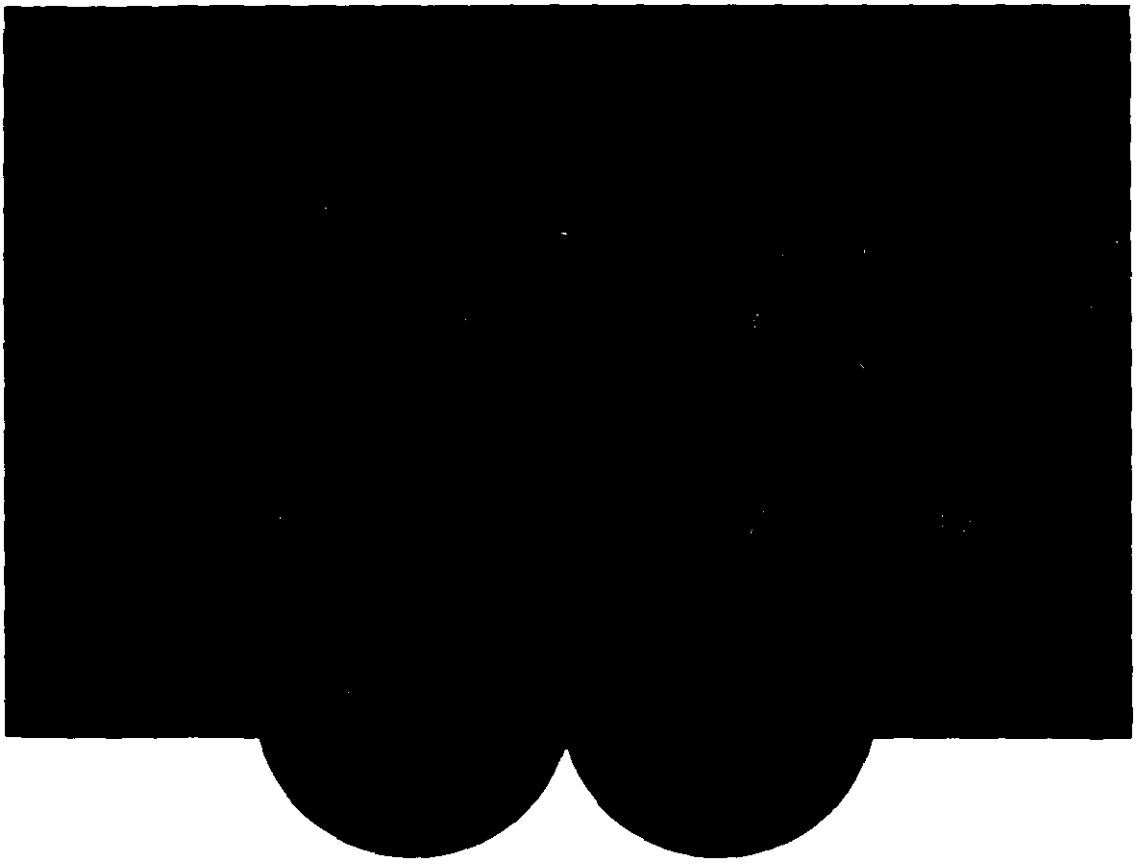

ON PROPERTIES OF Q-CHARTS FOR VARIABLES

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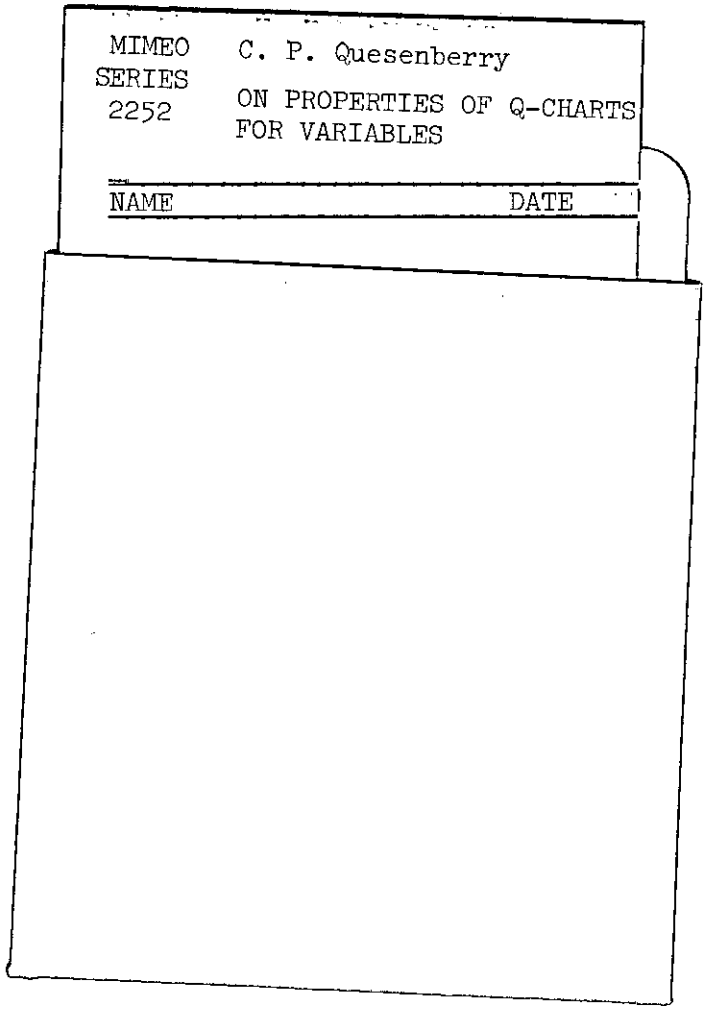
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On Properties of Q-Charts for Variables

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The sensitivity of four tests on Shewhart type Q-charts and of specially designed EWMA and CUSUM Q-charts to detect one-step permanent shifts of either a normal mean or standard deviation has been studied. The usual test that signals for one point beyond three standard deviations on a Shewhart chart is found to have poor sensitivity, generally. The test that signals when four out of five consecutive points are beyond one standard deviation in the same direction is found to be a good omnibus test. The EWMA and CUSUM Q-charts are the most sensitive and are about comparable in overall performance.

Introduction

Quesenberry (1991), Q91, defined Q-statistics that can be computed in a number of situations, assuming a normal process distribution and either one or both parameters can be unknown. Under the assumption that the normal process distribution is stable, i.e., that the mean μ and standard deviation σ are constant, these statistics all have distributions that are either exactly or approximately standard normal distributions, and, moreover, are either exactly or approximately independent. Quesenberry suggested that these statistics be plotted on Shewhart charts with control limits at ± 3, and pointed out that tests such as those of Nelson (1984) can be made on these charts. It is also apparent that these Q-statistics can be used as the input data to plot EWMA and CUSUM charts. Some users of these charting methods have expressed an interest in having more guidance in choosing the particular types of charts and tests to be applied on the charts. Since, for a stable normal process distribution, the several types of Q-statistics given in Q91 all have either exactly or approximately standard normal distributions, we know exactly the type of point pattern that represents a stable process on a Shewhart Q-chart of these statistics. However, if either the process mean μ or standard deviation σ are not constant while the data are being taken, then we expect this, in general, to result in anomalous point patterns that are different from those of a stable process. Indeed, the ability to recognize many anomalous patterns of plotted values that signal identifiable assignable causes for a process is an extremely useful property of Shewhart charts, in general, and of Shewhart Q-charts, in particular. However, in order to study the effectiveness of Q-charts to detect parameter variation it is necessary to focus on a particular type of parameter variation. Perhaps the most important, and surely the easiest to study, is a one-step permanent shift in a parameter. We will also take some note of the ability to detect one-step temporary parameter shifts, which, in the case of the mean, is sometimes called the outlier problem.

Tests for Parameter Shifts

There are many possible tests that can be made on a Shewhart Q-chart to detect a shift in μ or σ. In this work we will consider four tests as follows. Three of these tests were discussed by Nelson (1984). Given a sequence of Q-statistics $Q_{p+1}, Q_{p+2}, \cdots, Q_t, \cdots$; these tests are defined as follows.

The 1-of-1 Test: When Q_t is plotted, the test signals an increase in μ if $Q_t > 3$, and signals a decrease in μ if $Q_t < -3$. This is, of course, the classical Shewhart chart test.

The 9-of-9 Test: When Q_t is plotted, this test signals an increase in μ if $Q_t, Q_{t-1}, \cdots, Q_{t-8}$ all exceed 0, and signals a decrease in μ if $Q_t, Q_{t-1}, \cdots, Q_{t-8}$ are all less than 0. This test can only be made when nine consecutive Q-statistics are available.

The 3-of-3 Test: When Q_t is plotted, this test signals an increase in μ if Q_t, Q_{t-1} and Q_{t-2} all exceed 1, and signals a decrease in μ if Q_t, Q_{t-1} and Q_{t-2} are all less than -1. This test can only be made when three consecutive Q-statistics are available.

The 4-of-5 Test: When Q_t is plotted, this chart signals an increase in μ if at least four of the five values $Q_t, Q_{t-1}, \cdots, Q_{t-4}$, exceed 1, and signals a decrease in μ if at least four of the five values Q_t, $Q_{t-1}, \cdots, Q_{t-4}$ are less than -1. This test can only be made when five consecutive Q-statistics are available.

In addition to the four above tests that are applied on Shewhart Q-charts, we also consider here an exponentially weighted moving average, EWMA, and a cumulative sum, CUSUM, chart computed from the sequence of Q-statistics. The EWMA statistic Z_t is given by

$$Z_t = \lambda Q_t + (1-\lambda)\, Z_{t-1} \text{ for } t = 1, 2, \cdots \tag{1}$$

with $Z_0 = 0$. These values are plotted on a chart with control limits at $\pm K\sqrt{\lambda/(2-\lambda)}$. A procedure of Crowder (1989) was used to obtain values of λ and K that give an in-control ARL of 372.6 and an ARL of 5.18 to detect a shift of 1.5 standard deviations in a normal process mean. The values obtained were $(\lambda, K) = (0.25, 2.90)$, which give control limits at $\pm$ 1.096. This same EWMA chart was used for all of the work reported here.

The EWMA Test: When Q_t and Z_t are computed, if $Z_t > \text{UCL}\ (Z_t) = 1.096$, an increase in μ is signaled, and if $Z_t < -1.096$, a decrease in the μ is signaled.

The CUSUM statistics S_t^+ and S_t^- are defined as follows:

$$S_t^+ = \text{Max}\left\{0,\ S_{t-1}^+ + Q_t - k_s\right\}$$

$$S_t^- = \text{Min}\left\{0,\ S_{t-1}^- + Q_t + k_s\right\} \tag{2}$$

with $S_t^+ = S_t^- = 0$.

The CUSUM Test: If $S_t^t > h_s$ an increase in μ is signaled, and if $S_t^- < -h_s$ a decrease in μ is signaled.

The CUSUM test is determined by the reference value k_s and the decision interval h_s. We have used the values of $k_s = 0.75$ and $h_s = 3.34$ for the CUSUM test used in this study. These values were chosen to give a CUSUM test with approximately the same ARL's as the EWMA above for the in-control case (for this CUSUM this is 370.5) and to detect a 1.5 standard deviation shift in a process mean (for this CUSUM this is also 5.18, the same as for the EWMA test). This same CUSUM chart was used for all of the work reported here.

