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Use of damage model and method by correlation of digital pictures for the necking detection

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ABSTRACT : Two approaches are detailed for the determination of Forming Limit Diagrams (F.L.D). The first one is experimental : a correlation technique was developed for the displacement field measurement on a sheet. The second approach is numerical : a necking criterion based on the Swift assumption is proposed. It includes the modified Gurson-Tvergaard damage model taking account the Hill's or Barlat and Lian's anisotropy. Finally, the F.L.D for the XES steel obtained by an experimental and theoretical way are compared.

1. INTRODUCTION

In order to obtain a F.L.D, an experimental test could be done. Nevertheless improvements, this work remains long and difficult. Also, necking criterion, based on the M-K model or on a instability analysis, must be used. But, the metal forming process is complex and the given results are not coincident [1]. So, it is necessary to improve the experimental conditions for validating new necking criterion in order to found only one solution to this problem.

2. EXPERIMENTAL DETERMINATION IMPROVEMENT OF F.L.D

The classical grid deposit on a sheet would not allowed an important reduction of time for the determination of a F.L.D. In this aim, a new correlation technique is developed [2]. A whole of pixel, called pattern, of an initial picture are directly compared with the pixels of the same picture obtained after distortions. The displacement field is obtained with an excellent accuracy : 1/60 th of a pixel and the measured strain could reach 80% between two pictures. Classical marking were replaced by a randomize pattern (speckle) put down, in few seconds, on the sheet with a paint spray : the determination time of a F.L.D is strongly reduced.

2.1. Mathematical principle of the correlation method

An picture of the studied body could be represented by a discrete function of its grey levels : value between 0 and 255 and noted f(x,y). This picture will be deformed and its function of grey level will become f*(x*,y*). The displacement field is supposed homogeneous inside the pattern. The theoretical relation between this two discrete functions could be written as :

$$f^*(x^*,y^*) - f(x+u(x,y),y+v(x,y)) = 0 \quad (1)$$

where u(x,y) and v(x,y) represent the displacement field for one pattern. Figure 1 presents different used definitions (for a best understanding, the initial picture and the deformed picture are represented in the same axis).

There are four points for a correlation technique. First, the mathematical definition for the displacement field on a pattern which has to include, at the same time, the distortion and the term of rigid body displacement :

$$\begin{cases} u(x,y) = au \cdot x + bu \cdot y + cu \cdot x \cdot y + du \\ v(x,y) = av \cdot x + bv \cdot y + cv \cdot x \cdot y + dv \end{cases} \quad (2)$$

The main reason for this choice is that the research of all the terms of this displacement field will be done by a bilinear interpolation.

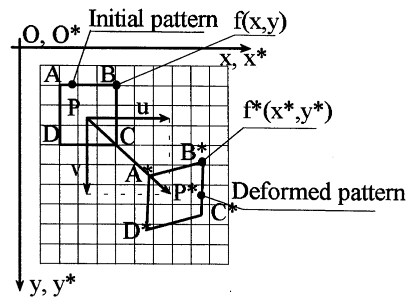

Figure 1 : Notation

Then, a mathematical correlation criterion between the two discrete functions f(x,y) and f*(x*,y*) must be held. The most used correlation coefficient for a correlation technique are the least square coefficient and the crossed coefficients [3] :

$$C_1 = \int_{\Delta M} \left(f(x,y) - f^*(x^*,y^*)\right)^2 \cdot dx \cdot dy \quad \text{or} \quad C_2 = 1 - \frac{\int_{\Delta M} f(x,y) \cdot f^*(x^*,y^*) \cdot dx \cdot dy}{\sqrt{\int_{\Delta M} f(x,y)^2 \cdot dx \cdot dy \cdot \int_{\Delta M} f^*(x^*,y^*)^2 \cdot dx \cdot dy}} \quad (3)$$

Where, ΔM represents the surface of the pattern in the initial picture. Then, an interpolation method for the grey level of the pictures is needed in order to reach a subpixel accuracy, and the mathematical solution for the determination of the elongation and the distortion terms for a pattern [4].

