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Limit analysis based on elastic compensation method of branch pipe tee connection under internal pressure and out-of-plane moment loading

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ABSTRACT :

A new method of calculating the limit load of a structure via a sequence of incompressible elastic finite element calculation with variable Young Modulus converging to the rigid perfectly plastic problem is used to study the limit load of branch pipe tee connections.

Several models of branch pipe tee connection are meshed with shell elements and submitted to internal pressure with end axial load effect or out of plane moment. Results are compared to lower and upper bound analytical solutions and experimental results that can be found in papers.

Computations with twenty noded cubic elements are also proposed to validate shell studies. A calculus of J integral by simplified method using the limit load is also treated on a very simple example of branch pipe tee connection with defect.

1- INTRODUCTION

The knowledge of accurate limit load for complex structures is interesting for design rules and, more recently, for defining reference stresses in the calculation of J Integral by simplified methods like R6 or A16 methods [1], [2] in fracture mechanics.

It is possible to easily obtain the limit load for a simple geometry subjected to a simple loading using lower or upper bound theorems. Unfortunately, when complex structures are considered, the problem becomes rapidly too difficult to be solved analytically. It is the case of branch pipe tee connections subjected to different loadings.

Recently, a new method has appeared , named the Elastic Compensation Method (ECM). This method enables to obtain the limit load using the Finite Element Method, and then to study problems with large degrees of freedom. This method has been first developed by Mackenzie and Boyle ([3]- [5]), and demonstrated by Ponter [6].

We propose to use this method implemented in CASTEM 2000 Finite Element Code [7] to find the limit load of branch-pipe tee connections submitted to internal pressure or out-of-plane moment using principally three noded shell elements. We shall validate results with lower and upper bound solutions and experimental results that can be found in papers. A calculus of J integral by simplified method is also presented on a very simple example of branch pipe tee connection with defect modelled with shell elements.

2- NOMENCLATURE

R_r, e_r, L_r represent respectively radius, thickness and ½ length of run (main pipe)
R_b, e_b, L_b represent respectively radius, thickness and height of branch (nozzle)
l_f is length of defect
σ_y is yield stress
J_{el} is elastic J integral, J_{ep} is elastoplastic J integral, J_s is simplified J integral

3- THE ELASTIC COMPENSATION METHOD

The elastic compensation method enables, via a sequence of incompressible elastic calculations with variable Young modulus in the structure to head towards the rigid perfectly plastic state using the algorithm shown in Figure 1.

It is suggested to the reader to refer to [6] for the complete theory of the technique and to [3] to [5] for the explanation of several applications.

This algorithm is of a great interest. We have tested a large number of examples ([8] to [10]). The result is always really accurate after few iterations. Moreover, this algorithm converges toward the solution in opposition to step by step programs used in perfectly plastic model. Indeed, as far as a step by step program is concerned, the solution in force is accurate when the code begins to diverge, which is less practical to analyze.

