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Application of structural reliability to multi-circumferential cracking in steam generator tubes

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ABSTRACT

The COMPROMIS Code, developed by Electricité de France, is a probabilistic software tool concerned with the assessment of probability of occurence of a Steam Generator (SG) tube rupture caused by multicircumferential cracking. It involves the calculation of low probabilies, for which Monte Carlo stratified sampling was selected. After a short description of the physical model, this paper presents the implementation of the numerical methods, some outputs of the code and sensitivity results of the rupture probability to input parameters.

1. INTRODUCTION

Of the main components of the primary coolant system of a pressurized water reactor (PWR), the tube bundles of the Steam Generators (SG) are undoubtedly the most sensitive to damage and premature ageing [1]. As in almost all heat exchangers, the boundary between the 'hot' and 'cold' fluids is exposed to severe thermal and mechanical stresses, in an environment that is often hostile. One of the causes of tube damage is stress corrosion cracking in the roll transition zone, that may result in multicircumferential cracking under both bending and pressure stresses. The main objective of the SG tube maintenance is to keep the probability of a tube rupture at a very low level ; therefore, the current rule is to plug a tube whenever a circumferential crack is detected. This criterion is adapted to the operating conditions as far as up to now, only few circumferential cracks have been detected.

The COMPROMIS Code was developed by Electricité de France to assess the probability of occurence of a Steam Generator (SG) tube rupture caused by longitudinal and multicircumferential cracking and also other degradation mechanisms. The current version, developed on workstation, consits of three basic modules. This paper is concerned with the module dedicated to multicircumferential cracking due to internal stress corrosion It involves the calculation of low probabilies, for which the Monte Carlo method using stratified sampling was selected.

This paper firstly gives a short description of the physical model. Then it presents the numerical methods implemented. Some outputs of the code are shown hereafter. Finally it is made a presentation of sensitivity results : the most important input parameters on the SG tube rupture probability are identified, and it is focussed on the critical crack size for which a comparison with the results of the PROBAN code is possible.

2. BASIC FEATURES OF THE COMPROMIS CODE VERSION 2

The physical model is based on Probabilistc Fracture Mechanics. It involves three main aspects :

1. reconstitution of initial circumferential crack size distribution (random variables : number ν, initial crack size β (degrees), ligament size λ(degrees));
2. evolution of crack size distribution versus time with propagation including possibility of coalescence
3. failure analysis

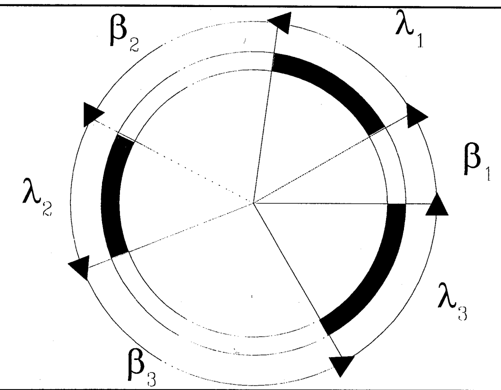

Figure 1. Initial multicircumferential crack distribution on an SG tube section

For the first part, figure 1 represents an example of the initial distribution of cracks on an SG tube section.

Note that the generated initial configurations are submitted to a consistency test: $\sum_{i=1}^{\nu}\beta_i + \sum_{i=1}^{\nu-1}\lambda_i \leq 2\gamma$, where 2γ is the size of the cracked zone. This modelization problem led to propose 6 input sequence choices to the user for the geometric variables. They are summarized in table 1.

If many non valid configurations are generated by the selected distributions, the user is given a warning message because the real distributions are different from the expected ones and the computation time may grow significantly. The so called simplectic laws allow a systematic consistency, but lack physical meaning. The choices 1 and 2 are simple, the choices 3 and 4 take account of user defined correlations between flaws and ligaments. The available law types are the following : gamma, Weibull, trunc-normal, lognormal, uniform, exponential, beta, triangular, Gumbel. If ν is fixed, configurations of 30 cracks can be treated.

