ABSTRACT

JONES, HERBERT LEE. A Fractional Integration Framework for Pricing Weather Derivatives in the United States. (Under the direction of Brandon M. McConnell and Negash Medhin.)

Firms in agriculture, energy and tourism, among other fields, are exposed to weather risk and seek to hedge their exposure to that risk. Weather derivatives based on temperature observations are an alternative to insurance to provide such a hedge. In this dissertation, we develop a temperature index based degree day forecasting framework that captures the trend, seasonality and persistence of the series of daily maximum and daily minimum temperatures for cities in the US for which exchange-traded weather derivatives are offered. We examine the effect of including one-day and two-day external temperature forecasts on degree day forecasts over a month. We then provide an example of hedging temperature risk with degree day future contracts through the use of the degree day forecasting framework.

We find that a degree day forecasting framework that combines linear and seasonal trends with a fractional ARIMA model is sensitive to the addition of external temperature forecasts. We also find that the framework is suitable for temperature risk hedging through the use of degree day future contracts on cities in the United States.

A Fractional Integration Framework for Pricing Weather Derivatives in the United States

by
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DEDICATION

To my mother, my wife, and all of the teachers and friends who have helped me get to where I am today.

BIOGRAPHY

Herbert Lee Jones, Jr. was born in Lansing, Michigan. He received his B.S. in mathematics from Wayne State University in April 2013 after which he began the Ph.D. program in operations research at North Carolina State University. His research interests include stochastic modeling, simulation and risk management.

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Chapter 1

Introduction

This portion of the document serves to provide contextual reference and foundation for the dissertation. Weather risk is defined and some examples of its importance in risk management are given. Weather derivatives traded on the Chicago Mercantile Exchange (CME) are introduced along with some discussion of the challenges in pricing those derivatives. The introduction concludes with a detail of the structure of the dissertation.

1.1 Weather risk

Organizations strive to minimize their exposure to risk while attempting to achieve their objectives. The U.S. Department of Energy has estimated that one-seventh of the U.S. economy is subject to weather risk. For companies in many industries, understanding and hedging their exposure to weather risk is an important component of their risk management strategy [24]. There are many classes of weather risk, such as risk due to wind, rain, snow, hurricane, cloud cover and earthquake [16, 34, 50].

Accurate quantification of weather risk is a challenge both for entities attempting to hedge their exposure to such risk and insurance providers that offer products in this area. Hedge opportunities can be in the form of products designed for events with low probability and high risk, such as insurance and catastrophe bonds, or products designed for events with low risk and high probability, that is weather derivatives. For this dissertation, use of the term *weather derivatives* refers to futures contracts on temperature outcomes and option contracts on those futures.

1.1.1 Parties exposed to weather risk

As mentioned before, organizations seek to minimize their risk exposure. Multiple industries have particularly high exposure to risk in the form of daily temperatures.

Energy producers are clearly exposed to weather risk. Warmer days can lead to an increase in energy consumption due to customers turning on air conditioning; cooler days can lead to an increase in energy consumption to due to customers turning on heating systems. Electricity is difficult and expensive to store; given that, producers attempt to accurately forecast the demand on their systems in order to produce sufficient output at a minimum cost [44, 69].

Agricultural output and quality may be affected by temperatures. Non-catastrophic but unusual temperatures can have an impact on crop yields, quality of specimens and costs of operation for industrial agricultural outfits. As such, agricultural firms seek to hedge their risk to weather phenomena, including unseasonably warm or cold daily temperatures [23, 43, 70].

The retail and tourism sectors are also sensitive to weather risk. Sites that rely on lower temperatures, such as ski resorts, stand to bear substantial revenue loss if their facilities do not feature a sufficient amount of snow. Additionally, unseasonably warm temperatures can lead to a decrease in traffic for destinations [27, 60, 65].

Compared to the situation for large industries, that of small- to medium-sized enterprises with exposure to weather risk is exacerbated by their relative lack of sophisticated forecasting and pricing resources [7].

1.2 Managing weather risk

Parties that are exposed to weather risk face a unique challenge in that weather, per se, is not tradable on the market. A party may choose to hold reserves to cover potential weather risk losses, purchase insurance against losses incurred by weather risk, or purchase products on the market that can, in turn, be used to hedge their weather risk [34]. In the following subsection we discuss weather derivatives, a market-traded product that can be and is used by parties to hedge weather risk.

1.2.1 Weather derivatives

A derivative is defined as a financial instrument whose value depends on the values of other, more basic, underlying variables [33]. Some commonly encountered derivatives are forward contracts (forwards), futures, and options. A forward is an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price. A future is similar to a forward, however the types of contracts are standardized and futures are sold on an exchange. An option gives the holder the right, but not the obligation, to buy (in the case of a *call option*) or sell (in the case of a *put option*) the underlying asset of the option by a certain date referred to as the strike date for a certain price referred to as the strike price. Options are traded on an exchange or in the over-the-counter market. By their nature, options allow an investor higher leverage opportunities than those offered by outright purchase of the underlying asset.

In particular, weather derivatives are instruments whose value depends on weather-related phenomena such as daily temperature, rainfall, snowfall or hurricanes. Such derivatives may be purchased by speculative investors or parties looking to reduce their exposure to risk due to weather-related events [54].

Parties exposed to temperature-related weather risk may find weather derivatives more attractive than weather insurance. The holder of an insurance contract must prove that they have incurred a financial loss to seek compensation. Proving such a weather-related incurred loss can be difficult. Parties looking to hedge their temperature-related weather risk have the opportunity to benefit from more advantageous pricing on the market compared to prices offered by companies that offer weather risk insurance [36, 62].

Investors may buy weather derivatives in seeking opportunities of a projected risk vs. return profile. Additionally, buying weather derivatives may be even more attractive to an investor who seeks opportunities that are uncorrelated with other holdings in their portfolio [34].

Market-exchanged weather derivatives based on temperature outcomes are futures contracts (futures) or option contracts (options) on such futures. Such products were introduced in 1996 and have been traded on the Chicago Mercantile Exchange (CME) since 1999[36]. Figure 1.1 shows weather futures and options monthly trading volume on the CME from January 2019 to November 2020.

Two of the most popular types of weather derivatives traded on the CME are heating degree day (HDD) and cooling degree day (CDD) futures contracts for several cities in

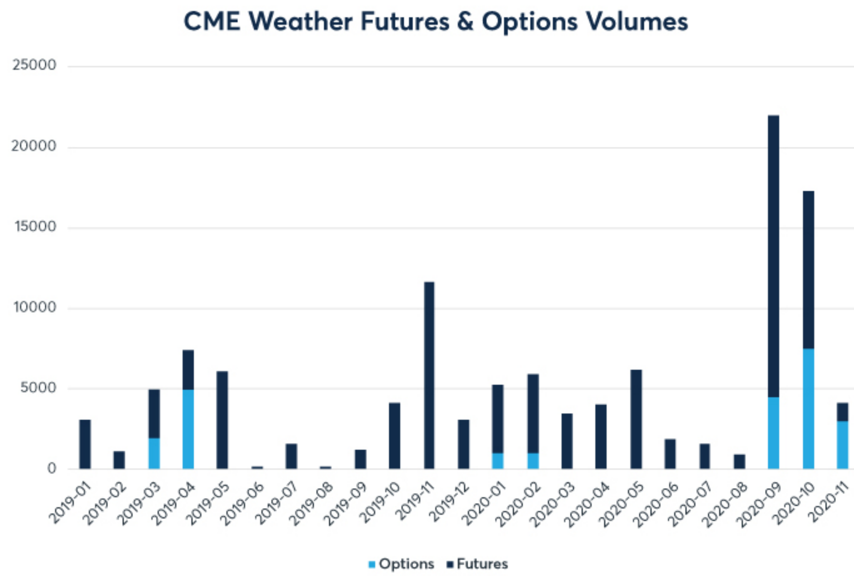

Figure 1.1: CME Weather futures and options monthly trading volume, 2019-2020 [66].

