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Nonlinear vibration of doubly-curved orthotropic shallow shells under a thermal gradient

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ABSTRACT : The present investigation aims at the finding of the effect of a thermal gradient on a doubly-curved orthotropic shallow shell. The basic governing differential equations have been derived under the effect of a temperature distribution varying linearly and uniaxially.

1 INTRODUCTION

In high-speed space vehicles and in nuclear reactors certain parts have to be operated under elevated temperature. Hoff (1958) has suggested that under this condition modulus of elasticity of material becomes functions of space variables, and as such vibrational characteristics of continuous elastic media must then be based on non-homogeneous elastic theory.

Compared to the linear theory nonlinear analysis of shallow shells appears to be rare. The investigations related to the dynamic response to orthotropic shallow shells under a thermal gradient are far below the present day need, since thermally induced vibration of structures play a major role in the design of modern nuclear technology. This is quite evident from the dearth of literature in this field.

Investigations on doubly-curved shallow shells has been made by El-Zaouk and Dym (1973) following suggestions made by the works of Leissa and adi (1971). Nath et al. (1987) have studied the nonlinear dynamic response of a doubly-curved shallow shell on an elastic foundation. As it appears, no literature is available on the dynamic response of such structures under a thermal gradient.

The present study is a part of a major project related to vibration of shallow shells. In the forthgoing sections, derivation of the governing differential equations and method of such highly nonlinear equations will be discussed with an illustration.

2 DERIVATION OF GOVERNING EQUATIONS

Considered here is a doubly curved shallow shell as shown in Figure 1. R* and R representing the meridional radius of curvature and the radius of a parallel circle at the equator, (x,y) denoting the axial and circumferential coordinates, the strain-displacement relations, following Hutchinson (1967) can be written as

$$\epsilon_x = (\partial u/\partial x) - (w/R^*) + \tfrac{1}{2}(\partial w/\partial x)^2 - z(\partial^2 w/\partial x^2) ,$$

$$\epsilon_y = (\partial v/\partial y) - (w/R) + \tfrac{1}{2}(\partial w/\partial y)^2 - z(\partial^2 w/\partial y^2) ,$$

$$\gamma_{xy} = (\partial u/\partial y) + (\partial v/\partial x) + (\partial w/\partial x)(\partial w/\partial y) - 2z(\partial^2 w/\partial x\partial y) . \tag{1}$$

where u, v and w are the usual shell displacements. For a two-dimensional orthotropic continuam, the stress-strain relations can be written as

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix} = c_{11} \begin{pmatrix} 1 & \alpha & 0 \\ \alpha & \beta & 0 \\ 0 & & \gamma \end{pmatrix} \begin{pmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{pmatrix} \tag{2}$$

where,

$$\alpha = c_{12}/c_{11}, \quad \beta = c_{22}/c_{11}, \quad \gamma = G_{12}/c_{11} \tag{3}$$

For simplicity it is assumed that a temperature distribution be inducted in such a way that it is a linear function of 'x' while the modulus of elasticity becoming a linear function of temperature, e.g.,

$$T = T_o\, f(x) , \quad c_{11} = c_{oo}\,(1 - m\, T) \tag{4}$$

T_o being the reference temperature, c_{oo} the reference modulus of elasticity, 'm' = the slope of variation of c_{11} with 'T'. It is important to note here that α, β and γ being dimensionless quantities, the variation of c_{11} with 'T' will only be considered, that is the values of α, β and γ will remain unaffected by heat input f(x) defines the law of variation of temperature along the x-axis.

The kinetic energy 'K' of the system can be expressed as

$$K = \tfrac{1}{2}(\rho\, h) \int_0^L \int_0^{2\pi R} \left[(\partial u/\partial t)^2 + (\partial v/\partial t)^2 + (\partial w/\partial t)^2 \right] dx\, dy \tag{5}$$

while the strain energy, 'V' is given by

$$V = \tfrac{1}{2} \int_0^L \int_0^{2\pi R} \int_{-h/2}^{h/2} (\sigma_{xx}\, \epsilon_x + \sigma_y\, \epsilon_y + \sigma_{xy}\, \gamma_{xy})\, dz\, dy\, dx \tag{6}$$

Applying Hamilton's principle and using Euler's variational equations to the Lagrangian function (K - V) and in-plane inertia being neglected, one can obtain the following equations of motions

$$(\partial N_x/\partial x) + (\partial N_{xy}/\partial y) = 0, \tag{7}$$

$$(\partial N_{xy}/\partial x) + (\partial N_y/\partial y) = 0 , \tag{8}$$

$$(\partial^2 M_x/\partial x^2) + 2(\partial^2 M_{xy}/\partial x\partial y) + (\partial^2 M_y/\partial y^2) + (\partial/\partial x)(N_x w_{,x} + N_{xy}\, w_{,y})$$
$$+ (\partial/\partial y)(N_{xy}\, w_{,x} + N_y\, w_{,y}) + (N_x/R^*) + (N_y/R) - \rho\, h\, w_{,tt} = 0. \tag{9}$$

where (N_x, N_y, N_{xy}), (M_x, M_y, M_{xy}) are the usual stress and moment resultants of their corresponding stress components σ_x, σ_y and σ_{xy}, respectively. Assuming that the vibrations take place principally in the direction perpendicular to the median surface, one may represent the stress function satisfying the equations of equilibrium (7) and (8), as

$$\sigma_x = \partial^2\phi/\partial y^2 = \phi_{,yy} , \quad \sigma_y = \phi_{,xx} , \quad \sigma_{xy} = - \phi_{,xy} \tag{10}$$

