UNSTEADY THERMAL STRESSES IN AN ORTHOTROPIC COMPOSITE DISK DUE TO ASYMMETRICAL HEATINGS

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SUMMARY

In recent years the thermoelastic problems of composite cylinder have received considerable attentions because of their use in the nuclear industry, in construction engineering, and others. Then, in our previous papers[1,2], we have solved the transient thermal stress problems in an isotropic, composite, hollow, circular cylinder made of different materials due to symmetrical and asymmetrical heating. Moreover, considering the trend of the recent practical nuclear application, it can be seen that the laminated composite media or bonded orthotropic composite cylinder are used extensively in high temperature environments.

According to this tendency, in the present paper, we have discussed the effects of transient thermal stresses in an orthotropic composite circular disk due to asymmetrical heat generation or heating. In this paper, we have analyzed for the different two types of asymmetrical temperature distribution, one of which is concerned with an instantaneous point heat source as the illustrative example of the asymmetrical heat generation, and others is related to the prescribed arbitrary temperature distribution $f(\theta)$ at the outer boundary standing for the asymmetrical heating.

As for the heat conduction problems, we have taken into account all the orthotropic properties of the material in polar coordinate system introduced in the heat conduction equation. The first example of a point heat source is developed analytically with the aid of the method of separation of variables, while the second example of asymmetrical heating is analyzed using with the method of Laplace transforms.

Next, we have analyzed the transient thermoelastic problems with the aid of the stress function approach of the orthotropic material which is correspond to the Airy's stress function method for the isotropic material. And, taking into account all the orthotropic properties included in the fundamental equation, the stress analysis carried out for the abovementioned different two temperature fields.

Numerical examples are carried out for several cases, varying with the elastic constants, to examine the effect of the orthotropic property on the temperature distribution, stress dis-distributions, displacements, and are compared with the case of the isotropic material.

From the results obtained, it is concluded that the temperature and stress distributions of the composite disk become too complicated in accordance with the variety of the orthotropic property of the material. Especially, the hoop stress distribution $\sigma_{\theta\theta}$ exhibit the complicated behavior remarkably.

1. Introduction

In recent years, thermal stress problems of composite cylinder made of different materials have received considerable attentions because of their use in the nuclear industry, in construction engineerings and chemical engineerings and others. In our previous papers[1,2], we have treated of the transient thermal stress problems in an isotropic composite cylinder due to symmetrical or asymmetrical temperature field by heating. Now, considering the trend of the recent nuclear industry and others, it can be seen that the laminated composite media or bonded orthotropic composite cylinder are used extensively in high temperature environments.

Therefore, in this paper, we shall analyze the transient thermal stress problems in an orthotropic composite disk due to asymmetrical temperature distributions, one of which is concerned with an instantaneous point heat source as the illustrative example of asymmetrical heat generation, and the other one is related to a different temperature field that is given by arbitrary function $f(\theta)$ at the outer boundary as the example of asymmetrical heating.

In the first place, considering all the orthotropic properties of the materials in polar coordinate system, unsteady-state heat conduction problems under the above-mentioned two different heat conditions will be analyzed, namely, the first example of a point heat source is developed analytically with the aid of the method of separation of variables, while the second example of asymmetrical heating is analyzed using with the method of Laplace transformation.

Secondary, the thermal stress problems associated with the above-mentioned two heat conditions will be developed analytically using with the stress function approach for the orthotropic material, which is corresponds to the Airy's stress function method for the isotropic material. In these treatments for thermal stress problems, all the orthotropic properties of the material are also taken into consideration, therefore these characteristics are included in the fundamental differential equation.

And then, numerical calculations are carried out for several cases, varying with the orthotropical elastic constants, to examine the effect of the orthotropic property on the distributions of the temperature, stresses and displacements. And these results obtained for the first example of a point heat source will be shown in figures as the variations along radial or circumferential directions aganist the nondimensional times.

2. Theoretical formulation

Consider a composite, hollow, circular disk, as shown in Fig.1, of its radius of inner, interface, outer surface given by a, b and c, respectively. And we suppose that the composite disk is made of the materials with the orthotropical characteristics in polar coordinate system. Throughout the paper, subscript Latin indicies are associated with inner and outer disks, respectively. We shall now analyze the transient heat conduction and thermal stresses associated with the heat generation of a instantaneous point heat source. And then, we shall explain the theoretical development of the similar one associated with the heat supply on the outer boundary, briefly.

2.1 Theoretical development for heat generation of an instantaneous point heat source

2.1.1 Heat conduction problem

The transient heat conduction equation of the composite disk, having orthotropic characteristics in polar coordinate, with no internal heat generation takes the form

$$\beta_i^{-2}\cdot\partial T_i/\partial t = \alpha_i^{-2}(\partial^2 T_i/\partial r^2 + r^{-1}\cdot\partial T_i/\partial r) + r^{-2}\cdot\partial^2 T_i/\partial\theta^2,\ (i=1,2) \tag{1}$$

where $T_i=T_i(r,\theta,t)$ denotes temperature change, $\alpha_i{}^2$ and $\beta_i{}^2$ means the ratio of conductivities and thermal diffusivity in θ-direction, respectively, and are given by

$$\alpha_i{}^2=\lambda_{\theta i}/\lambda_{ri},\quad \beta_i{}^2=\lambda_{\theta i}/c_i\gamma_i \tag{2}$$

in which λ_r, λ_θ are the thermal conductivites in r- and θ-directions, c_i and γ_i are specific heat and density.