the United States. For some given date, let T_{max} represent the maximum temperature in degrees Fahrenheit, T_{min} represent the minimum temperature in degrees Fahrenheit, and let T represent the average of said maximum temperature and minimum temperatures. That is,

$$T = \frac{T_{max} + T_{min}}{2}. \tag{1.1}$$

Given a particular day and the average temperature T for that day, a heating degree day (HDD) is defined by

$$\text{HDD} = \max(65 - T, 0) \tag{1.2}$$

and a cooling degree day (CDD) is defined by

$$\text{CDD} = \max(T - 65, 0). \tag{1.3}$$

The following weather futures are offered on the CME:

- *Seasonal Strip Weather Heating Degree Day*, with minimum contract length of two consecutive months and maximum contract length of seven consecutive calendar months spanning from October to April;

- *Seasonal Strip Weather Cooling Degree Day*, with minimum contract length of two consecutive months and maximum contract length of seven consecutive calendar months spanning from April to October;
- *Monthly Weather Heating Degree Day*, offered for November, December, January, February and March plus October and April; and
- *Monthly Weather Cooling Degree Day*, offered for May, June, July, August and September.

The listed contracts are currently available for each of the following U.S. cities: Atlanta, Chicago, Cincinnati, Dallas, Las Vegas, Minneapolis, New York, Portland and Sacramento as seen in Figure 1.2.

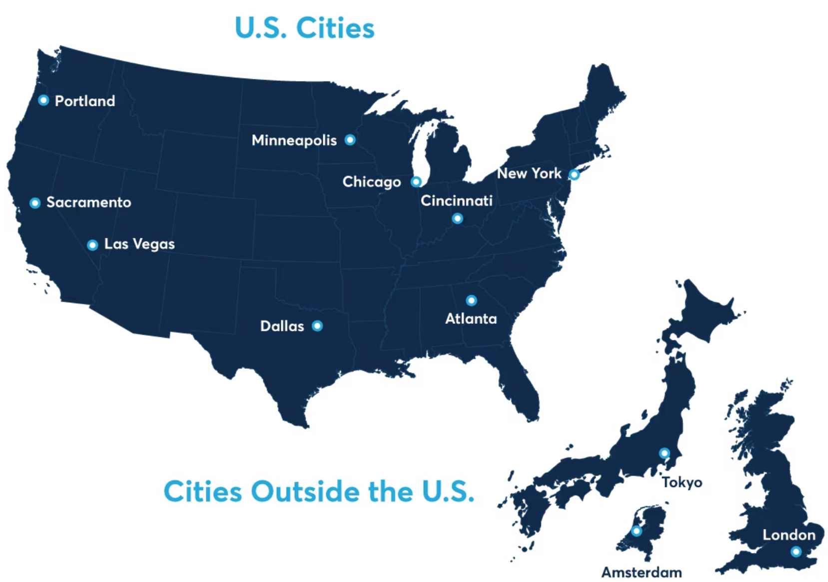

Figure 1.2: CME weather derivative cities [66].

Let HDD_i and CDD_i represent the number of heating degree days and cooling degree days of a weather derivative on day i of its duration, respectively. In general, the payoff at expiry for a heating degree day future contract of length n days and tick size   is

$$\sum_{i=1}^{n} \mathrm{HDD}_i \tag{1.4}$$

and the payoff at expiry for a CDD contract of length n days and tick size   is

$$\sum_{i=1}^{n} \mathrm{CDD}_i. \tag{1.5}$$

Heating degree day and cooling degree day future contracts offered on the CME feature a tick size of   = \$20. A long position in such a heating (cooling) contract pays \$20 per HDD (CDD) for the accumulated HDD (CDD) over the specified time period. European-style options on heating degree day and cooling degree day futures are offered on the CME as well. These options give the buyer the option, but not the obligation, to purchase (in the case of a call option) or sell (in the case of a put option) a weather future contract at a specified date and strike price.

As an example, consider a Monthly Weather CDD future contract for June 2016 in Atlanta. According to the CME U.S. Monthly Temperature File for Atlanta, over the period June 2016 there were 479.0 total CDD that month for the city. The total CDD for that month may be computed by summing the entries in the column labeled CDD of Table 1.1. Thus, a long position in the CDD future contract for Atlanta over June 2016 paid \$20   479.0, or \$9,580 after the month ended.

For years 2014 to 2018, the HDD for Atlanta are presented graphically in Figure 1.3 and the CDD are presented in Figure 1.4.

Municipalities and parties in agriculture [64] and tourism [27, 60] have shown increased interest in incorporating weather derivatives into their hedging strategies. However, the parties associated most with trading of weather derivatives are energy producers [8]. Through the use of weather derivatives, an energy producer may hedge against exposure to financial risk associated with having to meet unexpectedly high energy demands due to very warm or very cold days. Speculative investors also may explore weather derivatives as a component of their overall strategy. More generally, weather derivatives offer higher flexibility over extreme event weather insurance for all parties involved [1]. Weather derivatives have become more popular in recent year, as shown in the average open interest on the CME show in Figure 1.5.

Table 1.1: Atlanta Cooling Degree Days in degrees Fahrenheit, June 2016.

Date	CDD	Date	CDD	Date	CDD
June 01, 2016	16.5	June 11, 2016	19.0	June 21, 2016	15.5
June 02, 2016	13.5	June 12, 2016	20.0	June 22, 2016	17.5
June 03, 2016	16.0	June 13, 2016	21.0	June 23, 2016	17.5
June 04, 2016	14.5	June 14, 2016	19.0	June 24, 2016	20.5
June 05, 2016	12.5	June 15, 2016	15.5	June 25, 2016	23.0
June 06, 2016	12.0	June 16, 2016	17.5	June 26, 2016	15.5
June 07, 2016	14.5	June 17, 2016	18.5	June 27, 2016	17.0
June 08, 2016	9.5	June 18, 2016	11.5	June 28, 2016	17.0
June 09, 2016	10.5	June 19, 2016	11.0	June 29, 2016	17.5
June 10, 2016	17.5	June 20, 2016	10.5	June 30, 2016	17.5

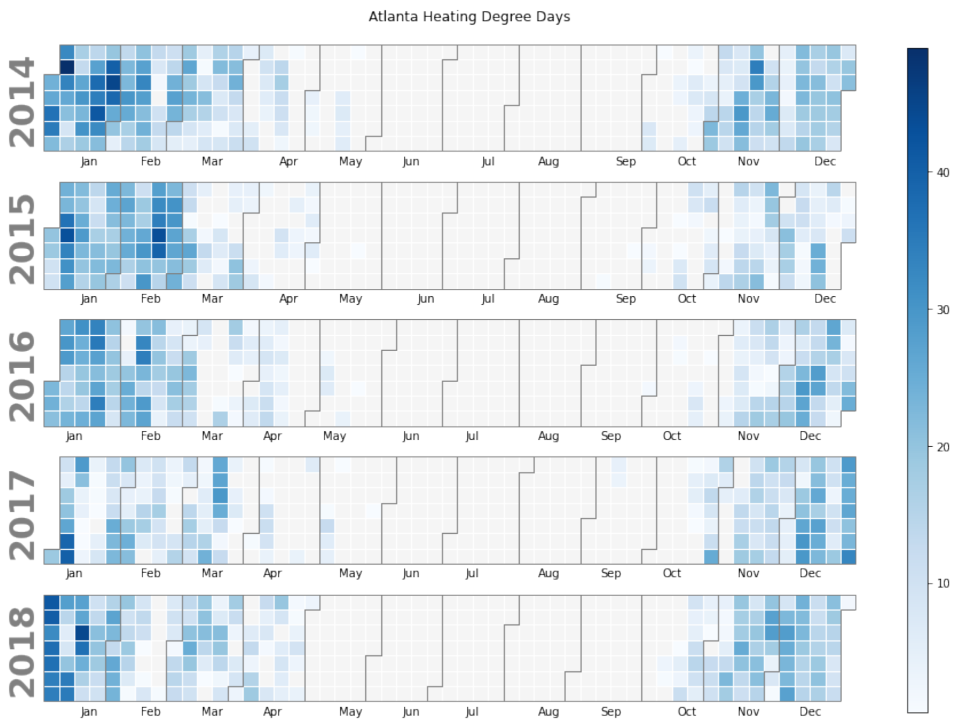

Figure 1.3: Atlanta Heating Degree Days in degrees Fahrenheit (2014–2018).

