COMPUTER CODES FOR THE CALCULATION OF THE FREQUENCY RESPONSE AND TIME BEHAVIOUR OF SHELL-AND-TUBE TYPE COUNTERFLOW HEAT EXCHANGERS WITH RESPECT TO PRIMARY AND SECONDARY INLET TEMPERATURE AND MASS FLOW PERTURBATIONS (TUMHEX IN COMBINATION WITH FRETI)

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SUMMARY

A digital code, named TUMHEX, will be presented which is based on a linear analytical one-dimensional model calculating the frequency response behaviour of coolant temperature and/or coolant mass flow on both (primary and secondary) outlet sides of a counterflow shell-and-tube type heat exchanger with respect to perturbations in the corresponding temperature and mass flow parameters at the inlet to the primary and/or secondary side, respectively.

To make the heat exchanger model as comprehensive as possible, mass-flow dependent heat transfer coefficients, an exact solution of the heat conduction through the tube walls and (at least with respect to the coolant mass flow rates) temperature-dependent coolant densities had been taken into account.

TUMHEX has been tested against frequency response measurements on an industrial-size heat exchanger and has shown good results.

An auxiliary program FRETI which permits a retransformation from the frequency into the time domain in a numerical way will enable to calculate from given frequency response curves the time response behaviour of important heat exchanger parameters against both step and/or any system perturbing signals.

1. Introduction

The digital code TUMHEX (an abbreviation for "Techn. Univ. Munich, Heat Exchanger Model") will be presented. It is based on a linear analytical one-dimensional model and calculates the frequency response behaviour of fluid temperature and/or fluid mass flow on both (primary and secondary) outlet sides of a counterflow shell- and-tube type heat exchanger with respect to perturbations in the corresponding temperature and mass flow parameters at the inlet to the primary and/or secondary side, respectively.

An auxiliary program FRETI (an abbreviation for "frequency-time transformation") will permit a retransformation from the frequency into the time domain in a numerical way, i.e., give the possibility to calculate from given frequency response curves the time response behaviour of important heat exchanger parameters against both step and/or any system perturbing signals.

With the development of more and more complex plant systems having different heat removal circuits heat exchangers play often a significant part among the whole system performance. From the necessity to control the whole plant in an adequate way the question concerning the dynamic behaviour of each individual element arose and became more and more important. Especially in the reactor industry the stability behaviour of large nuclear power plants had to be taken into consideration in addition to the problems of controlling these plants. Whereas much effort has been undertaken to calculate the dynamic behaviour of the reactor core as exactly as possible, the equally important question with respect to the conventional parts of such a plant (e.g., heat exchangers, evaporators, steam turbines) are very often not as carefully handled.

However, the literature about the dynamic behaviour of a shell- and -tube type heat exchanger has increased in recent years. Starting from the already classic papers of Profos [1] and Takahashi [2] the first of the theoretical models have been developed only for the inlet-temperature perturbed case (i.e., having constant mass flow), assuming temperature-independent physical fluid and tube wall properties, constant heat transfer coefficients and negligible heat transport delay through the tube wall (i.e., thin tube walls). Thus, the fluid dynamic behaviour could be characterized by uncomplicated partial differential equations following an energy balance consideration. Law [3] presented a frequency response model which took also mass flow perturbation into account. He assumed the heat transfer coefficient to vary proportionally to the fluid velocity and simplified the resulting transfer functions by taking advantage of the "resonance effect" behaviour. Good agreement with experimentally measured dynamic data could be stated. Pütter [4] established a similar model with respect to inlet fluid temperature and inlet fluid mass flow perturbations for both co- and countercurrent water-cooled heat-exchangers, adding half of the stationary tube wall heat resistance value to the corresponding heat transfer coefficients on each exchanger side. Calculations compared to oscillator-experiments performed on an industrial-size heat exchanger showed good agreement. Remarkable was the fact that the neglection of the fluid-velocity-dependency of the heat transfer coefficients can cause substantial errors. (Two of these measurements had been also used for the comparison with the theoretical heat exchanger model being presented in the next chapter).