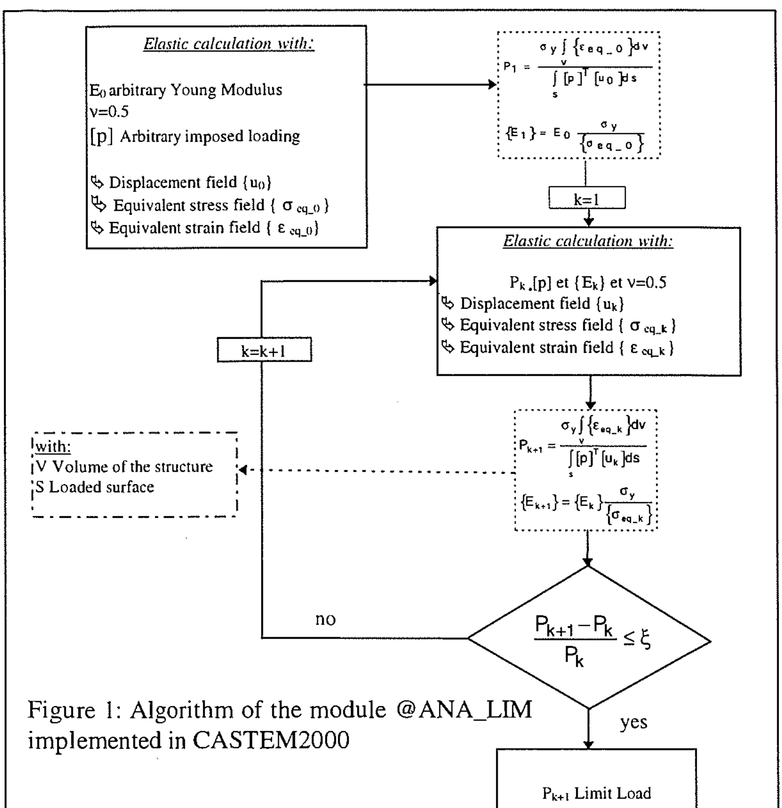

Figure 1: Algorithm of the module @ANA_LIM implemented in CASTEM2000

When shell elements are used, the Illiouchine Yield condition [11] is used and defined as :

If N_{ij} and M_{ij} are stretching and bending stress resultants and t the thickness, (1)

$$n_{ij} = \frac{N_{ij}}{\sigma_y t} \text{ and } m_{ij} = \frac{4M_{ij}}{\sigma_y t^2}, (n_{11}^2 + n_{22}^2 - n_{11}n_{22} + 3n_{12}^2) + (m_{11}^2 + m_{22}^2 - m_{11}m_{22} + 3m_{12}^2) \leq 1$$

As displacement Finite Element Method is the support of the technique, the result is always an upper bound. It seems to be difficult to extract any lower bound because stress fields obtained by finite element analysis don't verify equilibrium equations in the element. However, the solution is all the closer to the real solution that the mesh is refined.

As @ANA_LIM has been developed as part of a thesis on 'Limit analysis of branch pipe connection without and with defects' [12], we propose to study limit pressure and out-of-plane limit moment of different specimens existing in literature.

4- BRANCH PIPE CONNECTION UNDER INTERNAL PRESSURE

4-1 Comparison to Nadarajah & al study :

In order to validate the implementation of the algorithm in Castem 2000 Finite Element Code and in order to validate the use of shell elements, a comparison is done with the study in reference [5]. The limit pressure of a branch pipe connection is evaluated by Nadarajah & al with the Elastic Compensation Method and modelled with eight noded brick elements. Dimensions and specifications are presented table 1.

RUN	BRANCH
R_r= 505 mm	R_b= 105 mm
e_r= 10 mm	e_b=10 mm
L_r= 1000mm	L_b =700 mm
σ_y= 300 MPa	σ_y= 300 MPa

Table 1 : Specifications

We used three noded shell elements with Illiouchine Yield criterion.

At the same time, we tested a simple approximate limit pressure defined by Cloud and Rodabaugh [13] as follows:

If $e_b \leq e_r$

$$P_l = \frac{\frac{3}{2}\frac{e_b^2}{R_b^2} + \frac{e_r^2}{2R_b^2}(1+\frac{2}{h\sqrt{g}}) + \frac{38}{9g}\frac{e_b}{R_b} + (\frac{26}{9hg} + \frac{47}{54h^2 g\sqrt{g}})\frac{e_r}{R_b}}{\frac{38}{9g} + \frac{38}{9h^2 g} + \frac{76}{27h^3 g\sqrt{g}} + \frac{2}{h\sqrt{g}}}\sigma_y \qquad (2)$$

$$h = \frac{R_b}{R_r} \text{ and } g = \frac{2R_r}{e_r}$$

Results are presented table 2.