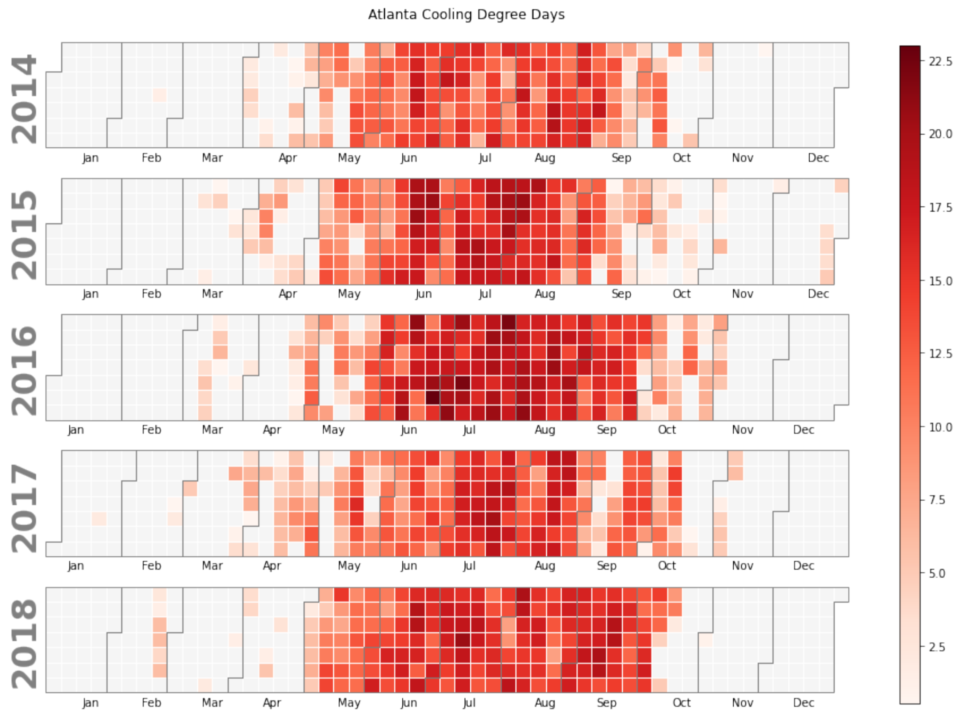

Figure 1.4: Atlanta Cooling Degree Days in degrees Fahrenheit (2014–2018).

1.3 Challenges

Typically, derivatives are priced using information about the market price of their associated underlying asset [33]. Pricing weather derivatives is an unusual task due to the fact that their underlying variable (i.e., daily temperatures) cannot be traded. Furthermore, there is evidence that indicates that the statistical dependence between surface temperature observations may decay slowly [19, 58, 59] or feature antipersistence [22, 28].

Accurate weather derivative pricing is further complicated due to the difficulty in capturing the characteristics of daily temperature observations. Deterministic trends and dependencies between observations may be tricky to distinguish from noise [45]. Additionally, there is evidence that supports the claim that there are significantly different dynamics between the progression of daily maximum and daily minimum temperature observations [41, 42]. As such, a forecasting and pricing model must have this taken into account upon attempting to identify sources and magnitudes of errors or fluctuations.

The foundation of any temperature index based weather derivative pricing model is the method used to forecast daily temperatures. A proposed method should be judged

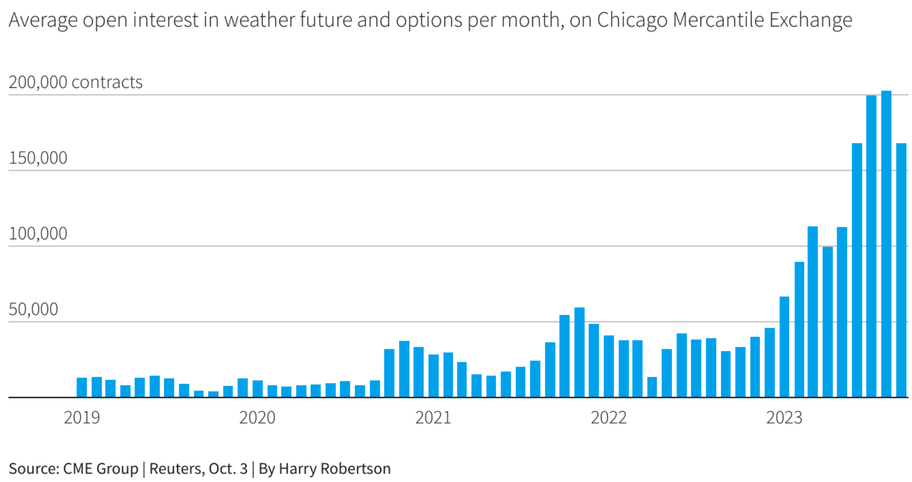

Figure 1.5: Average open interest in weather future and options per month, on Chicago Mercantile Exchange, 2019-2023 [40].

on its accuracy, flexibility, tractability and robustness. We begin Chapter 3 with work to characterize and parameterize the persistence of daily maximum and daily minimum observations, seasonal trend and deterministic trend.

1.4 Objective

The objective of this study is to respond to the challenges detailed above by developing a general framework for weather derivative pricing that may be used by speculators or parties that want to hedge their exposure to weather risk. The framework allows for estimates of the expected return associated with holding a weather derivative.

1.5 Structure of dissertation

In Chapter 2, we present a literature review covering various methods for daily temperature forecasting and weather derivative pricing. In Chapter 3, we analyze historical daily temperature data and propose a degree day estimation framework based on forecasts of daily maxima and daily minima over a month. In Chapter 4 we incorporate external temperature forecasts to explore their effect on the performance of the forecasting framework. In Chapter 5 we apply the framework to give an example of hedging temperature risk with weather derivative futures contracts. In Chapter 6 we summarize our results and propose future research work.

Chapter 2

Literature Review

In this chapter we present a survey of publications that pertain to modeling daily temperature data and weather derivative pricing, summarize the collection of work and state our objective for this dissertation.

2.1 Weather derivative pricing models

Degree days are illiquid in that they are inherently unable to be warehoused; thus, it is impossible to buy or sell the event underlying the weather derivative. However, market stresses unrelated to the weather can affect the prices of weather derivatives [68]. Because of this, developing adequate models for pricing weather derivatives presents a relatively unique challenge.

The predominant approaches are the burn-rate modeling, stochastic differential equation modeling, and index modeling [8, 36, 61].