2. Theoretical model:

In this chapter only a brief review of the theoretical frequency response model will be given (For more details see ref. [6]).

The following notations had been applied:

All quantities for which no time (or frequency) dependency is explicitely indicated are steady state values. Time dependent deviations around the steady state will be denoted by δ, relative time dependent deviations (with respect to their steady state value) by δ_r:

$$T(z,t) = T(z) + \delta T(z,t) = T(z)\left[1 + \delta_r T(z,t)\right] \qquad (1)$$

The corresponding Laplace - Transformation is abbreviated (and defined) by:

$$\overline{\delta T}(z,s) = \mathcal{L}\left\{\delta T(z,t)\right\} = \int_0^\infty e^{-st}\, \delta T(z,t)\, dt \qquad (2)$$

Subscripts:

A, E	outlet, inlet
M	mean value
T	tube wall
I, II	shell-side (outer) and tube-side (inner) fluid

Because of the different demands on a heat exchanger in a reactor power plant (mostly because of different structural materials and working fluid media, e.g., the combination water - sodium) the model had to be derived under much more serious restrictions than done in the literature before.

2.1. Assumptions

The underlying (single-pass) heat exchanger had been assumed to consist of a bundle of N_T parallel tubes (r_I, r_{II}: outer and inner radius) with the fluid I flowing on the outerside, fluid II in the opposite direction on the innerside of the tubes. The heat fluxes ($q_I(z,t)$, $q_{II}(z,t)$) were considered to be directed towards the corresponding fluid media, thus permitting negative values as well.

Since the model should include any medium (e.g., water, steam, sodium, organic substances) a temperature dependency of the fluid density had to be taken into account, approximating it by a linear function of the form

$$\rho\,[T(z,t)] \cong \rho_E \cdot [1 - \sigma_\rho \frac{T(z,t) - T_E}{T_E}] \qquad (3)$$

Similarly, the temperature-dependency of the fluid enthalpy could be also approximated by a linear function

$$h\,[T(z,t)] \cong h_E + c \cdot [T(z,t) - T_E] \qquad (4)$$

(c = specific heat of the fluid)

A "one-dimensional" local representation of the characteristic flow variables had been applied by assuming the variables to be only axial- (and time-) dependent, but being averaged values over the flow area.

The heat flux in radial direction through the tube wall had been calculated by using the exact Fourier equation, contrary to other authors which considered infinite heat conduction or negligible tube wall thickness. (Axial heat conduction could be neglected). The heat conduction coefficient λ_T, the specific heat c_T and the tube wall density ρ_T had been assumed to be (axially and radially) constant along and/or through the tube wall. In accordance with Law [3] and Pütter [4] the time behaviour of the heat transfer coefficients (α_I, α_{II}) had been taken as linearly dependent on possible (inlet) fluid mass flow variations, i.e.,

$$\alpha(t) \cong \alpha \cdot [1 + \sigma_\alpha\, \delta_r m_E(t)] \qquad (5)$$

Heat conduction to the outer shell had been (and could be) neglected.

Both the steady state and the non-steady state situation had been considered. In the non-steady state case a linearization procedure had to be applied in order to solve the complicated differential and integral equations, so that after going over to the Laplace domain exact analytical solutions of the above mentioned equations could be established. This procedure restricts of course the validity of the formulas to only "small" parameter variations around their steady state values. However, a validity in the range of 10 percent and more relative variation is satisfactory enough for the most problems being looked at.

