EFFECTS OF GAP SIZES ON STRESSES AND DEFLECTIONS OF A FUEL ELEMENT

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The transverse temperature gradient resulting from thermal-hydraulic variations in coolant subchannels surrounding a fuel element will cause lateral deflection of the element. If this lateral deflection is restricted by the neighboring elements or other associated components, appreciable stress is introduced in the fuel element. The magnitude of the stress is related to the size of the gap existing between a fuel element and its adjacent member. Should this gap be large enough that the element is allowed to bend freely, no flexural stress is expected. But excessive deflection, on the other hand, tends to reduce the subchannel area on the hotter side of the fuel element, resulting even higher temperature gradient in that direction. The gap between fuel elements thus plays a major role on the stresses and deflections, and its effects are of interest both from a structural and a thermal-hydraulic feedback point of view.

Based on the simple beam theory and the principle of superposition, the elastic lateral deflection and flexural stress of wire-wrapped fuel elements were studied. In wire-wrapped fuel elements, the space wire provides potential contact points for neighboring fuel elements. The actual points of contact, however, depend on the deflection of the fuel element. The model used in this investigation consists of a cantilever beam with intermediate stops located at a distance g from the axis of the beam, where g is the gap size. Three values of g -- 0,0.051 mm (2 mil), and 0.127 mm (5 mil) -- were used in this investigation.

Results from two wire-wrapped fuel elements under their respective transverse thermal gradients show that both the number of contact points and the magnitude of the maximum reaction force are reduced as the value of g increases. In all cases considered, the maximum reaction force is acting at the stop farthest from the fixed end, and the maximum deflection occurs in the vicinity where thermal gradient had its peak value. Another expected result is that the maximum lateral deflection increases with the gap size. The largest deflection for all cases studied, however, is still much smaller than the radius of a typical space wire. This suggests that, under the conditions such as the ones considered here, direct contact between a fuel element and its adjacent member is very unlikely.

The maximum flexural stresses with and without gap differ only slightly, and the relation between the stress and the gap size is not monotonic. Since a larger gap will only reduce stress slightly, but will cause larger deflection and therefore more positive thermal-hydraulic feedback, a smaller gap that is able to keep the maximum stress within the design limit should be favorably considered.

*Work supported by the U.S. Department of Energy.

1. Introduction

When the temperature of coolant surrounding a fuel element is not uniform, the element will be subjected to a transverse thermal gradient and tends to deform in the lateral direction. In current LMFBR fuel element designs, such lateral deformation is restricted either by grids or by spacer wires. The investigation presented here is mainly for spacer wired fuel elements.

In spacer wired fuel elements, a wire is wound helically around each element to serve equal separation between the elements, and to provide the mixing of coolant within the subassembly. The maximum clearance between such a fuel element and its adjacent member, which could be either the subassembly wall or another fuel element, is equal to the wire diameter plus any designed gap. With the gap, flexural stress of the fuel element may be lowered, while the lateral deflection is expected to be larger than when there is no gap. Excessive deflection, on the other hand, may cause drastic change of coolant flow area from its designed value, and therefore the fuel element could be subjected to higher temperature and temperature gradient. Any such positive feedback is undesirable and should be kept at as minimum as possible.

Fig. 1 shows schematically the longitudinal section of a bundle of spacer wired fuel elements. Structurally these elements can be divided into two categories: the center elements and the edge elements. A center element is surrounded on all sides by other fuel elements and, since spacer wires are providing lateral constraints, is supported by paired stops distributed along the element axis. An edge element, on the other hand, has the subassembly wall as one of its neighbors, and the number of supporting stops is different on different sides of such an element. Usually the temperature field of a fuel element bundle is not symmetric about the axis of the bundle. Edge elements such as the two shown in Fig. 1 will have different thermal conditions and therefore should be treated separately.

In this study, the fuel element is modeled as a cantilever beam, with the lateral restraints offered by spacer wires interpreted as intermediate stops. Small distances simulating the designed gaps are assumed to exist between the intermediate stops and the beam axis. With these gaps, the beam may or may not contact the stops. The actual points of contact are part of the unknowns, and are dependent on the fuel element deflection.