2.1.1 Burn-rate modeling

Burn-rate models are popular due to the ease in implementing them by analyzing historical payoff data for the instrument in question. With a burn-rate model approach, a weather derivative is priced based on the historical average of the payoffs of comparable contracts. However, they are limited in their flexibility and inherently not robust. Under such a framework there is little, if any, consideration for up-to-date information about upcoming weather dynamics. Consequently, such a model is unsuitable for accurate forecasting and hedging. For more on the history of application of burn-rate models to weather derivative pricing, we refer the reader to Richards et al. (2004) [55], Dorfleitner and Wimmer (2010) [25], Schiller et al. (2012) [61] and chapter 3 of ***Weather Derivative Valuation*** by Jewson and Brix [36].

2.1.2 Stochastic differential equation modeling

Modeling a stochastic process with a stochastic differential equation (SDE) can provide many advantages, in particular in cases where the SDE has a known, closed form solution. In the case of a SDE with no known closed form solution, quantities of interest may be estimated via Monte Carlo methods [33, 34, 57].

A famous example of a SDE approach to derivative pricing is the Black-Scholes model [12]. Under the Black-Scholes model, the price of an option is related to the price of the underlying asset, the strike price, the time to option maturity, the risk-free interest rate, asset price volatility, and the income expected from the asset during the life of the option [33, 34]. Via the associated stochastic differential equation (SDE), a practitioner may derive closed form expressions for many quantities of interest such as the change in the price of the contract with respect to the change in price of the underlying asset. However, a couple assumptions of the Black-Scholes valuation formula do not hold in the context of weather derivatives. First, it is assumed that the volatility of the rate of return on the underlying asset is constant. This assumption, though attractive in pursuit of a closed-form solution, is unreasonable in the context of weather derivatives. Second, it is assumed that it is possible to borrow any fraction of the price of a security to buy it or hold it. This is not possible since neither heating degree days nor cooling degree days are tradable commodities. Brody et al. (2002) [18] propose a function that satisfies a parabolic partial differential equation with fractional Brownian motion to produce a closed form expression for the value function of a degree day contract in degrees Celsius. The authors note that their model performs well for long-range dependence between daily temperature observations but underperforms in capturing short-range correlations. The authors characterize the dynamics of the price process of a degree day contract as an open question.

Alaton et al. (2002) [1] propose a SDE that features a deterministic component for the seasonal dynamics of average daily temperature and a random component to model daily average temperature fluctuations. The solution of the SDE permits approximation of solutions through the use of Monte Carlo simulation. Estimation of the volatility parameter $_t$ and the mean-reversion parameter a is through historical analysis of monthly data. This approach provides a straightforward approach for Monte Carlo simulation of the modeled quantity of interest.

Richards et al. (2004) [55] compare a traditional burn-rate model, a Black-Scholes model and an equilibrium Monte Carlo simulation model based on the work of Lucas (1978) [56] to price weather derivatives. The authors find that the best results come from using an equilibrium Monte Carlo simulation model of a mean-reverting Brownian motion process with lognormal jumps. They apply the equilibrium Monte Carlo simulation model to price CDD calls and puts as risk hedging tools for California specialty crop growers. They go on to argue that a risk-neutral assumption is unsuitable for weather derivative pricing and that the assumption of zero market risk could lead to significant pricing errors.

Oetomo and Stevenson (2005) [52] use data from 1979 to 2002 for 10 cities in the US to compare six extant models for weather derivative pricing; three Monte Carlo (i.e., SDE) models and three Autoregressive Moving Average (ARMA) models. They find that no one model uniformly performs better than the others and that model performance depends on location, season, forecast sample and type of contract (HDD or CDD). They also find that forecast accuracy, in particular beyond 30 days ahead, is poor.

Ali et al. (2023) [4] apply methods of Alexandridis et al. [2] and Benth et al. [9] to price degree day contracts for European cities by using a truncated Fourier series based on historical daily temperature data to estimate the volatility term $_t$ of an Ornstein-Uhlenbeck process [67].

Alfonsi and Nerea (2024) [3] modify a Gaussian SDE approach through the addition of a deterministic time-dependent volatility parameter to model the evolution of daily average temperatures. The authors use both Monte Carlo and truncated Fourier series transform techniques to estimate the average payoff of weather derivatives for several cities in Europe.

2.1.3 Index modeling

An index model for weather derivatives is a stochastic time-series model fit to daily temperature observations.

An index model for pricing weather derivatives is based on a statistical model of the series of heating degree days and cooling degree days. Such models may be parametric or non-parametric. The challenge of implementing an index model lies in the selection of parameters and application of the resultant statistical model. An index model can be fit to the series of daily maxima and minima, daily average, daily heating degree days, daily cooling degree days, or the contract payoff [36].

A time series-based index model features parameters based on historical temperature observations and, thus, offers sensitivity to temperature trends and seasonality [61, 25]. Bloomfield (1992) [13] shows that yearly temperatures feature a positive trend. There is a yearly seasonal trend to daily temperatures; Jewson and Caballero (2003) [37] explore the impact of seasonality on weather derivative pricing and shows that it is material.

Beyond the impact of seasonality, Caballero et al. (2002) [19], Rybski et al. (2006) [58] and Mann (2011) [48] discuss long-range persistence (i.e., long-memory) exhibited in daily temperatures. Benth and Lempa (2023) [10] use a seasonal continuous-time autoregressive moving average (CARMA) approach to model temperature dynamics.

For more information on the application of index models to weather derivatives the reader may consult Muller and Grandi (2000) [49], Oetomo and Stevenson (2005) [52], Brocket et al. (2005) [17], Bemš and Aydin (2022) [8], and Chapter 4 of ***Weather Derivative Valuation*** [36].

2.2 Long-memory processes

A mean-reversion parameter and the effort to adequately measure it is a common theme in stochastic differential equation (SDE) and index modeling papers. In this section we review the literature of long-term memory processes and their application to problems in the natural sciences and economics.

2.2.1 Theoretical foundation

The application of long-memory processes to problems in economics and the social sciences has its foundation in Hurst's 1956 paper on optimal long-term storage for a Nile river reservoir [35]. In the paper, Hurst considered the series of yearly measurements of the depth of water in the reservoir and noted that the series exhibited long-range persistence. Hurst continued by proposing a method to estimate the parameter from the series of observations, then considered several reservoir policies in light of his observations and analysis.

Mandelbrot and Van Ness (1968) [47] advance the topic of the use of long-memory models in the sciences by introducing fractional Brownian motion. The authors begin by noting that empirical studies of random phenomena often do not exhibit independence between observations; rather, they exhibit interdependence between distance samples. They continue by extending the work of Lévy [46] by defining fractional Brownian motion as a fractional integral of Brownian motion.

Granger and Joyeux (1980) [29] discuss the long-memory property of time series and the application of fractional autoregressive integrated moving average (ARIMA) models. In their work, they discuss how the analysis of the spectrum of a time series may be used to detect the long-memory process. They go on to propose methods to estimate the value of the differencing parameter of the fractional ARIMA model from data. Sowell (1992) [63] applies a fractional ARIMA model to model quarterly real GDP in the United States from the first quarter of 1947 to the fourth quarter of 1989. Sowell concludes by noting the difficulty in definitively distinguishing between unit root behavior and long-memory in time series data. Beveridge and Oickle (1993) [11] propose a Gaussian maximum likelihood method to estimate the differencing parameter of a long-memory time series. Baillie (1996) [6] extends the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model with a fractionally integrated parameter to consider econometric series. This Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) model shows promise in modeling series with long-term dependency between the variation of error terms between observations.

2.2.2 Persistence in daily temperature observations

The long-memory property in daily temperature observations is discussed in the literature.

Caballero et. al (2002) [19] discuss a number of ways to model long-term dependency in temperature data along with methods of applying the models to the problem of weather derivative pricing. Rybski et. al (2006) [58] analyze temperature data from six locations in the Northern Hemisphere and find that the series all exhibit long-term persistence. Mann (2010) [48] generates synthetic temperature data and argues that the differencing parameter can be understood as a linear response to past natural effects. Capparelli et. al (2011) [21] note the long-memory property in daily temperature time series from locations in the US with fluctuations on time scales in the range of three to fifteen years.