Q-Statistics from Individual Measurements

In this paper we consider again the Q-statistics for normal process variables of Q91. The Q-statistics for data given in the individual measurements format and for data organized in subgroups were treated separately in Q91, and the same organization is used here. We also use the same notation for distribution functions of Q91, and it is given in Table 1 for convenience.

TABLE 1

Notation for Distribution Functions

$\Phi(\cdot)$ – The standard normal distribution function.
$\Phi^{-1}(\cdot)$ – The inverse of the standard normal distribution function.
$G_\nu(\cdot)$ – The Student-t distribution function with ν degrees of freedom.
$H_\nu(\cdot)$ – The chi-squared distribution function with ν degrees of freedom.
$F_{\nu_1,\nu_2}(\cdot)$ – The Snedecor-F distribution function with (ν_1, ν_2) degrees of freedom.

Let $X_1, X_2, \cdots, X_r, \cdots$ denote measurements made on a sequence of parts as they are produced in time, and assume that these values are independently and identically distributed random variables with a normal $N(\mu, \sigma^2)$ process distribution. Define the following (sequential) sample statistics:

$$\bar{X}_r = \frac{1}{r}\sum_{j=1}^{r} X_j, \quad S_r^2 = \frac{1}{r-1}\sum_{j=1}^{r}(X_j - \bar{X}_r)^2 \qquad r = 2, 3, \cdots \tag{3}$$

We now consider transforming the individual measurements to obtain standard normal process mean Q-statistics for four cases. The Q-statistics for the process mean for the four cases of μ and σ known and unknown are defined as follows.

Q-Statistics for the Process Mean μ; Individual Measurements

Case KK: $\mu = \mu_o$, $\sigma = \sigma_o$, μ known and σ known; KK

$$Q_r(X_r) = \frac{X_r - \mu_o}{\sigma_o}, \; r = 1, 2, \cdots \tag{4}$$

Case UK: μ unknown, $\sigma = \sigma_o$ known; UK

$$Q_r(X_r) = \left(\frac{r-1}{r}\right)^{\frac{1}{2}} \frac{(X_r - \bar{X}_{r-1})}{\sigma_o}, \; r=2, 3, \cdots \tag{5}$$

Case KU: $\mu = \mu_o$ known, σ^2 unknown; KU

For this case put

$$S_{o,r}^2 = \frac{1}{r}\sum_{j=1}^{r}(X_j - \mu_o)^2$$

and

$$Q_r(X_r) = \Phi^{-1}\left\{G_{r-1}\left(\frac{X_r - \mu_0}{S_{o,r-1}}\right)\right\}, \; r = 2, 3, \cdots \tag{6}$$

Case UU: μ and σ^2 both unknown; UU

$$Q_r(X_r) = \Phi^{-1}\left\{G_{r-2}\left[\left(\frac{r-1}{r}\right)^{\frac{1}{2}}\left(\frac{X_r - \bar{X}_{r-1}}{S_{r-1}}\right)\right]\right\}, \; r = 3, 4, \cdots \tag{7}$$

If p denotes the number of unknown parameters in the model for a case, then we obtain the Q-statistics $Q_{p+1}, Q_{p+2}, \cdots$. The Q-statistics for the process variance are given next for the case when the variance has a known value σ_o^2, and when it has an unknown value. Only the cases for μ unknown are given.

Q-Statistics for the Process Variance σ^2; Individual Observations

Put $R_r = X_r - X_{r-1}$

Case K: μ unknown and $\sigma = \sigma_o$ known.

$$Q(R_r) = \Phi^{-1}\left\{H_1\left(\frac{R_r^2}{2\sigma_o^2}\right)\right\} \text{ for } r = 2, 4, 6, \cdots \tag{8}$$

Case U: μ unknown and σ unknown.

$$Q(R_r)=\Phi^{-1}\left\{F_{1,\nu}\left(\frac{\nu R_r^2}{R_2^2+R_4^2+\cdots+R_{r-2}^2}\right)\right\} \text{ for } r = 4, 6, \cdots; \quad \nu=\frac{r}{2}-1. \tag{9}$$

It was shown in Q91 that the Q-statistics of equations (4) – (9) are sequences of independently and identically distributed N(0, 1) random variables. Thus if these statistics are plotted on a Shewhart control chart with control limits at ± 3, we know exactly what point pattern to expect, *when the process is stable and the parameters μ and σ are constant.* The points will appear in the classical pattern for a normal distribution. However, if either μ or σ are not constant, then the normal pattern will be disturbed for these charts, in general. We would like to recognize the types of patterns that will result from particular parameter shifts, and to decide what types of chart tests are best for detecting particular parameter shifts.

Tests for μ, Individual Measurements

We consider now the performance of the six tests given above when used in conjunction with the Q-statistics in equations (4), (5), (6) and (7) for the cases of μ known and σ known, KK; μ unknown and σ known, UK; μ known and σ unknown, KU; and μ unknown and σ unknown, UU. We consider the effect of a shift of μ from a nominal value of $\mu = \mu_0$ to a new value $\mu = \mu_0 + \delta\sigma$, i.e., a shift of δ standard deviations in the process mean. By examining the formulas, it is readily seen that the effect of the mean shift on the statistics of the cases KK, UK and KU will be to increase the mean of the Q-statistics, when δ is positive, and decrease the mean when δ is negative, for all values of r. However, a point that should be noted for the UU case is that the shift will not only cause the value of the numerator $(X_r - \overline{X}_{r-1})$ to shift in the direction of the sign of δ, on points after the shift, it will also affect the value of the denominator, S_{r-1}. If the shift occurs at the value $r = c$, say, then from (3) we see that values of S_r^2 for $r = c+2, c+3, \cdots$ will tend to be inflated due to the shift, and the inflation increases with r. This tendency to decrease the denominator of the argument in turn decreases the Q-statistic itself and decreases the probability of detecting the shift by the 1-of-1 test. Wade and

Woodall (1992) also noted this point. This is a particular instance of a general phenomenon noted in Q91 where it was pointed out that for all of the cases with unknown parameters, the charts will eventually settle into the pattern of a stable process, provided, of course, there are no further parameter shifts.

The details of a simulation study of the performance of Q-charts made using the statistics of equations (4), (5), (6) and (7) will be given in this work, however, we would like to first observe that important information can be deduced directly by studying those transformation equations. First, observe that the Q-statistic for the KK case of equation (4) is exactly the statistic that one would use to test the hypothesis that a normal distribution with known standard deviation $\sigma = \sigma_0$ has a mean of μ_0 *using a sample of size one.* The Q-statistic for the UK case of equation (5) is the test statistic one would use to test that a sample of size one and a sample of size $r-1$ are from distributions with the same mean, when the distributions are assumed to have the same known variance $\sigma^2 = \sigma_0^2$. The argument of equation (6) is the test statistic to test that a sample of size one is from a distribution with mean $\mu = \mu_0$, when a second sample of size $r-1$ is available from a distribution with mean known to be μ_0, and the variance of all observations is the same, but unknown.

The argument for the UU case of equation (7) is the two-sample t statistic to test that a sample of size one and a sample of size $r-1$ have the same unknown mean when the samples are assumed to be from normal distributions with the same unknown variance. For this statistic note that for the first statistic computed, *viz*, Q_3, this t-test statistic will have just one degree of freedom, for Q_4 it has two degrees of freedom, for Q_r it has $r-2$ degrees of freedom. From this it is clear that the power of t-tests using these statistics, and, correspondingly, the sensitivity of Q-charts to detect changes in μ will not approach that for the known parameters case until the degrees of freedom becomes large. We will not discuss the many other cases of charts for variances and subgroups Q-statistics (given below) in detail, but clearly similar considerations obtain in all cases that have unknown parameters.

After a shift in μ, the Q-statistics for the three cases when at least one parameter is unknown; *viz.*, UK, KU and UU; are no longer independent standard normal statistics. If μ shifts to $\mu = \mu_0 + \delta\sigma$ immediately after $r = c$, then the distribution for these cases, and of the six tests above, will depend on c. In order to study the performance of these tests we have performed a simulation study. For each value of $c \in \{5, 10, 20, 40, 70, 100\}$ and $\delta \in \{0.0, 0.5, 1.0, 1.5, 2, 3, 4, 5, 6\}$, c values were generated from a $N(0, 1)$ distribution and then 30 additional values from a $N(\delta, 1)$ distribution. The above test statistics were computed as soon as enough values were available to define the particular tests. This

was repeated 5,000 times and the number of signals obtained on the points from $r = 1$ to c and from $r = c + 1$ to $c + 30$ counted. The proportions of times a signal was observed from $c + 1$ to $c + 30$, for the first time, are given in Tables A.1.1, A.1.2 and A.1.3 of appendix A. Four rows are given for each value of δ. The first row is for the case KK, the second is for UK, the third for KU and the fourth for UU.

Note first that there are six sets of entries for $\delta = 0.0$, i.e., one for each value of c. This is the stable case, and all of the Q-statistics are i.i.d. U(0, 1) for this case. Thus for each of these six values of c the four values given for each test statistic are estimating the probability that test will give at least one false signal on 30 consecutive points, with one exception. The exception is the 9-of-9 test with $c = 5$, and for this case the four values estimate the probability of at least one false alarm on 26 consecutive points. The exact probability being estimated for the 1-of-1 test is readily calculated as $1 - \{0.99865\}^{30} = 0.03972$. The four values given for each value of δ and each test statistic are all computed from the same 5,000 samples and are not independent estimates.

Note also that for a fixed value of δ the test values for the KK row, the first row, are all estimates of the same value and that value does not depend upon the value of c. For example, the six values for the 1-of-1 test for $\delta = 1$ and $c = 5, 10, \cdots, 100$ are: 0.490, 0.502, 0.515, 0.497, 0.506, and 0.495; and all are estimates of the same value from different sets of 5,000 samples.

The purpose of these tables is to compare the performance of these several tests, and to study the relative performance of each test for the cases KK, UK, KU and UU. It is, we hope, clear from the above discussion that cases with known values of parameters will have better performance that corresponding cases with unknown parameters, provided, of course, the "known" values are actually the correct ones. Also, the performance of each test should inprove with increasing c, and for c large the performance will approach that for the KK case.

First, note that for the stable case of $\delta = 0$ the tables give estimated rates of false alarms on 30 consecutive points. The EWMA and CUSUM tests both have false alarm rates about the same as the 1-of-1 test, as expected, since they were designed to have the same ARL's as the 1-of-1 test for $\delta = 0$. The 9-of-9 test has false alarm rates about 0.73 times those of the 1-of-1 test, and the 4-of-5 test has false alarm rates of about 1.20 times those of the 1-of-1 test. The false alarm rate for the 3-of-3 test is about 2.3 times that of the 1-of-1 test.

Consider next the information in Tables A.1.1, A.1.2 and A.1.3 with regard to the ability of the tests to detect changes in a process mean. The first point to note is that for all cases of $\delta > 0$ the

classical 1-of-1 test is not competitive with the other tests. This is not surprising since the 1-of-1 test utilizes information from only one point, whereas the other tests all utilize information from several points.

Among the other five tests there is no test that is better for all cases, however, some observations are possible. First, while there are a few cases where either the EWMA or the CUSUM test are better than the other, we feel that the overall performance of these two tests are reasonably comparable. Either of these tests would be a good overall omnibus test, although there are some cases where they are not as good as one of the Shewhart chart tests, for example $c = 5$ and $\delta \in \{1, 1.5, 2\}$ the 9-of-9 and 4-of-5 tests are better than the EWMA and CUSUM tests. The performance of the EWMA and CUSUM tests relative to the Shewhart charts tests tends to be best for the KK case, and not as good for some cases with parameters unknown, especially the UU case.