Taking into account the heat transfer on inner and outer boundaries, we may suppose for the heat conduction of instantaneous point heat source that the temperature field results in a constant value of 0° as the time tend to infinity, and then, by the method of separation of variables, the fundamental solution of eq.(1) can be obtained as

$$T_i=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}exp(-\alpha_i^{-2}\beta_i{}^2p_{nsi}^2t)\{A_{si}J_{\alpha_i n}(p_{nsi}r)+B_{si}Y_{\alpha_i n}(p_{nsi}r)\}\{C_{ni}\sin n\theta+D_{ni}\cos n\theta\} \tag{3}$$

in which A_{si}, B_{si}, C_{ni} and D_{ni} are the unknown constants, and p_{nsi} denotes the eigenvalues, these quantities are determined so as to satisfy the boundary conditions.

The boundary conditions for the composite disk are given by

$$\left.\begin{array}{llll}\partial T_1/\partial r-h_aT_1=0 & \text{on } r=a, & \partial T_2/\partial r+h_cT_2=0 & \text{on } r=c,\\ T_1=T_2, & \lambda_{r1}\cdot\partial T_1/\partial r=\lambda_{r2}\cdot\partial T_2/\partial r & & \text{on } r=b.\end{array}\right\} \tag{4}$$

where h_a and h_c are the relative heat transfer coefficients on inner and outer boundaries.

Substituting eq.(3) into the conditions at the inner and outer boundaries given by eq.(4), temperature solution can be written as

$$T(\rho,\theta,\tau)=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}exp(-\omega_{ns}^2\tau)A(\omega_{ns}\rho)\{C_{ns}\sin n\theta+D_{ns}\cos n\theta\} \tag{5}$$

in which the unknown constants C_{ns} and D_{ns} are to be determined from the initial condition. And the following expressions and notations are use in eq.(5),

$$A(\omega_{ns}\rho)=\begin{cases}U_1(\omega_{ns}\rho)\bar{\lambda}\{\alpha_2nb^{-1}U_2(\omega_{ns}\bar{b})-\kappa\omega_{ns}a^{-1}V_2(\omega_{ns}\bar{b})\}, & (1\leqq\rho\leqq\bar{b})\\ U_2(\omega_{ns}\rho)\{\alpha_1nb^{-1}U_1(\omega_{ns}\bar{b})-\omega_{ns}a^{-1}V_1(\omega_{ns}\bar{b})\}, & (\bar{b}\leqq\rho\leqq\bar{c})\end{cases}$$

$$U_1(\omega_{ns}\rho)=\{(\alpha_1na^{-1}-h_a)Y_{\alpha_1n}(\omega_{ns})-\omega_{ns}a^{-1}Y_{\alpha_1n+1}(\omega_{ns})\}J_{\alpha_1n}(\omega_{ns}\rho)-\{(\alpha_1na^{-1}-h_a)J_{\alpha_1n}(\omega_{ns})-\omega_{ns}a^{-1}\times J_{\alpha_1n+1}(\omega_{ns})\}Y_{\alpha_1n}(\omega_{ns}\rho),$$

$$U_2(\omega_{ns}\rho)=\{(\alpha_2nc^{-1}+h_c)Y_{\alpha_2n}(\kappa\omega_{ns}\bar{c})-\kappa\omega_{ns}a^{-1}Y_{\alpha_2n+1}(\kappa\omega_{ns}\bar{c})\}J_{\alpha_2n}(\kappa\omega_{ns}\rho)-\{(\alpha_2nc^{-1}+h_c)J_{\alpha_2n}(\kappa\omega_{ns}\bar{c})-\kappa\omega_{ns}a^{-1}J_{\alpha_2n+1}(\kappa\omega_{ns}\bar{c})\}Y_{\alpha_2n}(\kappa\omega_{ns}\rho),$$

$$V_1(\omega_{ns}\rho)=\{(\alpha_1na^{-1}-h_a)Y_{\alpha_1n}(\omega_{ns})-\omega_{ns}a^{-1}Y_{\alpha_1n+1}(\omega_{ns})\}J_{\alpha_1n+1}(\omega_{ns}\rho)-\{(\alpha_1na^{-1}-h_a)J_{\alpha_1n}(\omega_{ns})-\omega_{ns}a^{-1}J_{\alpha_1n+1}(\omega_{ns})\}Y_{\alpha_1n+1}(\omega_{ns}\rho),$$

$$V_2(\omega_{ns}\rho)=\{(\alpha_2nc^{-1}+h_c)Y_{\alpha_2n}(\kappa\omega_{ns}\bar{c})-\kappa\omega_{ns}a^{-1}Y_{\alpha_2n+1}(\kappa\omega_{ns}\bar{c})\}J_{\alpha_2n+1}(\kappa\omega_{ns}\rho)-\{(\alpha_2nc^{-1}+h_c)J_{\alpha_2n}(\kappa\omega_{ns}\bar{c})-\kappa\omega_{ns}a^{-1}J_{\alpha_2n+1}(\kappa\omega_{ns}\bar{c})\}Y_{\alpha_2n+1}(\kappa\omega_{ns}\rho),$$

$$\omega_{ns}=p_{ns1}a,\quad \tau=\beta_1^2\alpha_1^{-2}a^{-2}t,\quad p_{ns2}=\kappa p_{ns1},\quad \kappa=\beta_1\beta_2^{-1}\alpha_2\alpha_1^{-1},\quad \rho=r/a,\quad \bar{b}=b/a,\quad \bar{c}=c/a,\quad \bar{\lambda}=\lambda r_2/\lambda r_1. \tag{6}$$

Similarly as before, from eq.(3) and the heat condition at the interface given by eq.(4), the eigenvalues ω_{ns} for each value of n $(n=1,2,\ldots)$ can be determined as the positive root of the following trascendental equation.

$$\bar{\lambda}U_1(\omega\bar{b})\{\alpha_2nb^{-1}U_2(\omega\bar{b})-\kappa\omega a^{-1}V_2(\omega\bar{b})\}-U_2(\omega\bar{b})\{\alpha_1nb^{-1}U_1(\omega\bar{b})-\omega a^{-1}V_1(\omega\bar{b})\}=0 \tag{7}$$