Table 1. The 6 input choices for the initial geometrical variables

input choice	Joint distribution type
choice 1	Independant $p(\nu)$, $p(\beta)$, $p(\lambda)$
choice 2	Marginal conditional $p(\nu)$, $p(\beta/\nu)$, $p(\lambda/\nu)$
choice 3	Marginal conditional $p(\nu)$, $p(\beta/\nu)$, $p(\lambda/\nu,\beta)$
choice 4	Marginal conditional $p(\nu)$, $p(\beta/\nu)$,$p(\lambda/\nu,\Sigma\beta)$
choice 5	Simplectic $p(\nu)$,$p(\Sigma\beta/\nu)$ (β and λ simplectic)
choice 6	Simplectic $p(\nu)$, $p(\beta/\nu)$ (λ simplectic)

In the second phase, the crack size distribution is submitted to propagation according to Paris law giving the propagation rate $d\beta/dt$ in term of stress intensity factors (bending and pressure) : $\frac{d\beta}{dt} = C \cdot (K_{bending}(\beta) + K_{pressure}(\beta))^n$ {1}. For each crack tip a particular value of $K_{bending}$ is sampled, whereas $K_{pressure}$ is sampled for all the configuration. To every value of β corresponds a distribution of stress intensity factors. Coalescence phenomenon may appear : 2 close cracks may link up.

At the end of the propagation period, a comparison is made between $\Sigma\beta$(propagated) and $\Sigma\beta_c$, the critical size distribution, assumed to be known in this paper. This yields the fracture probability : $P(SGTR) = P(\Sigma\beta_T > \Sigma\beta_c)$, where T is the propagation time and SGTR stands for 'Steam Generator Tube Rupture'.

3. NUMERICAL METHODS OF PROBABILITY CALCULATION

The FORM/SORM reliability methods [2] did not seem to be relevant to the problem because of too many random variables and, most of all, coalescence that makes random variables disappear and causes strong nonlinearity during the propagation. Monte Carlo stratified sampling [3, 4] was selected because of the expected correlation between the result and some basic variables. 3 kinds of strata definition were adopted, corresponding to particular geometrical input choices. They are presented in table 2.

Table 2. Strata definition used in COMPROMIS

method	Stratification variables	Default number of strata	Input choice
method 1	$\Sigma\beta$ / K_1, K_2, K_p	5 * 3 = 15	5
method 2	$\Sigma\beta$ / K_1, K_2, K_p	5 * 3 = 15	3, 4, 6
method 3	$\Sigma\beta$, max λ / K_1, K_2, K_p	3 * 3 * 3 = 27	1, 2

Each stratum is of the type : $\Omega_i \otimes \Omega'_j$, where Ω_i is defined by geometric variables and Ω'_j by the fractiles of kinetics variables.

The estimate of the failure probability is based on the dimension reduction technique:

$$P_f = \frac{1}{N_S} \sum_{i=1}^{N_S} \left(\frac{1}{N} \sum_{j=1}^{N} F_{tc}(\Sigma\beta_j) \right) p(\Omega_i) \quad \{2\}$$

where F_{tc} stands for the cumulative distribution function of the critical crack size. A confidence interval for a given sample size is estimated. It is based on the following formula [3] :

$$E(\varepsilon^2)=\sum_{i=1}^{N_s}\frac{p_i^2 * \sigma_i^2}{n_i}, \quad \{3\}$$

where the first member is the mean square error, σ_i^2 is the estimated variance of the conditional failure probability on the stratum Ω_i, p_i the probability of stratum Ω_i, n_i the sample size from stratum Ω_i, Ns the number of strata. The 95% confidence interval bounds are as follows : C.I. = $[P_{est} - 1.96*\sqrt{E(\varepsilon^2)}\ ;\ P_{est} + 1.96*\sqrt{E(\varepsilon^2)}]$ {4}.

However, this formula is relevant to independant sample values ; and the way the sample procedure of the input values in the strata has been implemented causes correlation between the sample values. The following figure shows the sampling procedure in the strata in the default case of 3 geometrical strata and 3 kinetics strata. It can be seen that one geometrical value will be used several times in the same stratum, and also will be repeated in the other kinetics strata. This implementation is not so good as independant sampling but it is due to the forward version, in which the sampling of geometrical initial configurations and propagation variables were separate.

