THERMAL STRESSES AND THERMAL BUCKLING OF A RECTANGULAR PLATE WITH A CENTRAL CIRCULAR HOLE

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SUMMARY

When a plate is subjected to heating, cooler supporting members usually resist to the thermal expansion of the plate at edges, and the compressive thermal stresses will cause the buckling of the plate.

The thermal buckling of a rectangular plate subjected to non-uniform temperature distribution has already been treated by many authors. However, in the practical structural elements, a rectangular plate with a central circular hole has often been used from the requirement for the functions of the systems and the reduction of the weight of the structures. Owing to the complexity of the analyses, we have as yet very little information as to the thermal buckling of a multiply-connected plate so far as known to the present authors.

Therefore, in this paper, thermal stresses and thermal buckling of a rectangular plate with a central circular hole, which is clamped along the outer rectangular boundary and free from surface tractions along the inner circular boundary, are treated when the plate is subjected to two-dimensional steady temperature distibution. The two-dimensional thermal stresses of the clamped rectangular plate with a free circular hole may be solved by the complex variable method using the analytic continuation and the point-matching method, whenever the boundaries of a plate are clamped or, on the contrary, free from surface tractions. Therefore we use this method to obtain the thermal stress field in the prebuckling state.

Furthermore, expressing the deflection mode of the buckled plate into the trigonometric double series that satisfy the conditions of the clamped rectangular boundary, the thermal buckling criterion and the corresponding deflection mode of the plate are obtained by the Ritz energy method under the assumption that the thermal stresses acting in the middle plane of the plate remain constant during the deflection. In this treatment the boundary conditions for the inner circular boundary are satisfied as the natural boundary conditions.

Numerical calculations are carried out for the rectangular plate of dimensions a by b with a central circular hole of radius c, and the effects of the circular hole and the aspect ratio b/a on the critical temperature of the plate are studied and the following results are obtained: (1) For the aspect ratio $b/a=2/3$, the critical temperature decreases according to the increase of the diameter ratio $2c/a$ of the circular hole. (2) For the ratio $2c/a=1/3$, the critical temperature decreases according to the increase of the aspect ratio b/a of the plate for the case $b/a=1/2\sim4/3$.

1. Introduction

When a plate is subjected to heating, cooler supporting members usually resist to thermal expansion of the plate at edges, and the compressive thermal stresses will cause the thermal buckling of the plate.

The thermal buckling of an unpierced rectangular plate subjected to non-uniform temperature distribution has already been treated by N. J. Hoff [1], T. Hayashi [2], J. M. Klosner and M. J. Forray [3] and K. Miura [4]. However, in the practical structural elements, a rectangular plate with a central circular hole has often been used from the requirement of the functions of the systems or the reduction of the weight of the structures. But, we have as yet very little informations as to the thermal buckling of a multiply-connected plate so far as the present authors known.

Therefore, in this paper, thermal stresses and thermal buckling of a rectangular plate with a central circular hole, which is clamped along the outer rectangular boundary and free from surface tractions along the inner circular boundary, are treated when the plate is under the steady state of two-dimensional temperature distribution. In the analysis, the two-dimensional thermal stresses in the prebuckling state of the plate are obtained by the complex variable method utilizing the analytic continuation and the point-matching method. That is, generalizing the Muskhelishvili's basic equations for isothermal plane elasticity [5] into thermoelasticity and applying the Buchwald's and Davies' approach of analytic continuation [6], one of two complex potentials is continued analytically into the inverse region of the problem with respect to the inner circular boundary. Then a combination of the two complex potentials is obtained which satisfies the boundary condition on this circular boundary identically, leaving one of the potentials arbitrary. This potential is then determined so as to satisfy the condition on the outer rectangular boundary approximately, using the Laurent series expansion and the point-matching method.

Furthermore, expressing the deflection mode of the buckled plate into the trigonometric double series that satisfy the supporting condition of the clamped rectangular boundary, the thermal buckling criterion and the corresponding deflection mode of the plate are obtained by the Ritz energy method under the assumption that the thermal stresses obtained in the pre-buckling state of the plate remain constant during the beginning of the deflection. In this treatment, the boundary condition on the inner circular boundary is satisfied as the natural boundary condition.