2.3 Weather forecasts and weather derivative pricing

Our objective is to provide a framework that is extensible with respect to inclusion of meteorological forecasts. Incorporation of meteorological forecasts in weather derivative pricing is discussed in chapters 9 and 10 of *Weather Derivative Valuation* [36]. Forecasts and their incorporation into weather derivative pricing is further explored in Jewson and Caballero (2003) [38] and Jewson and Ziehrmann (2003) [39]. Campbell and Diebold (2005) [20] discuss the difficulty in obtaining weather forecasts that are designed with weather derivative pricing in mind. Jewson and Caballero discuss the advantage of using stochastic time-series models over numerical weather prediction models; however, they do not explore the incorporation of forecasts for upcoming days into such a model. In Jewson and Ziehrmann's paper, they discuss incorporation of ensemble forecasts to predict forecast change and, subsequently, explore the application of ensemble forecasts to weather swap values at risk. Palmer (2002) [53] discusses the use of ensemble forecasts of numerical weather prediction models in several fields, including weather-risk finance, but comes short of presenting a general weather derivative framework.

2.4 Summary

Our objective, restated, is to develop a framework that is sensitive to temperature forecasts and suitable for pricing weather derivative contracts with durations of up to five months.

As discussed in this chapter, there has been work done on incorporation of forecasts and the modeling of seasonal and linear components of daily temperature observations. The Black-Scholes model is popular but insufficient for the task at hand. Despite the wealth of publications on weather derivative pricing models, we find that, to this point, there is no single framework that allows for our desired flexibility and ease of deployment.

Our objective is to develop a weather derivative pricing model that incorporates historical temperature data, is sensitive to recent observations and allows for incorporation of forecasted temperatures. In Chapter 3, we analyze the series of daily temperature observations for the studied cities to verify that those series exhibit the long-memory property. From there, further analysis is carried out to construct a suitable time series model for weather derivative pricing that allows for the incorporation of external temperature forecasts. In Chapter 4 we examine the differences in forecasts of maxima and minima at different point of a month, with and without external weather forecasts. In Chapter 5, the model is used to price weather derivative contracts for a weather risk hedging exercise.

Chapter 3

Temperature index model development

In this chapter we present a model for producing temperature forecasts that captures long-term trends and persistence in daily temperature observations. The trends are modeled with a constant and linear term along with a sinusoidal term for seasonality. The error terms are modeled as a fractionally integrated autoregressive moving average process. We use historical data to estimate parameters of the models.

3.1 Seasonal and linear detrending

We begin by finding estimates of the seasonal and linear parameters of our model. The linear trend component of the series is modeled by

$$c_0 + c_1 t. \tag{3.1}$$

The seasonal component of the series is modeled by a sinusoidal function

$$A \sin \quad \frac{2}{365} t + \quad . \tag{3.2}$$

To simplify the seasonal component of our model, leap days are removed. It is very likely that there is be a non-negligible linear trend to the data; this phenomenon is best described in Bloomfield's paper about global temperature trends [13].

The parameters $(c_0, c_1, A, \ )$ are estimated by least squares. Table 3.1 details the

Table 3.1: Estimates of trend and seasonal parameters for Atlanta daily minima.

Start Date	5/27/2004
End Date	6/10/2019
No. of observations	5475
$\hat{c_0}$	$5{:}2982 \quad 10^{1}$
$\hat{c_1}$	$5{:}3435 \quad 10^{\ \ 4}$
$\hat{A}$	$1{:}3492 \quad 10^{1}$
$\hat{\ }$	$1{:}2872 \quad 10^{1}$

estimates for the series of daily minimum temperature observations.

By combining Equations (3.1) and (3.2), the model for the seasonal and linear trends of the daily temperature observations is

$$y_t = c_0 + c_1 t + A \sin \quad \frac{2}{365} t + \qquad + "_t{:} \tag{3.3}$$

3.2 Evidence of persistence in the series of daily temperature observations

In this section, we determine if the series for the cities in question exhibit the long-memory property through applying the rescaled range analysis of Hurst.

For those series that exhibit the long-memory property, we determine whether the series exhibits persistence or antipersistence. A time series is said to be persistent if the function of the autocorrelation of the observations of the series decays in a hyperbolic manner with positive correlations. A persistent time series features observations that tend to be closely related to previous values; that is, positive (negative) shocks to the system have a higher positive (negative) influence subsequent observations. A time series is said to be antipersistent if the function of the autocorrelation of the observations of the series decays in a hyperbolic manner with negative correlations. An antipersistent time series also features observations that tend to be closely related to previous values, however positive (negative) shocks tend to have higher negative (positive) influence on subsequent observations [5].

We estimate the parameters of a fractionally integrated model by the process suggested by Box and Jenkins [15].

3.2.1 Estimating the Hurst coefficient

Here, we present the method of Hurst's rescaled range analysis which can be used to detect the long-memory property of a time series. This approach was first used by Hurst in the study of water levels in reservoirs [35]. Rescaled range analysis has gone on to be used in applications ranging from the natural sciences to finance [19, 29, 32].

Let process $\{y_t\}_{t=1}^{T}$ be given. The observations will be broken down into non-overlapping, consecutive subseries of length τ; we refer to the observations in a subseries of length τ as a period. Denote the average over a given period τ, $\langle y \rangle_\tau$, as

$$\langle y \rangle_\tau = \frac{1}{\tau} \sum_{k=1}^{\tau} y_k. \tag{3.4}$$

The accumulated deviation from the mean at t over period τ, denoted $X(t, \tau)$, is

$$X(t, \tau) = \sum_{u=1}^{t} \{y_u - \langle y \rangle_\tau\}. \tag{3.5}$$

The range over period τ, denoted $R(\tau)$, is

$$R(\tau) = \max_{1 \le k \le \tau} X(t, \tau) - \min_{1 \le k \le \tau} X(t, \tau). \tag{3.6}$$

Define S by

$$S = \left(\frac{1}{\tau} \sum_{k=1}^{\tau} \{y_k - \langle y \rangle_\tau\}^2 \right)^{\frac{1}{2}}. \tag{3.7}$$

The quantity S is an estimate of the standard deviation over τ observations. Per Hurst's work, the rescaled range R/S may be described by the relation

$$R/S = (\tau/2)^H, \tag{3.8}$$

where H is referred to as the *Hurst exponent* or *Hurst coefficient* [26, 35]. For a time series generated by a statistically independent process with finite variance, the rescaled range should be such that R/S is asymptotically proportional to $\sqrt{\tau}$ with

$$R/S = (\pi\tau/2)^{1/2},$$

[35]. To estimate H, we create a log-log plot of the subseries size against $R{=}S$ and produce a least-squares linear fit. The slope of that least-squares linear fit is the estimate of H. For processes that feature antipersistence we have that $0 < H < 1{=}2$ and for processes that the long-memory property we have that $1{=}2 < H < 1$.

Table 3.2: Rescaled range data for Atlanta daily maxima.

Size	$R{=}S$	Log Size	Log $R{=}S$
7300	7.0088	8.8956	1.9472
3650	6.4230	8.2025	1.8599
1825	6.3093	7.5093	1.8420
912	5.6870	6.8156	1.7382
456	5.5047	6.1225	1.7056
228	5.1875	5.4293	1.6463
114	4.4451	4.7362	1.4918
57	4.3602	4.0431	1.4725

Table 3.3: Rescaled range data for Atlanta daily minima.