(,) notation in equations (9) and (10) denotes partial differentiation. The moment resultants in terms of displacement function may be written as

$$\begin{pmatrix} M_x \\ M_y \\ M_{xy} \end{pmatrix} = - \frac{c_{11}h^3}{12} \begin{pmatrix} 1 & \alpha & 0 \\ \alpha & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{pmatrix} \tag{11}$$

and the stress resultants are given by

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix} = c_{11}h \begin{pmatrix} 1 & \alpha & 0 \\ \alpha & \beta & \\ 0 & 0 & \end{pmatrix} \begin{pmatrix} u_{,x} - w/R^* + \frac{1}{2} w_{,x}^2 \\ v_{,y} - w/R + \frac{1}{2} w_{,y}^2 \\ u_{,y} + v_{,x} + w_{,x} w_{,y} \end{pmatrix} \tag{12}$$

Equation (9) will now reduce to

$$\begin{aligned} & - c_{11}(h^2/12) \left[w_{,xxxx} + 2(\alpha + 2\gamma) w_{,xxyy} + \beta w_{,yyyy} \right] \\ & - (h^2/12)\, 2(\partial c_{11}/\partial x) \left(w_{,xxy} + \alpha w_{,yyy} + 2\gamma w_{,xyy} \right) \\ & - (h^2/12)(\partial^2 c_{11}/\partial x^2) \left(w_{,xx} + \alpha w_{,yy} \right) + (\phi_{,yy}/R^*) + (\phi_{,xx}/R) \\ & + D(w,\phi) - \rho h w_{,tt} = 0 \end{aligned} \tag{13}$$

The variables (u,v) are now elimanated from equation (12) using the stress-strain component relations, establishing the following compatibility equation :

$$\begin{aligned} & \Big[c_{11}^2 \left\{ \phi_{,xxxx} + \left(\frac{\beta - \alpha^2}{\gamma} - 2\alpha \right) \phi_{,xxyy} + \beta \phi_{,yyyy} \right\} \\ & - c_{11} (\partial c_{11}/\partial x) \left\{ 2 \phi_{,xxx} + \left(\frac{\beta - \alpha^2}{\gamma} - 2\alpha \right) \phi_{,xyy} \right\} \\ & - \left\{ c_{11}(\partial^2 c_{11}/\partial x^2) + 2 (\partial c_{11}/\partial x)^2 \right\} \left\{ \phi_{,xx} - \alpha \phi_{,yy} \right\} \Big] c_{11}^3 (\beta - \alpha^2) \\ & = \left[- \tfrac{1}{2} D(w,w) - (w_{,yy}/R^*) - (w_{,xx}/R) \right] \end{aligned} \tag{14}$$

where the nonlinear and partial differential operator 'D' is defined by

$$D(a,b) = (a_{,xx}\, b_{,yy} - 2\, a_{,xy}\, b_{,xy} + a_{,yy}\, b_{,xx}) \tag{15}$$

and employed in equations (13) and (14).

Equations (13) and (14) now constitute the two fundamental field equations. In the next section an illustration will be cited with the method for finding an approximate solutions for them for further discussions.

3 ILLUSTRATION

Considered here is a **simply**- supported shallow shell of the nature as described earlier. The proper boundary conditions are then,

$$v = w = N_x = M_x = 0, \text{ at } x = 0, L. \tag{16}$$

The method of solution will be what has been followed by Nowinski (1963) for the analysis of nonlinear transverse vibrations of orthotropic cylindrical shells.
The first step to be followed is that one shall have to select the deflection function in a separable form

$$w(x,y,t) = f(t) \sin(m\pi \cdot x/L) . \sin(n/R)y + f_0(t) \tag{17}$$

where 'm' and 'n' represent the numbers of half-waves along the x- and y-directions. Inserting this value into the compatibility equation (14) a particular integral can be obtained in the form :

$$\phi(x,y,t) = (A \cos 2m\pi x/L + B \cos 2ny/R) f^2(t) + C \sin m \; x/L . \sin ny/R \tag{18}$$

where A,B and C are known quantities. The use of the closure property

$$\int_0^{2\pi R} (\partial v/\partial y)\, dy = 0 \tag{19}$$

will then yield

$$f_0(t) = n^2 f^2(t)/8R \tag{20}$$

Finally, a Galerkin procedure is applied to the equation of motion (13) when the following time differential equation may be obtained :

$$(d^2f/dt^2) + P f(t) + Q f^3(t) = 0 \tag{21}$$

which is a well-known time equation and can be solved in the form of cosine type Jacobian elliptic function.

The observation being carried on will be presented for a particular type of orthotropy. The governing differential equations can coveniently be employed for further studies. However, the equations are very highly nonlinear, one must use an appropriate approximate method.

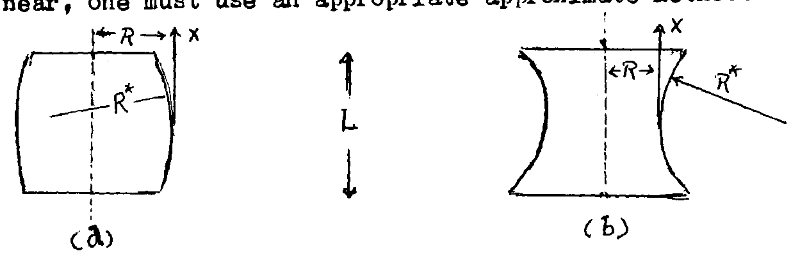

Figure-1. Geometry of the shell:(a) positive (b) negative Gaussian Curvature

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