If we assume that the initial condition is biven by the following relation

$$T(r,\theta,t)=f(r,\theta)\qquad at\ t=0 \tag{8}$$

then the unknown constants C_{ns} and D_{ns} included in eq.(5) can be determined from eqs.(5) and (8), then we have the relation from these equations as follows,

$$\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}A(\omega_{ns}\rho)\{C_{ns}\sin n\theta+D_{ns}\cos n\theta\}=f(r,\theta) \tag{9}$$

Now, for eq.(9), we can operate the orthogonal transformation with respect to the trigonometric functions and Bessel functions with the aid of the transcendental equation (7), thus the unknown terms C_{ns} and D_{ns} are determined, and for instance D_{ns} can be written as

$$D_{ns}=\frac{1}{\varepsilon_n\pi I}[U_2(\omega_{ns}\bar{b})\int_a^b\int_{-\pi}^{\pi} rU_1(\omega_{ns}\rho)f(r,\theta)\cos n\theta drd\theta+\bar{\lambda}\kappa^2U_1(\omega_{ns}\bar{b})\int_b^c\int_{-\pi}^{\pi} rU_2(\omega_{ns}\rho)f(r,\theta)\cos n\theta drd\theta], \tag{10}$$

in which $\varepsilon_o=2$, $\varepsilon_n=1$ $(n\geq 2)$ and

$$I=\bar{\lambda}U_2(\omega_{ns}\bar{b})\{\frac{\alpha_2 n}{b}U_2(\omega_{ns}\bar{b})-\frac{\kappa\omega_{ns}}{a}V_2(\omega_{ns}\bar{b})\}\int_a^b rU_1^2(\omega_{ns}\rho)dr$$
$$+\bar{\lambda}\kappa^2U_1(\omega_{ns}\bar{b})\{\frac{\alpha_1 n}{b}U_1(\omega_{ns}\bar{b})-\omega_{ns}/a.V_1(\omega_{ns}\bar{b})\}\int_b^c rU_2^2(\omega_{ns}\rho)dr, \tag{11}$$

Next, assuming that the initial temperature field $f(r,\theta)$ is represented by the instantaneous point heat source of its intensity S, assigned to arbitrary position (r_o,θ_o), then $f(r,\theta)$ take the form

$$f(r,\theta)=S.\delta(r-r_o).\delta(\theta-\theta_o)/r \tag{12}$$

in which δ denotes Dirac's delta function. Substituting eq.(12) into eq.(10), the temperature solution for the case of instantaneous point heat source can be written as follows

$$T(\rho,\theta,\tau)=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\exp(-\omega_{ns}^2\tau)A(\omega_{ns}\rho)\frac{S}{\varepsilon_n\pi I}C(\omega_{ns}\rho_o)\cos n\theta \tag{13}$$

in which θ_o is assumed to be zero and

$C(\omega_{ns}\rho_o)=U_2(\omega_{ns}\bar{b})U_1(\omega_{ns}\rho_o)$ for $1\leq\rho_o\leq\bar{b}$, $=\bar{\lambda}\kappa^2U_1(\omega_{ns}\bar{b})U_2(\omega_{ns}\rho_o)$ for $\bar{b}\leq\rho_o\leq\bar{c}$, $\rho_o=r_o/a$.

2.1.2 Thermal stress problem

Transient thermal stress problem associated with the above-mentioned temperature field can be developed theoretically by the stress function approach for the orthotropic medium which is correspond to the Airy's stress function method for the isotropic materials. First, the stress-strain relations for orthotropic medium in polar coordinate can be written as

$$\varepsilon_{rri}=a_{11i}\sigma_{rri}+a_{12i}\sigma_{\theta\theta i}+\alpha_{ri}T_i,\quad \varepsilon_{\theta\theta i}=a_{12i}\sigma_{rri}+a_{22i}\sigma_{\theta\theta i}+\alpha_{\theta i}T_i,\quad \varepsilon_{r\theta i}=a_{66i}\sigma_{r\theta i} \tag{14}$$

in which $\sigma_{\alpha\beta}$ and $\varepsilon_{\alpha\beta}$ are the stress and strain tensors respectively, and $a_{\alpha\beta}$ the elastic constants, α_r and α_θ the coefficient of thermal expansion in r- and θ-directions.
And then, the only compativility condition to be satisfied in terms of strain components is

$$2\frac{\partial^2(r\varepsilon_{r\theta i})}{\partial r\partial\theta}=\frac{\partial}{\partial r}\{r^2\frac{\partial\varepsilon_{\theta\theta i}}{\partial r}\}+\frac{\partial^2\varepsilon_{rri}}{\partial\theta^2}-r\frac{\partial\varepsilon_{rri}}{\partial r} \tag{15}$$

On the other hand, stress components $\sigma_{\alpha\beta}$ can be given by the stress function $\chi(r,\theta)$ as follows

$$\sigma_{rr}=r^{-2}.\partial^2\chi/\partial\theta^2+r^{-1}.\partial\chi/\partial r,\quad \sigma_{\theta\theta}=\partial^2\chi/\partial r^2,\quad \sigma_{r\theta}=-\partial(r^{-1}.\partial\chi/\partial\theta)/\partial r. \tag{16}$$

Substituting eqs.(14) and (16) into eq.(15), the fundamental differential equation in terms of stress function $\chi(r,\theta)$ in orthotropic medium is obtained as follows

$$\{\partial^4/\partial r^4+2r^{-1}.\partial^3/\partial r^3-\beta_{1i}r^{-2}.\partial^2/\partial r^2+\beta_{1i}r^{-3}.\partial/\partial r-2\beta_{2i}r^{-3}.\partial^3/\partial r\partial\theta^2+2\beta_{2i}r^{-2}.\partial^4/\partial r^2\partial\theta^2$$
$$+2(\beta_{1i}+\beta_{2i})r^{-4}.\partial^2/\partial\theta^2+\beta_{1i}r^{-4}.\partial^4/\partial\theta^4\}\chi_i=-\nu_i\{\partial^2/\partial r^2+(2-\mu_i)r^{-1}.\partial/\partial r+\mu_i r^{-2}.\partial^2/\partial\theta^2\}T_i, \tag{17}$$

where, $\beta_{1i}=a_{11i}/a_{22i}$, $\beta_{2i}=(a_{12i}+a_{66i})/a_{22i}$, $\mu_i=\alpha_{ri}/\alpha_{\theta i}$, $\nu_i=\alpha_{\theta i}/a_{22i}$.