2.2. Research of the displacement field

The originality of this method is the following way of researching the displacement field. Consider a pattern centred in P and tops A, B, C, D (figure 1). The correlation calculations will be done on the fours patterns centred in A, B, C, and D with the following assumption : their strain field are identical to the one centred in P. These calculations are going to determine the rigid body displacement of the point A, B, C, and D which will be used to obtain all the terms of the strain field for the pattern centred in P. The exact solution from a numeric point of view, will be determined with the help of an iterative process : the rigid body displacement of the points A, B, C, and D will be searched during the step 'i' with the help of the components of the strain field obtained at the step '(i-1)'. At the beginning of the iterative process, a rough initial solution for the two components of the displacement field is given for the pattern. The strain field inside this pattern centred in P is therefore reduced, at the first iteration, to the rigid body displacement terms only, because the strain terms are considered initially equal to zero. The new position for the points A*, B*, C*, and D* in the final picture are searched successively for this first iteration with the help of the previous strain field. This research is done by scanning following x and y axis. The accurate position of the new coordinates of the points is obtained by the decreasing evolution simultaneously of the scan size area and the scan step. These displacements which allow to obtain a minimal correlation coefficients denoted du_{A1} and dv_{A1} for the point A and respectively, du_{B1}, dv_{B1}, du_{C1}, dv_{C1}, du_{D1}, dv_{D1}, for the points B, C, D. With this eight values, the displacement field in P is obtained by a bilinear interpolation. The rigid body displacement calculation is continued until the following stability is reached :

$$\left|du_{Pn} - du_{P(n-1)}\right| \le \varepsilon \quad \text{and} \quad \left|dv_{Pn} - dv_{P(n-1)}\right| \le \varepsilon \quad \text{with} \quad \varepsilon = 0.01 \text{ pixel} \tag{4}$$

In the case where the iterative process would not converge, it is stopped at the 10 th iteration and the pattern is wrong and will not be taken into account for the strain calculation. Then, an interpolation allows to find the missing points after the displacement field calculations. The displacement field being obtained for this first pattern, the solution for the neighbours has to be searched in order to obtain the solution for the whole picture. Consequently, all the patterns surrounding this first pattern, will take its displacement field as initial solution. The displacement field being obtained, the distortion field must be evaluated. The distortions calculations could be done either for one pattern either for a set of pattern. This calculation use the deformation gradient F and the Green-Lagrangian tensor E.

3. MATERIAL AND EXPERIMENTS

This correlation technique for the strain limit determination on a sheet requires the following equipment : a CCD camera (1024*1024 pixels) and a acquisition card. A constant light during the metal forming must be held. The cover field by the camera is 15x15 mm².

Due to the Marciniak test (special shape of the punch), the necking will occur on this field, so a correlation technique will be easily used. A new apparatus was developed for the Marciniak test [5]. During the punch stroke, the sheet metal is stamped on the fixed punch. An inclined mirror at 45° restores the picture on the camera placed in front of the machine. The grid step could be changed at the beginning of the calculations. For the strain measurement on a sheet tested in metal forming, a pattern will be constituted by 8 or 12 pixels square (grid size between 2 and 3.5 .mm). Sheets are cut with different widths in order to cover all the strain path. An area is taken perpendicularly to the necking line. An initial picture is captured as reference, at the necking occurrence, an other picture is taken

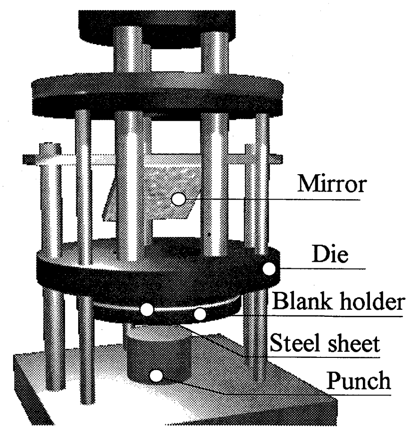

Figure 2 : New metal forming apparatus

The advantages of this correlation technique for this application are various. First, classical marking are not needed which represents a big saving of time. Then, the computation time are strongly reduced due to the correlation algorithm. Also, the accuracy on the limit strain is improved.