Size	$R{=}S$	Log Size	Log $R{=}S$
7300	8.0699	8.8956	2.0881
3650	7.0483	8.2025	1.9528
1825	7.0587	7.5093	1.9543
912	6.5899	6.8156	1.8855
456	6.4152	6.1225	1.8587
228	5.9727	5.4293	1.7872
114	5.0360	4.7362	1.6166
57	4.5901	4.0431	1.5239

Figure 3.1 is a log-log plot of $R{=}S$ against size for the series of daily maxima for Atlanta with a least-squares line with slope 0.0993. Figure 3.2 is a log-log plot of $R{=}S$ against size for the series of daily minima for Atlanta with a least-squares line with slope 0.1058. The resultant estimates of H from the least-square line slope values suggest

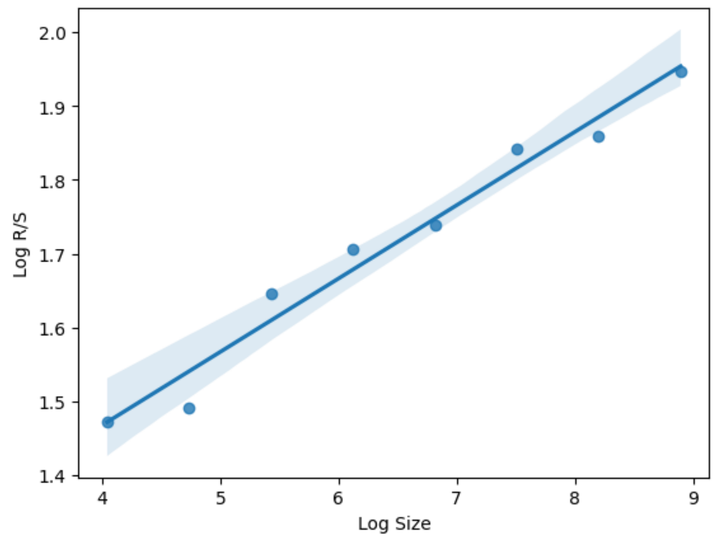

Figure 3.1: $R{=}S$ plot for Atlanta daily maxima, least-squares slope 0.0993.

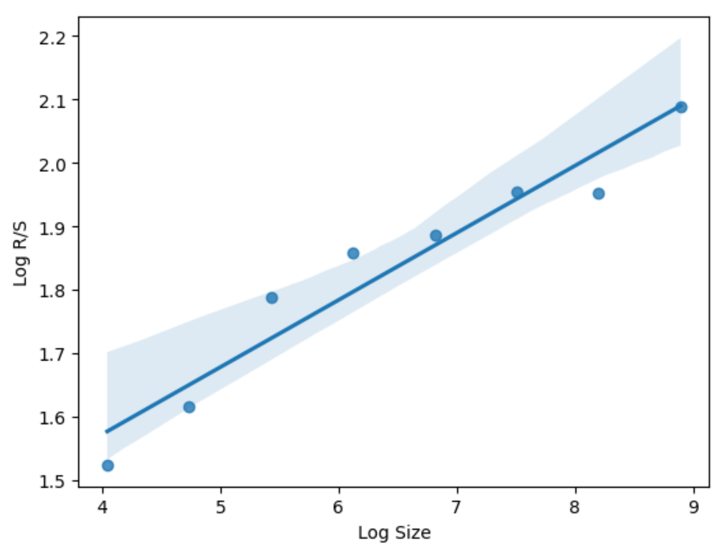

Figure 3.2: $R{=}S$ plot for Atlanta daily minima, least-squares slope 0.1058.

that the series exhibit antipersistence.

3.2.2 Modeling persistence

An appropriate model to capture persistence in a time series is the fractionally integrated model, or ARFIMA$(0, d, 0)$ model [15, 29, 32], given by:

$$(1 - B)^d y_t = \varepsilon_t, \tag{3.9}$$

with $-0.5 < d < 0.5$, where B is the backward-shift operator on the series of observations $\{y_t\}_{t \geq 0}$ with the property

$$B(y_t) = y_{t-1}, \quad t \geq 1,$$

$\{\varepsilon_t\}_{t \geq 1}$ is a discrete time white noise process, and

$$\begin{aligned}(1 - B)^d &= \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k \\ &= 1 - dB - \frac{1}{2}d(1-d)B^2 - \frac{1}{6}d(1-d)(2-d)B^3 - \dots .\end{aligned}$$

For $d \in (-0.5, 0)$ the ARFIMA$(0, d, 0)$ model is appropriate for modeling antipersistence and for $d \in (0, 0.5)$ the ARFIMA$(0, d, 0)$ model is appropriate for modeling long-range persistence [26, 32, 47].

3.2.3 Fitting the fractionally integrated model

Recall that from Equation (3.9), the ARFIMA$(0, d, 0)$ model may be expressed as

$$(1 - B)^d y_t = \varepsilon_t,$$

where $\{y_t\}$ is the series of observations and $\{\varepsilon_t\}$ is a discrete white noise process. We can express the *fractionally differenced* component, $(1 - B)^d$, as

$$(1 - B)^d = \sum_{j=0}^{\infty} \pi_j B^j, \tag{3.10}$$

where $\pi_0 = 1$ and

$$\pi_j = \frac{\Gamma(j - d)}{\Gamma(j + 1)\Gamma(-d)}, \tag{3.11}$$

with

$$\Gamma(n) = \int_0^{\infty} x^n e^{-x} dx \quad n > 0. \tag{3.12}$$

To estimate ARFIMA$(0, d, 0)$ parameters d, we use a method suggested by Box and Jenkins [15]. Consider the autoregressive (AR) representation of a stationary ARFIMA$(0, d, 0)$ process given by

$$y_t - \sum_{j=1}^{\infty} \pi_j y_{t-j} = \varepsilon_t, \tag{3.13}$$

where

$$\pi(B) = 1 - \sum_{j=1}^{\infty} \pi_j B^j = (1 - B)^d.$$

One may use implied recursive relationships to compute the π weights.

Given a representation as in Equation (3.13), we consider the series of truncated errors

$$\varepsilon_t(d) = y_t - \sum_{j=1}^{t-1} \pi_j y_{t-j} \quad t = 1, \ldots, n,$$

as a function of d. A maximum likelihood estimate $\hat{d}$ of d may be computed by minimizing the sum of squares

$$S(d) = \sum_{i=1}^{n} \varepsilon_t^2(d). \tag{3.14}$$

To accomplish this, the parameter d is selected from a discretized range of $-0.5 < d < 0.5$. For each value d, the resultant sum of squares is computed. The value $\hat{d}$ that minimizes the sum of squares expression is selected for the ARFIMA$(0, d, 0)$ model.

Given $\hat{d}$, and for n sufficiently large, the l-step ahead forecast of y_{t+l} for series $\{y_i\}_{i=1}^n$ is

$$\hat{y}_n(l) = \sum_{j=1}^{n+l-1} \pi_j \hat{y}_t(l - j), \tag{3.15}$$

where $\hat{y}_t(l - j) = y_{t+l-j}$ for $j \geq l$. Additionally, the forecast error $e_t(l)$ is given by

$$e_t(l) = \sum_{j=0}^{t-1} \psi_j \varepsilon_{t+l-j}, \tag{3.16}$$

with variance

$$\mathrm{var}[e_t(l)] = \sigma^2(l) = \sigma_\varepsilon^2 \sum_{j=0}^{l-1} \psi_j^2, \tag{3.17}$$

where

$$\psi_j = \frac{\Gamma(j+d)}{\Gamma(j+1)\Gamma(d)}. \tag{3.18}$$

3.2.4 Numerical results: Atlanta parameter estimation

Table 3.5 features the estimation results for the series of daily maxima for Atlanta showing $\hat{d} = -0.17$ and Table 3.6 features the estimation results for the series of daily minima for Atlanta with $\hat{d} = -0.18$. As the values for $\hat{d}$ fall in the interval $(-0.5, 0)$, an ARFIMA$(0, d, 0)$ model is appropriate for modeling the antipersistence we detected through the rescaled range analysis.