Study	Limit Pressure (MPa)
Cloud approximated value	5.46
ECM with 8 noded brick elements ([5])	5.43
ECM with 3 noded shell elements (@ANA_LIM)	5.2

Table 2 : Limit pressure

It can be seen that differences are small. It can be also noticed that the expression of the limit pressure by Cloud is quite accurate and extremely simple.

4-2 Comparison to Schroeder & al study

J. Schroeder [14] tested a tee in order to validate an upper bound he presented in his article. Dimensions and specifications are presented table 3. The tee is submitted to internal pressure with end axial load effect.

RUN	BRANCH
R_r=43.815mm	R_b=27.6 mm
e_r=2.54 mm	e_b=1.78mm
L_r=½ length=245 mm	L_b=height=260mm
σ_y=282 MPa	σ_y=282 MPa

Table 3 : Specifications

We compare three results (an upper bound, an experimental result and an approximate solution ([13])) to the solution of the ECM when a branch pipe tee connection is modelled with three noded shell elements and with 20 noded brick elements as it is shown on Figure 2 and Figure 3.

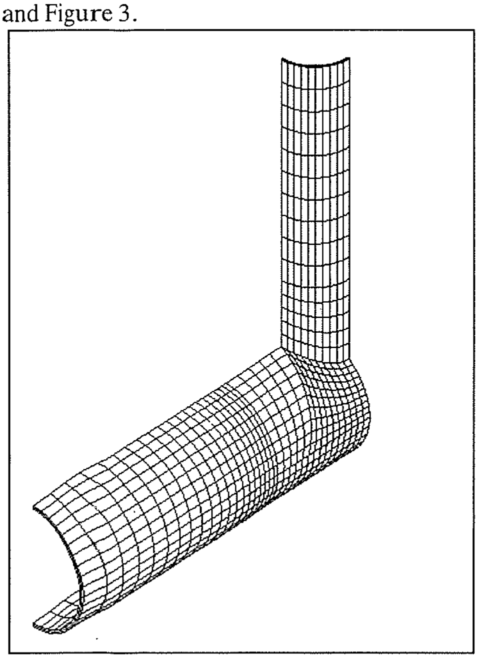

Figure 2 : Brick model

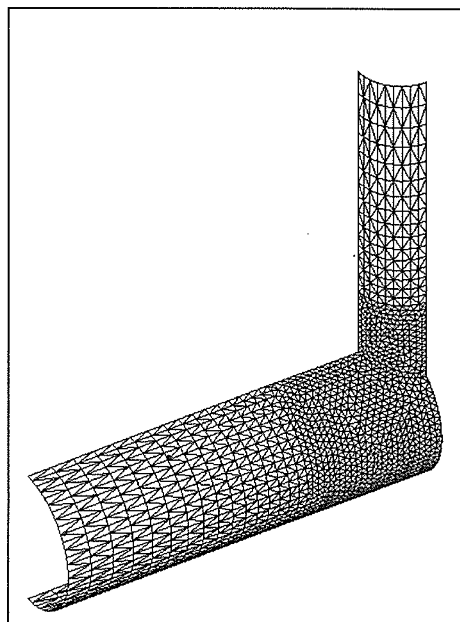

Figure 3 : Shell model

Study	Limit Pressure (MPa)
Experimental	~9.65
Schroeder Upper Bound	11.38
Cloud approximated value	9.29
ECM with 20 noded brick elements	10.06
ECM with 3 noded shell elements	9.32

Table 4 : Limit pressure

It must be emphasised that results presented table 4 are very close to each other even for the experimental solution .

4-3 Comparison to parametric and incremental studies

Results obtained by several authors for a branch pipe tee connection are gathered in [15]. Characteristics of the branch pipe connection are :

- $R_r=25$, $R_r/e_r=25$, $R_re_b/R_be_r=1$, R_b/R_r from 0.1 to 0.9

We did the same study both with the ECM (@ANA_LIM) and with the incremental module of CASTEM 2000 named PASAPAS. Results presented figure 4 are always in accordance with the theory.

