Non-Linear Seismic Analysis of Structures Coupled with Fluid

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SUMMARY

Seismic analysis of a liquid metal fast breeder reactor is generally made with a linear dynamic model. The resulting loading is used to evaluate the resistance of the different parts of the structure and if they include thin shells the maximum pressures determined by this method are used for stability calculation. Separate quasi static elasto-plastic models are used for this purpose.

Buckling behaviour is highly non-linear and is dependent on thermal stresses and geometrical defects. Furthermore at this stage, the permanent and seismic loads must be added. As a consequence of large displacements and plasticity effects, the dynamic behaviour of shells is modified and the seismic loading may not be correctly evaluated by only a linear elastic model.

This paper presents a method to calculate non-linear structure behaviour under horizontal and vertical seismic excitation, making possible the full non-linear seismic analysis of a reactor vessel.

A pseudo forces method is used to introduce non linear effects and the problem is solved by superposition. Two steps are used in the method :

- Linear calculation of the complete model
- Non linear analysis of thin shell elements and calculation of seismic induced pressure originating from linear and non linear effects, including permanent loads and thermal stresses.

Basic aspects of the mathematical formulation are developed. It has been applied to axi-symmetric shell element using a Fourier series solution.

For the fluid interaction effect, a comparison is made with a dynamic test. In an example of application, the displacement and pressure time history are given.

1. Introduction

The seismic analysis of a nuclear reactor structure involves the following four steps :

- Construction of a dynamic model
- Response analysis either by the time history or the response spectrum method.
- Determination of seismic loads to be combined with permanent and transient loads.
- Stress analysis

More generally the licensing authorities require that the structure be protected against different failure modes, including excessive strain and elastic or elasto plastic instability resulting from permanent or cyclic loads.

The reactor vessel of a breeder reactor contains thin shells submitted to thermal gradients and, during seismic events, to fluctuating external pressure. In such a situation instability or buckling is more likely to be the failure mode.

Then the problem arises of how to properly determine the pressure loads to be applied to a shell structure without being overly conservative. In a simple approach, use is made of a linear structural model. The maximum pressure applied to the structure can be calculated at each point and used to define the load. Difficulties arise from two facts : first, the maximums are not reached in different points at the same time, and second, in the response spectrum method, the direction of the pressure is not determined. To circumvent these deficiencies, several loadings may be used and the worst result taken into consideration.

A more satisfactory method is possible with a time history analysis if a loading criterion is available. When the criterion reaches a maximum, the forces applied to the structure reach also a maximum at that time and a single set of data may be used for design.

However this procedure is not entirely satisfactory for further static evaluation of thin shell stability under seismic loads. The two main concerns are :

- In the dynamic model geometrical defects are generally not introduced and the structure behaviour is assumed to be linear elastic.
  When the defects are introduced afterwards, in a stability model, the structure stiffness properties may be significantly reduced. This modification increases the deformations and affects the load path in the whole structure.

- Higher modes of deformation are neglected in the dynamic analysis. Of particular interest is the dynamic response of the buckling modes. Depending on the geometry, the corresponding frequencies are above or below seismic excitation range. In the first case, static calculation can be expected to provide an acceptable estimate. In the second case, it may be reasonably assumed that the method overestimates the seismic requirements. No reliable evaluation can be made for the intermediate situation.

Starting from a development already made for dynamic non linear anlysis of axi-symmetric shells, (Réf.1), a method has been developed to perform seismic analysis of imperfect shells, including :

- Large displacements
- Mechanical and thermal permanent loads
- Plasticity

The next section describes the mathematical formulation.

2. Formulation

2.1. Non-linear shell element

The principle of virtual work can be stated by the following equation :

$$\int_V \rho \delta u^T \ddot{u} \, dv + \int_V \delta \varepsilon^T \sigma \, dV = \int_A \delta u^T f \, dA \qquad (1)$$

ρ mass density
$\ddot{u}$ acceleretion
σ Piola-Kirchhoff stresses
f external force
δu virtual displacement $\delta \varepsilon$ virtual deformation
dV volume element
dA volume or surface element

Stresses are related to deformations by Hooke's law :

$$\sigma = D \varepsilon_e$$

D elasticity matrix based on material properties
starting from an underformed structure $\varepsilon = \varepsilon_0 + \varepsilon_t + \varepsilon_p + \varepsilon_e$
ε Green strains ε_0 initial strain
ε_t thermal "
ε_p plastic "

Finite element formulation is derived by expressing ε and u through shape functions N and nodal displacement U.

$$u = N U$$

Virtual displacements and deformations can be expressed as :

$$\delta u = N \delta U \qquad \delta \varepsilon = B \delta U$$

substituting in equation (1) yields

$$M \ddot{U} + \int_V B^T D \varepsilon \, dV = F + Q_{IN} \qquad (2)$$

where :

$$M = \int_V N^T \rho N \, dV \quad \text{mass matrix}$$

$$F = \int_A N^T p \, dA \quad \text{external force}$$

$$Q_{IN} = \int_V B^T D(\varepsilon_0 + \varepsilon_t + \varepsilon_p) dV \quad \text{inelastic force}$$