Comparisons among the three Shewhart chart tests (9-of-9 , 3-of-3 and 4-of-5) are complicated by having different false alarm rates, as given above. Also, the false alarm rates of the 9-of-9 tests are constant only for $c \geq 8$, and have varying smaller values for $c < 8$. The estimated value of this rate from Table 2.1, $c = 5$ and $\delta = 0.0$, is approximately 0.026. For $c \geq 8$ this rate is approximately 0.030. The relative magnitudes of the false alarm rates of the 9-of-9, 4-of-5 and 3-of-3 tests are approximately $3 : 5 : 10$. Keeping these values in mind, we feel the 9-of-9 test is preferred among these Shewhart tests for small c (say $c \leq 15$), and also is better than the EWMA and CUSUM tests for these values. For larger values of c the 4-of-5 is a good test for the Shewhart chart, and is a reasonable general omnibus test for all cases.

Tables A.1.1, A.1.2 and A.1.3 also give information on the effect of c, i.e., the size of sample available before a shift in μ occurs, on the sensitivity of the charts for the UK, KU, and UU cases. Table 2 displays this information for a typical case, $\delta = 1$, in a form convenient for easy comparison. The values are given for the 4-of-5, EWMA and CUSUM tests. The KK row gives the average values for the tests from the six cases of 5,000 replications each. For each value of c the values for UK, KU and UU cases are given for these three statistics. For the smallest value of $c = 5$, the drop-off is significant, and similar, for all three statistics. It is, perhaps, of some interest to observe that the UK case generally has lower power to detect the mean shift than the KU case. This shows that on these tests to detect a shift in the process mean, it is more helpful to know the nominal value μ_0 of the mean before the shift, than to know the value of σ. Also, the UU case is always the least sensitive, as we anticipated from the above discussion of the Student-t distribution test. Finally, the reader should understand that the tests for cases KK, UK, KU and UU are not competing tests. We will always use all known parameters, of course.

Table 2: Prior Sample Size Effects, $\delta = 1$

c		4-of-5	EWMA	CUSUM
	KK:	0.895	0.983	0.975
5	UK:	0.258	0.311	0.291
	KU:	0.614	0.728	0.701
	UU:	0.165	0.143	0.129
10	UK:	0.383	0.505	0.467
	KU:	0.689	0.828	0.802
	UU:	0.265	0.308	0.273
20	UK:	0.542	0.720	0.674
	KU:	0.746	0.906	0.888
	UU:	0.424	0.538	0.493
40	UK:	0.683	0.846	0.812
	KU:	0.810	0.945	0.931
	UU:	0.594	0.750	0.715
70	UK:	0.767	0.916	0.892
	KU:	0.847	0.961	0.950
	UU:	0.709	0.875	0.837
100	UK:	0.818	0.939	0.917
	KU:	0.866	0.970	0.958
	UU:	0.773	0.910	0.891

Q-Statistics from Subgrouped Data

It will sometimes be useful to form Q-charts from sample means $\bar{X}_i$ and sample variances S_i^2 for data grouped into samples of size n_i as in Table 3.

TABLE 3
Sample and Statistics Notation

Sample			Sample Mean	Sample Variance
X_{11}	X_{12} ⋯	X_{1n_1}	$\bar{X}_1$	S_1^2
X_{21}	X_{22} ⋯	X_{2n_2}	$\bar{X}_2$	S_2^2
⋮	⋮	⋮	⋮	⋮
X_{r1}	X_{r2} ⋯	X_{rn_r}	$\bar{X}_r$	S_r^2

Put $\bar{\bar{X}}_r = \frac{n_1\bar{X}_1 + \cdots + n_r\bar{X}_r}{n_1 + \cdots + n_r}$, and $S^2_{p,r} = \frac{(n_1-1)S_1^2 + \cdots + (n_r-1)S_r^2}{n_1 + \cdots + n_r - r}$

Note particularly that the sample mean $\bar{X}_r$ and variance $S^2_{p,r}$ are defined differently from $\bar{X}_r$ and S_r^2 of equation (2). The Q transformations for the sample mean $\bar{X}_r$ to control μ are given in the following for the same four cases considered above for individual measurements.

Q-Statistics for the Process Mean μ; Subgrouped Data

Case KK: $\mu = \mu_o$, $\sigma = \sigma_o$, μ known and σ known; KK

$$Q_r(\bar{X}_r) = \frac{\sqrt{n_r}\,(\bar{X}_r - \mu_o)}{\sigma_o} \quad r = 1, 2, \cdots \tag{10}$$

Case UK: μ unknown, $\sigma = \sigma_o$ known; UK

$$Q_r(\bar{X}_r) = \sqrt{\frac{n_r(n_1 + \cdots + n_{r-1})}{n_1 + \cdots + n_r}}\left(\frac{\bar{X}_r - \bar{\bar{X}}_{r-1}}{\sigma_o}\right) \quad r = 2, 3, \cdots \tag{11}$$

Case KU: $\mu = \mu_o$ known, σ^2 unknown; KU

For this case put

$$S^2_{o,r} = \frac{\sum_{\alpha=1}^{r}\sum_{j=1}^{n_\alpha}(X_{\alpha j} - \mu_o)^2}{n_1 + \cdots + n_r}$$

$$Q_r(\bar{X}_r) = \Phi^{-1}\left[G_{n_1+\cdots+n_r}\left(\frac{\sqrt{n_r}\,(\bar{X}_r - \mu_o)}{S_{o,r}}\right)\right] \quad r = 2, 3, \cdots \tag{12}$$

Case UU: μ unknown, σ unknown; UU

$$Q_r(\bar{X}_r) = \Phi^{-1}\left\{G_{n_1+\cdots+n_r-r}\left[\sqrt{\frac{n_r(n_1 + \cdots + n_{r-1})}{n_1 + \cdots + n_r}}\left(\frac{\bar{X}_r - \bar{\bar{X}}_{r-1}}{S_{p,r}}\right)\right]\right\} \quad r = 2, 3, \cdots \tag{13}$$

The Q-statistics from equations (10) and (11) for $\bar{X}_r$ were shown in Q91 to be independently and identically distributed standard normal random variables, N(0, 1), under the stable or in-control normality assumptions, and are approximately so for many other process distributions. The Q-

statistics from equations (12) and (13) for $\bar{X}_r$ are also standard normal random variables, and are approximately independent as the number $n_1 + \cdots + n_{r-1}$ becomes large. As we commented above, by studying these formulas it can be seen that these statistics are essentially the test statistics that would be used to test that a mean is constant under various different conditions. The Q-statistic of equation (10) is the statistic to test that the mean of the r^{th} sample has a particular value $\mu = \mu_0$ when $\sigma = \sigma_0$ is known. The other Q-statistics are also readily recognized as the appropriate test statistics for particular hypothesis testing problems. For example, for $r = 2$ the argument of equation (13) is essentially the Student-t statistic with $2n - 2$ degrees of freedom to test that the first and second samples are from distributions with the same mean, assuming that the distributions are normal with the same (unknown) variance. For all values of r this Student-t statistic will have degrees of freedom $(n - 1)r$, and the test and chart performance will approach that of the Q-statistic in (10) , that assumes known parameters, as r increases. Of course, for r small the power of the t-test and the corresponding sensitivity of the Q-chart using just the 1-of-1 test will be much less than that for for the case with parameters known. For $n = 5$, say, the degrees of freedom for the consecutive t-statistics are 8, 12, 16, $\cdots$; and the sensitivity of the charts increases much more rapidly than for the individual measurements chart.

The Q-statistics for controlling the process variance for data from samples as set out in Table 3 are as follows.

Q-Statistics for the Process Variance σ^2; Subgrouped Data

Case K: $\sigma = \sigma_o$ known.

$$Q_r(S_r^2) = \Phi^{-1}\left\{H_{n_r-1}\left[\frac{(n_r - 1)S_r^2}{\sigma_o^2}\right]\right\}, \; r = 1, 2, \cdots \tag{14}$$

Case U: σ unknown.

Put

$$w_r = \frac{(n_1 + \cdots + n_{r-1} - r + 1)S_r^2}{(n_1 - 1)\,S_1^2 + \cdots + (n_{r-1} - 1)\,S_{r-1}^2}\;, \; r = 2, 3, \cdots$$

and

$$Q_r(S_r^2) = \Phi^{-1}\left[F_{n_r-1,\; n_1+\cdots+n_{r-1}-r+1}(w_r)\right], \; r = 2, 3, \cdots \tag{15}$$

Under the assumption that the values in sample r are independent observations from $N(\mu_r, \sigma^2)$ distributions, the Q_r statistics of both (14) or (15) are sequences of independent normal N(0, 1) random

variables. These Q-statistics can also be identified with well known test statistics. The argument of the transformation in equation (14), *viz.* $(n_r - 1)\ S_r^2/\sigma_0^2$, is the chi-squared distributed test statistic to test that the sample variance S_r^2 is from a normal distribution with variance σ_0^2. The w_r statistic of equation (15) is the Snedecor F ratio statistic, with n_r-1 and $n_1+\cdots+n_{r-1}-r+1$ degrees of freedom for testing that the r^{th} sample is from a distribution with the same variance as the $r-1$ preceding samples. With increasing r, the power of the hypothesis test and the performance of the corresponding chart for the U case will both approach those of the K case.

Tests for μ, Subgrouped Data

We have performed simulation studies of charts for μ based on subgrouped data that are similar to the study for individual measurements data of the last section. Studies were made for two sample sizes, $n = 2$ and $n = 5$.

Sample Size n = 2 Results:

We consider the performance of the same six tests used with the Q-statistics for the formulas (10), (11), (12) and (13); and, as before, we refer to these cases as KK, UK, KU and UU. For a sequence of samples $r = 1, 2, \cdots, c, \cdots$ of size $n = 2$ each; suppose the mean shifts from $\mu = \mu_0$ to $\mu = \mu_0 + \delta\sigma$ immediately after sample number $r = c$. After the shift, the Q-statistics for the cases that involve at least one unknown parameter are no longer independent. To study the performance of the above tests a simulation study was performed as follows. For the same values of δ as studied before for the individual measurements case, and for each $c \in \{1, 5, 10, 20, 35, 50\}$, we first generated c samples of size 2 from a $N(0, 1)$ distribution and fifteen more samples of size 2 from a $N(\delta, 1)$ distribution. Note that this was a total of 30 observations taken after the shift. The Q-statistics for the four cases were computed and the six test statistics computed from them. This was replicated 5,000 times and the data are given in Tables A.2.1, A.2.2 and A.2.3. These are the proportions of times a signal occurs, for the first time, on the fifteen samples immediately after the mean shift.

Sample Size n = 5 Results:

A simulation study similar to that just described for $n = 2$ was performed also for samples of size $n = 5$. For this study, c was a value in the set $\{1, 2, 4, 8, 14, 20\}$, and in each case six samples of size five each were generated after the parameter shift. Again, this was a total of 30 observations after the shift. The results are given in Tables A.3.1, A.3.2 and A.3.3.

The purpose of these simulations is, again, to compare the relative performance of the tests and to study the effect of estimating parameters upon performance. The values in Table A.2.1 for $\delta = 0$ are the estimated false alarm rates on fifteen points, and, for example, the values for the 1-of-1 test are estimates of $1 - 0.99865^{15} = 0.020$. The values in Table A.3.1 for $\delta = 0$ are the estimated false alarm rates on six points. Thus the results in the tables for individual observations and for subgrouped data with n = 2 and n = 5 are not directly comparable.