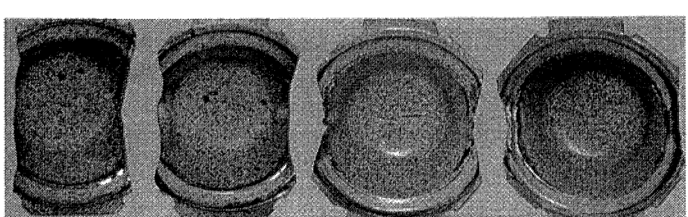

Figure 3 : Drawing sheets until the necking occurrence with speckle deposit

4. THEORETICAL DETERMINATION OF F.L.D.

Concurrently to this experimental approach, a numerical study for the necking detection of thin sheet is developed. This criterion is based on the Swift assumption.

4.1. Gurson-Tvergaard damage model

The Gurson-Tvergaard [6-7] yield condition taking account an damage process is written as :

$$\Phi = q^2/\sigma_y^2 + 2q_1 f^* \cosh(-3q_2\, p/2\sigma_y - (1+q_3 f^{*2}) = 0 \tag{5}$$

Where p is the hydrostatic pressure and f^* represents the porosity of the microvoided material. In order to make the predictions of the Gurson model agree with numerical studies of ordered voided materials in plane strain tensile fields, Tvergaard [8] introduced the constant $q_1 = 1.5$, $q_2 = 1$, $q_3 = 2.25$. The damage model takes into account the three main phases of damage evolution : nucleation, growth and coalescence of microvoids. The microvoid volume fraction is given by :

$$df = df_N + df_G + df_C \tag{6}$$

Considering a random distribution of second phase particles, microvoid volume fraction increment due to nucleation is expressed by :

$$df_N = (f_N/S_N\sqrt{2\pi})\cdot \exp\left\{-\left(\bar{\varepsilon}^p - \varepsilon_N\right)^2/(2S_N^2)\right\} d\bar{\varepsilon}^p \tag{7}$$

The normal distribution of the nucleation strain has a mean value ε_N, standard deviation S_N and nucleates voids with volume fraction f_N. Growth of existing voids is based on the apparent volume change and law of conversation of mass and is expressed as :

$$df_G = (1-f)\left(d\varepsilon_{11}^P + d\varepsilon_{22}^P + d\varepsilon_{33}^P\right) \tag{8}$$

Finally, the modification of the yield condition to account for coalescence and final material failure is introduced trough the function $f^*(f)$ specified by [9] :

$$\begin{cases} f^* = f & f < f_c \\ f^* = f_c + \delta(f - f_c) & f \geq f_c \end{cases} \quad \text{with} \quad \delta = \frac{f_u^* - f_c}{f_f - f_c} \tag{9}$$

Where $f_u^* = 1/q_1$ is the ultimate value of f^* at ductile rupture, f_c is a critical value of the void volume fraction when the coalescence of microvoids occurs and the stress-carrying capability of the material sharply drops and finally, f_f is the void volume fraction for which the stress-capability totally vanishes (final failure). q is the effective stress of the macroscopic Cauchy stress tensor σ which, instead of the original Mises stress, is replaced by the orthotropic Hill effective stress or by any others general quadratic or non quadratic loci description as Barlat and Lian [10].