Numerical calculations are carried out for various configuration of the plates and the effects of circular hole on the thermal stresses and the thermal buckling criterion of the plates are examined.

2. Thermal stresses in a rectangular plate with a central circular hole

2.1 Basic equations

Under the steady state of heat flow without any heat generation at the inner points, the two-dimensional temperature function $\tau(x,y)$ is a harmonic function. Denote by $G(z)$ the analytic function of the complex variable $z=x+iy$, the real part being $\tau(x,y)$, and put

$$g(z)=\alpha\int G(z)dz \tag{1}$$

Then, generalizing Muskhelishvili's basic equations for isothermal plane elasticity [5] into thermoelasticity, the thermal stresses and displacements are given in terms of $g(z)$ and two complex potentials, $\phi(z)$ and $\psi(z)$, as follows [7]:

$$\sigma_{xx}+\sigma_{yy}=2[\phi'(z)+\overline{\phi'(z)}] \tag{2}$$

$$\sigma_{yy}-\sigma_{xx}+2i\sigma_{xy}=2[\bar{z}\phi''(z)+\psi'(z)] \tag{3}$$

$$u+iv=(1+\nu)/E\cdot[\kappa\phi(z)-z\overline{\phi'(z)}-\overline{\psi'(z)}]+g(z) \tag{4}$$

where

α=coefficient of linear thermal expansion

ν=Poisson's ratio

E=Young's modulus

$\kappa=(3-\nu)/(1+\nu)$

From the physical viewpoint, the temperature and the stresses must be single-valued in the region. Therefore the general forms of $g(z)$, $\phi(z)$ and $\psi(z)$ that assure the single-valuedness of the temperature and the stresses may be given by

$$g(z)=L^*\cdot z\,log\,z+M^*\cdot log\,z+g^*(z) \tag{5}$$

$$\phi(z)=L\cdot z\,log\,z+M\cdot log\,z+\phi^*(z) \tag{6}$$

$$\psi(z)=N\cdot z\,log\,z+K\cdot log\,z+\psi^*(z) \tag{7}$$

where

L^*, L=real constants

M^*, M, N, K=complex constants

$g^*(z)$, $\phi^*(z)$, $\psi^*(z)$=holomorphic functions of z

Then the increments of rotation ω_z, displacements (u,v), resultant forces (X,Y) in the directions of x- and y-axes, respectively, and resultant moment M_0 along an arbitrary closed contour, surrounding the inner circular boundary, may be given by

$$[\omega_z]_A^A = 2\pi/E\cdot[4L+EL^*] \tag{8}$$

$$[(u+\omega_z y)+i(v-\omega_z x)]_A^A = 2\pi i/E\cdot[(3-\nu)M+(1+\nu)\overline{N}+EM^*] \tag{9}$$

$$[X+iY]_A^A = 2\pi[-M+\overline{N}] \tag{10}$$

$$M_0 = 2\pi\,\mathcal{I}m\{K\}. \tag{11}$$

That is, the constants L^*, M^*, L, M, N, K involved in eqs.(5)–(7) represent the single-valuedness of the functions $g(z)$, $\phi(z)$ and $\psi(z)$, and the physical meanings of these constants are given by eqs.(8)–(11).

2.2 Thermal stresses in a rectangular plate with a central circular hole

Now consider a rectangular plate of dimensions a by b with a central circular hole of radius c as shown in Fig.1.

If the temterature on the inner circular and the outer rectangular boundaries are τ_i and zero respectively, the temperature distribution is symmetrical about x- and y-axes. Hence only a quarter part of the plate will be considered. Then the temperature which satisfies the condition on the inner circular boundary can be represented as

$$\tau=\tau_i+A_0 \log(r/c)+\sum_{n=1}^{\infty} A_{2n}(r^{-2n}-c^{-4n}r^{2n})\cos 2n\theta \tag{12}$$

The substitution of eq.(12) into the boundary condition at all points on the outer rectangular boundary results in the infinite number of linear equations for the infinite number of unknown coefficients. However, since the outer rectangular boundary does not coincide with coordinate lines r=const. or θ= const., it is difficult to find an exact solution for the temperature function (12). In order to obtain an approximate solution, the infinite series in eq.(12) is truncated at the N-th term, leaving N+1 coefficients to be determined. If we choose N+1 points along the outer quarter boundary and apply the truncated series (12) to the boundary conditions at these points, N+1 equations are obtained for N+1 unknown coefficients. Solving these set of equations, N+1 unknown coefficients are determined and the approximate solution for the temperature function is obtained.