The relative performances of the tests for subgrouped data are similar to those discussed above for individual measurements. The 1-of-1 test is not competitive, and the EWMA and CUSUM tests tend to give the best performance and are quite comparable in overall performance. The 4-of-5 test is a reasonable omnibus test to use on Shewhart Q-charts for the process mean, and much superior to the classical 1-of-1 test. The effect of having the number of samples c before the shift small is the same as before when c was the number of observations. Having a limited number of observations to estimate unknown parameters lowers the sensitivity of all of the tests, of course.

Tests for Changes in σ

Consider using the Q-statistics given in equations (14) and (15) to make control chart tests to detect changes in σ. Suppose that immediately after sample c is observed that σ shifts from $\sigma = \sigma_0$ to $\sigma = \delta\sigma_0$, $\delta > 0$. Since the Q-statistics of (14) and (15) are strictly monotone functions of S_i, we can observe the effect of this shift on the distributions of the Q-statistics by studying the effect on the distribution of S_i. Now, the mean and standard deviation of S_i are $E(S_i) = c_4\sigma$ and $SD(S_i) = \sigma\sqrt{1 - c_4^2}$. Thus when σ changes from $\sigma = \sigma_0$ to $\sigma = \delta\sigma_0$, then $E(S_i)$ shifts from $c_4\sigma_0$ to $c_4\delta\sigma_0$ and $SD(S_i)$ shifts from $SD(S_i) = \sigma_0\sqrt{1 = c_4^2}$ to $SD(S_i) = \delta\sigma_0\sqrt{1 - c_4^2}$. Thus if σ increases, so that $\delta > 1$, then both the mean and standard deviation of the distribution of S_i and of Q_i will increase. On the other hand, if σ decreases, so that $0 < \delta < 1$, then both the mean and standard deviation of the distribution of S_i and of Q_i will decrease.

This development implies that a Q-chart, or, for that matter, also a classical S^2 probability chart, can be expected to have poor sensitivity to detect a *decrease* in σ. Simply put, a decrease in σ causes the mean of the plotted statistic to decrease, but at the same time the distribution becomes more concentrated about the mean rather than placing correspondingly more probability below a lower control limit. From this development, we expect a classical S^2 probability chart and the Q-charts to have poor sensitivity to detect decreases in σ by the 1-of-1 test. However, it is apparent that many of the other tests should perform well to detect decreases in σ.

We have conducted a simulation to study the performance of the above six tests to detect changes in σ. The values of $\delta \in \{0.1, 0.5, 0.9, 1, 1.5, 2, 3, 5, 10\}$ were used and studies for $n = 2$ and $n = 5$ conducted. For $n = 2$ the values of $c \in \{3, 5, 10, 20, 35, 50\}$ were used. It should be noted that for $n = 2$ the formulas from (14) and (15) are the same as those from (8) and (9), so separate studies are not needed for individual measurements and subgroups Q-charts. The results of this study are given in Tables A.4.1, A.4.2 and A.4.3. These tables also have four rows for each pair of values of δ and c. A set of c samples was generated from a N(0, 1) distribution and then fifteen additional samples of size 2 generated from a N(0, δ) distribution. The six test statistics are computed from the two sets of Q-statistics from equations (14) and (15). This was replicated 5,000 times and the proportion of times that the tests signal a decrease from the equation (14) statistics is given in the first row, the proportion of decreases signaled from the (15) statistics is given in the second row, the proportion of increases signaled by (14) is given in the third row, and the number of increases signaled by (15) statistics is given in the fourth row.

To illustrate reading the table for a particular case, consider the entry in Table A.4.1 for $c = 5$ and $\delta = 0.1$. This value of δ represents, of course, a ten-fold decrease in σ. The first row of values, *i.e.,* (0.184, 1.000, 1.000, 1.000, 0.997, 1.000), gives the estimated probability that the decrease will be signaled on the fifteen samples of size 2 following the decrease in σ by each of the tests, when $\sigma = \sigma_0$ is known. The second row of values, *viz.*, (0.127, 0.984, 0.945, 0.943, 0.964, 0.972), gives the estimated probabilities of correctly signaling the decrease from $\sigma = \sigma_0$ to $\sigma = 0.1\sigma_0$ when σ_0 is not known but there are $c = 5$ samples of size 2 each before the shift occurs. The values in row three, all zeros, are the estimated probabilities of (falsely) signaling an increase in σ when $\sigma = \sigma_0$ is known, and row four gives these estimated probabilities (also all zeros) when σ_0 is unknown but $c = 5$ samples are taken before the shift.

Tables A.5.1, A.5.2 and A.5.3 give the same information when sample size is $n = 5$. For these tables c is one of the values in the set {1, 2, 4, 8, 14, 20}.

The 9-of-9 chart is sensitive to detect moderate to large decreases in σ for $n = 2$, but it is poor for $n = 5$. The EWMA and CUSUM charts are overall quite good to detect both decreases and increases in σ. As one test to detect all changes in σ we prefer the EWMA chart. Of the tests made on the Shewhart Q-chart, with false alarm rates reasonably comparable, the 4-of-5 test is a reasonable choice. Readers will note that these are the same tests recomended above for charts to control the process mean, both for individual measurements and for subgrouped data.

The 3-of-3 test often has good sensitivity to detect shifts in σ, but it also has false alarm rates

that are roughly 2 to 2.5 times those of most of the other tests. It may be worthwhile in some applications, especially when σ is unknown and c is small, to accept this increased false alarm rate in order to have increased sensitivity to detect shifts in σ.

Remarks on Data Deletion and Outliers

In the foregoing, we have been concerned with detecting one-step permanent shifts in either a process mean or variance. One-step temporary shifts in a parameter, usually the mean, are sometimes a major concern. The observations produced by such shifts are often called "outliers" or "fliers." By this we mean that a process that is producing stably suddenly produces a small number, usually only one or two, values that are much larger or smaller than their preceding fellows. The fact that a process is outlier prone will likely be known by engineers and technicians familiar with the process. There are many factors that can cause outliers in a process. The Q-statistics of equations (4)-(8) can be conveniently used in many problems to screen outliers in a data screen, or, essentially equivalent statistics are used in Quesenberry (1986), Q86, to establish processes to screen outliers.

If outliers are automatically removed and not used in subsequent computations to compute Q-statistics, then the sensitivities to detect parameter shifts will be improved. However, the distribution theory for the stable case will also be disturbed. In many cases this disturbance of the stable case distribution theory is only slight while the improvement of the chart sensitivity can be large. See Q86 for a more detailed discussion of these issues.

Summary and Recommendations

We have studied the sensitivity of four tests made on Shewhart type Q-charts to detect a one-step permanent shift in a process mean or a process standard deviation. Specially designed EWMA and CUSUM charts were also studied as competitors to the Shewhart chart tests.

- The overall performances of the EWMA and CUSUM charts were comparable with each other and better than the Shewhart chart tests to determine one-step permanent parameter shifts. It would be good practice to regularly plot one of these charts. Although there is little difference in these two charts in performance, the personal preference of this author is the EWMA chart. The particular chart used here would work well in charting programs.

• The 4-of-5 test on the Shewhart chart is considered to be the best overall test of the Shewhart chart tests. We recommend that this test be made on Shewhart Q-charts for both the mean and standard deviation to detect parameter shifts.

• The classical 1-of-1 test is a poor competitor for many of the cases considered. However, it is essentially the only test for detecting a single outlier on a chart for μ.

• If the increased rate of false alarms is considered tolerable in order to improve sensitivity to shifts, then the 3-of-3 test is a reasonable choice of test. This can be a reasonable trade-off in a start-up process before many data are available to improve sensitivity, *i.e.*, while c is small.

APPENDIX A, TABLES

Table A.1.1: Individual Measurements Charts for the Mean

δ	c = 5						c = 10					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.0	0.038	0.026	0.098	0.051	0.040	0.045	0.041	0.030	0.102	0.051	0.043	0.042
	0.041	0.027	0.094	0.049	0.038	0.042	0.037	0.032	0.095	0.049	0.039	0.039
	0.044	0.026	0.096	0.048	0.039	0.043	0.039	0.030	0.103	0.051	0.040	0.040
	0.044	0.026	0.096	0.048	0.040	0.045	0.039	0.032	0.098	0.050	0.040	0.041
0.5	0.173	0.271	0.475	0.381	0.522	0.455	0.174	0.278	0.474	0.387	0.513	0.452
	0.068	0.065	0.187	0.116	0.122	0.106	0.086	0.100	0.248	0.162	0.191	0.162
	0.086	0.270	0.385	0.291	0.347	0.280	0.096	0.278	0.399	0.308	0.379	0.317
	0.049	0.064	0.165	0.098	0.093	0.087	0.064	0.102	0.224	0.137	0.142	0.128
1.0	0.490	0.768	0.911	0.896	0.979	0.969	0.502	0.769	0.911	0.896	0.983	0.973
	0.122	0.162	0.340	0.258	0.311	0.291	0.182	0.269	0.475	0.383	0.505	0.467
	0.074	0.767	0.676	0.614	0.728	0.701	0.118	0.769	0.751	0.689	0.828	0.802
	0.051	0.158	0.244	0.165	0.143	0.129	0.083	0.271	0.365	0.265	0.308	0.273
1.5	0.876	0.979	0.998	0.997	1.000	1.000	0.872	0.980	0.999	0.999	1.000	1.000
	0.226	0.299	0.558	0.469	0.597	0.581	0.338	0.514	0.749	0.695	0.848	0.836
	0.057	0.979	0.862	0.838	0.907	0.958	0.117	0.980	0.935	0.920	0.977	0.989
	0.045	0.294	0.332	0.243	0.195	0.194	0.082	0.515	0.538	0.454	0.496	0.514
2.0	0.995	1.000	1.000	1.000	1.000	1.000	0.994	0.999	1.000	1.000	1.000	1.000
	0.392	0.478	0.751	0.713	0.851	0.860	0.579	0.740	0.915	0.905	0.979	0.982
	0.057	1.000	0.949	0.947	0.965	0.999	0.158	0.999	0.985	0.986	0.999	1.000
	0.042	0.473	0.410	0.317	0.243	0.278	0.109	0.741	0.689	0.625	0.667	0.754
3.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.792	0.778	0.964	0.957	0.994	0.996	0.946	0.962	0.999	0.999	1.000	1.000
	0.134	1.000	0.994	0.993	0.997	1.000	0.342	1.000	1.000	1.000	1.000	1.000
	0.072	0.776	0.525	0.415	0.281	0.472	0.254	0.962	0.890	0.859	0.886	0.982
4.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.981	0.935	0.999	0.999	1.000	1.000	0.999	0.996	1.000	1.000	1.000	1.000
	0.279	1.000	1.000	1.000	1.000	1.000	0.487	0.996	0.964	0.953	0.961	1.000
	0.148	0.934	0.631	0.518	0.305	0.666	0.612	1.000	1.000	1.000	1.000	1.000
5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	0.985	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.471	1.000	1.000	1.000	1.000	1.000	0.842	1.000	1.000	1.000	1.000	1.000
	0.259	0.985	0.701	0.582	0.344	0.807	0.746	1.000	0.988	0.980	0.988	1.000
6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.664	1.000	1.000	1.000	1.000	1.000	0.956	1.000	1.000	1.000	1.000	1.000
	0.403	0.998	0.758	0.617	0.375	0.895	0.901	1.000	0.997	0.995	0.997	1.000