2. Structural-Analysis Model

The fuel element is idealized as a beam clamped at the end $x = 0$, with a number of immovable stops located at $x = a_i$. These stops are at a distance either GA_i or GB_i from the beam axis (Fig. 2). Physically GA_i and GB_i are the gap sizes, respectively, for the gaps on the hot side and cold side of a fuel element. The beam is initially straight and obeys Hooke's law. Both Young's modulus E and the coefficient of linear thermal expansion α are constant throughout the entire beam. The diameter of the spacer wire is assumed to be small such that it could be neglected when evaluating the stresses and deflections of the fuel element. The beam therefore has a uniform circular cross-section, whose outside diameter is denoted by d. In addition to a temperature field, the beam may also be under the action of concentrated forces F_j applied at $x = b_j$. These concentrated forces are either applied externally or the reaction forces when the beam is in contact with the stops. The temperature T varies with the axial coordinate x as well as a diametral coordinate y. For simplicity, T is assumed to be linear in y,

$$T = \frac{y}{d} T_o(x) + C(x), \tag{1}$$

where $T_o(x)$ is the maximum diametral temperature difference at a given section. The simple form given in Eq. (1) implies that all the maximum diametral temperature difference are in y-direction. If the externally applied forces are also in the same direction as the maximum diametral temperature difference, only the y-direction deflections and reactions have to be considered.

3. Method of Approach

Using the principle of superposition, the deflection $V(x)$ of a cantilever beam subjected to concentrated forces F_j at $x = b_j$ and a temperature distribution as given by Eq. (1) can be written as

$$V(x) = -\int_0^x \int_0^{x_1} \frac{\alpha}{d} T_o(x_2)\, dx_2\, dx_1 + \sum_j \frac{F_j}{6EI} \left[\langle x - b_j \rangle^3 - x^3 + 3x^2 b_j \right], \tag{2}$$

where I is the area moment of inertia of the beam. The expression with the pointed bracket has the meaning that [1], for any $n \geq 0$

$$\langle x - b_j \rangle^n = \begin{cases} 0 & \text{for } x \leq b_j \\ (x - b_j)^n & \text{for } x \geq b_j. \end{cases} \tag{3}$$

If the deflections $V(a_i)$ at all potential contact points calculated from Eq. (2) are less than the corresponding gaps at $x = a_i$, the beam does not touch any of the intermediate stops, and Eq. (2) gives the final result for the problem under consideration.

Should any one of the deflections $V(a_i)$ overlap the gap at the corresponding stop, a reaction force which is yet to be determined should appear at that stop. To determine the reaction force R_k at $x = a_k$, the total beam deflection at each of the contact points due to the temperature, the external forces F_j and all reaction forces is required to equal exactly the proper gap size at the contact point $x = a_k$, that is

$$V(a_k) + \sum_m \frac{R_m}{6EI} \left[\langle a_k - a_m \rangle^3 - a_k^3 + 3a_k^2 a_m \right] = \begin{cases} GA_k & \text{if } V(a_k) \geq 0 \\ GB_k & \text{if } V(a_k) < 0 \end{cases} \tag{4}$$

where $V(a_k)$ is calculated from Eq. (2), and includes only the deflection caused by temperature T and external forces F_j. Equation (4) is the set of linear algebraic equations for determining the reaction forces R_m. Since the intermediate stops can only offer compressive reactions, any tensile force R_m obtained from Eq. (4) should be discarded, and new reaction forces are to be solved from the reduced set of Eq. (4). This process continues until all reaction forces R_m solved from Eq. (4) are compressive.

With the addition of these reaction forces, the beam may contact stops different from those determined previously. A new deformed configuration of the beam is obtained using again Eq. (2), but with the reactions forces R_m just solved from Eq. (4) as part of the external forces. If the new deflections $V'(a_i)$ at $x = a_i$ do not exceed the corresponding gap sizes, the true deformed configuration of the beam has been obtained, and one can proceed to evaluate the flexural stresses through the standard formulas. Otherwise Eq. (4) should be employed again to obtain a new set of reaction forces. The final result is the one that all the reaction forces at the contacting stops are compressive, and that deflections at all stops do not exceed the corresponding gap sizes.

4. Sample Problems

The first example considered is a fuel element of length 214.63 cm (84.5 in.) under a temperature distribution [2] whose maximum diametral difference $T_o(x)$ has the profile of a triangle. $T_o(x)$ is zero between x = 0 and x = 85.09 cm (33.5 in.), raises to peak difference of 94.5°C (170°F) at x = 170.18 cm (67 in.), and drops back to zero at the end x = 214.63 cm (84.5 in.). Values for E and α are taken to be 1.72×10^5 MPa (25×10^6 psi) and 18×10^{-6} (1/°C) (10×10^{-6} 1/°F) respectively. The fuel element has inner radius of 0.254 cm (0.1 in.) and outer radius of 0.292 cm (0.115 in.). The first stop is at 1.27 cm (0.5 in.) from the clamped end, and the succeeding stops are spaced 15.24 cm (6 in.) apart. The three types of beam supports investigated are shown in Fig. 2, where Type 1 is for center elements, and Types 2 and 3 are for edge elements. This same $T_o(x)$ was used for all three types of supports, and for each type three different gap sizes, 0, 5.08×10^{-3} cm (2 mils) and 0.0127 cm (5 mils) were used in the numerical evaluations. The resulting deflections and reaction forces are shown in Figs. 3-5.