Then, the function g(z) defined by eq.(1) is given by

$$g(z)=\alpha[A_0 z\log\frac{z}{c} + (\tau_i-A_0)z + \sum_{n=1}^{N} A_{2n}(\frac{z^{-2n+1}}{-2n+1} - c^{-4n}\frac{z^{2n+1}}{2n+1})] \tag{13}$$

where the terms related to rigid body displacements are omitted. Comparing eq.(13) with eq.(5), we obtain

$$L^*=\alpha A_0 \ , \qquad M^*=0 \tag{14}$$

Next, we will consider the state of stresses and displacements in the prebuckling state of the plate, where the inner circular boundary is free from surface tractions ($\sigma_{rr}=\sigma_{r\theta}=0$) and the outer rectangular boundary is fixed (u=v=0). In this case, it is simpler to take a modified form of the boundary condition on the circular boundary. It can be shown [5] that the resultant force P on arc AB of the boundary is given by

$$P=X+iY=i[\phi(z)+z\overline{\phi'(z)}+\overline{\psi(z)}]_A^B \tag{15}$$

The boundary condition to zero stress on the circle, therefore, can be written as

$$\phi(z)+z\overline{\phi'(z)}+\overline{\psi(z)}=0 \tag{16}$$

On the other hand, the boundary condition on the rectangular boundary is given by eq.(4) as follows :

$$-\kappa\phi(z)+z\overline{\phi'(z)}+\overline{\psi(z)}=E/(1+\nu)\cdot g(z) \tag{17}$$

In order to find $\phi(z)$ and $\psi(z)$ from these equations, we introdece the analytic continuation approach proposed by Buchwald and Davies [6]. Denote by S^* the inverse of the given region S with respect to the circle |z|=c, and define the analytic continuation of $\phi(z)$ from S to S^* by the equation

$$\phi(z)=-z\overline{\phi}'(\frac{c^2}{z})-\overline{\psi}(\frac{c^2}{z}) \ , \qquad z\in S^* \tag{18}$$

$$\psi'(z)=-\frac{c^2}{z}\phi'(z)-\bar{\phi}\left(\frac{c^2}{z}\right) \qquad z\in S \tag{19}$$

Substituting eq.(19) into the left hand side of eq.(16), it is obvious that the condition on the inner circular boundary is identically satisfied if, and only if, $\phi(z)$ is analytic in both S and S^* and also continuous across the circle $|z|=c$. On the other hand, substituting eq.(19) into eq.(17), we can write down the boundary condition on the outer rectangular boundary in terms of $\phi(z)$ only as follows :

$$-\kappa\phi(z)-\phi\left(\frac{c^2}{\bar{z}}\right)+\left(z-\frac{c^2}{\bar{z}}\right)\overline{\phi'(z)}=\frac{\alpha E}{1+\nu}g(z) \tag{20}$$

From the physical viewpoint of the problem, assuming eqs.(8)–(11) equal to zero, and considering eq.(14), we obtain

$$L=-\frac{EL^*}{4}=-\frac{\alpha E}{4}A_0 \quad , \quad M=N=0 \quad , \quad Im\{K\}=0 \tag{21}$$

Therefore, expanding $\phi^*(z)$ in eq.(6) as a Laurent series in $S+S^*$, and using symmetry arguments of the problem, we obtain the formal expansion of $\phi(z)$ as follows :

$$\phi(z)=-\frac{\alpha E}{4}A_0 z\,log\,z+\sum_{-\infty}^{\infty}B_{2n+1}z^{2n+1} \tag{22}$$

where all the coefficients are real. Then, substituting eq.(22) into the boundary condition (20) for the outer rectangular boundary, we obtain

$$\sum_{-\infty}^{\infty}B_{2n+1}\left\{-\kappa z^{2n+1}-\left(\frac{c^{4n+2}}{\bar{z}^{2n+1}}\right)+(2n+1)(r^2-c^2)\bar{z}^{2n-1}\right\}$$
$$=-\frac{\alpha E}{4}A_0\left\{\kappa z\,log\,z-z\,log\,\bar{z}+\frac{2c^2}{\bar{z}}\,log\,c-(r^2-c^2)/\bar{z}\right\}+\frac{\alpha E}{1+\nu}g(z) \tag{23}$$