Referring to temperature solution (13), and assuming that the stress function $\chi_i(\rho,\theta,\tau)$ take the form

$$\chi_i(\rho,\theta,\tau)=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\exp(-\omega_{ns}^2\tau)R_{ni}(\rho)\frac{S}{\varepsilon_n\pi I}C(\omega_{ns}\rho_o)\cos n\theta \tag{18}$$

it can be seen that the expression for $R_{ni}(\rho)$ is determined from eq.(17) using with eq.(13), and shown as

1) When $n=0$ and $\beta_{1i}=1$,

$$R_{0i}(\rho)=C_{01i}+C_{02i}\rho^2+C_{03i}\ln\rho+C_{04i}\rho^2\ln\rho+\frac{\nu_i}{4}[-\int_{\rho_i}^{\rho}\rho^3(1-\ln\rho)f_{0i}(\rho)d\rho+\rho^2\int_{\rho_i}^{\rho}\rho(1+\ln\rho)f_{0i}(\rho)d\rho$$
$$-\ln\rho\int_{\rho_i}^{\rho}\rho^3f_{0i}(\rho)d\rho-\rho^2\ln\rho\int_{\rho_i}^{\rho}\rho f_{0i}(\rho)d\rho],$$

2) When $n=0$ and $\beta_{1i}\neq 1$,

$$R_{0i}(\rho)=C_{01i}+C_{02i}\rho^{1+\xi_i}+C_{03i}\rho^{1-\xi_i}+C_{04i}\rho^2+\frac{\nu_i}{2(1-\beta_{1i})}[\int_{\rho_i}^{\rho}\rho^3f_{0i}(\rho)d\rho+\xi_i^{-1}\rho^{1+\xi_i}\int_{\rho_i}^{\rho}\rho^{2-\xi_i}f_{0i}(\rho)d\rho$$

$$-\xi_i^{-1}\rho^{1-\xi_i}\int_{\rho_i}^{\rho}\rho^{2+\xi_i}f_{0i}(\rho)d\rho - \rho^2\int_{\rho_i}^{\rho}\rho f_{0i}(\rho)d\rho], \qquad \xi_i^2=\beta_{1i}$$

3) When $n=1$,

$$R_{1i}(\rho)=C_{11i}\rho+C_{12i}\rho^{1-\lambda_i}+C_{13i}\rho^{1+\lambda_i}+C_{14i}\rho\ln\rho+\frac{\nu_i}{\lambda_i^2}[-\rho\int_{\rho_i}^{\rho}\rho^2\ln\rho f_{1i}(\rho)d\rho+\frac{1}{2\lambda_i}\rho^{1-\lambda_i}\int_{\rho_i}^{\rho}\rho^{2+\lambda_i}\times f_{1i}(\rho)d\rho-\frac{1}{2\lambda_i}\rho^{1+\lambda_i}\int_{\rho_i}^{\rho}\rho^{2-\lambda_i}f_{1i}(\rho)d\rho+\rho\ln\rho\int_{\rho_i}^{\rho}\rho^2 f_{1i}(\rho)d\rho], \quad \lambda_i^2=1+\beta_{1i}+2\beta_{2i}.$$

4) When $n\geq 2$,

$$R_{ni}(\rho)=C_{n1i}\rho^{1+m_{1i}}+C_{n2i}\rho^{1+m_{2i}}+C_{n3i}\rho^{1-m_{1i}}+C_{n4i}\rho^{1-m_{2i}}+\frac{\nu_i}{2(m_{1i}^2-m_{2i}^2)}[-m_{1i}^{-1}\rho^{1+m_{1i}}\int_{\rho_i}^{\rho}\rho^{2-m_{1i}}\times f_{ni}(\rho)d\rho+m_{2i}^{-1}\rho^{1+m_{2i}}\int_{\rho_i}^{\rho}\rho^{2-m_{2i}}f_{ni}(\rho)d\rho+m_{1i}^{-1}\rho^{1-m_{1i}}\int_{\rho_i}^{\rho}\rho^{2+m_{1i}}f_{ni}(\rho)d\rho-m_{2i}^{-1}\rho^{1-m_{2i}}\int_{\rho_i}^{\rho}\rho^{2+m_{2i}}f_{ni}(\rho)d\rho],$$

m_{1i}^2 and m_{2i}^2 are the roots of the equation $m^4-(1+\beta_{1i}+2\beta_{2i}n^2)m^2+\beta_{1i}(n^2-1)^2=0.$ (19)

The modified Michell's conditions for an orthotropic medium under a nonuniform temperature distribution, which is the physical conditions that the displacements and rotation must be single-valued, have already described [3]. Then, substituting $\chi(\rho,\theta,\tau)$ into the modified Michell's conditions, we find

$$\left.\begin{aligned} C_{04i}&=-\frac{\nu_i a^2}{4}[\rho\frac{\partial A(\omega_{0s}\rho)}{\partial\rho}+(1-\mu_i)A(\omega_{0s}\rho)]\Big|_{\rho=\rho_i} && for\ \ \beta_{1i}=1\\ C_{04i}&=-\frac{\nu_i a^2}{2(1-\beta_{1i})}[\rho\frac{\partial A(\omega_{0s}\rho)}{\partial\rho}+(1-\mu_i)A(\omega_{0s}\rho)]\Big|_{\rho=\rho_i} && for\ \ \beta_{1i}\neq 1\\ C_{14i}&=\frac{\nu_i a^2}{\lambda_i^2}[\rho^2\frac{\partial A(\omega_{1s}\rho)}{\partial\rho}-\mu_i\rho A(\omega_{1s}\rho)]\Big|_{\rho=\rho_i} \end{aligned}\right\} \quad (20)$$