Table A.1.2: Individual Measurements Charts for the Mean

δ	c = 20						c = 40					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.0	0.040	0.030	0.098	0.052	0.041	0.042	0.039	0.029	0.093	0.051	0.038	0.042
	0.042	0.033	0.093	0.051	0.040	0.043	0.041	0.025	0.093	0.051	0.040	0.041
	0.044	0.030	0.095	0.047	0.041	0.042	0.043	0.029	0.094	0.053	0.040	0.045
	0.045	0.033	0.091	0.052	0.038	0.042	0.040	0.025	0.092	0.053	0.040	0.044
0.5	0.171	0.270	0.479	0.383	0.513	0.449	0.182	0.269	0.471	0.378	0.512	0.457
	0.103	0.140	0.296	0.210	0.248	0.222	0.131	0.173	0.341	0.253	0.331	0.289
	0.116	0.270	0.425	0.333	0.421	0.360	0.136	0.269	0.437	0.341	0.441	0.380
	0.083	0.140	0.278	0.191	0.210	0.183	0.108	0.173	0.318	0.231	0.284	0.250
1.0	0.515	0.774	0.912	0.889	0.986	0.978	0.497	0.774	0.904	0.892	0.984	0.977
	0.252	0.412	0.620	0.542	0.720	0.674	0.320	0.539	0.730	0.683	0.846	0.812
	0.175	0.774	0.798	0.746	0.906	0.888	0.259	0.774	0.841	0.810	0.945	0.931
	0.119	0.412	0.526	0.424	0.538	0.493	0.183	0.539	0.658	0.594	0.750	0.715
1.5	0.870	0.981	0.997	0.998	1.000	1.000	0.874	0.980	0.998	0.998	1.000	1.000
	0.485	0.717	0.886	0.868	0.970	0.964	0.638	0.865	0.959	0.956	0.997	0.995
	0.237	0.981	0.970	0.967	0.997	1.000	0.408	0.980	0.986	0.986	1.000	1.000
	0.172	0.717	0.751	0.709	0.838	0.848	0.308	0.865	0.908	0.891	0.975	0.976
2.0	0.993	0.999	1.000	1.000	1.000	1.000	0.994	0.999	1.000	1.000	1.000	1.000
	0.768	0.919	0.987	0.987	1.000	1.000	0.901	0.979	0.999	0.999	1.000	1.000
	0.334	0.999	0.997	0.998	1.000	1.000	0.571	0.999	1.000	1.000	1.000	1.000
	0.253	0.919	0.915	0.903	0.969	0.986	0.462	0.979	0.989	0.988	1.000	1.000
3.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.994	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.623	1.000	1.000	1.000	1.000	1.000	0.875	1.000	1.000	1.000	1.000	1.000
	0.516	0.997	0.994	0.993	0.999	1.000	0.804	1.000	1.000	1.000	1.000	1.000
4.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.887	1.000	1.000	1.000	1.000	1.000	0.989	1.000	1.000	1.000	1.000	1.000
	0.816	1.000	1.000	1.000	1.000	1.000	0.977	1.000	1.000	1.000	1.000	1.000
5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.986	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.962	1.000	1.000	1.000	1.000	1.000	0.998	1.000	1.000	1.000	1.000	1.000
6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table A.1.3: Individual Measurements Charts for the Mean

δ	c = 70						c = 100					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.0	0.039	0.033	0.094	0.049	0.041	0.045	0.043	0.026	0.105	0.047	0.041	0.040
	0.039	0.031	0.093	0.049	0.039	0.040	0.043	0.027	0.099	0.048	0.036	0.040
	0.040	0.033	0.093	0.051	0.038	0.040	0.042	0.026	0.101	0.048	0.039	0.041
	0.039	0.031	0.091	0.047	0.037	0.038	0.042	0.027	0.100	0.051	0.036	0.038
0.5	0.168	0.282	0.468	0.381	0.520	0.456	0.174	0.276	0.473	0.386	0.517	0.457
	0.137	0.214	0.391	0.301	0.394	0.344	0.153	0.228	0.410	0.320	0.425	0.364
	0.138	0.282	0.450	0.349	0.480	0.406	0.150	0.276	0.458	0.366	0.486	0.426
	0.113	0.214	0.377	0.281	0.359	0.318	0.133	0.228	0.394	0.308	0.394	0.337
1.0	0.506	0.777	0.916	0.897	0.982	0.975	0.495	0.774	0.914	0.903	0.982	0.975
	0.380	0.637	0.799	0.767	0.916	0.892	0.394	0.661	0.850	0.818	0.939	0.917
	0.329	0.777	0.876	0.847	0.961	0.950	0.352	0.773	0.892	0.866	0.970	0.958
	0.259	0.637	0.754	0.709	0.875	0.837	0.295	0.661	0.812	0.773	0.910	0.891
1.5	0.869	0.976	0.998	0.998	1.000	1.000	0.864	0.977	0.997	0.996	1.000	1.000
	0.714	0.917	0.983	0.983	0.999	0.999	0.748	0.940	0.990	0.989	1.000	1.000
	0.549	0.976	0.993	0.991	1.000	1.000	0.614	0.977	0.994	0.993	1.000	1.000
	0.446	0.917	0.966	0.961	0.997	0.997	0.532	0.940	0.977	0.976	0.999	0.999
2.0	0.994	1.000	1.000	1.000	1.000	1.000	0.991	1.000	1.000	1.000	1.000	1.000
	0.945	0.994	1.000	1.000	1.000	1.000	0.968	0.996	1.000	1.000	1.000	1.000
	0.751	1.000	1.000	1.000	1.000	1.000	0.850	1.000	1.000	1.000	1.000	1.000
	0.648	0.994	0.998	0.998	1.000	1.000	0.771	0.996	1.000	1.000	1.000	1.000
3.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.979	1.000	1.000	1.000	1.000	1.000	0.996	1.000	1.000	1.000	1.000	1.000
	0.951	1.000	1.000	1.000	1.000	1.000	0.988	1.000	1.000	1.000	1.000	1.000
4.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table A.2.1: Charts for the Mean, Subgroups of Size n = 2

δ	c = 1						c = 5					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.0	0.022	0.010	0.051	0.026	0.021	0.023	0.022	0.014	0.048	0.026	0.019	0.019
	0.020	0.009	0.053	0.023	0.017	0.019	0.023	0.013	0.047	0.025	0.021	0.019
	0.019	0.010	0.058	0.036	0.024	0.030	0.019	0.014	0.057	0.035	0.027	0.034
	0.020	0.009	0.065	0.039	0.027	0.034	0.020	0.013	0.058	0.036	0.027	0.032
0.5	0.147	0.238	0.417	0.366	0.503	0.459	0.153	0.245	0.428	0.363	0.514	0.473
	0.051	0.043	0.131	0.076	0.089	0.084	0.054	0.073	0.171	0.118	0.137	0.119
	0.079	0.238	0.414	0.368	0.428	0.391	0.096	0.245	0.424	0.372	0.450	0.415
	0.032	0.043	0.149	0.099	0.091	0.095	0.040	0.073	0.177	0.136	0.133	0.131
1.0	0.584	0.743	0.933	0.926	0.989	0.989	0.580	0.735	0.924	0.921	0.989	0.987
	0.119	0.112	0.320	0.238	0.305	0.307	0.173	0.215	0.438	0.368	0.477	0.474
	0.291	0.743	0.906	0.895	0.949	0.937	0.312	0.735	0.898	0.888	0.950	0.944
	0.064	0.112	0.330	0.257	0.258	0.256	0.094	0.215	0.431	0.367	0.381	0.390
1.5	0.962	0.960	1.000	1.000	1.000	1.000	0.960	0.954	0.998	0.999	1.000	1.000
	0.284	0.265	0.563	0.483	0.648	0.668	0.408	0.422	0.734	0.691	0.847	0.863
	0.584	0.960	0.996	0.997	1.000	1.000	0.636	0.954	0.998	0.998	1.000	1.000
	0.101	0.265	0.530	0.467	0.476	0.489	0.174	0.422	0.695	0.647	0.710	0.722
2.0	1.000	0.995	1.000	1.000	1.000	1.000	1.000	0.995	1.000	1.000	1.000	1.000
	0.532	0.401	0.787	0.736	0.899	0.924	0.709	0.648	0.926	0.913	0.986	0.990
	0.848	0.995	1.000	1.000	1.000	1.000	0.878	0.995	1.000	1.000	1.000	1.000
	0.154	0.401	0.741	0.686	0.717	0.742	0.297	0.648	0.896	0.879	0.916	0.936
3.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.938	0.708	0.976	0.966	0.999	1.000	0.990	0.881	0.998	0.998	1.000	1.000
	0.996	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000
	0.336	0.708	0.956	0.943	0.957	0.975	0.609	0.881	0.997	0.996	0.998	0.999
4.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.999	0.877	0.999	0.999	1.000	1.000	1.000	0.976	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.534	0.877	0.997	0.996	0.998	1.000	0.843	0.976	1.000	1.000	1.000	1.000
5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	0.953	1.000	1.000	1.000	1.000	1.000	0.996	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.718	0.953	1.000	1.000	1.000	1.000	0.963	0.996	1.000	1.000	1.000	1.000
6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	0.984	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.855	0.984	1.000	1.000	1.000	1.000	0.994	0.999	1.000	1.000	1.000	1.000

Table A.2.2: Charts for the Mean, Subgroups of Size n = 2

δ	c = 10						c = 20					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.0	0.019	0.013	0.052	0.027	0.020	0.020	0.018	0.018	0.051	0.023	0.019	0.018
	0.020	0.015	0.052	0.025	0.021	0.021	0.017	0.020	0.049	0.024	0.019	0.020
	0.018	0.013	0.057	0.031	0.022	0.022	0.020	0.018	0.057	0.028	0.023	0.026
	0.016	0.015	0.059	0.033	0.022	0.028	0.020	0.020	0.053	0.027	0.023	0.026
0.5	0.160	0.245	0.439	0.374	0.521	0.478	0.151	0.247	0.421	0.362	0.503	0.461
	0.083	0.118	0.226	0.168	0.215	0.196	0.095	0.153	0.289	0.231	0.295	0.267
	0.115	0.245	0.435	0.371	0.467	0.431	0.115	0.247	0.410	0.360	0.451	0.410
	0.064	0.118	0.231	0.181	0.204	0.187	0.075	0.153	0.278	0.225	0.268	0.246
1.0	0.581	0.745	0.929	0.924	0.990	0.988	0.598	0.742	0.924	0.922	0.989	0.989
	0.245	0.366	0.603	0.553	0.710	0.705	0.362	0.511	0.747	0.712	0.871	0.868
	0.365	0.745	0.915	0.908	0.970	0.965	0.443	0.742	0.911	0.910	0.973	0.971
	0.159	0.366	0.586	0.541	0.616	0.617	0.273	0.511	0.728	0.700	0.812	0.813
1.5	0.958	0.961	0.999	0.999	1.000	1.000	0.958	0.951	0.999	1.000	1.000	1.000
	0.579	0.667	0.900	0.894	0.977	0.983	0.747	0.821	0.968	0.970	0.998	0.999
	0.715	0.961	0.998	0.998	1.000	1.000	0.812	0.951	0.999	1.000	1.000	1.000
	0.332	0.667	0.875	0.866	0.926	0.938	0.553	0.821	0.956	0.960	0.991	0.993
2.0	1.000	0.995	1.000	1.000	1.000	1.000	1.000	0.996	1.000	1.000	1.000	1.000
	0.879	0.866	0.989	0.991	0.999	1.000	0.970	0.951	1.000	1.000	1.000	1.000
	0.937	0.995	1.000	1.000	1.000	1.000	0.978	0.996	1.000	1.000	1.000	1.000
	0.556	0.866	0.984	0.985	0.996	0.998	0.830	0.951	0.999	0.999	1.000	1.000
3.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	0.984	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.910	0.984	1.000	1.000	1.000	1.000	0.996	0.999	1.000	1.000	1.000	1.000
4.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.994	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table A.2.3: Charts for the Mean, Subgroups of Size n = 2