Table 3.4 features $\hat{d}$ estimates for the series of maxima and minima for Atlanta, Chicago, Cincinnati, Las Vegas, Minneapolis, New York and Portland. Though the set of estimates for $\hat{d}$ over the cities detailed features some small variation, in all scenarios $\hat{d} < 0$ which indicates that the series feature antipersistence.

3.3 Summary

We now have a way to characterize the linear, seasonal and antipersistence components of the series of daily maxima and daily minima. The combined trend and ARFIMA model is the foundation upon which we perform our forecasting and hedging example work in the coming chapters.

In Chapter 4 we use the parameters and associated models to compute estimates of the resultant weather derivative valuation under a selection of weather forecast scenarios. In Chapter 5 we use real temperature and weather derivative future price information for a weather risk hedging exercise.

Table 3.4: Parameter d estimates and characterizations of persistence for Atlanta, Chicago, Cincinnati, Las Vegas, Minneapolis, New York and Portland.

City	Minima $\hat{d}$	Characterization	Maxima $\hat{d}$	Characterization
Atlanta	-0.18	Antipersistent	-0.17	Antipersistent
Chicago	-0.17	Antipersistent	-0.17	Antipersistent
Cincinnati	-0.19	Antipersistent	-0.18	Antipersistent
Las Vegas	-0.18	Antipersistent	-0.18	Antipersistent
Minneapolis	-0.16	Antipersistent	-0.17	Antipersistent
New York	-0.17	Antipersistent	-0.17	Antipersistent
Portland	-0.17	Antipersistent	-0.17	Antipersistent

Table 3.5: Parameter d estimation Atlanta daily maxima. $\hat{d} =$ 0:17.

d	$S(d)$	d	$S(d)$
-0.50	740.5504088	-0.25	47.3540347
-0.49	660.5715501	-0.24	44.24409587
-0.48	589.0059611	-0.23	41.71797551
-0.47	524.9861074	-0.22	39.71677902
-0.46	467.735609	-0.21	38.18743473
-0.45	416.5594592	-0.20	37.08210696
-0.44	370.8353144	-0.19	36.35766963
-0.43	330.0057347	-0.18	35.975234
-0.42	293.5712702	**-0.17**	**35.89972489**
-0.41	261.0842983	-0.16	36.09950031
-0.40	232.1435297	-0.15	36.54600989
-0.39	206.3891089	-0.14	37.21348811
-0.38	183.498243	-0.13	38.07867871
-0.37	163.1813015	-0.12	39.12058702
-0.36	145.1783353	-0.11	40.32025731
-0.35	129.2559687	-0.10	41.6605726
-0.34	115.204623	-0.09	43.12607459
-0.33	102.8360379	-0.08	44.70280161
-0.32	91.98105475	-0.07	46.37814286
-0.31	82.48763694	-0.06	48.14070712
-0.30	74.21909866	-0.05	49.9802045
-0.29	67.0525216	-0.04	51.88734002
-0.28	60.87733827	-0.03	53.8537176
-0.27	55.59406436	-0.02	55.87175362
-0.26	51.11316417	-0.01	57.93459892

Table 3.6: Parameter d estimation Atlanta daily minima. $\hat{d} =$ 0:18.

d	$S(d)$	d	$S(d)$
-0.50	548.1825199	-0.25	38.41434311
-0.49	487.1648273	-0.24	36.37025036
-0.48	432.9545105	-0.23	34.74482515
-0.47	384.7881906	-0.22	33.49633363
-0.46	341.9915268	-0.21	32.58728404
-0.45	303.9684874	-0.20	31.9839715
-0.44	270.1919466	-0.19	31.65607399
-0.43	240.1954386	**-0.18**	**31.57629354**
-0.42	213.5659258	-0.17	31.72003749
-0.41	189.937452	-0.16	32.06513497
-0.40	168.9855719	-0.15	32.59158478
-0.39	150.422458	-0.14	33.2813308
-0.38	133.9926023	-0.13	34.11806201
-0.37	119.4690379	-0.12	35.08703416
-0.36	106.6500166	-0.11	36.17491069
-0.35	95.35608602	-0.10	37.36962076
-0.34	85.42751659	-0.09	38.66023237
-0.33	76.72203634	-0.08	40.03683904
-0.32	69.11283469	-0.07	41.49045831
-0.31	62.48680315	-0.06	43.01294106
-0.30	56.7429836	-0.05	44.5968901
-0.29	51.79119915	-0.04	46.23558732
-0.28	47.55084543	-0.03	47.92292833
-0.27	43.94982278	-0.02	49.65336372
-0.26	40.92359266	-0.01	51.42184638

Chapter 4

Inclusion of external daily temperature forecasts

In Chapter 3, we used historical data to develop a model that accounts for long-term trend and antipersistence of daily temperature observations. However, our model does not take into account information about atmospheric conditions and what that information could imply for future temperature observations. A natural question is: can we combine the model with weather forecasts to improve our degree day forecasting accuracy? Weather forecasts themselves are imprecise, in particular for times further into the future. We attempt to strike a balance between the uncertainty inherent in our daily temperature model and the weather forecasts by examining the effect of including up to several days of weather forecasts in our degree day forecasting method.

In this chapter, we present a method to incorporate externally sourced weather forecasts into our model with the goal of examining the resultant change in heating degree day and cooling degree day future contracts. To this end, we perform an incremental analysis which consists of the following scenarios: no forecasts included, one-day forecasts included, and two-day forecasts included. We then examine their impact on our weather derivative pricing estimates. This analysis highlights the flexibility of our model and its utility for parties that deal in weather derivatives for hedging purposes.

4.1 Differences in weather forecasts

Meteorologists use up-to-date weather data about along with understanding and assumptions about the evolution of atmospheric conditions and computation to generate forecasts. The practice is inherently limited by the frequency and quality of atmospheric data, the validity of atmospheric dynamic assumptions and the availability of computing resources to generate forecasts. As a result, different weather forecasting services may produce different weather forecasts even if the forecasting services use the same atmospheric data.

4.2 Producing degree day forecasts

As in Chapter 3, let $\{y_t\}$ denote a series of daily temperature observations and $\{\varepsilon\}$ denote a discrete white noise process. Suppose that a fractional differencing parameter $\hat{d}$ has been found for the series. From Equation 3.15, we have that the l-step ahead forecast of y_{t+l} for series $\{y_i\}_{i=1}^{n}$ is

$$\hat{y}_n(l) = \sum_{j=1}^{n+l-1} \pi_j \hat{y}_t(l-j)$$

where $\hat{y}_t(l-j) = y_{t+l-j}$ for $j \geq l$. From Equation 3.11, π_j is given by

$$\pi_j = \frac{\Gamma\left(j-\hat{d}\right)}{\Gamma(j+1)\Gamma\left(-\hat{d}\right)}.$$

Thus, given a one-month degree day contract and a date during the contract period for which we have historical daily maxima and minima, we may forecast daily maxima and daily minima for every day to the end of the contract period. From the forecasted daily maxima and minima we may forecast degree days. By combining the actual degree days to the date in question along with the forecasted degree days, in turn we may estimate the payoff of the one-month degree day contract in question.

In this chapter, we consider Minneapolis in November 2023. Two periods are used for our analysis: November 1, 2023 to November 30, 2023 and November 15, 2023 to November 30, 2023. We examine the effect of including one-day and two-day external temperature forecasts on our model heating degree day (HDD) forecasts.