2.2. Heat transfer through the tube wall

The heat fluxes $q_I(z,t)$ and $q_{II}(z,t)$ can be calculated from the wellknown Fourier heat conduction equation

$$\frac{\rho_T c_T}{\lambda_T} \frac{\partial T_T(z,r,t)}{\partial t} = \frac{\partial^2 T_T(z,r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial T_T(z,r,t)}{\partial r} \tag{6}$$

with the initial condition

$$\delta T_T(z,r,t=0) = 0 \tag{7}$$

and the boundary (and/or heat transfer) conditions

$$-\lambda_T \frac{\partial T_T(z,r,t)}{\partial r}\bigg|_{r=r_{II}} = -\frac{q_{II}(z,t)}{2\pi r_{II}} = \alpha_{II}(t)\left[T_{II}(z,t) - T_T(z,r_{II},t)\right] \tag{8}$$

$$-\lambda_T \frac{\partial T_T(z,r,t)}{\partial r}\bigg|_{r=r_I} = \frac{q_I(z,t)}{2\pi r_I} = \alpha_I(t)\left[T_T(z,r_I,t) - T_I(z,t)\right] \tag{9}$$

This yields

$$q_{II}(z) = -q_I(z) = 2\pi r_{II} \alpha_{TII}\left[T_I(z) - T_{II}(z)\right] \tag{10}$$

with the "overall heat transfer coefficient"

$$\frac{1}{\alpha_{TII}} = \frac{1}{\alpha_{II}} + \frac{r_{II}}{\lambda_T} \ln \frac{r_{II}}{r_I} + \frac{r_{II}}{r_I} \frac{1}{\alpha_I} \tag{11}$$

and

$$\overline{\delta q_I}(z,s) = -2\pi r_I \alpha_I \left\{ Y_{TII}(s)\left[\overline{\delta T_I}(z,s) + T_{\alpha I}(z)\,\overline{\delta_r m_{IE}}(s)\right] + Y_{TIII}(s)\left[\overline{\delta T_{II}}(z,s) + T_{\alpha II}(z)\,\overline{\delta_r m_{IIE}}(s)\right]\right\} \tag{12}$$

with

$$Y_{TII}(s) = -\Omega_I Y_E(s)\left[I_1(\Lambda_I) Y_B(s) + K_1(\Lambda_I) Y_A(s)\right] \tag{13}$$

$$Y_{TIII}(s) = \Omega_I Y_E(s)\left[I_1(\Lambda_I) Y_D(s) + K_1(\Lambda_I) Y_C(s)\right] \tag{14}$$

or

$$\overline{\delta q_{II}}(z,s) = 2\pi r_{II} \alpha_{II} \left\{ Y_{TIII}(s)\left[\overline{\delta T_I}(z,s) + T_{\alpha I}(z)\,\overline{\delta_r m_{IE}}(s)\right] + Y_{TIIII}(s)\left[\overline{\delta T_{II}}(z,s) + T_{\alpha II}(z)\,\overline{\delta_r m_{IIE}}(s)\right]\right\} \tag{15}$$

with

$$Y_{TIII}(s) = -\Omega_{II} Y_E(s)\left[I_1(\Lambda_{II}) Y_B(s) + K_1(\Lambda_{II}) Y_A(s)\right] \tag{16}$$

$$Y_{TIIII}(s) = \Omega_{II} Y_E(s)\left[I_1(\Lambda_{II}) Y_D(s) + K_1(\Lambda_{II}) Y_C(s)\right] \tag{17}$$

and the abbreviations

$$\Lambda_I = r_I \sqrt{\frac{\rho_T c_T}{\lambda_T} s} \qquad \Lambda_{II} = r_{II} \sqrt{\frac{\rho_T c_T}{\lambda_T} s} \tag{18}$$

$$\Omega_I = \frac{\lambda_T}{\alpha_I r_I} \Lambda_I \qquad \Omega_{II} = \frac{\alpha_I}{\alpha_{II}} \Lambda_I \tag{19}$$

$$Y_A(s) = I_0(\Lambda_{II}) - \Omega_{II} I_1(\Lambda_{II}) \tag{20}$$

$$Y_B(s) = K_0(\Lambda_{II}) + \Omega_{II} K_1(\Lambda_{II}) \tag{21}$$

$$Y_C(s) = I_0(\Lambda_I) + \Omega_I I_1(\Lambda_I) \tag{22}$$

$$Y_D(s) = K_0(\Lambda_I) - \Omega_I K_1(\Lambda_I) \tag{23}$$

$$Y_E(s) = \left[Y_A(s) Y_D(s) - Y_B(s) Y_C(s) \right]^{-1} \tag{24}$$

$$T_{\alpha I}(z) = - \frac{q_I(z)}{2\pi r_I \alpha_I} \sigma_{\alpha I} \tag{25}$$

$$T_{\alpha II}(z) = - \frac{q_{II}(z)}{2\pi r_{II} \alpha_{II}} \sigma_{\alpha II} \tag{26}$$

