Thermal Ratchetting of a Clamped Circular Plate Subjected to Steady Transverse Pressure and Cyclic Radial Temperature Variation

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Abstract

The shakedown limit for a clamped circular plate subjected to steady transverse pressure load and cyclic radial temperature variation is obtained from finite element elastic-plastic solutions. This computed limit is compared with an upper bound solution which proves to be very unsafe.

Two very different ratchetting mechanisms were identified; one is dominantly flexural and involves large incremental deformation; the other mechanism is associated with very small incremental deformation within the cyclic plasticity zone which develops when the thermal loads are large.

The design implications of the solutions are discussed.

1. Introduction

Many components in power and chemical plant are subjected to cyclic loading. Under severe loading conditions incremental strain accumulation (ratchetting) and/or cyclic plasticity may occur. This behaviour may lead to failure due to fatigue or excessive deformation. It is very useful, at the design stage, to know the shakedown limit of a component and its behaviour for loads both within and in excess of its shakedown limit. This information is provided in this paper for a simple idealisation of an above core structure in liquid metal fast breeder reactors.

2. Problem Definition and Analytical Solutions

The uniform thickness circular plate, $R/h = 20$, is clamped at its outside edge. The plate is loaded by a steady uniform transverse pressure p and its temperature is cycled between a uniform state and:-

$$T = \Delta T(1 - r/R) \qquad (1)$$

where T is the temperature relative to the uniform temperature.

The plate material is elastic - perfectly plastic. Yield and plastic deformation are governed by the Von-Mises effective stress criterion and the Prandtl-Reuss flow rule. Poissons ratio is 0.3. The material behaviour is assumed to be independent of temperature.

Using Ponter's [1] and Ponter and Karadeniz's [2] simplification of the upper bound shakedown theorem, it can be shown, by assuming a transverse displacement field:-

$$w = w_o(1 - r/R) \tag{2}$$

together with the thermo-elastic stresses that, for $\sigma_t < 2\sigma_y$, an upper bound shakedown limit is:-

$$1.1085 = p/p_L + 0.1412\ \sigma_t/\sigma_y \tag{3}$$

Where p_L is the limit load for a Von-Mises thin plate, obtained by Hopkins and Wang[3], and σ_t is the maximum thermo-elastic stress.

ie
$$p_L = 3.125\ \sigma_y(h/R)^2 \tag{4}$$
and
$$\sigma_t = (2 - \nu)\ E\alpha\Delta T/[3(1 - \nu)] \tag{5}$$

When $\sigma_t > 2\sigma_y$ a zone of cyclic plasticity develops at the centre of the plate. The extent of this zone, obtained from the thermo-elastic stresses and the Von-Mises yield condition, is:-

$$\frac{r}{R} = \frac{1}{6}\left(\frac{2 - \nu}{1 - \nu}\right)\left\{3 - \left(48\left(\frac{\sigma y}{\sigma t}\right)^2 - 3\right)^{\frac{1}{2}}\right\} \tag{6}$$

Following the arguments in references [1] and [2], the steady transverse pressure load must be supported by the stresses within the cyclic plasticity zone. Thus, in the steady cyclic state, the mean stresses at each point cannot be zero and incremental deformation must occur when $\sigma_t > 2\sigma_y$.

3. Finite Element Analysis

The plate was modelled with a uniform 10 (radial) X 4 (through thickness) mesh of 8-noded isoparametric axisymmetric elements. At the clamped edge the radial displacements were restrained and a uniform transverse shear loading was imposed.

The elastic-plastic behaviour of the plate was investigated for the combinations of steady pressure and cyclic thermal loads associated with the points plotted in the σ_t/σ_y, p/p_L load space diagram, Figure 1. For each combination of loads, the cyclic calculations were continuued generally until either shakedown or a steady cyclic state was achieved.

4. Results

4.1 Accuracy of Finite Element Model

The accuracy of the finite element model was assessed from some preliminary solutions for transverse pressure loading. The model gave accurate solutions for elastic displacements and stress with only small discontinuities in the radial variation of transverse shear stress near the outside radius. The largest pressure for which a converged elastic-plastic solution was obtained was 4% above the value of limit load for a thin, Von-Mises plate, equation (4), at this pressure the maximum displacement of the plate for small increases in the pressure load was approximately 550 times the elastic value indicating that the model was very close to collapse.

