{\varepsilon_R}$ and ε, ε_R are the given strain and fracture strain, respectively. From this assumption it ensues (see Eq.(18)) that

(26) $$\frac{d\hat{\varepsilon}}{d\sigma} = \varphi_{11}(\sigma)(1-\omega)^{-r}$$

Hence, r may be found as the slope of the curve in coordinates $tg\left(\frac{d\hat{\varepsilon}}{d\sigma}\right) - tg(1-\hat{\varepsilon})$ in the point which corresponds to the given strain $\hat{\varepsilon}$.

4 SUMMATION OF "STATIC" AND "FATIGUE" DAMAGE

Let us solve Eq.(18) for any i-th cycle taking into account the assumulation of "static" and "fatigue" damage in the foregoing cycles, i.e. when the initial condition is $t = 0$, $\omega = \omega_s^{(i-1)} + \omega_f^{(i-1)}$. Then we have

(27) $$\omega^{(i)} = 1 - \left[\left(1 - \omega_s^{(i-1)} - \omega_f^{(i-1)}\right)^{r+1} - R\right]^{\frac{1}{r+1}}.$$

According to the initial assumption about damage parameter in fracture ($i = N_R$)

(28) $$\omega^{(N_R)} = 1.$$

Comparing Eq,(27) and (28), the fracture criterion may be written as

$$(29)\qquad (1-\omega_s^{(i-1)}-\omega_f^{(i-1)})^{r+1}\leqslant R,\ (i=N_R).$$

Here a formulation may be given for another case of fracture criterion. "Static" and "fatigue" damage increment per each cycle is

$$(30)\qquad \Delta\omega_s^{(i)}=\omega_s^{(i)}-\omega_s^{(i-1)},\quad \Delta\omega_f^{(i)}=\omega_f^{(i)}-\omega_f^{(i-1)}.$$

So, the limiting value of damage parameter is decreased per each cycle by $\Delta\omega_s^{(i)}+\Delta\omega_f^{(i)}$. Therefore, the equation for the total damage parameter may be written as

$$(31)\qquad \omega^{(i)}=1-[(1-\Delta\omega_s^{(i)}-\Delta\omega_f^{(i)}-\omega^{(i-1)})^{r+1}-R]^{\frac{1}{r+1}}$$

and the appropriate fracture criterion takes such a form

$$(32)\qquad (1-\Delta\omega_s^{(i)}-\Delta\omega_f^{(i)}-\omega^{(i-1)})^{r+1}\leqslant R,\ (i=N_R).$$

Cycles to fracture are calculated through verification of conditions (29) or (32) starting from i = 1 and using Eqs.(17) and (23). The latter ones suggest independent variations in ω_s and ω_f. However, if they are assumed to interfere with each other, then the appropriate changes take place in the initial conditions of integration of Eqs.(1) and (18), and instead of (17) and (29) the following equations should be employed

$$(33)\qquad \omega_s^{(i)}=1-[(1-\omega_s^{(i-1)}-\omega_f^{(i-1)})^{s+1}-K]^{\frac{1}{s+1}},$$

$$(34)\qquad \omega_f^{(i)}=1-[(1-\omega_f^{(i-1)}-\omega_s^{(i-1)})^{r+1}-R]^{\frac{1}{r+1}}.$$

5 EXPERIMENTAL VERIFICATION

Upon austenization (1323 K, hold-time 30 min., cooling in water) type OKh16N15M3B steel was tested in creep to failure and low-cycle fatigue under symmetrical strain-controlled alternating loading with hold-times at maximum strain amplitudes. Tests were conducted at T=923 K, in unirradiated and in-pile conditions (φ_t = $6{,}9\cdot10^{18}$ n·m^{-2} sec^{-1}, $E\geqslant0.1$ MeV). Under unirradiated conditions hold periods lasted 5 and 30 min. and in in-reactor experiments t_g = 1, 5, 10 and 30 min. During hold times specimens were in the intermediate creep-to-relaxation regime.

It was taken into account that coefficient $s = n + 1$, where n is the power index for the long-term strength equation (5).

The value of K was calculated from (14). The real relaxation curve was approximated by Eq.(12), therefore the rate of the decrease in stress was determined as

$$\sigma=\frac{\Delta\sigma}{t_g}$$

where $\Delta\sigma$ is the absolute value for the decrease in stress during the hold time.

To evaluate B according to (16), an experiment was conducted at a constant rate of the increase in stress from σ_0 to the fracture at stress σ_R.

Table. The values for experimental and predicted cyclic life of irradiated type 0Kh16N15M3B steel determined from criteria (29) and (32) using Eq.(17) and (23)