($I_0(\Lambda)$, $I_1(\Lambda)$, $K_0(\Lambda)$ and $K_1(\Lambda)$ are modified Bessel and Mc Donald functions of 0-th and 1-st order)

2.3. Mass flow behaviour

The temperature-dependent axial mass flow variations can be calculated from a mass balance equation, e.g., given in a differential form

$$A_I \frac{\partial}{\partial t} \{ \rho_I [T_I(z,t)] \} - \frac{\partial}{\partial z} \{ m_I(z,t) \} = 0 \tag{27}$$

$$A_{II} \frac{\partial}{\partial t} \{ \rho_{II} [T_{II}(z,t)] \} + \frac{\partial}{\partial z} \{ m_{II}(z,t) \} = 0 \tag{28}$$

with the initial

$$\delta T_I(z, t=0) = 0 \qquad \delta m_I(z, t=0) = 0 \tag{29}$$

$$\delta T_{II}(z, t=0) = 0 \qquad \delta m_{II}(z, t=0) = 0 \tag{30}$$

and boundary conditions

$$T_I(z = z_T, t) = T_{IE}(t) \qquad m_I(z = z_T, t) = m_{IE}(t) \tag{31}$$

$$T_{II}(z = 0, t) = T_{IIE}(t) \qquad m_{II}(z = 0, t) = m_{IIE}(t) \tag{32}$$

Thus,

$$m_I(z) = A_I \rho_I [T_I(z)] v_I(z) = m_{IE} = m_{IA} \tag{33}$$

$$\dot{m}_{II}(z) = A_{II}\, \rho_{II}\left[T_{II}(z)\right] v_{II}(z) = \dot{m}_{IIE} = \dot{m}_{IIA} \qquad (34)$$

and

$$\overline{\delta_r \dot{m}_{IA}}(s) = \overline{\delta_r \dot{m}_{IE}}(s) + s\,\theta_{ITM}\, \overline{\delta_r T_{IM}}(s) \qquad (35)$$

$$\overline{\delta_r \dot{m}_{IIA}}(s) = \overline{\delta_r \dot{m}_{IIE}}(s) + s\,\theta_{IITM}\, \overline{\delta_r T_{IIM}}(s) \qquad (36)$$

with

$$\theta_{ITM} = \frac{z_T A_I \rho_{IE}}{\dot{m}_{IE}} \frac{T_{IM}}{T_{IE}} \sigma_{\rho I} \qquad (37)$$

$$\theta_{IITM} = \frac{z_T A_{II} \rho_{IIE}}{\dot{m}_{IIE}} \frac{T_{IIM}}{T_{IIE}} \sigma_{\rho II} \qquad (38)$$

2.4. Fluid temperature behaviour

As presented in an earlier report [5] the dynamic fluid temperature behaviour can be calculated from an energy balance equation which is now written in integral form and differs from those given by other authors. It can be derived due to the fact that the enthalpy of a differential fluid volume element (at a certain position z and time t) can be thought of as the sum of its entrance enthalpy a certain fluid travelling time before plus the heating up of the fluid element during its way up to the position z. After some rearrangements and the application of the mass balance equation this leads - applied to our case - to the following integral equations:

$$T_I(z,t) - T_{IE}\left[t - \tau_I(z_T, z, t)\right] = \frac{N_T}{c_I} \int_z^{z_T} \left.\frac{q_I(z_1,t_1)}{\dot{m}_I(z_1,t_1)}\right|_{t_1 = t - \tau_I(z_1, z, t)} dz_1 \qquad (39)$$