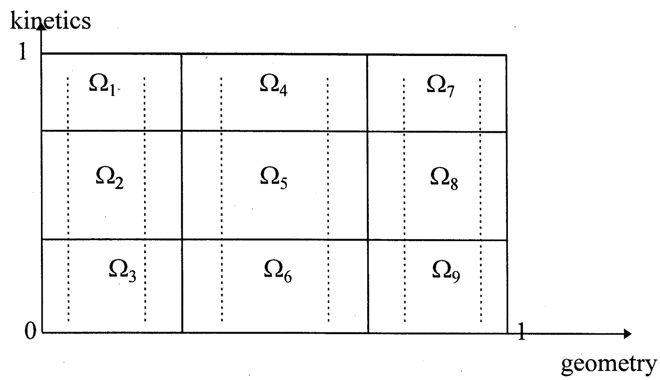

Figure 2 - Illustration of the general strata definition

A 3 step sampling procedure has been implemented. In the first part, it is done a first selection of strata after 50 sampled values per stratum. Then, 50% of the residual sample size is used for a predefined stratification corresponding to the current approach, giving a first estimate of the conditional probabilities used to calculate the optimal sample repartition. This optimized procedure has been implemented in a test software before modifying the original code. The comparative results have already been presented [5].

4. OUTPUTS OF COMPROMIS

In this part are presented some output screens relative to the case described hereafter (cf figure 3). It corresponds to a target reliability value about 10^{-3} without controls. The reference distributions are as follows :

- those presented in §5.1 for the critical sizes ;
- the input case 2 for the geometrical configurations (cf table 3) ;
- for the propagation parameters, the parameters of the stress intensity factors are given by tabulated formulas calculated for every 5 degree histogram class, and the Paris law constants are C= 1.5 10^{-5}, m = 1.

Table 3 - Reference distributions of the input geometrical variables

ν value	p(ν)	conditional density p(β/ν)	conditional density p(λ/ν)
1	0.5	N_trunc(198 ; 35 ; 0. ; 360)	-
2	0.3	N_trunc(99 ; 25 ; 0. ; 360)	N_trunc(30 ; 15 ; 0. ; 360)
3	0.2	N_trunc(66 ; 18 ; 0. ; 360)	N_trunc(20 ; 10 ; 0. ; 360)

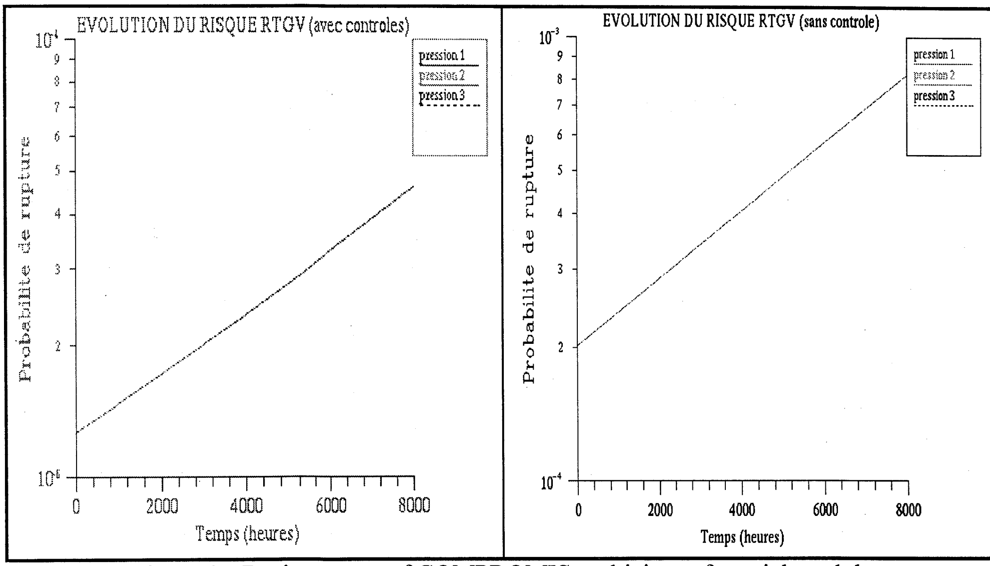

Figure 3 - Basic outputs of COMPROMIS multicircumferential module

An evolution of the SGTR probability is presented. It is calculated on 3 points : 0, 4000 and 8000 hours, which corresponds to the approximate duration of an operating cycle. It can take into account the control policy. The controls are modelized through POD (probability of detection) and probability of obturation. In this case the probability is 2 decades lower with control.