4.2. Mathematical form of the necking criterion

A necking criterion is proposed within the framework of the modified force maximum criterion which includes the determination of local necking for arbitrary strain paths for anisotropic or isotropic materials. Different orientation between the principal stresses axes and the orthotropic axes are taken account, see figure 4. x and y are the in-plane orthotropic axes and $\sigma_1 > \sigma_2$ are the principal component of stresses [7]. The Considere's assumption of unstable plastic flow occurring at the maximum load was used by Swift. The proposed criterion uses this theory. At the onset of load instability :

$$d\sigma_1/d\varepsilon_1 \le \sigma_1 \qquad (10)$$

If internal damage is taken account, σ_1 is replaced by $\sigma_1\sigma_y/q$. σ_1 is the main major Cauchy stresses and ε_1 is the main major strain. Note that without damage with the incompressibility hypothesis ($d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$), the criterion is :

$$d\sigma_1/\sigma_1 + d\varepsilon_2 + d\varepsilon_3 \le 0 \qquad (11)$$

Now, a stress dependency $\sigma_1(\varepsilon_1,\beta)$ is taken account which was introduced by Hora [11]. The strain ratio $\beta = \Delta\varepsilon_2/\Delta\varepsilon_1$ has an evident influence on the internal damage of sheets metals. At the same level of strain, it is generally noted that the damage increment is the greatest at plane strain

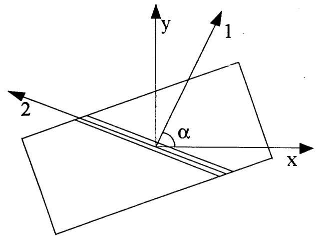

Figure 4 : Orthotropic and principal stresses axes

This indicates that plane strain is the most dangerous strain state at which strain localization is most likely to appear. Gradually forming of plane strain state after load instability (diffuse necking) may be the common origin of leading to strain localization at tension-tension and tension-compression states. In both regions, the strain evolves towards the plane strain state such that $\Delta\varepsilon_{22} = 0$ when the localized necking occurs. Then the necking criterion takes the following form :

$$\partial\sigma_1/\partial\varepsilon_1 + (\partial\sigma_1/\partial\beta)\cdot(d\beta/d\varepsilon_1) \le \sigma_1 \qquad (12)$$

With :

$$\partial\sigma_1/\partial\varepsilon_1 = (\partial\sigma_1/\partial q)\cdot(\partial q/\partial\bar{\sigma})\cdot(\partial\bar{\sigma}/\partial\bar{\varepsilon})\cdot(\partial\bar{\varepsilon}/d\varepsilon_1) \text{ and } d\sigma_1 = \partial\sigma_1/\partial\varepsilon_1\cdot d\varepsilon_1 + \partial\sigma_1/\partial\beta\cdot d\beta \qquad (13)$$

There are the six following terms : $\partial\sigma_1/\partial q$, $\partial q/\partial\bar{\sigma}$, $\partial\bar{\sigma}/\partial\bar{\varepsilon}$, $\partial\bar{\varepsilon}/d\varepsilon_1$, $\partial\sigma_1/\partial\beta$, $d\beta/d\varepsilon_1$ to be determined in order to write the necking criterion.

$\partial q/\partial\bar{\sigma}$ is determined by the following manner. The Gurson-Tvergaard yield condition gives :

$$q = \bar{\sigma}\left[1 - 2q_1f^*\cosh(-3q_2\,p/2\bar{\sigma}) + q_3f^{*2}\right]^{\frac{1}{2}} \qquad (14)$$

So :

$$\frac{\partial q}{\partial\bar{\sigma}} = \left[1 - 2q_1f\cosh(-3q_2\frac{p}{2\bar{\sigma}}) + q_3f^2\right]^{\frac{1}{2}}$$

$$+\bar{\sigma}\frac{1}{2}\left[1 - 2q_1f\cosh(-3q_2\frac{p}{2\bar{\sigma}}) + q_3f^2\right]^{-\frac{1}{2}} * -2q_1f\sinh(-3q_2\frac{p}{2\bar{\sigma}}) * 3q_2\frac{p}{2\bar{\sigma}^2} \qquad (15)$$

$\partial\bar{\sigma}/\partial\bar{\varepsilon}$ is given by the yield curve of the fully dense matrix material $\bar{\sigma} = f(\bar{\varepsilon})$, in the common form $h' = d\bar{\sigma}/d\bar{\varepsilon}$.