and

$$T_{II}(z,t) - T_{IIE}\left[t - \tau_{II}(z,0,t)\right] = \frac{N_T}{c_{II}} \int_0^{z} \left.\frac{q_{II}(z_1,t_1)}{\dot{m}_{II}(z_1,t_1)}\right|_{t_1 = t - \tau_{II}(z, z_1, t)} dz_1 \qquad (40)$$

with the travelling times

$$\tau_I(z_1, z, t) = \int_z^{z_1} \frac{dz_2}{v_I\left[z_2, t - \tau_I(z_2, z, t)\right]} \qquad (41)$$

$$\tau_{II}(z, z_1, t) = \int_{z_1}^{z} \frac{dz_2}{v_{II}\left[z_2, t - \tau_{II}(z, z_2, t)\right]} \qquad (42)$$

Inserting for the heat flux and mass flow from the equations derived before, differentiating with respect to z and going over to the Laplace domain leads to a system of two ordinary differential equations whose solutions are given by

$$T_{II}(z) = T_{IIE} + \frac{1 - e^{-(c_N z / z_T)}}{1 - c_M\, e^{-c_N}} \left(T_{IE} - T_{IIE}\right) \qquad (43)$$

$$T_I(z) = T_{IE} - C_M \left[T_{IIA} - T_{II}(z) \right] \tag{44}$$

with

$$C_M = \frac{m_{IIE}\, c_{II}}{m_{IE}\, c_I} \tag{45}$$

$$C_N = (1 - C_M) \frac{2\pi r_{II} \alpha_{TII} N_T z_T}{m_{IIE}\, c_{II}} \tag{46}$$

and

$$q_{II}(z) = - q_I(z) = q_N e^{-(C_N z / z_T)} \tag{47}$$

with

$$q_N = q_I(z=0) = \frac{m_{IIE}\, c_{II}}{z_T N_T} \frac{C_N}{1 - C_M e^{-C_N}} (T_{IE} - T_{IIE}) \tag{48}$$

Similarly, the relative variation of the fluid temperature at any axial position is given with respect to its inlet perturbations by

$$\begin{aligned} \overline{\delta_r T_{II}(z,s)} = {} & Y_{T2T1}(z,s)\,\overline{\delta_r T_{IE}(s)} + Y_{T2T2}(z,s)\,\overline{\delta_r T_{IIE}(s)} + \\ & + Y_{T2M1}(z,s)\,\overline{\delta_r m_{IE}(s)} + Y_{T2M2}(z,s)\,\overline{\delta_r m_{IIE}(s)} \end{aligned} \tag{49}$$

$$\begin{aligned} \overline{\delta_r T_I(z,s)} = {} & Y_{T1T1}(z,s)\,\overline{\delta_r T_{IE}(s)} + Y_{T1T2}(z,s)\,\overline{\delta_r T_{IIE}(s)} + \\ & + Y_{T1M1}(z,s)\,\overline{\delta_r m_{IE}(s)} + Y_{T1M2}(z,s)\,\overline{\delta_r m_{IIE}(s)} \end{aligned} \tag{50}$$

The formulae for the corresponding transfer functions can be found in ref. [6] .

Based on these results the equations of other important and the heat exchanger characterizing parameters can be now derived at once, e.g., the relative variation of the mean fluid temperature, the local and total heat flux, the mean tube wall temperature (which can be of interest in connection with the temperature induced elongation of the tube length causing stresses in the structural material), etc.

A similar frequency response model for a U-tube heat exchanger is in preparation.

3. Digital code TUMHEX

Based on this analytical model by substituting in the equations above for $s = j\omega$ $(j = \sqrt{-1})$ the FORTRAN-IV digital code TUMHEX could be established. It permits the calculation of the frequency response behaviour of characteristic parameters (such as primary and secondary fluid outlet and fluid mean temperatures, fluid outlet mass flows, heat fluxes and the mean tube wall temperature) of a shell- and -tube type counterflow heat exchanger with respect to primary and secondary inlet temperature and mass flow perturbations. (A detailed program description is given in [7]).