In eqs.(19) and (20), ρ_i are chosen as

$\rho_1=1 \quad for \quad 1\leq\rho\leq\bar{b}, \qquad \rho_2=\bar{c} \quad for \quad \bar{b}\leq\rho\leq\bar{c}$ (21)

Thus, the stress components $\sigma_{\alpha\beta}$ can be calculated from eq.(16) using with $\chi(\rho,\theta,\tau)$ given by eqs.(18) and (19), and the integral constants are determined so as to satisfy the boundary conditions. For the composite medium, the boundary conditions at the interface are given by the stress components and the displacements. Then, we have to formulate the expressions for displacements. The stress-strain relations for the orthotropic medium are given by eq.(14), whereas strain components in terms of displacements are

$\varepsilon_{rri}=\partial u_{ri}/\partial r, \quad \varepsilon_{\theta\theta i}=r^{-1}.u_{ri}+r^{-1}.\partial u_{\theta i}/\partial\theta, \quad 2\varepsilon_{r\theta i}=r^{-1}.\partial u_{ri}/\partial\theta-r^{-1}.u_{\theta i}+\partial u_{\theta i}/\partial r$ (22)

Substituting eq.(22) into eq.(14), and using the expressions for the stress components given by eq.(16), displacements u_{ri} and $u_{\theta i}$ can be obtained.

Next, we consider the boundary conditions determining the integral constants. For the composite medium, we can assume that the composite disk is bonded at the interface. It follows from this assumption that at the interface the stress normal to the interface and the shear stress along the one, and the displacements are continuous. Then, the boundary conditions available for the asymmetrical thermoelastic problems are

$$\left.\begin{aligned} &\sigma_{rr1}=0,\quad \sigma_{r\theta 1}=0 \quad at\ r=a, \qquad \sigma_{rr2}=0,\quad \sigma_{r\theta 2}=0 \quad at\ r=c,\\ &\sigma_{rr1}=\sigma_{rr2},\quad \sigma_{r\theta 1}=\sigma_{r\theta 2},\quad u_{r1}=u_{r2},\quad u_{\theta 1}=u_{\theta 2} \quad at\ r=b. \end{aligned}\right\} \quad (23)$$

Thus, all the integral constants become determined values, and the distributions for the stress components and displacements can be calculated numerically.

2.2 Theoretical development for heating on the outer boundary

2.2.1 Heat conduction problem

We consider the transient heat conduction problem under an arbitrary temperature distribution due to a heat supply on the outer surface. Now, we assume that the temperature distribution of the outer surrounding medium can be expressed as an arbitrary function $f(\theta)$ of coordinate θ, and can be expanded into the Foufier series as shown in the following form.

$$f(\theta)=\sum_{n=0}^{\infty} c_n \cos n\theta, \quad c_o=\frac{1}{\pi}\int_0^{\pi} f(\theta)d\theta, \quad c_n=\frac{2}{\pi}\int_0^{\pi} f(\theta)\cos n\theta d\theta, \quad (n \geq 1) \tag{24}$$

On the other hand, the initial and boundary conditions for the composite disk are

$$T_1=0, \quad T_2=0 \qquad at\ t=0. \tag{25}$$

$$\left.\begin{aligned} &\partial T_1/\partial r - h_a T_1 = 0 \quad at\ r=a, \qquad \partial T_2/\partial r + h_c T_2 = h_c f(\theta) \quad at\ r=c, \\ &T_1=T_2\ , \quad \lambda_{r1}.\partial T_1/\partial r = \lambda_{r2}.\partial T_2/\partial r \qquad at\ r=b. \end{aligned}\right\} \tag{26}$$

In order to obtain the temperature solution of fundamental equation (1), we may introduce the Laplace transformation. Performing the Laplace transform on eq.(1) and boundary conditions (26) under the consideration of the initial condition (25), and considering the symmetry for $\theta=0$, then the temperature solution of eq.(1), with the aid of the separation of variables, can be obtained in the subsidiary space. Using the residue theorem, we may perform the inversion of Laplace transform, we obtain the complete temperature solution, and expressed as

$$T_i(\rho,\theta,\tau)=\sum_{n=0}^{\infty}\sum_{s=0}^{\infty} exp(-\omega_{ns}^2\tau)B_{ns}^{i}/H_{ns}.\cos n\theta \tag{27}$$