4.2 Shakedown Limit and Ratchetting Mechanisms

The computed shakedown boundary, shown in Figure 1, defines the load for zero cyclic steady state increment of centre line displacement, obtained by extrapolation of graphs of load against the cyclic steady state increment of displacement. Also shown in the figure are lines of constant cyclic steady state centre line displacement increment obtained by interpolating the load displacement graphs; the upper bound, equation (3), and $\sigma_t = 2\sigma_y$ shakedown limits are also included.

Examination of the finite element solutions showed that two quite distinct ratchetting mechanisms may occur. The ratchetting behaviour for two load cases which illustrate the two mechanisms is described below.

(i) $\underline{p/p_L = 0.832;\ \sigma_t/\sigma_y = 1.5}$

The yield zones for the initial application of the pressure load and for the application and removal of the thermal load in the first cycle are shown in Figure 2. During successive thermal cycles the behaviour of the plate approaches a cyclic steady state, for which the development of the yield zone for the application of the thermal load is shown in Figure 3. For the second half of the thermal cycle the growth of the plastic zone is a mirror image, about z = 0 plane, of that shown in Figure 3. The first and steady state cyclic increments of transverse displacement of the plate are shown in Figure 4; the steady state is approached asymtotically with practically no further reduction in incremental displacement after the tenth cycle. The steady state cyclic increments of strain adjacent to the top surface of the plate are shown in Figure 5. The increment of radial strain increases rapidly in the element adjacent to the clamped boundary, indicating that a plastic hinge occurs at the outside radius. The hoop and radial strain increments are zero on the centre plane of the plate, z = 0, and vary linearly through the plate thickness. The cyclic increments of transverse strain are generally at least an order of magnitude less than the hoop and radial components. Thus, for this load combination, the ratchetting behaviour of the plate is almost purely bending.

(ii) $\underline{p/p_L = 0.104;\ \sigma_t/\sigma_y = 2.3}$

The initial pressure loading is elastic. In the first half thermal cycle yield initiates on the top surface at the centre of the plate and spreads radially outwards and through the thickness to approximately r/R = 0.7 on the top surface and r/R = 0.8 on the bottom surface. During the second half of every thermal cycle and for each application of thermal load after the second, yielding is practically confined to the cyclic plasticity zone, r/R < 0.22, given by equation (6). Each thermal cycle produces transverse displacement, and shakedown practically occurs after the second cycle. The transverse displacement at the centre of the plate for the first cycle is 0.033h; this value is very much larger than the second cycle value of 2.02×10^{-4}h and the subsequent cyclic steady state value of 0.9×10^{-4}h.

In the cyclic plasticity zone the alternating plastic radial and hoop strains vary linearly from approximately 0.56 ε_y at the centre to zero at r/R = 0.22; the steady state incremental direct and transverse shear strains are approximately an order of magnitude less than these alternating strains. The incremental deformation within the cyclic plasticity zone is illustrated in Figure 6; the distorted mesh has been obtained from the incremental displacements using a magnification factor or of approximately 3000. It can be seen that the incremental deformation involves complex direct and shear deformation and is very different from the bending type deformation associated with the behaviour for $p/p_L = 0.832$ and $\sigma_t/\sigma_y = 1.5$.

5. Discussions and Conclusion

For $\sigma_t < 2\sigma_y$, the computed shakedown limit, obtained from the finite element solutions is approximately:

$$1.04 = p/p_L + 0.26\ \sigma_t/\sigma_y \tag{7}$$

It can be seen, Figure 1, that the upper bound shakedown limit is unsafe particularly when the thermal load is significant. The difference between the upper bound and the computed shakedown limits arise, mainly because the assumed incremental deformation, equation (2), used to obtain the upper bound, inadequately represents the actual deformation, shown in Figure 4. The upper bound shakedown limit, equation (3), would be of little use for design purposes. Generally optimised displacement assumptions need to be used to provide useful upper bound shakedown solutions; these solutions should also be checked with a limited number of numerical solutions.

For $\sigma_t > 2\sigma_y$, ratchetting should occur for all non-zero values of pressure load because the transverse pressure load induces an incremental deformation mechanism in the cyclic plasticity zone. The computed shakedown boundary, obtained from the $p/p_L = 0.104$ and 0.416 solutions, is somewhat greater than $\sigma_t = 2\sigma_y$. The failure to accurately predict this shakedown limit could be due to a number of factors which include:- discretisation errors of the finite element model, errors due to the accuracy criteria used in the iterative elastic-plastic procedures, and incorrectly assuming that the ratchet displacements continue to vary linearly with σ_t as the shakedown limit is approached.