4. Comparison theory - experiment

The validity of the computer code (and thus the analytical model) could be proofed against oscillator tests performed with conventional heat exchanger types. Two runs from recent tests performed by Pütter [4] at the "Techn. Hochschule Aachen" on a conventional water-cooled industrial-size counterflow heat exchanger had been chosen as a basis for comparative calculations with TUMHEX. Fig. 1 shows, together with some important input data, the relative frequency response behaviour of both outlet temperatures against the relative variation of the primary inlet mass flow, Fig. 2 against the relative variation of the primary inlet mass flow temperature (Note: Medium II is the primary fluid flowing on the innerside, medium I the secondary fluid flowing on the outerside of the tubes).

Whereas in the mass flow perturbed case (Fig.1) good agreement could be stated, it was astonishing that in the temperature perturbed case (Fig.2) good agreement could be found only on the secondary side, with the primary side showing considerable discrepancies between theoretical and experimental behaviour. Pütter [4] found similar results with his model. A very reasonable explanation for these discrepancies seems to come from the fact, that the primary fluid flows on the innerside of several parallel tubes. Small differences in the flow path and the pressure differences can cause phase differences which after being mixed together can interfere and result with increasing frequency in considerable different behaviour than being calculated from exactly parallel channels. The secondary fluid (on the outerside of the tubes) flows practically through one single channel, thus radial discrepancies can be eliminated at once during its way up the tubes and no difference between theory and experiment is to be expected. For the mass flow pertubed experiments the different flow pathes to the different tubes does not play any role and eventual differing pressure differences can be neglected. This explains the good agreement between theory and experiment for this case. (The consideration of the heat transfer coefficient as being dependent from mass flow has a great influence on the theoretical frequency response behaviour, i.e., setting $\sigma_{\alpha II} = 0$ yielded complet different and wrong frequency response curves).

5. Digital code FRETI

At present a generally applicable digital code (named FRETI) is in development which will permit a retransformation from the frequency into the time domain in a numerical way. This auxiliary program should thus enable to determine from given frequency response curves (e.g., calculated by means of TUMHEX) the transient time response behaviour of important heat exchanger parameters against both step and/or any system perturbing signals.

6. References

[1] P. PROFOS: "Die Behandlung von Regelproblemen mittels des Frequenzganges des Regelkreises", Ph. D. Thesis, Technical University Zürich, 1943

[2] Y. TAKAHASHI: "Transfer function analysis of heat exchange processes" in "Automatic and manual control", ed. by A. Tustin (Granfield-Conf. 1951), pp 235-48, Butterworth Scientific Publ., London, 1952

[3] W.M. LAW: "The dynamic response of shell- and -tube heat exchangers to flow changes", Neue Technik 1/2, 34-44, 1962

[4] L. PÜTTER: "Einfache dynamische Modelle für Doppelrohr und Rohrbündelwärmeaustauscher", Dissertation 1972, T.H. Aachen

[5] A. HOELD, D. STAHL: "Analytisches Modell zur Beschreibung des Frequenzgang- und Stabilitätsverhaltens eines leichtwassermoderierten Druckwasserreaktors", Atomkernenergie 17(3), 167-174, April 1971

[6] A. HOELD: "Linear analytical one-dimensional model describing the frequency response behaviour of shell- and -tube type counterflow heat exchangers with respect to primary and secondary inlet temperature and mass flow pertubations". To be published in Nucl. Eng. and Design, 1973

[7] A. HOELD: "TUMHEX - A computer code for the calculation of the frequency response behaviour of shell- and -tube type counterflow heat exchangers with respect to primary and secondary inlet temperature and mass flow pertubations", MRR-P-2 (Technical University Munich), March 1973

[8] A. HOELD: "FRETI - A computer code for the calculation of transient system behaviour from given frequency response values". To be published (Technical University Munich), 1973