When the gap of Type 1 arrangement (center element) is zero, the fuel element is simply a continuous beam with intermediate hinged supports. The cases with non-zero gaps simulate the situation where the supports are not in close contact with the beam. For all the nine cases considered, maximum reaction force occurs at the last stop, with its magnitude decreasing with the gap size. The gap sizes considered have only a slight effect on the maximum flexural stress, which occurs at x = 168.91 cm (66.5 in.) for all cases.

The fuel element of the second example has length 60.96 cm (24 in.), inner diameter 0.29 cm (0.114 in.) and outer diameter 0.442 cm (0.174 in.). E and α are the same as the previous example, while the temperature difference $T_o(x)$ raises almost linearly from zero at the clamped end to 48.6°C (87.5°F) at x = 29.21 cm (11.5 in.), drops to 17.8°C (32°F) at x = 53.34 cm (21 in.), and practically holds constant from there on. Stops are spaced 7.62 cm (3 in.) apart starting from the clamped end. Since the previous example shows that both the maximum stress and deflection for a given gap size are found in the Type 2 arrangement, only the results for such support arrangement are presented and are shown in Fig. 6. Deflection curves in Fig. 6 generally have the same appearance as Figs. 3-5, except the case of zero gap where the part of fuel element near the last stop deforms toward the hot side. This could be resulted from the fact that the free thermal deflection of this example is smaller than the previous example.

5. Conclusion

Based on the model of a cantilever beam with a number of intermediate stops at a uniform distance from the beam axis, the thermal responses of two fuel elements under different support arrangements and gap sizes are presented in Figs. 3-5 and Fig. 6. Effects of gap sizes on stresses and deflections are summarized:

(1) For a given gap size, Type 2 of the three types studied gives highest values in stress and deflection. Both the maximum stress and deflection occur in the vicinity of maximum temperature difference.

(2) When a fuel element has more lateral restraints on the hot side than the cold side (Type 3) or equal number of restraints on both sides (Type 1), the maximum deflection is at approximately one third of the element length from the fixed end. At that location, the element deforms toward the cold side. Maximum stress, however, is still at the vicinity of maximum temperature difference.

(3) From all the cases investigated, the maximum deflection increases with the gap size. Maximum stress, however, changes only slightly for the gap sizes studied, and the relation between stress and gap size is not necessarily monotonic.

(4) Larger gap tends to reduce the number of contact points and the magnitude of maximum reaction force, which occurs at the farthest stop from the clamped end.

(5) The maximum deflection of all cases studied here is 0.034 cm (13.22 mils) (Fig. 4), which is smaller than the radius of a typical spacer wire. Direct contact between a spacer wired fuel element and its adjacent member seems very unlikely.

Results presented here show that a larger gap will change the fuel element stress only slightly, but will introduce larger deflection, which in turn could cause positive thermal-hydraulic feedback. Smaller gap which could keep the maximum stress within the design limit should therefore receive favorable considerations.

References

[1] Pilkey, W. D., "Clebach's Method for Beam Deflections," J. Eng. Educ., 54, 170-174 (1964).

[2] Betten, P. R., Argonne National Laboratory, private communication (1978).