The point-matching method is used again here to obtain B_{2n+1}. If we replace $\sum_{-\infty}^{\infty}$ in eq.(22) by $\sum_{-M}^{M-1}$, eq.(22) contains 2M unknown coefficients. Considering eq.(23) for M+1 points on the quarter part of the rectangular boundary and taking real and imaginary parts of eq.(23), 2M equations are obtained, since the imaginary part of eq.(23) is identically satisfied on x- ans y-axes. Solving these 2M equations for 2M unknown coefficients, B_{2n+1} are determined and an approximate solution for $\phi(z)$ is obtained.

In the present treatment, as $\psi(z)$ is represented in terms $\phi(z)$ by eq.(19) it is unnecessary to obtain $\psi(z)$. Namely, by substituting eq.(19) into eqs. (2)–(4), the stresses and displacements are expressed by $\phi(z)$ only as follows:

$$\sigma_{xx}+\sigma_{yy}=2[\phi'(z)+\overline{\phi'(z)}] \tag{24}$$

$$\sigma_{yy}-\sigma_{xx}+2i\sigma_{xy}=2\left[\frac{z\bar{z}-c^2}{z}\phi''(z)+\frac{c^2}{z^2}\left\{\phi'(z)+\bar{\phi}'\left(\frac{c^2}{z}\right)\right\}\right] \tag{25}$$

$$u+iv=\frac{1+\nu}{E}\left[\kappa\phi(z)+\phi\left(\frac{c^2}{\bar{z}}\right)-\frac{z\bar{z}-c^2}{\bar{z}}\overline{\phi'(z)}\right]+g(z) \tag{26}$$

3. Thermal buckling of a rectangular plate with a circular hole

When the temperature of the plate rises to some level, the plate which is subjected to the thermal stresses undergoes some lateral bending consistent with given supporting conditions.

The Ritz energy method can be used for the present investigation of the thermal buckling problems of pierced rectangular plates. Under the assumption that the thermal stresses acting in the middle plane of the plate remain constant during the beginning of the deflection, the total change of the potential energy $\Delta\Pi$ of the plate at buckling is given as the sum of the energy of bending ΔU_b of the plate and the total work ΔV done by the forces acting in the middle plane of the plate, that is

$$\Delta\Pi=\Delta U_b+\Delta V$$

$$=\frac{D}{2}\iint_S \left[\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2-2(1-\nu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right\}\right]dxdy$$

$$+\frac{h}{2}\iint_S\left[\sigma_{xx}\left(\frac{\partial w}{\partial x}\right)^2+2\sigma_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+\sigma_{yy}\left(\frac{\partial w}{\partial y}\right)^2\right]dxdy \tag{27}$$

As for the supporting conditions of the plate discussed in the preceeding section, we will treat the case where the inner circular boundary is free and the outer rectangular boundary is clamped. Then, considering symmetric deflection with respect to x- and y-axes, the deflection of the buckled plate can be represented by the trigonometric double series that satisfy the supporting condition on the clamped rectangular boundary, as follows :

$$w=h\sum_m\sum_n a_{mn}\left\{\cos\frac{2m\pi}{a}x-(-1)^m\right\}\left\{\cos\frac{2n\pi}{b}y-(-1)^n\right\}\ ,\quad (m,n=1,2,\cdots) \tag{28}$$

Since the total change of the potential energy at buckling must be stationary, then

$$\frac{\partial\Delta\Pi}{\partial a_{mn}}=0 \tag{29}$$

Using the notation

$$\sigma^*_{xx}=\sigma_{xx}/\alpha E\tau_i\ ,\quad \sigma^*_{yy}=\sigma_{yy}/\alpha E\tau_i\ ,\quad \sigma^*_{xy}=\sigma_{xy}/\alpha E\tau_i \tag{30}$$

$$\lambda=\alpha\tau_i a^2/\pi^2 h^2 \tag{31}$$

$$\beta=b/a \tag{32}$$

and substituting eq.(28) into eqs.(27) and (29), we obtain a system of homogeneous linear equations in a_{mn}.