5. SENSITIVITY STUDIES ON THE SG TUBE RUPTURE PROBABILITY

This part contains a presentation of sensitivity results : the most important input parameters on the SG tube rupture probability are identified, and it is focussed on the critical crack size distribution for which a comparative evaluation with the PROBAN code is possible.

5.1 Critical crack size distribution

The critical multicircumferential crack size distribution is calculated in an independant module ; the computation results are stored in a file where both the density and the cumulative distribution function for three selected pressure values are available. Various files corresponding to various simulations or input distribution parameters can thus be created and employed hereafter to make a complete risk evaluation. The sample size is 10,000 and the sampling procedure also involves stratification on each input variable. The following formula gives the physical model of the critical size :

$$\Sigma\beta_c = 360 + (c - 360) \times 2 \cdot \frac{\Delta P\, r_i}{2 \cdot t \cdot 0.94 \cdot k \cdot (R_e + R_m)} \quad \{5\}$$

The table 4 gives the meaning of the variables and their reference distribution selected in the example.

Table 4 - Reference input variables involved in the calculation of critical crack sizes.

Variable	Meaning and unit	Distribution
c	coefficient (°)	N_trunc(150 ; 60 ; 0 ; 360)
ΔP	Pressure difference between primary and secondary part (MPa)	10.
De	external diameter (mm)	N_trunc(22.22 ; 0.06 ; 22.04 ; 22.4)
Re+Rm	Elastic and mechanical strength (MPa)	N_trunc(956 ; 50.3 ; 800 ; 1100)
t	tube thickness (mm)	N_trunc(1.27 ; 0.04 ; 1.15 ; 1.39)
k	coefficient (-)	N_trunc(0.58 ; 0.01 ; 0.55 ; 0.61)

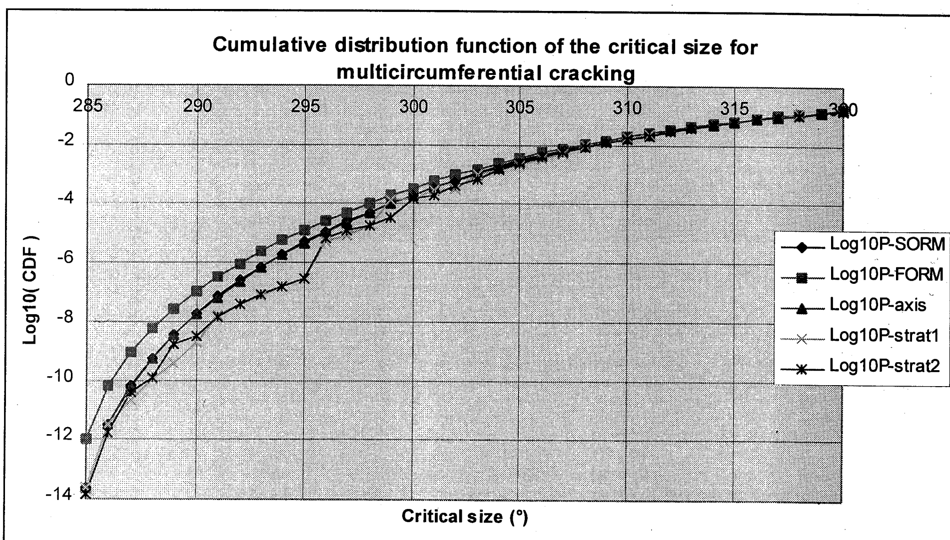

Figure 4 - Cumulative distribution function of the critical size for multicircumferential cracking

This model was also implemented in PROBAN [6], the general software of probability analysis used at EDF. The next figure presents the probability results obtained by COMPROMIS (2 examples of 10,000 simulations) and those of PROBAN using FORM, SORM and the so-called axis orthogonal method. The axis-orthogonal method provides a correction of the FORM approximation by making simulations around the design point calculated by FORM. Here the number of simulations was 1000 maximum and also submitted to a CPU time limit of 60 seconds.