Two distinct modes of ratchetting occur in the plate. With $\sigma_t < 2\sigma_y$ the ratchetting mechanism is almost exclusively bending. Small excursions outside the computed shakedown limit, equation (7), result in significant incremental deformation with, theoretically, infinite strain increments at the clamped boundary, due to the 'plastic hinge'. Thus the shakedown limit for this region, equation (7), is directly applicable for the determination of maximum acceptable loads.

For $\sigma_t > 2\sigma_y$, a ratchetting mechanism develops in the cyclic plasticity zone and there is no purely cyclic plasticity regime except for zero pressure load. However, for small pressure load, the ratchet deformations are extremely small, even for significant excusions into the ratchetting regime. Hence, it is possible that design loads of $\sigma_t > 2\sigma_y$ might be acceptable when the pressure load is small, provided the cyclic plastic strains are acceptable.

References:

[1] Ponter, A.R.S., "Shakedown and Ratchetting Below the Creep Range, Part II Theoretical Considerations", University of Leicester, Department of Engineering, Report 81-13 (1981).

[2] Ponter, A.R.S., and Karadeniz, S., "An Extended Shakedown Theory for Structures which suffer Cyclic Thermal Loading, Part I Theory", accepted by Jl. Appl. Mechs.

[3] Hopkins, H.G. and Wang, A.J., "Load Carrying Capacities of Perfectly-Plastic Materials with Arbitrary Yield Conditions", Jl. of Mech. & Phy. Solids, 3, 117-129 (1954).

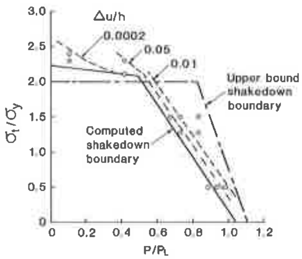

Fig. 1. Shakedown Boundaries for Clamped Circular Plate.

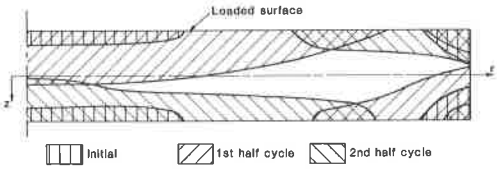

Fig. 2. Initial and First Cycle Yield Zones; $p/p_L = 0.832$, $\sigma_t/\sigma_y = 1.5$.

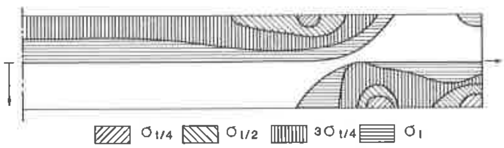

Fig. 3. Growth of Plastic Zone in First Half of Steady State Cycle; $p/p_L = 0.832$, $\sigma_t/\sigma_y = 1.5$.

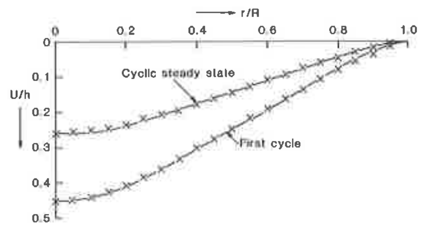

Fig. 4. First Cycle and Cyclic Steady State Displacement; $p/p_L = 0.832$, $\sigma_t/\sigma_y = 1.5$.

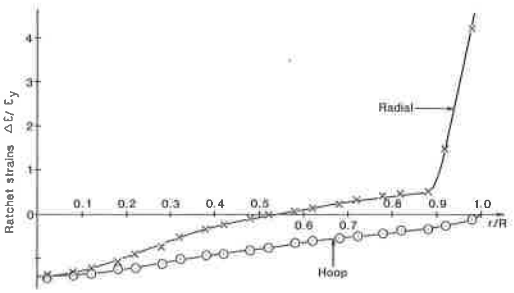

Fig. 5. Distribution of Steady State Ratchet Strains; $z/h = -0.4412$, $p/p_L = 0.832$, $\sigma_t/\sigma_y = 1.5$.

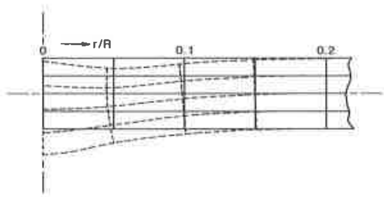

Fig. 6. Steady State Cyclic Incremental Deformation; $p/p_L = 0.104$, $\sigma_t/\sigma_y = 2.3$.