$d\beta/d\varepsilon_1$ is written as :

$$d\beta/d\varepsilon_1 = -d\varepsilon_2/d\varepsilon_1^2 = -\beta/d\varepsilon_1 \text{ because } \beta = d\varepsilon_2/d\varepsilon_1 \qquad (16)$$

The three others terms depend on the material behaviour. The calculations are given for the Hill'48 material behaviour.

4.3. Intrinsic formulation with the Hill'48 material behaviour

The criterion is formulated in terms of principal stress and their orientation in preference to the more usual components relative to the orthotropic directions. This type of decomposition is needed in particular when the deformation is viewed from the standpoint of a rectangular specimen cut from the sheet at any angles. The yield criterion is then written as :

$$q^2 = a(\alpha)\sigma_1^2 - 2c(\alpha)\sigma_1\sigma_2 + b(\alpha)\sigma_2^2 \qquad (17)$$

Where :

$$\begin{cases} a(\alpha) = \left[(F+G)+2N-2(F-G)\cos 2\alpha + (F+G+4H-2N)\cos^2 2\alpha\right]/4 \\ b(\alpha) = \left[(F+G)+2N+2(F-G)\cos 2\alpha + (F+G+4H-2N)\cos^2 2\alpha\right]/4 \\ c(\alpha) = -\left[(F+G)-2N-(F+G+4H-2N)\cos^2 2\alpha\right]/4 \end{cases} \qquad (18)$$

The anisotropic plastic behaviour of the metal is described by the anisotropy parameters G, H, F and N which are defined in terms of the Lankford parameters r_0, r_{90}, and r_{45} as :

$$H = r_0/(1+r_0) \quad G = 1-H \quad F = r_0/\left(r_{90}(1+r_0)\right) \quad N = \left((r_{90}+r_0)(2r_{45}+1)\right)/\left(2r_{90}(1+r_0)\right) \qquad (19)$$

The flow rule gives the normal components of the plastic strain increments on the principals axes of stress as :

$$\begin{cases} d\varepsilon_1 = d\lambda\, \partial q/\partial\sigma_1 = d\bar{\varepsilon}\left[a(\alpha)\sigma_1 - c(\alpha)\sigma_2\right]/q \\ d\varepsilon_2 = d\lambda\, \partial q/\partial\sigma_2 = d\bar{\varepsilon}\left[-c(\alpha)\sigma_1 + b(\alpha)\sigma_2\right]/q \end{cases} \quad \text{such that} \quad d\lambda = d\bar{\varepsilon} \qquad (20)$$

It is induced that the stress ratio :

$$\sigma_2 = \Omega(\beta,\alpha)\sigma_1 \quad \text{with} \quad \Omega(\beta,\alpha)_{hill} = \left(a(\alpha)\beta + c(\alpha)\right)/\left(b(\alpha)+\beta c(\alpha)\right) \qquad (21)$$

$\partial\sigma_1/\partial q$ is deduced by :

$$\sigma_1 = q\Big/\sqrt{a(\alpha)+b(\alpha)\Omega(\alpha,\beta)^2 - 2c(\alpha)\Omega} \quad \text{so} \quad \partial\sigma_1/\partial q = 1\Big/\sqrt{a(\alpha)+b(\alpha)\Omega(\alpha,\beta)^2 - 2c(\alpha)\Omega} \qquad (22)$$

$\partial\sigma_1/\partial\beta$ is given by :

$$\partial\sigma_1/\partial\beta = -q/2\left[a(\alpha)+b(\alpha)\Omega(\alpha,\beta)^2 - 2c(\alpha)\Omega\right]^{-\frac{3}{2}} \cdot \left[2b(\alpha)\Omega(\alpha,\beta)\cdot d\Omega/d\beta - 2c(\alpha)\cdot d\Omega/d\beta\right] \qquad (23)$$

With :

$$d\Omega/d\beta = \left(a(\alpha)b(\alpha) - c(\alpha)a(\alpha)\right)/\left(b(\alpha)+\beta c(\alpha)\right)^2 \qquad (24)$$