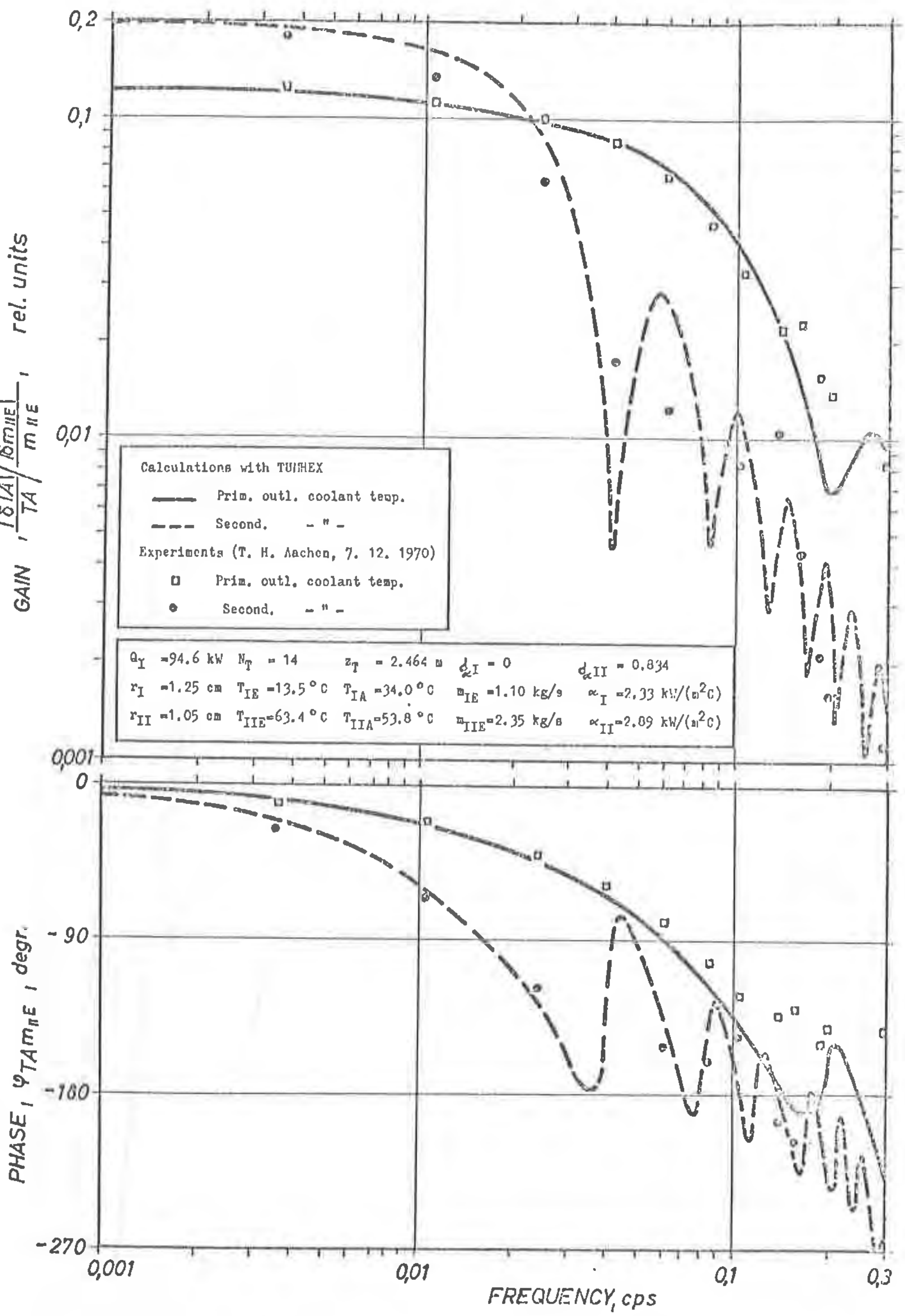

Fig.1: Frequency response behavior of an industrial-size counterflow heat exchanger. Primary inlet mass flow variation. (Comparison theory-experiment)