in which

$$\begin{aligned}
&B_{00}^1=-c_0\bar{c}B_c\bar{\lambda}\{1+B_a ln\rho\},\ B_{00}^2=-c_0\bar{c}B_c\{\bar{\lambda}(1+B_a ln\bar{b})+B_a ln\rho/\bar{b}\},\ H_{00}=-B_a(1-\bar{c}B_c ln\bar{b}/\bar{c})-\bar{\lambda}\bar{c}B_c(1+B_a\times \\
&ln\bar{b}),\ B_{n0}^1=-c_n\bar{c}B_c\bar{\lambda}\{(\alpha_1 n+B_a)\rho^{\alpha_1 n}+(\alpha_1 n-B_a)\rho^{-\alpha_1 n}\}/(2\alpha_1 n),\ B_{n0}^2=-c_n\bar{c}B_c\frac{1}{4}[\bar{\lambda}\{(\rho/\bar{b})^{\alpha_2 n}+(\rho/\bar{b})^{-\alpha_2 n} \\
&\}\{(\alpha_1 n+B_a)\bar{b}^{\alpha_1 n}+(\alpha_1 n-B_a)\bar{b}^{-\alpha_1 n}\}/(\alpha_1 n)+\{(\rho/\bar{b})^{\alpha_2 n}-(\rho/\bar{b})^{-\alpha_2 n}\}\{(\alpha_1 n+B_a)\bar{b}^{\alpha_1 n}-(\alpha_1 n-B_a)\bar{b}^{-\alpha_1 n}\}/\alpha_2 n], \\
&H_{n0}=-\frac{1}{4}[\bar{\lambda}\{(\alpha_1 n+B_a)\bar{b}^{\alpha_1 n}+(\alpha_1 n-B_a)\bar{b}^{-\alpha_1 n}\}\{(\alpha_2 n+\bar{c}B_c)(\bar{c}/\bar{b})^{\alpha_2 n}-(\alpha_2 n-\bar{c}B_c)(\bar{c}/\bar{b})^{-\alpha_2 n}\}/\alpha_1 n+\{(\alpha_1 n+ \\
&+B_a)\bar{b}^{\alpha_1 n}-(\alpha_1 n-B_a)\bar{b}^{-\alpha_1 n}\}\{(\alpha_2 n+\bar{c}B_c)(\bar{c}/\bar{b})^{\alpha_2 n}+(\alpha_2 n-\bar{c}B_c)(\bar{c}/\bar{b})^{-\alpha_2 n}\}/\alpha_2 n],\ B_{ns}^1=-c_n\bar{c}B_c\frac{4}{\pi}\bar{\lambda}\{-(\alpha_1 n \\
&-B_a)G_1+\omega_{ns}G_3\},\quad B_{ns}^2=2c_n\bar{c}B_c[-\bar{\lambda}\{-\alpha_2 nG_5+\kappa\bar{b}\omega_{ns}G_7\}\{-(\alpha_1 n-B_a)F_1+\omega_{ns}F_3\}-G_5\{-\alpha_1 n(\alpha_1 n-B_a)F_1+(\alpha_1 n \\
&-B_a)\bar{b}\omega_{ns}F_2+\alpha_1 n\omega_{ns}F_3-\bar{b}\omega_{ns}^2F_4\}],\quad H_{ns}=\bar{\lambda}[\{\omega_{ns}^2-2\alpha_1 n(\alpha_1 n-B_a)\}F_1+(\alpha_1 n-B_a)\bar{b}\omega_{ns}F_2+(\alpha_1 n-B_a)\omega_{ns}F_3 \\
&-\bar{b}\omega_{ns}^2F_4][\alpha_2 n(\alpha_2 n+\bar{c}B_c)F_5-\alpha_2 n\kappa\bar{c}\omega_{ns}F_6-(\alpha_2 n+\bar{c}B_c)\kappa\bar{b}\omega_{ns}F_7+\kappa^2\overline{bc}\omega_{ns}^2F_8]+\bar{\lambda}[-(\alpha_1 n-B_a)F_1+\omega_{ns}F_3][\{2\alpha_2^2\times \\
&n^2(\alpha_2 n+\bar{c}B_c)-(\alpha_2 n+\bar{c}B_c)\kappa^2\bar{b}^2\omega_{ns}^2-\alpha_2 n\kappa^2\bar{c}^2\omega_{ns}^2\}F_5+\kappa\bar{c}\omega_{ns}\{\kappa^2\bar{b}^2\omega_{ns}^2-\alpha_2 n(\alpha_2 n+\bar{c}B_c)\}F_6+\kappa\bar{b}\omega_{ns}\{\kappa^2\bar{c}^2\omega_{ns}^2 \\
&-\alpha_2 n(\alpha_2 n+\bar{c}B_c)\}F_7+\bar{c}B_c\kappa^2\overline{bc}\omega_{ns}^2F_8]+[\{-2\alpha_1^2n^2(\alpha_1 n-B_a)+(\alpha_1 n-B_a)\bar{b}^2\omega_{ns}^2+\alpha_1 n\omega_{ns}^2\}F_1+\bar{b}\omega_{ns}\{\alpha_1 n(\alpha_1 n-B_a) \\
&-\omega_{ns}^2\}F_2+\omega_{ns}\{\alpha_1 n(\alpha_1 n-B_a)-\bar{b}^2\omega_{ns}^2\}F_3+B_a\bar{b}\omega_{ns}^2F_4][-(\alpha_2 n+\bar{c}B_c)F_5+\kappa\bar{c}\omega_{ns}F_6]+[-\alpha_1 n(\alpha_1 n-B_a)F_1+(\alpha_1 n-B_a)\times \\
&\bar{b}\omega_{ns}F_2+\alpha_1 n\omega_{ns}F_3-\bar{b}\omega_{ns}^2F_4][\{-2\alpha_2 n(\alpha_2 n+\bar{c}B_c)+\kappa^2\bar{c}^2\omega_{ns}^2\}F_5+\kappa\bar{c}\omega_{ns}(\alpha_2 n+\bar{c}B_c)F_6+\kappa\bar{b}\omega_{ns}(\alpha_2 n+\bar{c}B_c)F_7-\kappa^2\overline{bc}\times \\
&\omega_{ns}^2F_8],\qquad B_a=ah_a,\ B_c=ah_c,\qquad G_1=J_{\alpha_1 n}(\omega_{ns})Y_{\alpha_1 n}(\rho\omega_{ns})-J_{\alpha_1 n}(\rho\omega_{ns})Y_{\alpha_1 n}(\omega_{ns}),\ G_3=J_{\alpha_1 n+1} \\
&(\omega_{ns})Y_{\alpha_1 n}(\rho\omega_{ns})-J_{\alpha_1 n}(\rho\omega_{ns})Y_{\alpha_1 n+1}(\omega_{ns}),\ G_5=J_{\alpha_2 n}(\kappa\bar{b}\omega_{ns})Y_{\alpha_2 n}(\kappa\rho\omega_{ns})-J_{\alpha_2 n}(\kappa\rho\omega_{ns})Y_{\alpha_2 n}(\kappa\bar{b}\omega_{ns}), \\
&G_7=J_{\alpha_2 n+1}(\kappa\bar{b}\omega_{ns})Y_{\alpha_2 n}(\kappa\rho\omega_{ns})-J_{\alpha_2 n}(\kappa\rho\omega_{ns})Y_{\alpha_2 n+1}(\kappa\bar{b}\omega_{ns}),\qquad F_1=G_1|_{\rho=\bar{b}}\ ,\quad F_3=G_3|_{\rho=\bar{b}}\ , \\
&F_5=G_5|_{\rho=\bar{c}}\ ,\quad F_7=G_7|_{\rho=\bar{c}}\ ,\ F_2=J_{\alpha_1 n}(\omega_{ns})Y_{\alpha_1 n+1}(\bar{b}\omega_{ns})-J_{\alpha_1 n+1}(\bar{b}\omega_{ns})Y_{\alpha_1 n}(\omega_{ns}),\ F_4=J_{\alpha_1 n+1}(\omega_{ns})\times \\
&Y_{\alpha_1 n+1}(\bar{b}\omega_{ns})-J_{\alpha_1 n+1}(\bar{b}\omega_{ns})Y_{\alpha_1 n+1}(\omega_{ns}),\ F_6=J_{\alpha_2 n}(\kappa\bar{b}\omega_{ns})Y_{\alpha_2 n+1}(\kappa\bar{c}\omega_{ns})-J_{\alpha_2 n+1}(\kappa\bar{c}\omega_{ns})Y_{\alpha_2 n}(\kappa\bar{b}\omega_{ns}), \\
&F_8=J_{\alpha_2 n+1}(\kappa\bar{b}\omega_{ns})Y_{\alpha_2 n+1}(\kappa\bar{c}\omega_{ns})-J_{\alpha_2 n+1}(\kappa\bar{c}\omega_{ns})Y_{\alpha_2 n+1}(\kappa\bar{b}\omega_{ns}).
\end{aligned} \tag{28}$$