$$\sum_m\sum_n(A^{kl}_{mn}-\lambda B^{kl}_{mn})a_{mn}=0\ ,\quad \text{for } k,l=1,2,\cdots \tag{33}$$

where A^{kl}_{mn} and B^{kl}_{mn} are given as follows :

$$A^{kl}_{mn}=m^2k^2J_1(m,k,n,l)+\left(\frac{n}{\beta}\right)^2\left(\frac{l}{\beta}\right)^2J_2(m,k,n,l)+\nu m^2\left(\frac{l}{\beta}\right)^2J_3(m,k,n,l)$$
$$+\nu k^2\left(\frac{n}{\beta}\right)^2J_3(k,m,l,n)+2(1-\nu)mk\left(\frac{n}{\beta}\right)\left(\frac{l}{\beta}\right)J_4(m,k,n,l) \tag{34}$$

$$B^{kl}_{mn}=-3(1-\nu^2)\left\{mkJ_5(m,k,n,l)+\left(\frac{n}{\beta}\right)\left(\frac{l}{\beta}\right)J_6(m,k,n,l)\right.$$
$$\left.+m\left(\frac{l}{\beta}\right)J_7(m,k,n,l)+k\left(\frac{n}{\beta}\right)J_7(k,m,l,n)\right\} \tag{35}$$

where

$$J_1(m,k,n,l)=\iint_S\cos\frac{2m\pi}{a}x\cdot\cos\frac{2k\pi}{a}x\left\{\cos\frac{2n\pi}{b}y-(-1)^n\right\}\left\{\cos\frac{2l\pi}{b}y-(-1)^l\right\}dxdy$$

$$J_2(m,k,n,l)=\iint_S\left\{\cos\frac{2m\pi}{a}x-(-1)^m\right\}\left\{\cos\frac{2k\pi}{a}x-(-1)^k\right\}\cos\frac{2n\pi}{b}y\cdot\cos\frac{2l\pi}{b}y\ dxdy$$

$$
\left.\begin{aligned}
J_3(m,k,n,l) &= \iint_S \cos\frac{2m\pi}{a}x\left\{\cos\frac{2k\pi}{a}x-(-1)^k\right\}\left\{\cos\frac{2n\pi}{b}y-(-1)^n\right\}\cos\frac{2l\pi}{b}y\ dxdy \\
J_4(m,k,n,l) &= \iint_S \sin\frac{2m\pi}{a}x\ \sin\frac{2k\pi}{a}x\ \sin\frac{2n\pi}{b}y\ \sin\frac{2l\pi}{b}y\ dxdy \\
J_5(m,k,n,l) &= \iint_S \sigma^*_{xx}\sin\frac{2m\pi}{a}x\ \sin\frac{2k\pi}{a}x\left\{\cos\frac{2n\pi}{b}y-(-1)^n\right\}\left\{\cos\frac{2l\pi}{b}y-(-1)^l\right\}dxdy \\
J_6(m,k,n,l) &= \iint_S \sigma^*_{yy}\left\{\cos\frac{2m\pi}{a}x-(-1)^m\right\}\left\{\cos\frac{2k\pi}{a}x-(-1)^k\right\}\sin\frac{2n\pi}{b}y\ \sin\frac{2l\pi}{b}y\ dxdy \\
J_7(m,k,n,l) &= \iint_S \sigma^*_{xy}\sin\frac{2m\pi}{a}x\left\{\cos\frac{2k\pi}{a}x-(-1)^k\right\}\left\{\cos\frac{2n\pi}{b}y-(-1)^n\right\}\sin\frac{2l\pi}{b}y\ dxdy
\end{aligned}\right\} \quad (36)
$$

The set of homogeneous linear equation (33) constitutes a eigenvalue problem, and the solution of which goves the thermal buckling parameter λ_{cr} as eigenvalues and sets of associated relative values of the coefficients a_{mn}, *i.e.* the buckling modes.

In this treatment, the boundary condition that the stresses are free on the inner circular boundary are automatically satisfied as the natural boundary condition.