δ	c = 35						c = 50					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.0	0.020	0.015	0.057	0.031	0.019	0.023	0.021	0.016	0.051	0.025	0.022	0.022
	0.021	0.015	0.058	0.031	0.020	0.025	0.019	0.015	0.055	0.028	0.022	0.021
	0.020	0.015	0.059	0.033	0.022	0.026	0.021	0.016	0.051	0.026	0.022	0.021
	0.021	0.015	0.059	0.034	0.022	0.025	0.018	0.015	0.051	0.027	0.022	0.022
0.5	0.147	0.228	0.430	0.366	0.497	0.454	0.154	0.233	0.430	0.364	0.501	0.469
	0.106	0.162	0.331	0.268	0.352	0.323	0.127	0.187	0.351	0.294	0.400	0.364
	0.123	0.228	0.414	0.354	0.461	0.418	0.137	0.233	0.419	0.359	0.487	0.438
	0.092	0.162	0.318	0.260	0.331	0.303	0.116	0.187	0.343	0.286	0.375	0.344
1.0	0.577	0.738	0.931	0.928	0.991	0.990	0.595	0.746	0.929	0.931	0.990	0.990
	0.413	0.590	0.823	0.808	0.940	0.935	0.474	0.636	0.851	0.844	0.958	0.953
	0.467	0.738	0.918	0.914	0.981	0.977	0.507	0.746	0.915	0.915	0.983	0.982
	0.334	0.590	0.805	0.796	0.905	0.903	0.402	0.636	0.837	0.828	0.937	0.935
1.5	0.958	0.961	0.999	0.999	1.000	1.000	0.963	0.959	0.999	1.000	1.000	1.000
	0.832	0.893	0.993	0.992	1.000	1.000	0.880	0.919	0.993	0.995	1.000	1.000
	0.861	0.961	0.999	0.999	1.000	1.000	0.901	0.959	0.999	0.999	1.000	1.000
	0.699	0.893	0.991	0.989	0.999	1.000	0.795	0.919	0.991	0.993	1.000	1.000
2.0	1.000	0.994	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	1.000	1.000
	0.991	0.977	1.000	1.000	1.000	1.000	0.997	0.985	1.000	1.000	1.000	1.000
	0.991	0.994	1.000	1.000	1.000	1.000	0.995	0.997	1.000	1.000	1.000	1.000
	0.937	0.977	1.000	1.000	1.000	1.000	0.977	0.985	1.000	1.000	1.000	1.000
3.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table A.3.1: Charts for the Mean, Subgroups of Size n = 5

δ	c = 1						c = 2					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.0	0.008	0.000	0.017	0.005	0.006	0.006	0.007	0.000	0.020	0.008	0.007	0.007
	0.008	0.000	0.015	0.005	0.004	0.005	0.007	0.000	0.017	0.005	0.004	0.005
	0.008	0.000	0.020	0.007	0.009	0.011	0.007	0.000	0.023	0.008	0.008	0.010
	0.010	0.000	0.017	0.007	0.006	0.007	0.008	0.000	0.018	0.005	0.007	0.008
0.5	0.162	0.000	0.404	0.341	0.447	0.443	0.174	0.000	0.409	0.358	0.447	0.455
	0.032	0.000	0.054	0.026	0.027	0.030	0.047	0.000	0.109	0.065	0.072	0.081
	0.107	0.000	0.394	0.336	0.397	0.391	0.121	0.000	0.398	0.347	0.406	0.405
	0.022	0.000	0.061	0.034	0.035	0.036	0.037	0.000	0.115	0.069	0.071	0.082
1.0	0.780	0.000	0.943	0.941	0.992	0.994	0.772	0.000	0.938	0.935	0.991	0.992
	0.121	0.000	0.175	0.107	0.159	0.183	0.206	0.000	0.358	0.278	0.375	0.423
	0.544	0.000	0.926	0.924	0.975	0.978	0.571	0.000	0.931	0.926	0.978	0.982
	0.062	0.000	0.176	0.110	0.130	0.146	0.134	0.000	0.360	0.281	0.325	0.367
1.5	0.998	0.000	1.000	1.000	1.000	1.000	0.998	0.000	0.999	0.999	1.000	1.000
	0.340	0.000	0.338	0.248	0.430	0.503	0.554	0.000	0.681	0.608	0.804	0.859
	0.924	0.000	0.999	0.999	1.000	1.000	0.948	0.000	0.999	0.999	1.000	1.000
	0.162	0.000	0.332	0.244	0.339	0.381	0.345	0.000	0.664	0.591	0.705	0.774
2.0	1.000	0.000	1.000	1.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000
	0.660	0.000	0.555	0.435	0.773	0.838	0.878	0.000	0.878	0.833	0.974	0.989
	0.998	0.000	1.000	1.000	1.000	1.000	0.999	0.000	1.000	1.000	1.000	1.000
	0.316	0.000	0.537	0.426	0.598	0.668	0.634	0.000	0.868	0.823	0.931	0.964
3.0	1.000	0.000	1.000	1.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000
	0.983	0.000	0.839	0.786	0.995	0.999	1.000	0.000	0.992	0.987	1.000	1.000
	1.000	0.000	1.000	1.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000
	0.728	0.000	0.831	0.771	0.942	0.974	0.964	0.000	0.991	0.986	0.999	1.000
4.0	1.000	0.000	1.000	1.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000
	0.999	0.000	0.967	0.938	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000
	1.000	0.000	1.000	1.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000
	0.952	0.000	0.963	0.932	0.997	0.999	1.000	0.000	1.000	1.000	1.000	1.000

Table A.3.2: Charts for the Mean, Subgroups of Size n = 5

δ	c = 4						c = 8					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.0	0.009	0.003	0.019	0.012	0.007	0.008	0.010	0.007	0.021	0.011	0.010	0.008
	0.007	0.002	0.017	0.008	0.005	0.007	0.009	0.005	0.022	0.012	0.009	0.011
	0.008	0.003	0.021	0.015	0.008	0.008	0.009	0.007	0.019	0.014	0.010	0.011
	0.009	0.002	0.018	0.009	0.007	0.009	0.009	0.005	0.026	0.014	0.012	0.014
0.5	0.165	0.053	0.404	0.355	0.449	0.457	0.170	0.063	0.417	0.368	0.469	0.475
	0.067	0.021	0.175	0.128	0.142	0.148	0.091	0.044	0.252	0.198	0.240	0.244
	0.130	0.053	0.399	0.353	0.414	0.420	0.145	0.063	0.413	0.363	0.436	0.444
	0.052	0.021	0.177	0.131	0.135	0.147	0.083	0.044	0.251	0.201	0.222	0.231
1.0	0.773	0.115	0.949	0.945	0.991	0.993	0.776	0.115	0.942	0.942	0.992	0.993
	0.321	0.062	0.588	0.527	0.647	0.702	0.462	0.096	0.757	0.733	0.844	0.884
	0.627	0.115	0.945	0.939	0.981	0.987	0.670	0.115	0.939	0.939	0.984	0.987
	0.247	0.062	0.577	0.525	0.589	0.646	0.383	0.096	0.745	0.718	0.803	0.853
1.5	0.998	0.120	0.999	0.999	1.000	1.000	0.998	0.122	1.000	1.000	1.000	1.000
	0.779	0.097	0.899	0.886	0.972	0.987	0.917	0.121	0.973	0.973	0.998	1.000
	0.968	0.120	1.000	0.999	1.000	1.000	0.987	0.122	0.999	0.999	1.000	1.000
	0.596	0.097	0.889	0.872	0.938	0.967	0.833	0.121	0.972	0.970	0.995	0.998
2.0	1.000	0.120	1.000	1.000	1.000	1.000	1.000	0.120	1.000	1.000	1.000	1.000
	0.981	0.117	0.988	0.986	1.000	1.000	0.998	0.126	0.999	0.999	1.000	1.000
	1.000	0.120	1.000	1.000	1.000	1.000	1.000	0.120	1.000	1.000	1.000	1.000
	0.902	0.117	0.985	0.982	0.998	1.000	0.990	0.126	0.998	0.998	1.000	1.000
3.0	1.000	0.123	1.000	1.000	1.000	1.000	1.000	0.123	1.000	1.000	1.000	1.000
	1.000	0.122	1.000	1.000	1.000	1.000	1.000	0.125	1.000	1.000	1.000	1.000
	1.000	0.123	1.000	1.000	1.000	1.000	1.000	0.123	1.000	1.000	1.000	1.000
	0.999	0.122	1.000	1.000	1.000	1.000	1.000	0.125	1.000	1.000	1.000	1.000
4.0	1.000	0.119	1.000	1.000	1.000	1.000	1.000	0.130	1.000	1.000	1.000	1.000
	1.000	0.126	1.000	1.000	1.000	1.000	1.000	0.125	1.000	1.000	1.000	1.000
	1.000	0.119	1.000	1.000	1.000	1.000	1.000	0.130	1.000	1.000	1.000	1.000
	1.000	0.126	1.000	1.000	1.000	1.000	1.000	0.125	1.000	1.000	1.000	1.000

Table A.3.3: Charts for the Mean, Subgroups of Size n = 5

δ	c = 14						c = 20					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.0	0.007	0.006	0.024	0.013	0.009	0.011	0.010	0.005	0.018	0.010	0.010	0.010
	0.007	0.005	0.019	0.010	0.009	0.009	0.012	0.005	0.018	0.010	0.010	0.007
	0.007	0.006	0.024	0.013	0.010	0.013	0.011	0.005	0.018	0.011	0.011	0.011
	0.008	0.005	0.024	0.011	0.010	0.013	0.011	0.005	0.019	0.011	0.010	0.008
0.5	0.162	0.060	0.402	0.355	0.444	0.462	0.176	0.065	0.406	0.358	0.463	0.473
	0.108	0.047	0.293	0.249	0.289	0.305	0.127	0.053	0.321	0.271	0.332	0.355
	0.146	0.060	0.400	0.355	0.430	0.441	0.164	0.065	0.400	0.351	0.443	0.455
	0.099	0.047	0.289	0.246	0.285	0.299	0.117	0.053	0.324	0.272	0.323	0.344
1.0	0.774	0.118	0.948	0.948	0.992	0.997	0.777	0.114	0.934	0.935	0.991	0.993
	0.565	0.107	0.839	0.827	0.929	0.949	0.626	0.105	0.868	0.859	0.952	0.964
	0.709	0.118	0.945	0.945	0.989	0.993	0.730	0.114	0.932	0.931	0.988	0.992
	0.494	0.107	0.836	0.829	0.913	0.932	0.574	0.105	0.865	0.858	0.941	0.959
1.5	0.997	0.131	1.000	0.999	1.000	1.000	0.998	0.125	0.999	1.000	1.000	1.000
	0.959	0.131	0.991	0.991	1.000	1.000	0.981	0.125	0.995	0.994	1.000	1.000
	0.991	0.131	1.000	0.999	1.000	1.000	0.993	0.125	0.999	0.999	1.000	1.000
	0.932	0.131	0.991	0.990	0.999	1.000	0.964	0.125	0.996	0.995	1.000	1.000
2.0	1.000	0.126	1.000	1.000	1.000	1.000	1.000	0.128	1.000	1.000	1.000	1.000
	1.000	0.127	1.000	1.000	1.000	1.000	1.000	0.129	1.000	1.000	1.000	1.000
	1.000	0.126	1.000	1.000	1.000	1.000	1.000	0.128	1.000	1.000	1.000	1.000
	0.999	0.127	1.000	1.000	1.000	1.000	1.000	0.129	1.000	1.000	1.000	1.000
3.0	1.000	0.124	1.000	1.000	1.000	1.000	1.000	0.126	1.000	1.000	1.000	1.000
	1.000	0.129	1.000	1.000	1.000	1.000	1.000	0.124	1.000	1.000	1.000	1.000
	1.000	0.124	1.000	1.000	1.000	1.000	1.000	0.126	1.000	1.000	1.000	1.000
	1.000	0.129	1.000	1.000	1.000	1.000	1.000	0.124	1.000	1.000	1.000	1.000
4.0	1.000	0.117	1.000	1.000	1.000	1.000	1.000	0.119	1.000	1.000	1.000	1.000
	1.000	0.121	1.000	1.000	1.000	1.000	1.000	0.126	1.000	1.000	1.000	1.000
	1.000	0.117	1.000	1.000	1.000	1.000	1.000	0.119	1.000	1.000	1.000	1.000
	1.000	0.121	1.000	1.000	1.000	1.000	1.000	0.126	1.000	1.000	1.000	1.000