The ω_{ns} values $(s=1,2,\ldots)$ for each values of n $(n=0,1,\ldots)$ can be determined as the positive root of the following transcendental equation

$$\begin{aligned}&\bar{\lambda}\{-(\alpha_1 n-B_a)F_1+\omega F_3\}\{\alpha_2 n(\alpha_2 n+\bar{c}B_c)F_5-\alpha_2 n\kappa\bar{c}\omega F_6-(\alpha_2 n+\bar{c}B_c)\kappa\bar{b}\omega F_7+\kappa^2\overline{bc}\omega^2F_8\} \\ &\quad+\{-\alpha_1 n(\alpha_1 n-B_a)F_1+(\alpha_1 n-B_a)\bar{b}\omega F_2+\alpha_1 n\omega F_3-\bar{b}\omega^2F_4\}\{-(\alpha_2 n+\bar{c}B_c)F_5+\kappa\bar{c}\omega F_6\}=0\end{aligned} \tag{29}$$

and the values of ω_{ns} for $s=0$ are zero.

2.2.2 Thermal stress problem

Now, we consider the thermal stress problem. Referring to the temperature solution given by eq.(27), we may develop the theoretical formulation in the similar manner as the above-mentioned procedure illustrated for the case of the heat generation. Then we shall leave out the descriptions for the thermal stress problem in this case.

3. Numerical results

For illustrative purpose to the foregoing analysis, numerical calculations are carried out for the first case of a instantaneous point heat source. The computation for the transient heat conduction is carried out for the following data

$$\bar{b} = 2.0, \quad \bar{c} = 3.0, \quad \bar{\lambda} = 2.0, \quad \kappa = 0.5, \quad \alpha_1 = \alpha_2 = 1.0, \quad h_a = h_c = \infty, \quad \rho_o = 2.0 \qquad (30)$$

And the numerical results for the thermal stresses and displacements are presented for the several cases of orthotropic material constants as shown in Table 1. Several numerical results for the temperature, thermal stresses and displacements are shown in Figs.2-6 for several dimensionless times in dimensionless forms. Fig.2 show the variations of the temperature distribution, and Figs.3 and 4 show the variations of the radial and hoop stress distributions, furthermore Figs.5 and 6 show the radial and circumferential displacements. And the isotropic results correspond to the case III shown in Fig.3.

From the results shown in these figures, it can be seen that the temperature, stress and displacement distributions of the composite disk become too complicated in accordance with the orthotropical material constants.

References

[1] TAKEUTI,Y. and *et al.*, "Transient Thermal Stresses in a Bonded Composite Hollow Circular Cylinder under Symmetrical Temperature Distribution", Nuclear Engineering and Design 41, 335-343 (1977).

[2] TAKEUTI,Y., TANIGAWA,Y., "Asymmetrical Transient Thermoelastic Problems in a composite Hollow Circular Cylinder", Nuclear Engineering and Design 45, 159-172 (1978).

[3] TAKEUTI,Y., NODA,N., "Plane Thermoelastic Problems in a Multiply Connected Orthotropic Body", Journal of Applied Mechanics 44, (3), 431-436 (1977).