4. Numerical results

The method of solution outlined in the foregoing sections is applied to investigate the thermal stress distribution and the thermal buckling criterion of a rectangular plate with a central circular hole under the steady state of temperature distribution. In this paper, Poisson's ratio ν is assumed to be 1/3. Using the point-matching method, the temperature function $\tau(x,y)$ and the complex potential $\phi(z)$ are approximately determined by taking N=19 and retaining 20 coefficients in eq.(12) and by taking M=16 and retaining 32 coefficients in eq.(22), respectively. For pierced rectangular plates with aspect ratio β=2/3, stress distributions on x-axes are shown in Fig.2 as a function of hole size, and for pierced rectangular plates with hole size c/a= 1/6, stress distributions on x-axes are shown in Fig.3 as a function of aspect ratio.

To calculate the critical temperature λ_{cr} from eq.(33), the integrals involved in eqs.(36) are numerically evaluated by Simpson double integral method. Table 1 and 2 show the critical temperatures when we take 4, 6 and 9 coefficients a_{mn} in the deflection function (28). From these tables, it is seen that a good approximation for the critical temperature will be obtained by taking only 9 coefficients a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33} in eq.(28). For pierced rectangular plates with aspect ratio β=2/3, critical temperatures are shown in Fig.4 as a function of hole size, and for pierced rectangular plates with hole size c/a=1/6, critical temperatures are shown in Fig.5 as a function of aspect ratio. Lastly, the typical deflection mode of a pierced rectangular plate with β=2/3 and c/a=1/6 is shown in Fig.6.

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Table 1. Critical values of λ in eq.(33) for pierced rectangular plates with aspect ratio β=2/3

hole size	Terms used in the deflection function (28)		
c/a	a_{11}, a_{12} a_{21}, a_{22}	a_{11}, a_{12}, a_{13} a_{21}, a_{22}, a_{23}	a_{11}, a_{12}, a_{13} a_{21}, a_{22}, a_{23} a_{31}, a_{32}, a_{33}
	λ_{cr}	λ_{cr}	λ_{cr}
1/12	1.69	1.68	1.67
1/9	1.73	1.71	1.70
1/6	2.16	2.14	2.12
1/5.1	2.55	2.47	2.46

Table 2. Critical values of λ in eq.(33) for pierced rectangular plates with hole size c/a=1/6

aspect ratio	Terms used in the deflection function (28)		
β	a_{11}, a_{12} a_{21}, a_{22}	a_{11}, a_{12}, a_{13} a_{21}, a_{22}, a_{23}	a_{11}, a_{12}, a_{13} a_{21}, a_{22}, a_{23} a_{31}, a_{32}, a_{33}
	λ_{cr}	λ_{cr}	λ_{cr}
3/6	3.88	3.80	3.71
4/6	2.16	2.14	2.12
6/6	1.09	1.08	1.08
8/6	0.813	0.811	0.801

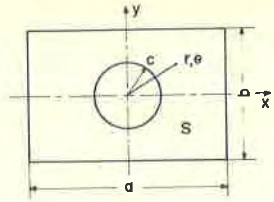

Fig.1 Rectangular plate with a circular hole

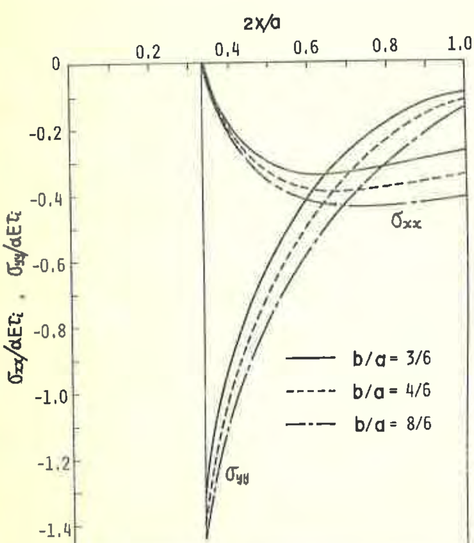

Fig.2 Thermal stress distributions on x-axis as function of hole size ($\beta=b/a=2/3$)