It must be emphasised that results of Cloud seem to be really close to the real result.

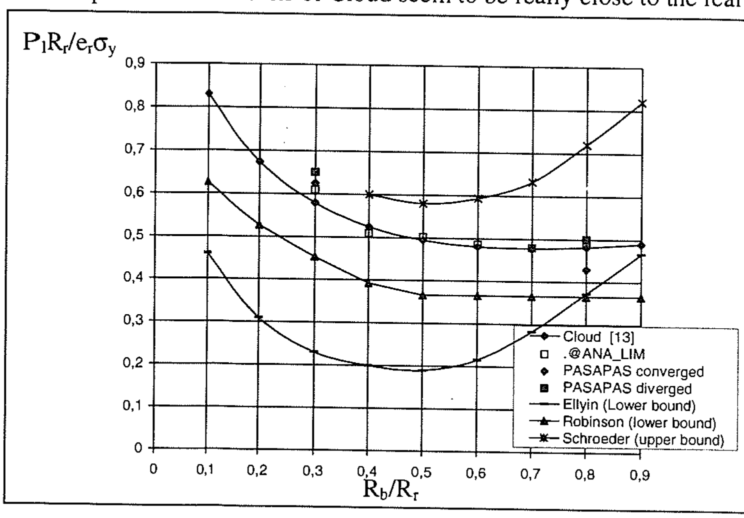

Figure 4 : Limit pressure

5- BRANCH PIPE CONNECTION SUBMITTED TO OUT-OF-PLANE MOMENT

An experimental investigation of out-of-plane limit moment for nozzles in cylindrical vessels has been reported in [16].

This study is really interesting for the limit analysis because Ellyin used a material with a quasi rigid perfectly plastic behaviour. We compared the ECM solution with his experimental limit load. Specimens are presented table 5 :

Specimen	R_r (mm)	R_b	e_r	e_b	½ L_r	L_b-R_r	χ (MPa)
B2	73.52	47.78	3.15	4.75	163.30	76.5	172
B4	72.77	48.11	6.35	2.54	163.5	79.65	169

Table 5 : Branch pipe specimens

Solutions of investigation are :

Specimen	M_l experiment *	M_l experiment **	M_l experiment ***	ECM
B2	2576	2283	2543	2440
B4	3040	3164	3108	3080

Table 6 : Limit moment (N.m)

* corresponding to the mean of 0.2 % offset strain gage placed near the junction
** corresponding to the mean of intersection of elastic behaviour with tangent to the plastic response of dial gages placed around the junction.
*** corresponding to the mean of intersection of a line with a slope 1/3 of elastic response of dial gage indication near the junction .
We can notice that shell model seems to be sufficient to model a pseudo realistic experiment.

6- J INTEGRAL OF A BRANCH PIPE TEE CONNECTION WITH LONGITUDINAL DEFECT UNDER INTERNAL PRESSURE BY SIMPLIFIED METHODS

In order to validate the R6 rule option 2 [1], a very simplified example of branch pipe with defect is studied. ¼ of a branch pipe is modelled with shell elements and submitted to internal pressure. Two symmetries are defined in order to model complete branch pipe. A defect is located longitudinally to the run at the intersection between the run and the branch (figure 5).

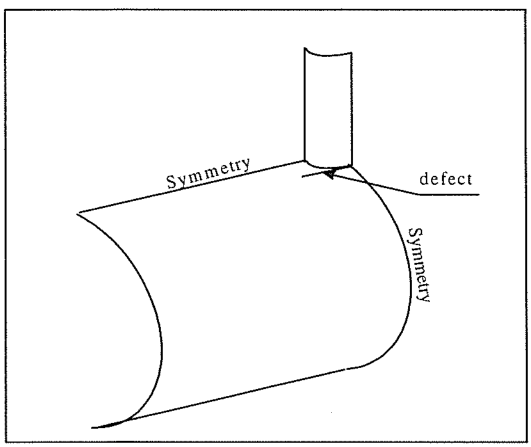

Figure 5 : Defect location

Ainsworth proposes a simplified formula of J Integral defined as follows :

$$J_s = J_{el}\left(\frac{\varepsilon(L_r\sigma_y)}{L_r \, {}^{\sigma_y}/_{E}} + \frac{1}{2}\frac{L_r^2}{1+L_r^2}\right) \qquad \text{with } Lr = \frac{P}{P_l} \qquad (3)$$