Table 1 Orthotropic material constants of composite disk

	β_{1i}	β_{2i}	μ_i	ν_2/ν_1	a_{222}/a_{221}
case I	0.6	1.0	1.0	2.0	2.0
case II	0.8	1.0	1.0	2.0	2.0
case III	1.0	1.0	1.0	2.0	2.0
case IV	1.0	1.2	1.0	2.0	2.0
case V	1.0	1.4	1.0	2.0	2.0

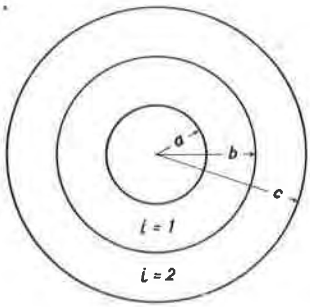

Figure 1 Composite hollow circular disk

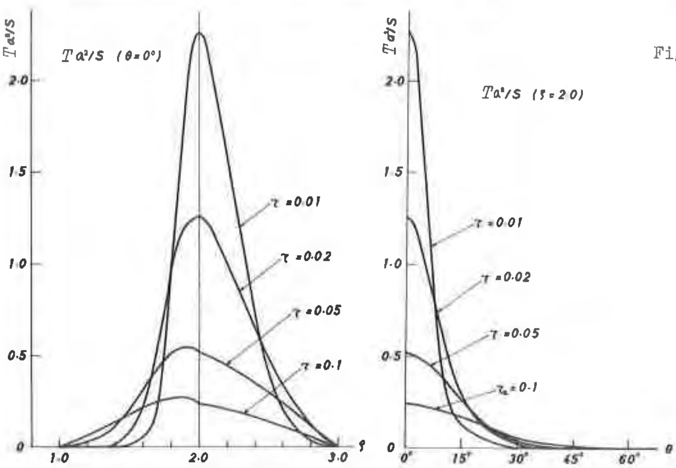

Figure 2 Temperature distributions along θ=0° and interface

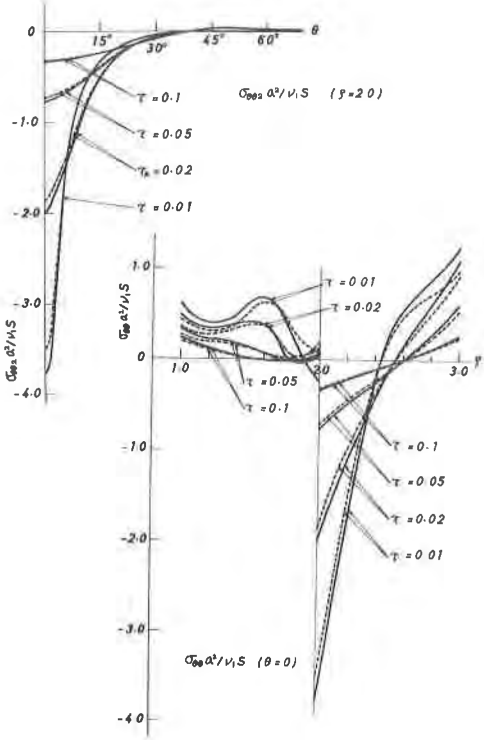

Figure 4 Hoop stress distributions along $\theta = 0°$ and interface

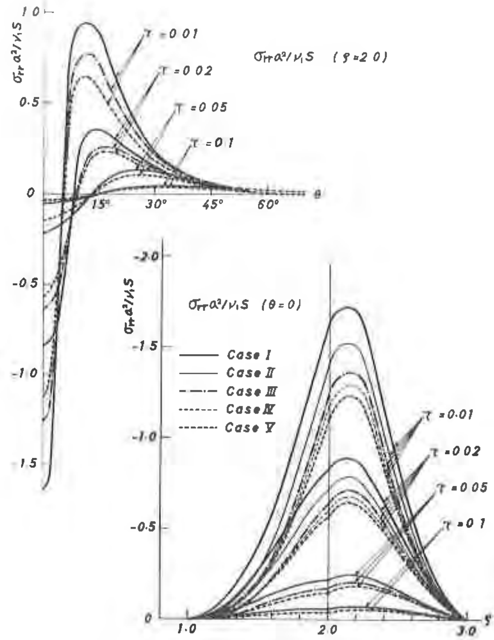

Figure 3 Radial stress distributions along $\theta = 0°$ and interface

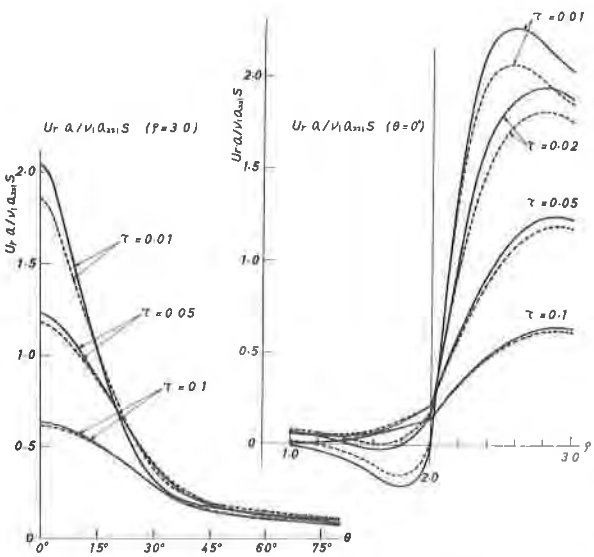

Figure 5 Distributions of radial displacement along $\theta = 0°$ and outer boundary

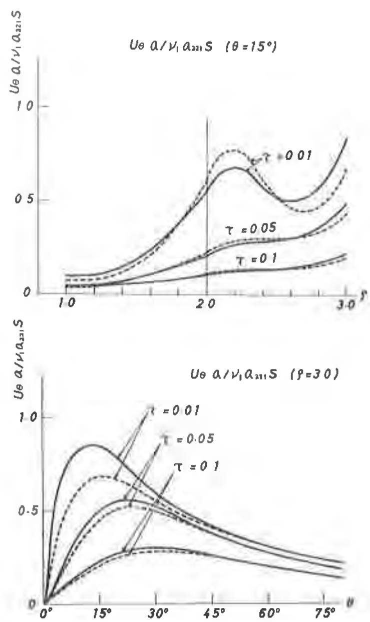

Figure 6 Distributions of circumferential displacement along $\theta = 15°$ and outer boundary

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