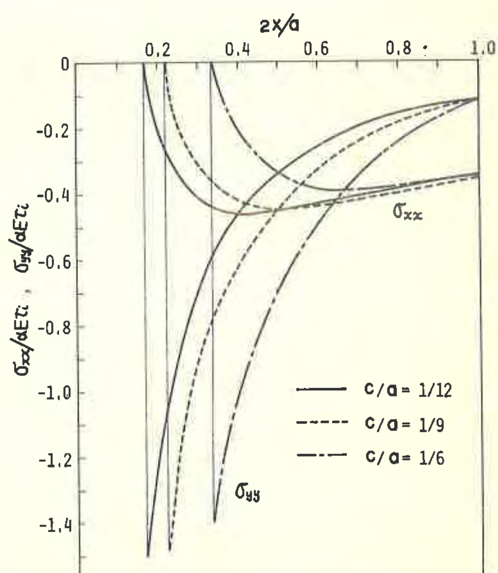

Fig.3 Thermal stress distributions on x-axis as function of aspect rati
($c/a=1/6$)

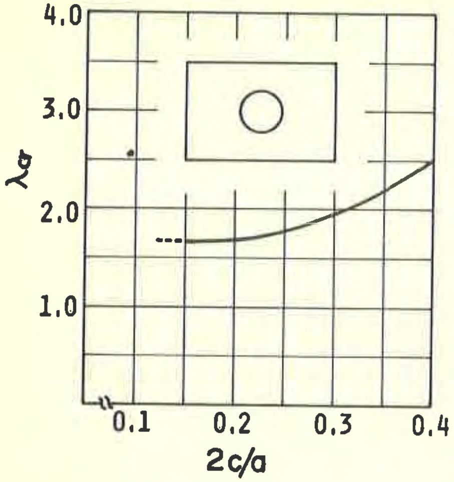

Fig.4 Critical temperature as function of hole size (β=b/a=2/3)

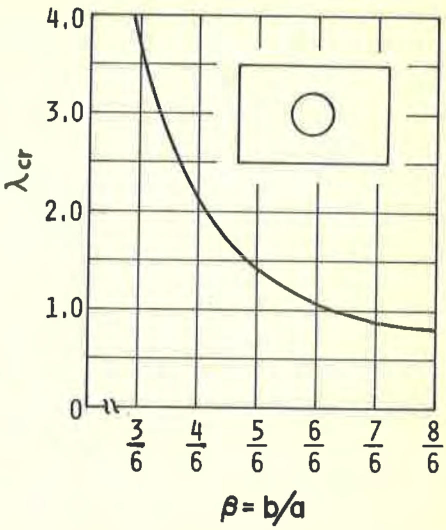

Fig.5 Critical temperature as function of aspect ratio (c/a=1/6)

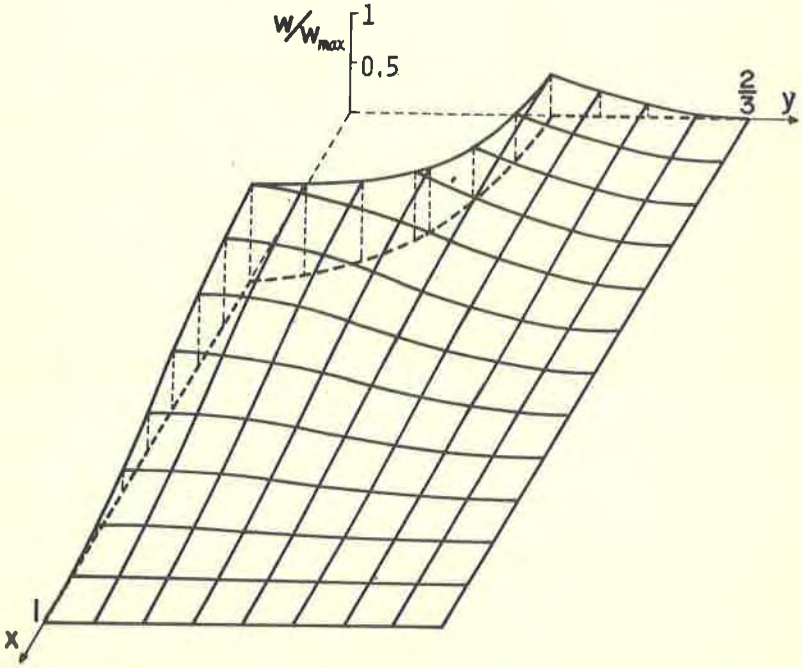

Fig.6 Deflection mode (β=b/a=2/3, c/a=1/6)