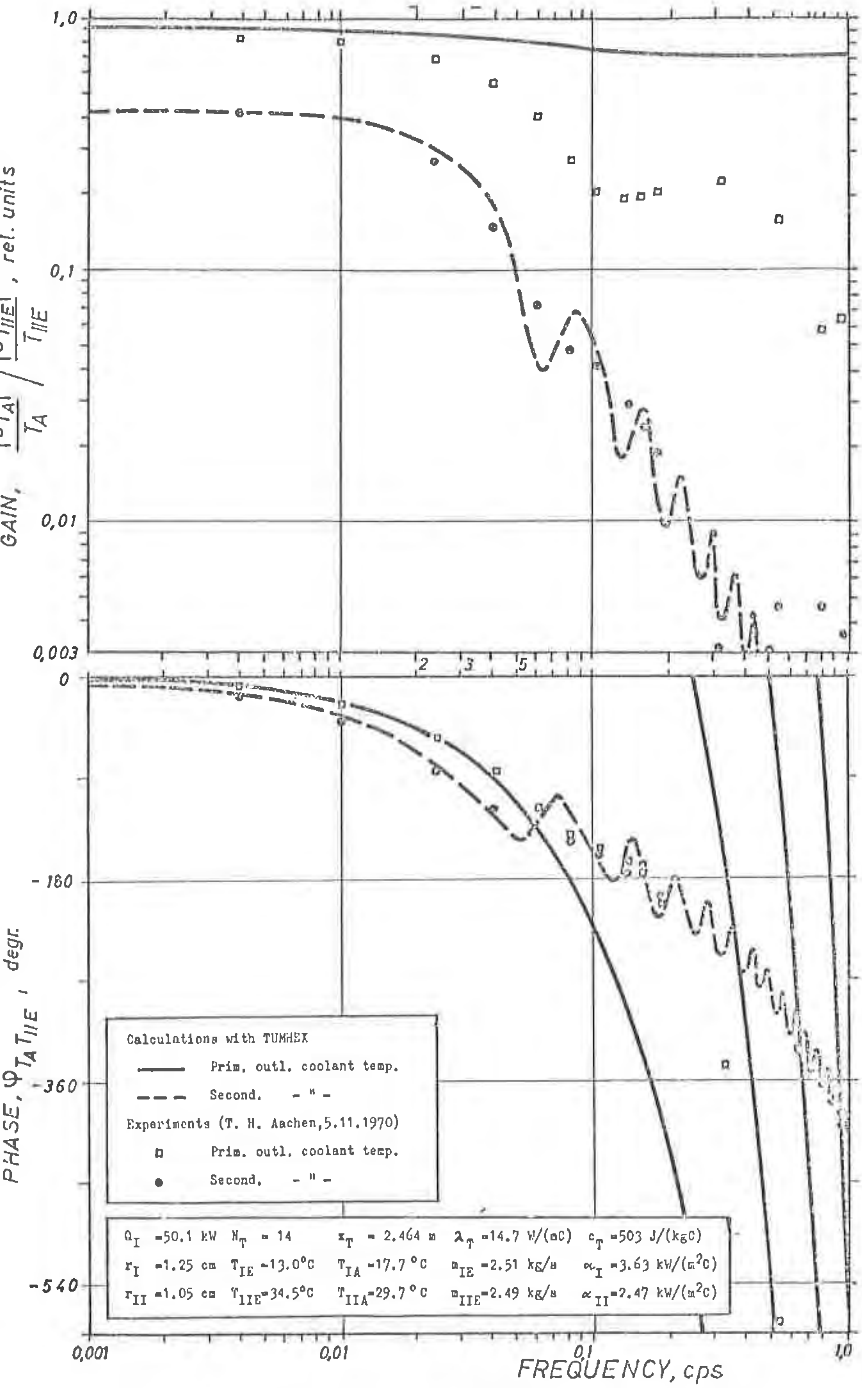

Fig.2: Frequency response behavior of an industrial-size counterflow heat exchanger. Primary inlet coolant temperature variation. (Comparison theory-experiment)