P is the applied pressure and P_l the limit pressure at σ_y. Results are compared to a numerical elastoplastic J integral by the G-theta method in accordance with the R6 rule option 3. Results are presented as $K_r(L_r)$ with $K_r = \sqrt{{}^{J_{el}}/_{J_{ep}}}$.

The elastoplastic constitutive equation is defined with a Ramberg- Osgood law (4).

$$\frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} + \alpha\left(\frac{\sigma}{\sigma_y}\right)^n \quad \text{with n=6, } \alpha=1 \text{ and } \sigma_y=163 \text{ MPa} \qquad (4)$$

J_{el} is obtained by the G-theta method.

The limit pressure of several specimens of branch pipe tee connections is presented on table 7, fixed parameters are : $R_r e_b/R_b E_r= 5$, $L_r=2R_r$ and $L_b=2R_r$

Specimen	A	B	C	D	E
R_r/R_b	4	4	4	2	4
R_r/e_r	25	25	25	50	50
$l_f/(R_r e_r)^{1/2}$	1,5	2	5	5	4
$PR_r/e_r\sigma_y$	8,75E-01	7,50E-01	5,28E-01	6,80E-01	6,27E-01

Table 7 : Limit pressure for different specimens

The comparison between K_r option 2 and option 3 for specimen B is presented on Figure 6

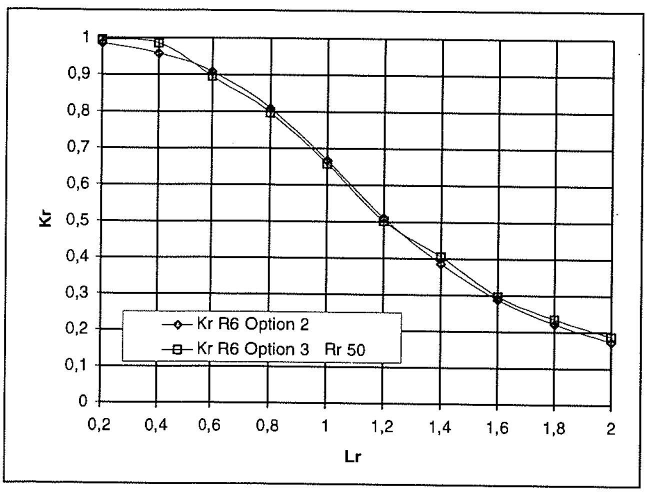

Figure 6 : K_r for branch pipe tee connection under internal pressure

It seems that J_s is guided by the limit load in the case of a branch pipe tee connection under internal pressure. Of course, the model used is extremely simplified : symmetries imposed two throughwall defects, shell elements don't consider transversal shear stress, K_r is the mean value in the thickness of shell. Results can't be used for design. Nevertheless, this study can be the first step toward a more accurate study with brick elements and non-throughwall defect of a complete branch pipe tee connection. Then, with only one calculus of the limit load, it would be possible to obtain J Integral whatever the intensity of applied load using simplified methods.

7- CONCLUSION

A new method of calculating limit state of a structure has been used to define the limit load of specimens of branch pipe tee connection. Validations have been done thanks to different limit loads obtained in the past. It must be emphasised that the Elastic Compensation Method can be easily implemented in any Finite Element Code and enables to swiftly obtain the limit load for any complex structure. The ECM can also be an interesting mean of evaluating reference stress in calculus of J integral by simplified methods.

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