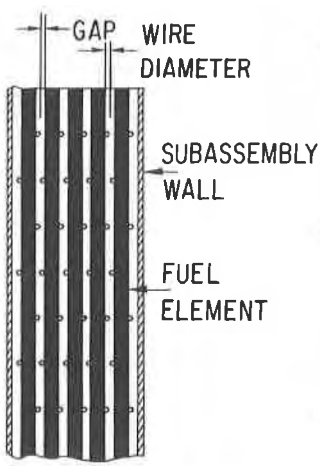

Fig. 1. Longitudinal Section of an Illustrative Subassembly

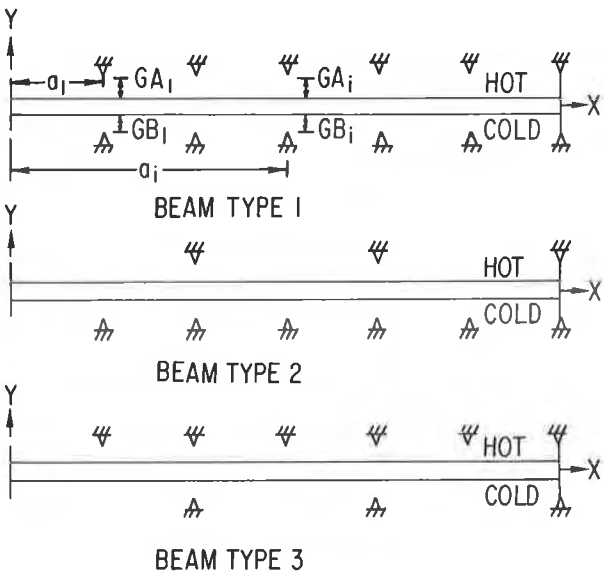

Fig. 2. Idealized Fuel Elements with Different Support Arrangements

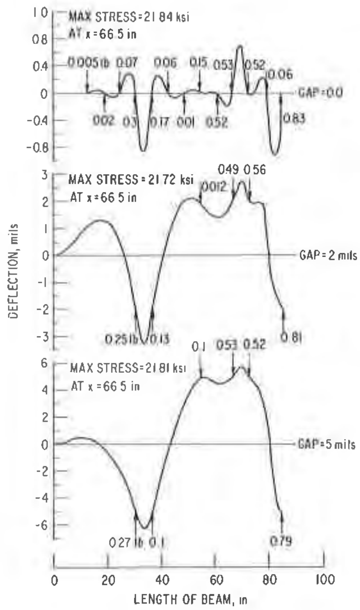

Fig. 3. Deflection of an 84.5 in. Fuel Element with Type 1 Supports.
Conversion Factors: 1 in. = 25.4 mm,
1 lbf = 4.45 N,
1 mil = 0.0254 mm,
1 ksi = 6.895 MPa

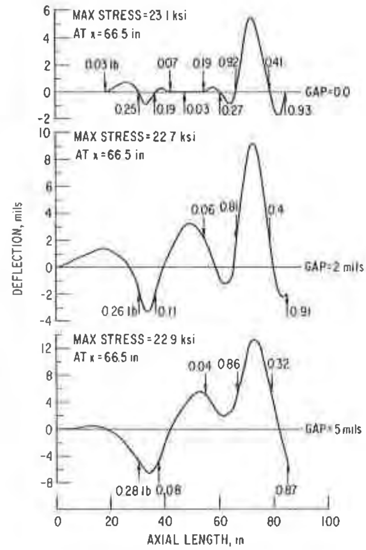

Fig. 4. Deflection of an 84.5 in. Fuel Element with Type 2 Supports.
Conversion Factors: 1 in. = 25.4 mm,
1 lbf = 4.45 N,
1 mil = 0.0254 mm,
1 ksi = 6.895 MPa

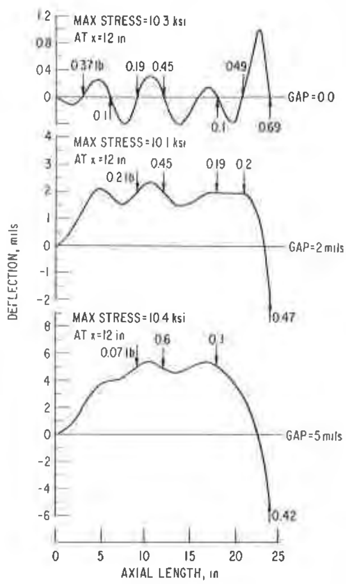

Fig. 5. Deflection of an 84.5 in. Fuel Element with Type 3 Supports.
Conversion Factors: 1 in. = 25.4 mm,
1 lbf = 4.45 N,
1 mil = 0.0254 mm,
1 ksi = 6.895 MPa

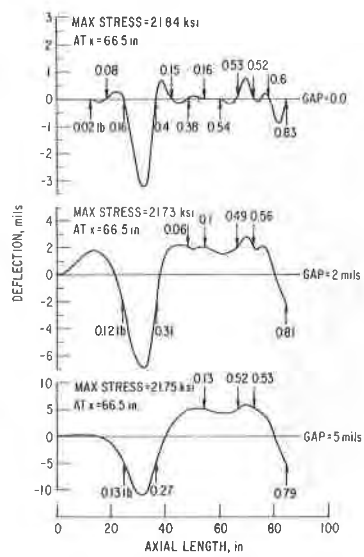

Fig. 6. Deflection of a 24 in. Fuel Element with Type 2 Supports.
Conversion Factors: 1 in. = 25.4 mm,
1 lbf = 4.45 N,
1 mil = 0.0254 mm,
1 ksi = 6.895MPa

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