It can be seen on the next figure that :

- the results given by the simulation methods are very close to each other after the threshold value 310 (C.D.F # 10^{-2}), that is why the C.D.F. values are calculated only before 320 ;

- the SORM and Axis-orthogonal corrections are consistent and can be considered as a reference in this case ;
- FORM results are overestimated ;
- the 2 COMPROMIS simulation results are close to each other ;
- they remain close to the reference results within 1 decade.

So one can conclude that the cumulative distribution function values of COMPROMIS are acceptable. Increasing the simulation number should better those results.

5.2 Other input parameters

Some sensitivity results have been calculated to identify the most important factors on the SGTR probability. Note that the conclusions strongly depend on the example selected, as there is an huge amount of possibilities of input parameter values. Some of the input distributions of the following case are not supposed to be realistic ; they only aim at illustrating the methodology. The reference distributions are those presented in §4 and 5.1. The sensitivity measure concerns the reliability index corresponding to the failure probability: $\boxed{m_\theta = \frac{\partial\beta}{\partial\theta} \cdot k \cdot \theta}$ {6}, where k is a coefficient equal to 0.1, θ is the parameter and $\frac{\partial\beta}{\partial\theta}$ is estimated by finite differences and 100,000 simulations for each calculation. This formula is the same as that used in PROBAN as « sensitivity measure » [6]. It enables comparisons between parameters without scale effect. Note that the result uncertainty due to simulation leads to take into account several parameter variations (6 trials). For each parameter are given the maximum and minimum absolute values of the sensitivity measure in table . The first parameter considered is the crack mean sum, $\mu_{\Sigma\beta}$. It is not a real input parameter, but in the example the initial crack mean sum is identical for each ν value. σ_i is the standard deviation of the crack size distribution for ν=i.

Table 5 - Sensitivity measures in the reference example

	Geometrical Parameters				Critical size		Kinetics	
Parameter	$\mu_{\Sigma\beta}$	σ_1	σ_2	σ_3	μ_c	μ_{Re+Rm}	C	m
m_θ^{upper}	-0.69	-0.19	-0.17	-0.033	0.091	0.094	-0.038	-0.17
m_θ^{lower}	-0.37	-0.13	-0.07	-0.015	0.031	0.034	-0.019	-0.035

It can be noticed that :

- although the 2 extreme values are quite different important variables can be identified and classified ;
- the geometrical and kinetics variables presented can be considered as stress variables (sensitivity measures are negative) [2, 7], whereas critical size variables are strength variables;
- the initial crack mean sum $\mu_{\Sigma\beta}$ is the most important parameter in this example ; it justifies the fact that stratification focusses on geometrical variables ;
- the geometrical variances can be ordered as follows : $\sigma_1>\sigma_2>\sigma_3$. This order reflects thi initial values of variances and of p(ν=i) ;
- the sensitivity to the exponent m has particularly scattered values : this parameter plays a strongly non linear part ;
- in the critical sizes, c and Re + Rm have the same importance.

6. CONCLUSIONS

This paper presented the COMPROMIS code, a probabilistic software tool developed by Electricité de France and concerned with the assessment of probability of occurence of a Steam Generator (SG) tube rupture caused by multicircumferential cracking, among other degradation mechanisms.

The numerical background was firstly detailed and it was shown how the Monte Carlo stratification methods were implemented. The basic outputs of COMPROMIS are the curves showing the evolution of the probability of SG tube rupture within an operating cycle, for which the control policy can be taken into account.

Then sensitivity studies and comparisons with other software results were carried out. The critical crack size distribution analysis and its comparison with PROBAN numerical results proved that COMPROMIS results were acceptable, even for very low probabilities : the different evaluations of the CDF do not exceed 1 decade. Moreover, the sensitivity studies carried out for the 3 kinds of input parameters showed that for a specified example it was possible to classify importance input variables and to clearly identify their type (stress/strength). The results confirmed the specific part played by geometrical variables in stratification.

It appears that the methodology of COMPROMIS should be used successfully on realistic examples. Improvements can be done in the error evaluation, and other sampling methods are to be implemented in a test software.

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