Table A.4.1: Charts for the Standard Deviation, Subgroups of Size n=2

δ	c = 3						c = 5					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.1	0.183	1.000	1.000	1.000	1.000	1.000	0.184	1.000	1.000	1.000	0.997	1.000
	0.105	0.938	0.872	0.863	0.886	0.902	0.127	0.984	0.945	0.943	0.964	0.972
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.042	0.399	0.262	0.200	0.277	0.220	0.042	0.390	0.271	0.206	0.281	0.220
	0.031	0.123	0.140	0.090	0.103	0.088	0.033	0.167	0.170	0.117	0.136	0.109
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.004	0.001	0.012	0.004	0.003	0.003	0.002	0.000	0.006	0.001	0.001	0.001
0.9	0.022	0.022	0.062	0.037	0.031	0.029	0.020	0.022	0.064	0.038	0.026	0.026
	0.020	0.012	0.058	0.029	0.024	0.021	0.019	0.016	0.057	0.033	0.027	0.026
	0.006	0.004	0.022	0.008	0.005	0.006	0.006	0.007	0.026	0.010	0.003	0.004
	0.015	0.005	0.043	0.017	0.017	0.016	0.014	0.010	0.040	0.019	0.013	0.012
1.0	0.021	0.009	0.046	0.025	0.020	0.019	0.020	0.012	0.056	0.030	0.024	0.023
	0.019	0.009	0.050	0.026	0.020	0.021	0.021	0.009	0.056	0.031	0.021	0.022
	0.016	0.011	0.048	0.023	0.017	0.018	0.017	0.010	0.049	0.025	0.017	0.020
	0.020	0.011	0.053	0.022	0.021	0.019	0.019	0.011	0.052	0.027	0.016	0.018
1.5	0.013	0.001	0.017	0.007	0.003	0.005	0.014	0.001	0.021	0.009	0.005	0.010
	0.019	0.003	0.036	0.015	0.006	0.011	0.020	0.002	0.039	0.017	0.007	0.014
	0.395	0.082	0.355	0.286	0.491	0.493	0.401	0.084	0.355	0.286	0.496	0.493
	0.046	0.020	0.100	0.053	0.051	0.048	0.062	0.026	0.120	0.074	0.075	0.073
2.0	0.009	0.000	0.006	0.002	0.002	0.004	0.007	0.000	0.009	0.003	0.002	0.002
	0.016	0.003	0.031	0.013	0.006	0.010	0.013	0.001	0.025	0.012	0.004	0.007
	0.824	0.194	0.649	0.609	0.901	0.906	0.819	0.186	0.646	0.602	0.895	0.903
	0.070	0.027	0.132	0.080	0.086	0.084	0.129	0.037	0.177	0.120	0.153	0.159
3.0	0.009	0.000	0.004	0.002	0.000	0.002	0.005	0.000	0.003	0.001	0.001	0.001
	0.018	0.002	0.027	0.010	0.004	0.008	0.012	0.001	0.022	0.009	0.003	0.007
	0.995	0.411	0.911	0.907	0.997	0.998	0.993	0.391	0.906	0.896	0.994	0.998
	0.135	0.036	0.196	0.124	0.160	0.164	0.291	0.062	0.262	0.188	0.329	0.350
5.0	0.005	0.000	0.002	0.000	0.001	0.002	0.005	0.000	0.001	0.001	0.000	0.000
	0.015	0.002	0.023	0.008	0.003	0.010	0.015	0.000	0.019	0.007	0.001	0.007
	1.000	0.634	0.988	0.991	0.998	1.000	1.000	0.637	0.992	0.992	0.996	1.000
	0.274	0.044	0.258	0.176	0.296	0.313	0.584	0.070	0.363	0.271	0.588	0.663
10.0	0.002	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.000
	0.013	0.002	0.019	0.007	0.001	0.008	0.015	0.001	0.017	0.006	0.000	0.007
	1.000	0.841	1.000	0.999	0.999	1.000	1.000	0.825	0.999	1.000	0.996	1.000
	0.560	0.048	0.330	0.229	0.489	0.563	0.883	0.095	0.428	0.346	0.808	0.932

Table A.4.2: Charts for the Standard Deviation, Subgroups of Size n=2

δ	c = 10						c = 20					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.1	0.198	1.000	1.000	1.000	0.990	1.000	0.180	1.000	1.000	1.000	0.976	1.000
	0.154	0.999	0.993	0.992	0.989	0.999	0.161	1.000	0.999	0.999	0.979	1.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.036	0.403	0.270	0.211	0.271	0.223	0.040	0.388	0.262	0.207	0.275	0.232
	0.031	0.235	0.189	0.140	0.169	0.140	0.036	0.289	0.219	0.165	0.208	0.173
	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
	0.000	0.001	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.9	0.024	0.024	0.064	0.038	0.031	0.027	0.020	0.029	0.068	0.037	0.032	0.031
	0.024	0.022	0.058	0.032	0.027	0.026	0.019	0.027	0.063	0.032	0.030	0.028
	0.006	0.011	0.024	0.013	0.006	0.006	0.004	0.008	0.022	0.008	0.003	0.004
	0.012	0.012	0.030	0.016	0.012	0.011	0.010	0.011	0.027	0.010	0.008	0.007
1.0	0.023	0.016	0.051	0.028	0.023	0.023	0.021	0.013	0.050	0.024	0.020	0.021
	0.023	0.014	0.049	0.028	0.022	0.022	0.022	0.014	0.050	0.026	0.020	0.020
	0.018	0.014	0.051	0.025	0.019	0.021	0.019	0.017	0.047	0.023	0.020	0.018
	0.021	0.016	0.054	0.027	0.020	0.021	0.018	0.016	0.048	0.020	0.014	0.019
1.5	0.015	0.003	0.016	0.008	0.002	0.006	0.011	0.002	0.018	0.006	0.002	0.005
	0.019	0.004	0.028	0.013	0.004	0.008	0.014	0.003	0.024	0.010	0.005	0.005
	0.398	0.086	0.353	0.283	0.488	0.486	0.396	0.087	0.337	0.269	0.463	0.481
	0.112	0.042	0.157	0.102	0.133	0.137	0.181	0.052	0.201	0.137	0.216	0.224
2.0	0.011	0.001	0.011	0.004	0.002	0.002	0.009	0.000	0.008	0.004	0.001	0.002
	0.015	0.002	0.020	0.008	0.003	0.007	0.013	0.001	0.017	0.006	0.001	0.004
	0.817	0.197	0.629	0.599	0.883	0.893	0.823	0.181	0.638	0.607	0.876	0.901
	0.272	0.070	0.264	0.191	0.324	0.346	0.455	0.082	0.371	0.297	0.546	0.586
3.0	0.007	0.001	0.003	0.002	0.001	0.001	0.006	0.001	0.004	0.001	0.001	0.001
	0.013	0.001	0.015	0.006	0.001	0.005	0.011	0.001	0.010	0.004	0.001	0.004
	0.995	0.406	0.909	0.904	0.986	0.998	0.993	0.413	0.903	0.902	0.973	0.998
	0.583	0.107	0.429	0.360	0.677	0.730	0.820	0.158	0.574	0.520	0.886	0.928
5.0	0.003	0.000	0.001	0.000	0.000	0.000	0.004	0.000	0.001	0.000	0.000	0.001
	0.011	0.000	0.012	0.004	0.000	0.004	0.010	0.000	0.007	0.000	0.000	0.002
	1.000	0.630	0.990	0.993	0.991	1.000	1.000	0.654	0.987	0.992	0.978	1.000
	0.903	0.132	0.543	0.478	0.917	0.971	0.989	0.238	0.712	0.680	0.974	0.998
10.0	0.002	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000
	0.010	0.000	0.010	0.003	0.000	0.004	0.010	0.000	0.009	0.002	0.000	0.003
	1.000	0.833	1.000	0.999	0.989	1.000	1.000	0.833	0.999	1.000	0.978	1.000
	0.996	0.157	0.617	0.556	0.978	1.000	1.000	0.251	0.789	0.762	0.980	1.000