2.2. Large displacements and pseudo forces

With the large displacements hypothesis, deformation is a quadratic expression of displacement derivatives.

$$\varepsilon_x = \frac{\partial u_x}{\partial x} + \frac{1}{2}\left(\frac{\partial u_z}{\partial x}\right)^2$$

$$\gamma_{xy} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial x} \cdot \frac{\partial u_z}{\partial y}$$

Therefore deformation will be expressed as the sum of a linear and a non linear term

$$\varepsilon = \varepsilon_L + \varepsilon_{NL} \qquad \delta\varepsilon = \delta\varepsilon_L + \delta\varepsilon_{NL}$$

and using shape functions.

$$\varepsilon = B_L U + \varepsilon_{NL} \qquad \delta\varepsilon = (B_L + B_{NL})\delta U$$

In a standard formulation the stiffness matrix is defined by the following expression :

$$K = \int_V B_L^T D B_L \, dV$$

The displacement is measured relative to the initial perfect unloaded structure and is the sum of three terms :

$$U = U_0 + U_S + U_D$$

U_0 geometrical defect, the corresponding deformation is :

$$\varepsilon_0 = B_L U_0 + \varepsilon_{NL0}$$

U_S initial displacement induced by permanent mechanical load Fs and thermal expansion ε_t

$$K U_S = F_S + \int_V B_L^T D \varepsilon_t \, dV$$

U_D is the dynamic displacement

similarly

$$F = F_S + F_D$$

After substitution in equation (2) the complete set of equation in matrix form is :

$$M \ddot{U}_D + K U_D = F_D + Q_{IN} + Q_{NL} \qquad (3)$$

$$Q_{IN} = \int_V B^T D \varepsilon_p \, dV$$

$$Q_{NL} = -\int_V B_{NL}^T D (\varepsilon - \varepsilon_0 - \varepsilon_t) dv - \int_V B_L^T D (\varepsilon_{NL} - \varepsilon_{NL0}) dv$$

$\varepsilon, \varepsilon_{NL}, B, B_{NL}$ are function of $U = U_0 + U_S + U_D$

In the usual expressions for a thin shell, only the non linear membrane deformation is used. Therefore, calculation of the pseudo forces due to geometric non linearities involves only a surface integration. However, the calculation of pseudo forces due to plasticity involves a volume integration.

For shells forming a surface of revolution, a Fourier series decomposition may be used. In that case the left hand side of equation (3) is uncoupled, all coupling is contained in the pseudo forces terms which must be computed at each time step.

When fluid is included using an added mass formulation the problem involves a full mass matrix (Réf. 2).

2.3. Linear superposition

In the linear seismic calculation ofaxy-symmetric shells, only the two first harmonics of the Fourier decomposition are used.

Considering non linearities, the mesh can be devided in two parts :

- Linear part with degrees of freedom U_1
- Non linear part with degrees of freedom U_2

Then the general formulation for the seismic analysis of a structure is (Réf. 3)

$$|M| \begin{vmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{vmatrix} + |K| \begin{vmatrix} U_1 \\ U_2 \end{vmatrix} = -|M| \begin{vmatrix} R_1 \\ R_2 \end{vmatrix} \ddot{U}_G + \begin{vmatrix} 0 \\ F_2 \end{vmatrix}$$

In the right hand of the equation, F_2 is the pseudo forces. This problem can be solved by superposition of two solutions :

- U_L the usual linear problem involving the first two Fourier terms.
- U_{NL} the non linear problem with several harmonics but restricted to a limited part of the structure.

Assuming, for simplicity, that the non linear effects are restricted to higher harmonics, the problem can be solved with previously presented method if $U_D = U_{2L} + U_{2NL}$ is used in the calculation of pseudo forces.

3. Example of application

3.1. Experimental test of the fluid structure interaction effect

The structure tested is shown in figures 1 and 2. It is simplified model of a LMFBR main vessel, scaled down by a factor of 30. The vessel was filled with water and bolted to a horizontal shaking table. A calculation has been made to determine the eigen-frequencies of the system for comparison with pressure gage measurements made during sinusoidal tests.

The following tabe gives the first four frequencies measured and calculated for the second harmonic of the Fourier decomposition. As a reference for fluid-structure interaction effect, calculated values are also given in air (frequencies are given in Hz).

N	calculation in water	test	difference %	calculation in air
1	26.6	26.5	+ 0.4	162.4
2	101.2	101	+ 0.2	307.6
3	148.5	152	- 2.3	642.4
4	165.8	164	+ 1.1	819.4

3.2. Effect of geometrical non linearities on seismic displacement and pressure

The shell shown on figure 3 is a thermal baffle part of a larger structure submitted to horizontal and certical seismic excitation.

The geometrical defect is introduced in the non linear part of the calculation and 4 Fourier harmonics are used : N = 0, 1, 3 and 11.