t_B min	ε_p %	$\Phi_f^B \cdot 10^{22}$ $n \cdot m^{-2}$	$R \cdot 10^3$	$K \cdot 10^3$	r	N_R^{exp} cycle	N_R using eq.(29) cycle	N_R using eq.(32) cycle
1	0.69	1.04	3.884	0.29	3.32	165	213	156
1	0.54	1.01	1.735	0.21	4.43	370	515	351
1	0.42	3.04	1.445	0.278	3.55	270	577	418
1	0.38	3.0	1.051	0.242	4.01	360	821	574
1	0.31	3.14	0.886	0.174	5.22	585	1043	686
1*	0.3	5.03	1.337	0.364	3.02	430	583	448
5	1.34	0.33	25.53	10.528	1.36	25	21	21
5	0.62	5.38	7.641	3.358	0.95	65	55	55
5	0.66	0.86	3.008	0.984	3.61	195	258	199
5	0.55	0.95	1.772	0.762	4.52	235	471	336
5	0.42	2.94	1.877	0.868	4.02	260	425	316
5	0.31	3.11	0.889	0.536	6.17	440	1011	658
5	0.42	1.58	1.123	0.536	6.12	480	808	528
5	0.33	2.28	0.778	0.388	8.05	570	1229	755
5	0.64	2.25	4.862	1.49	2.41	55	135	122
10	0.52	1.64	2.176	1.148	4.94	195	382	272
10	0.47	1.93	1.774	4.148	4.94	211	461	329
30	0.88	2.9	13.18	4.056	1.44	31	32	32
30	0.72	1.66	5.729	4.604	3.04	61	106	101
30	0.46	3.41	2.679	3.71	3.65	97	215	203
30	0.64	1.85	4.322	3.4	4.02	114	160	134
30	0.43	4.2	2.67	3.71	3.73	122	217	204
30	0.36	4.47	1.827	3.19	5.76	124	393	299
30	0.32	4.39	1.35	1.532	7.96	194	539	399
30	0.24	5.0	0.777	0.97	11.45	325	1011	652
30	0.19	9.18	0.906	1.064	10.57	424	906	573
30	0.21	6.08	0.73	0.846	12.93	233	1157	694
30	0.40	5.3	0.273	0.96	1.08	120	95	95

*Post-reactor experiments

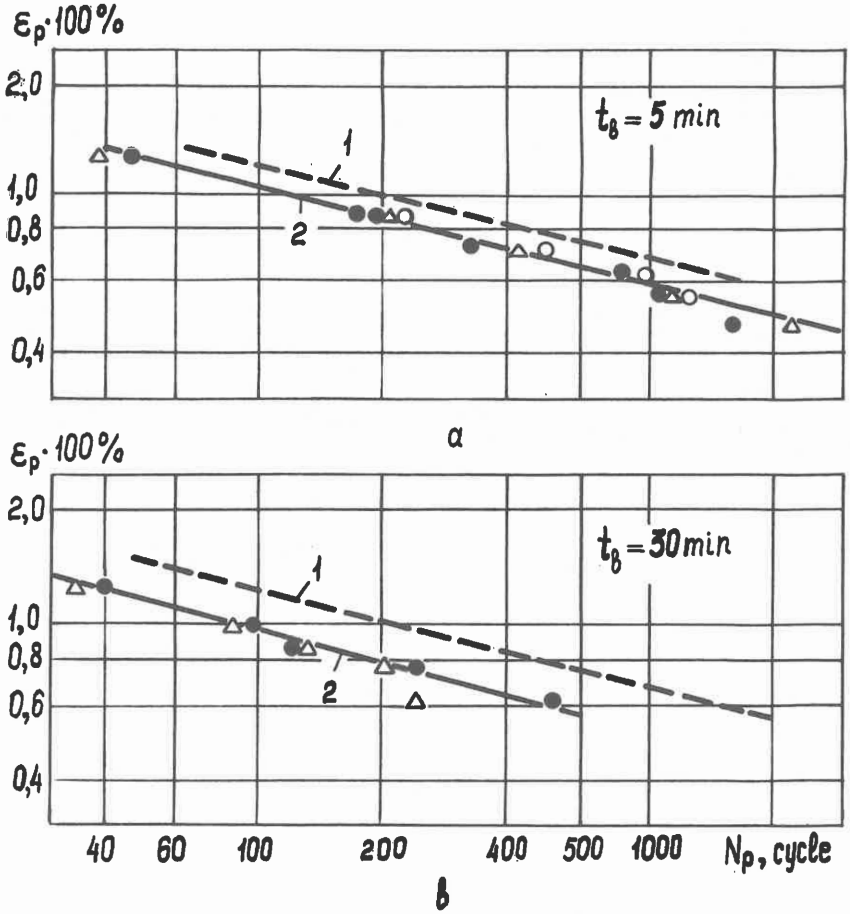

Figure. Cyclic life vs plastic strain amplitude for OKh16N15M3B steel at 650°C: 1 experimental curve for low--cycle fatigue ($t_в$ = 0); 2 predicted cyclic life (● experiment; ○ prediction based on criterion (29); △ prediction based on criterion (32)).

The value of K was duplicated because stress relaxation occurred twice per one cycle.

The procedure for determining r is indicated in Section 3. To evaluate ε_R , the real strain curve obtained upon stabilization of elasto-plastic hysteresis loop was approximated by the Eq. $\varepsilon = \varepsilon$ (σ) and then ε_R corresponding to σ_R was calculated. The value of σ_R was duly found from Eq.(24), provided N^o_R = 1. For in-reactor experiments the strain curve for irradiated steel was approximated with the account taken of fluence φ^B_f attained by the beginning of the experiment (Kiselevsky 1983).

Calculation of R was carried out by (25). In the case of in-reactor experiments the approximating equation for N^o_R versus ε_p (plastic strain amplitude) took account of the decrease in life due to fluence φ_f^e attained by the moment of fracture (Skripnik 1979).

The values for experimental and predicted cyclic life determined on the basis of (29) and (32) using (17) and (23) for unirradiated conditions are given in the Figure and for in-reactor conditions in the Table.

REFERENCES

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