Table A.4.3: Charts for the Standard Deviation, Subgroups of Size n=2

δ	c = 35						c = 50					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.1	0.192	1.000	1.000	1.000	0.957	1.000	0.185	1.000	1.000	1.000	0.933	1.000
	0.178	1.000	1.000	1.000	0.956	1.000	0.173	1.000	1.000	1.000	0.935	1.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.035	0.401	0.269	0.202	0.268	0.218	0.039	0.397	0.260	0.194	0.261	0.219
	0.034	0.331	0.233	0.170	0.225	0.180	0.036	0.347	0.236	0.177	0.230	0.188
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.9	0.022	0.026	0.068	0.035	0.028	0.025	0.020	0.029	0.067	0.034	0.031	0.029
	0.021	0.028	0.064	0.033	0.027	0.026	0.020	0.027	0.065	0.034	0.030	0.029
	0.008	0.009	0.024	0.011	0.004	0.006	0.006	0.009	0.026	0.011	0.005	0.006
	0.009	0.013	0.028	0.014	0.006	0.008	0.006	0.008	0.027	0.013	0.006	0.006
1.0	0.018	0.012	0.051	0.026	0.017	0.019	0.022	0.015	0.054	0.030	0.020	0.021
	0.018	0.014	0.048	0.024	0.017	0.020	0.023	0.016	0.055	0.029	0.018	0.019
	0.019	0.015	0.049	0.026	0.018	0.019	0.017	0.015	0.045	0.024	0.020	0.020
	0.020	0.015	0.051	0.024	0.020	0.019	0.018	0.012	0.046	0.024	0.016	0.019
1.5	0.012	0.003	0.017	0.009	0.003	0.006	0.017	0.003	0.019	0.010	0.003	0.007
	0.013	0.003	0.019	0.009	0.002	0.007	0.018	0.003	0.023	0.011	0.003	0.008
	0.390	0.084	0.330	0.278	0.465	0.488	0.385	0.084	0.337	0.283	0.456	0.499
	0.240	0.061	0.240	0.183	0.280	0.298	0.269	0.063	0.265	0.204	0.325	0.349
2.0	0.011	0.001	0.009	0.002	0.002	0.003	0.010	0.001	0.008	0.004	0.001	0.003
	0.013	0.001	0.010	0.003	0.002	0.004	0.012	0.002	0.012	0.005	0.002	0.004
	0.829	0.194	0.643	0.603	0.864	0.907	0.822	0.202	0.646	0.606	0.829	0.895
	0.583	0.112	0.445	0.384	0.689	0.733	0.636	0.132	0.497	0.437	0.715	0.776
3.0	0.010	0.001	0.003	0.001	0.002	0.002	0.007	0.000	0.004	0.001	0.001	0.001
	0.014	0.000	0.008	0.002	0.001	0.005	0.008	0.000	0.007	0.002	0.001	0.002
	0.995	0.390	0.902	0.898	0.954	0.997	0.994	0.402	0.905	0.895	0.934	0.998
	0.927	0.201	0.697	0.647	0.924	0.978	0.956	0.243	0.739	0.707	0.928	0.989
5.0	0.002	0.001	0.002	0.000	0.000	0.000	0.004	0.000	0.002	0.001	0.000	0.000
	0.006	0.000	0.007	0.001	0.000	0.001	0.007	0.000	0.004	0.002	0.000	0.000
	1.000	0.634	0.988	0.990	0.961	1.000	1.000	0.635	0.991	0.992	0.936	1.000
	0.998	0.284	0.834	0.812	0.962	1.000	1.000	0.346	0.886	0.879	0.935	1.000
10.0	0.001	0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	0.007	0.000	0.004	0.000	0.000	0.001	0.006	0.000	0.002	0.000	0.000	0.001
	1.000	0.838	1.000	1.000	0.955	1.000	1.000	0.839	1.000	1.000	0.940	1.000
	1.000	0.353	0.901	0.883	0.957	1.000	1.000	0.425	0.922	0.929	0.940	1.000

Table A.5.1: Charts for the Standard Deviation, Subgroups of Size n=5

δ	c = 1						c = 2					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.1	1.000	0.000	1.000	1.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000
	0.880	0.000	0.986	0.984	0.985	0.989	0.970	0.000	1.000	1.000	1.000	1.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.116	0.000	0.796	0.775	0.796	0.818	0.115	0.000	0.795	0.772	0.796	0.821
	0.043	0.000	0.215	0.161	0.166	0.175	0.057	0.000	0.354	0.291	0.271	0.299
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.003	0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000
0.9	0.012	0.000	0.040	0.018	0.017	0.015	0.010	0.000	0.047	0.023	0.019	0.016
	0.011	0.000	0.021	0.008	0.008	0.009	0.009	0.000	0.030	0.012	0.011	0.010
	0.001	0.000	0.003	0.000	0.000	0.000	0.001	0.000	0.007	0.001	0.001	0.001
	0.006	0.000	0.009	0.003	0.003	0.004	0.004	0.000	0.010	0.003	0.003	0.004
1.0	0.008	0.000	0.018	0.007	0.006	0.005	0.008	0.000	0.024	0.010	0.004	0.006
	0.008	0.000	0.014	0.005	0.004	0.004	0.009	0.000	0.019	0.009	0.006	0.007
	0.007	0.000	0.017	0.006	0.007	0.006	0.008	0.000	0.023	0.010	0.005	0.005
	0.008	0.000	0.016	0.004	0.004	0.004	0.008	0.000	0.017	0.006	0.005	0.005
1.5	0.001	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.002	0.000	0.000	0.001
	0.005	0.000	0.004	0.001	0.000	0.001	0.003	0.000	0.004	0.000	0.000	0.001
	0.448	0.000	0.446	0.392	0.614	0.622	0.459	0.000	0.450	0.393	0.624	0.642
	0.024	0.000	0.042	0.017	0.021	0.021	0.052	0.000	0.081	0.043	0.047	0.054
2.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.004	0.000	0.004	0.000	0.000	0.001	0.004	0.000	0.001	0.000	0.000	0.000
	0.923	0.000	0.828	0.809	0.970	0.973	0.928	0.000	0.832	0.825	0.974	0.979
	0.044	0.000	0.067	0.029	0.034	0.036	0.138	0.000	0.172	0.097	0.139	0.159
3.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.003	0.000	0.004	0.000	0.000	0.001	0.001	0.000	0.002	0.000	0.000	0.000
	1.000	0.000	0.983	0.984	1.000	1.000	1.000	0.000	0.986	0.986	0.999	1.000
	0.123	0.000	0.105	0.055	0.088	0.097	0.417	0.000	0.299	0.200	0.404	0.500
5.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.004	0.000	0.002	0.000	0.000	0.000	0.002	0.000	0.001	0.000	0.000	0.000
	1.000	0.000	0.999	0.999	1.000	1.000	1.000	0.000	0.999	0.999	1.000	1.000
	0.343	0.000	0.150	0.077	0.192	0.244	0.849	0.000	0.371	0.261	0.739	0.894
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.004	0.000	0.002	0.000	0.000	0.001	0.002	0.000	0.001	0.000	0.000	0.000
	1.000	0.000	1.000	1.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000
	0.812	0.000	0.176	0.101	0.403	0.590	0.997	0.000	0.401	0.289	0.954	0.999

Table A.5.2: Charts for the Standard Deviation, Subgroups of Size n=5

δ	c = 4						c = 8					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.1	1.000	0.124	1.000	1.000	0.998	1.000	1.000	0.125	1.000	1.000	0.993	1.000
	0.996	0.127	1.000	1.000	1.000	1.000	1.000	0.128	1.000	1.000	0.993	1.000
	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.108	0.127	0.794	0.772	0.788	0.815	0.114	0.124	0.790	0.771	0.781	0.826
	0.063	0.080	0.487	0.446	0.412	0.448	0.079	0.106	0.602	0.567	0.553	0.600
	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.9	0.012	0.008	0.045	0.029	0.019	0.020	0.012	0.016	0.050	0.030	0.019	0.019
	0.010	0.003	0.036	0.022	0.017	0.017	0.012	0.012	0.039	0.024	0.020	0.019
	0.001	0.000	0.006	0.003	0.001	0.001	0.001	0.003	0.004	0.002	0.001	0.001
	0.004	0.001	0.011	0.008	0.004	0.003	0.003	0.004	0.007	0.003	0.002	0.002
1.0	0.010	0.002	0.019	0.009	0.008	0.007	0.009	0.006	0.019	0.010	0.010	0.009
	0.011	0.001	0.017	0.006	0.007	0.006	0.010	0.005	0.022	0.011	0.008	0.008
	0.009	0.003	0.019	0.010	0.007	0.007	0.008	0.006	0.022	0.010	0.008	0.008
	0.008	0.002	0.024	0.013	0.006	0.006	0.008	0.007	0.022	0.008	0.009	0.008
1.5	0.002	0.000	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.000	0.000
	0.003	0.000	0.003	0.001	0.001	0.001	0.002	0.000	0.002	0.001	0.000	0.000
	0.447	0.043	0.442	0.392	0.603	0.628	0.451	0.055	0.450	0.392	0.606	0.638
	0.104	0.012	0.160	0.106	0.132	0.138	0.185	0.031	0.227	0.168	0.256	0.283
2.0	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	0.003	0.000	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.000	0.001
	0.924	0.088	0.824	0.826	0.976	0.982	0.925	0.080	0.824	0.816	0.965	0.977
	0.323	0.025	0.310	0.256	0.415	0.468	0.558	0.049	0.489	0.438	0.706	0.781
3.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	1.000	0.119	0.987	0.987	0.997	1.000	1.000	0.111	0.986	0.984	0.994	1.000
	0.778	0.038	0.517	0.457	0.836	0.926	0.951	0.071	0.727	0.693	0.978	0.995
5.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	1.000	0.116	1.000	0.999	0.997	1.000	1.000	0.125	1.000	1.000	0.993	1.000
	0.990	0.043	0.618	0.544	0.982	1.000	1.000	0.091	0.831	0.796	0.996	1.000
10.0	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1.000	0.128	1.000	1.000	0.998	1.000	1.000	0.130	1.000	1.000	0.991	1.000
	1.000	0.054	0.655	0.604	0.999	1.000	1.000	0.100	0.867	0.845	0.993	1.000

Table A.5.3: Charts for the Standard Deviation, Subgroups of Size n=5

δ	c = 14						c = 20					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.1	1.000	0.123	1.000	1.000	0.985	1.000	1.000	0.127	1.000	1.000	0.978	1.000
	1.000	0.125	1.000	1.000	0.985	1.000	1.000	0.130	1.000	1.000	0.980	1.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.114	0.115	0.792	0.779	0.786	0.832	0.103	0.111	0.795	0.783	0.773	0.826
	0.094	0.105	0.673	0.655	0.632	0.686	0.091	0.104	0.707	0.686	0.662	0.720
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.9	0.012	0.013	0.044	0.027	0.024	0.021	0.009	0.015	0.043	0.023	0.022	0.018
	0.012	0.015	0.041	0.025	0.023	0.017	0.010	0.015	0.041	0.021	0.020	0.018
	0.001	0.002	0.006	0.002	0.000	0.001	0.001	0.001	0.006	0.002	0.001	0.001
	0.002	0.002	0.004	0.002	0.001	0.001	0.001	0.002	0.004	0.003	0.002	0.003
1.0	0.007	0.004	0.020	0.010	0.006	0.009	0.008	0.006	0.018	0.009	0.008	0.008
	0.006	0.004	0.023	0.012	0.008	0.009	0.008	0.006	0.018	0.009	0.010	0.010
	0.007	0.006	0.020	0.009	0.009	0.007	0.008	0.005	0.022	0.010	0.007	0.008
	0.007	0.005	0.018	0.008	0.009	0.008	0.010	0.004	0.023	0.011	0.008	0.008
1.5	0.002	0.000	0.001	0.001	0.000	0.001	0.002	0.001	0.001	0.001	0.000	0.000
	0.003	0.000	0.001	0.001	0.001	0.000	0.003	0.000	0.001	0.001	0.000	0.001
	0.443	0.048	0.444	0.397	0.599	0.630	0.441	0.045	0.444	0.396	0.601	0.630
	0.255	0.038	0.290	0.238	0.365	0.400	0.287	0.036	0.317	0.271	0.412	0.454
2.0	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
	0.001	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	0.923	0.088	0.832	0.825	0.962	0.981	0.929	0.085	0.839	0.827	0.954	0.979
	0.711	0.062	0.618	0.577	0.847	0.893	0.788	0.068	0.665	0.638	0.886	0.930
3.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	1.000	0.118	0.983	0.983	0.985	1.000	1.000	0.120	0.987	0.988	0.974	1.000
	0.989	0.092	0.852	0.844	0.987	0.999	0.995	0.101	0.903	0.903	0.978	1.000
5.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1.000	0.121	1.000	1.000	0.988	1.000	1.000	0.129	0.999	1.000	0.975	1.000
	1.000	0.098	0.936	0.931	0.986	1.000	1.000	0.108	0.970	0.969	0.981	1.000
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1.000	0.125	1.000	1.000	0.987	1.000	1.000	0.118	1.000	1.000	0.978	1.000
	1.000	0.102	0.956	0.954	0.988	1.000	1.000	0.106	0.982	0.983	0.979	1.000

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