The last term $\partial\bar{\varepsilon}/\partial\varepsilon_1$ is obtained by :

$$\partial\bar{\varepsilon}/\partial\varepsilon_1 = 1/\Delta\cdot\sqrt{\gamma(\alpha,\beta)} \quad \text{with} \quad \begin{cases} \Delta = a(\alpha)b(\alpha) - c(\alpha)^2 \\ \gamma(\alpha,\beta) = a_1(\alpha) + a_2(\alpha)\beta + a_3(\alpha)\beta^2 \end{cases} \qquad (25)$$

In which :

$$
\begin{cases}
a_1(\alpha) = a(\alpha)b(\alpha)^2 - b(\alpha)c(\alpha)^2 \\
a_2(\alpha) = 2\left[a(\alpha)b(\alpha)c(\alpha) - c(\alpha)^3\right] \\
a_3(\alpha) = b(\alpha)a(\alpha)^2 - a(\alpha)c(\alpha)^2
\end{cases}
\quad (26)
$$

5. COMPARISON A F.L.D FOR A XES OBTAINED BY EXPERIMENT (CORRELATION) AND BY THE PROPOSED NECKING CRITERION

As an example, the comparison between the proposed necking criterion and our experiments have been obtained from a mild steel named XES (thickness = 0.67 mm, Elastic modulus = 198 GPa, Poisson's ratio = 0.3) The coefficients of the true-stress and logarithmic-strain uniaxial curve are : $\sigma = B\left(C + \bar{\varepsilon}^P\right)^n$, with $B = 551.14$, $C = 9.54E^{-03}$ and $n = 0.2797$. The Lankford coefficients are : $R_0 = 2.2$, $R_{90} = 1.6$ and $R_{45} = 1.9$. The following figure shows the F.L.D calculated using the proposed criterion where the given strain ration varies from -0.5 to 1 and is imposed along a linear strain path, the stress state is determined incrementally by integration until the criterion is satisfied. The F.L.D has been obtained with internal damage coupled. An initial porosity of $f_0 = 0.001$ is introduced in conjunction with the following damage parameters $S_N = 0.1, \varepsilon_N = 0.5, F_N = 0.04$, $f_c = 0.04$ and $\delta = 5$.

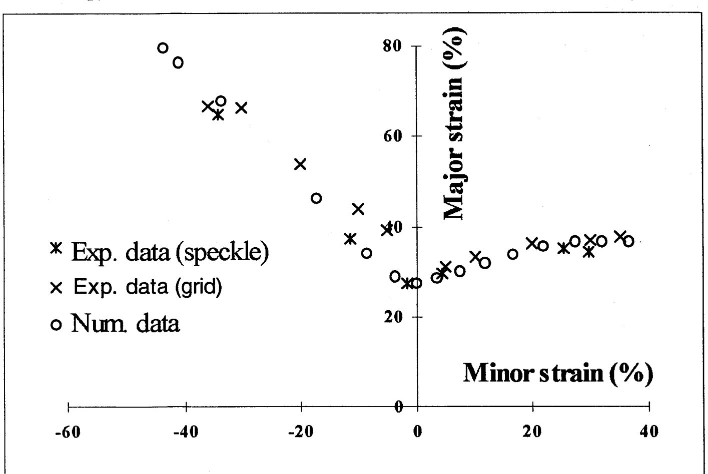

Figure 5 : Experimental and calculated Forming Limit Diagram for a XES steel

5. Conclusion

A necking criterion is proposed based on the fact that the strain state becomes a plane strain state in the neck localized by the damage variable. The criterion is written in an intrinsic form where the angle of the orthotropic axes with respect to the principal stresses axes is introduced. The given criterion is both applicable to linear and non-linear strain paths. A damage process is incorporated in the constitutive laws. The correlation technique allow to valid the necking criterion easily due to important experimental improvement (less time, easily strain measurement without hard grid deposit, better accuracy).

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