Figures 4 to 7 give the time histories of displacement and pressure, linear and non linear part for OBE conditions.

5. Conclusions

- Fluid structure interation can be introduced in the seismic analysis of a breeder reactor. A test performed on a vessel-like structure, demonstrates that a good accuracy can be achieved.
- A method has been devised to introduce in the calculation of thin shell behaviour, geometrical and material non linearities, including the influence of defects, mechannical loads and temperature distribution.

References

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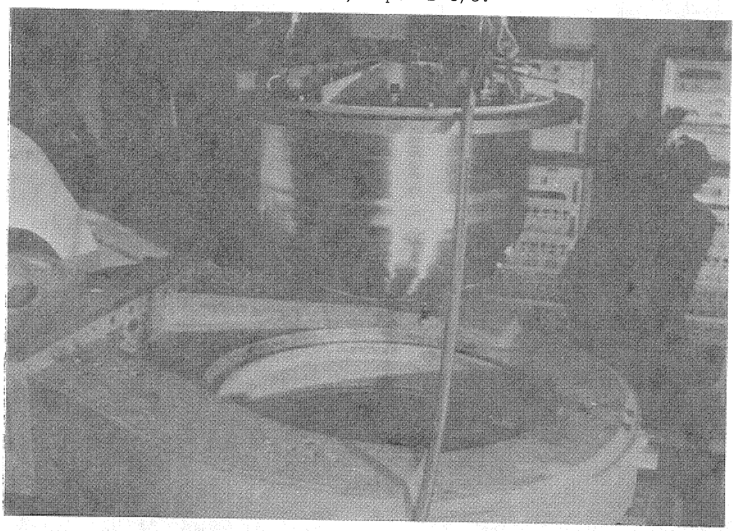

FIGURE 1 : Main vessel scale model test

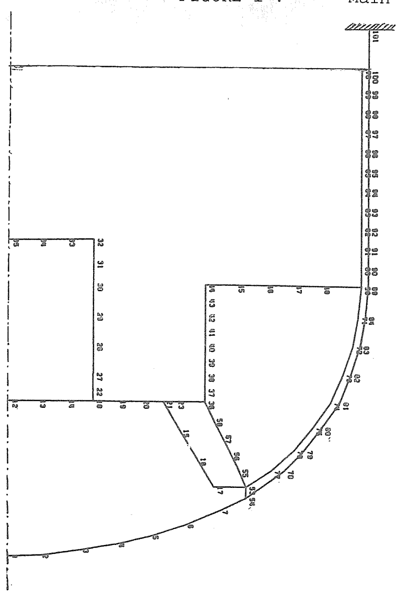

FIGURE 2 : Test model mesh

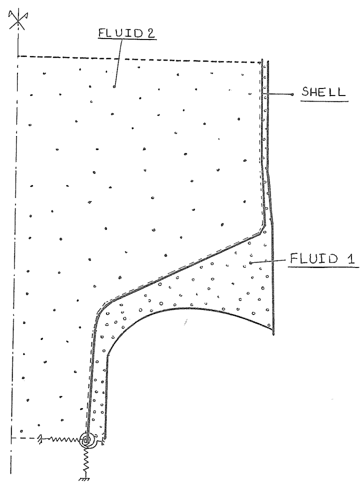

FIGURE 3 : Thermal baffle

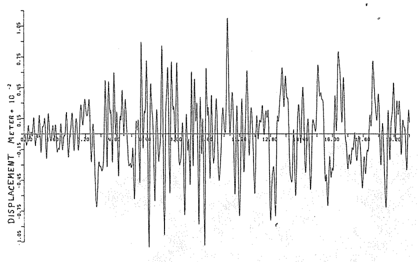

FIGURE 4 : Plot of linear radial displacement at top of baffle

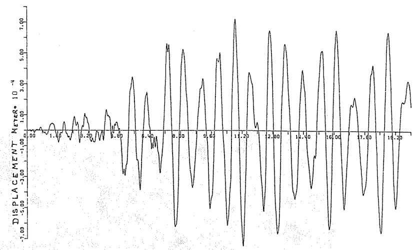

FIGURE 5 : Plot of non linear radial displacement at top of baffle

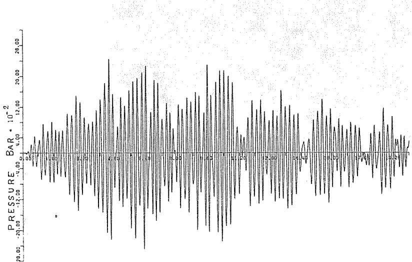

FIGURE 6 : Plot of linear differential pressure

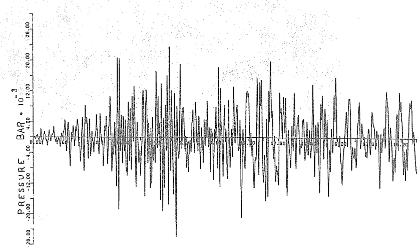

FIGURE 7 : Plot of non linear differential pressure