4.3 Degree day estimations with inclusion of weather forecasts

We begin by producing model forecasts with no external temperature forecasts. This is a base case from which begin our analysis of the inclusion of external temperature forecasts.

The external temperature forecasts are collected daily from the National Oceanic and Atmospheric Administration (NOAA) National Weather Service Forecast Office web site [51]. There, one may obtain hourly temperature forecasts for a location for a 48-hour period. We use these 48-hour period hourly temperature forecasts to find the external forecast daily maximum and daily minimum for the two days past the start date of a given period.

To include the external temperature forecasts, the residual between our sinusoidal model estimated up the start date of the period in question and the external temperature forecast are fed into the model as error terms. Thus, the model output agrees with the external forecasts for the days over which they are used, and subsequent model forecasted error terms include the residual(s) in calculation.

The actual daily maximum temperature, minimum temperature, average temperature and heating degree day values for Minneapolis for November 2023 may be found Table 4.1.

4.3.1 Minneapolis: November 1, 2023 to November 30, 2023

Temperatures in Minneapolis in November 2023 were unseasonably warm for much of the month. This is noted in the overestimate of HDD. The deviation from typical daily maximum and daily minimum values leads to consistent model overestimation of HDD even when including external one-day and external two-day temperature forecasts. The results for November 1, 2023 to November 30, 2023 are discussed in the following paragraphs and are summarized in Table 4.2.

Table 4.3 details our model forecasts for November 2023 with no external daily temperature forecasts. Table 4.4 combines the actual HDD from Table 4.1 with the model forecast HDD from Table 4.3, and shows that our model overpredicts HDD over the period by 100.5. Figure 4.1 provides a graphical comparison of the cumulative HDD forecast over the period against the actual cumulative HDD.

Table 4.5, Table 4.6 and Figure 4.2 show that our model produces poorer overpredic-

Table 4.1: Daily actual maxima, minima, average temperature and heating degree days for Minneapolis in degrees Fahrenheit. November 1, 2023 to November 30, 2023.

Date	Max	Min	Average	HDD
2023-11-01	38	23	30.5	34.5
2023-11-02	43	27	35.0	30.0
2023-11-03	53	32	42.5	22.5
2023-11-04	51	28	39.5	25.5
2023-11-05	58	37	47.5	17.5
2023-11-06	58	40	49.0	16.0
2023-11-07	46	39	42.5	22.5
2023-11-08	51	40	45.5	19.5
2023-11-09	49	33	41.0	24.0
2023-11-10	40	36	38.0	27.0
2023-11-11	48	32	40.0	25.0
2023-11-12	59	37	48.0	17.0
2023-11-13	57	37	47.0	18.0
2023-11-14	66	43	54.5	10.5
2023-11-15	61	39	50.0	15.0
2023-11-16	69	36	52.5	12.5
2023-11-17	46	28	37.0	28.0
2023-11-18	57	32	44.5	20.5
2023-11-19	55	32	43.5	21.5
2023-11-20	46	40	43.0	22.0
2023-11-21	43	29	36.0	29.0
2023-11-22	47	26	36.5	28.5
2023-11-23	37	20	28.5	36.5
2023-11-24	29	13	21.0	44.0
2023-11-25	32	23	27.5	37.5
2023-11-26	34	21	27.5	37.5
2023-11-27	21	12	16.5	48.5
2023-11-28	32	8	20.0	45.0
2023-11-29	48	26	37.0	28.0
2023-11-30	42	24	33.0	32.0

tion of 916 HDD after including the one-day external temperature forecast.

Table 4.7, Table 4.8 and Figure 4.3 show that our model produces an even poorer overprediction of 938.5 HDD after including the two-day external temperature forecast.

Table 4.2: Summary of HDD forecasts from model with no external daily temperature forecast, one-day external daily temperature forecast and two-day external daily temperature forecast. November 1, 2023 to November 30, 2023. The error is the forecast HDD minus actual HDD. Actual HDD: 795.5.

Scenario	Forecast HDD	Error
No external forecast	896.0	100.5
One-day external forecast	916.0	120.5
Two-day external forecast	938.5	143.0

4.3.2 Minneapolis: November 15, 2023 to November 30, 2023

Over the period from November 15, 2023 to November 30, 2023, our model produces with forecasts of total HDD that are closer to the actual total HDD upon including external temperature forecasts. The results are discussed in the following paragraphs and are summarized in Table 4.9.

The model overestimates total HDD over the period by 46, or a percent difference of about 9.5%. This is shown in Table 4.10 and Table 4.11.

The values in Table 4.12 and Table 4.13 show that inclusion of the one-day external temperature forecast leads to a reduction of HDD overestimation over the period to 15.5.

Of note, we see that the inclusion of the external two-day forecast results in zero error for total HDD over the period. This information may be found in Table 4.14 and Table 4.15.

4.3.3 Using multiple external forecasts

To this point we have included temperature forecasts from the NOAA. However, a practitioner could include information from multiple external temperature forecast sources. To do so, the practitioner would need a weighting method for the multiple external forecasts and could then weigh the resultant framework degree day forecasts accordingly.

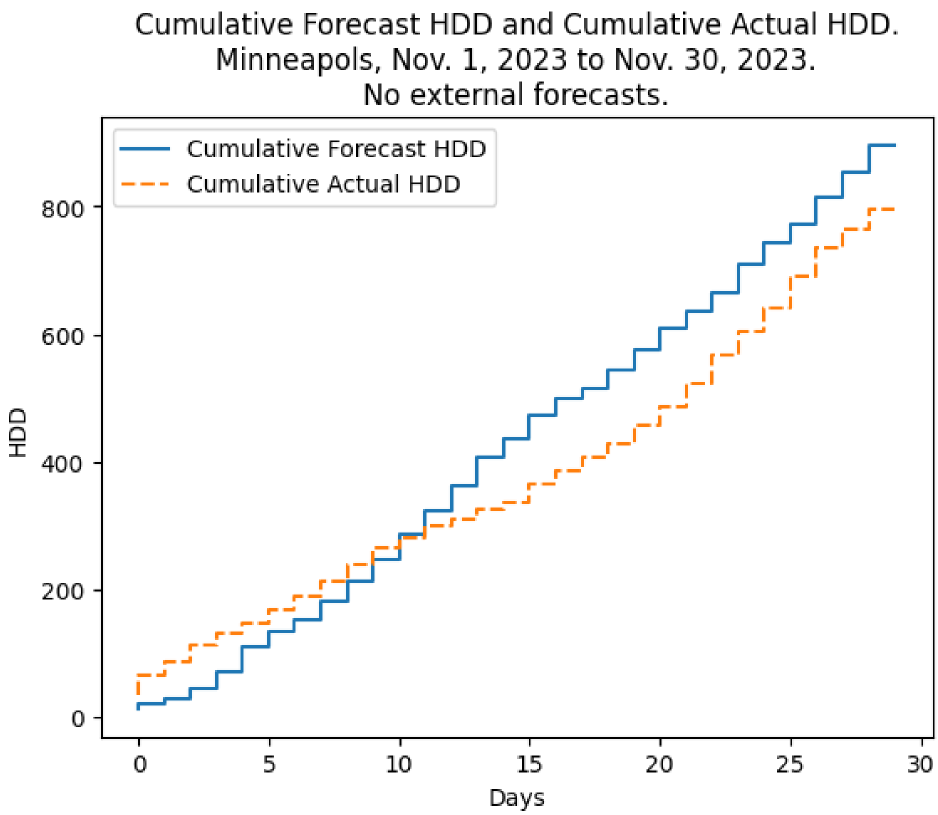

Figure 4.1: Cumulative Forecast HDD and Cumulative Actual HDD. Minneapolis, November 1, 2023 to November 30, 2023. No external forecasts.

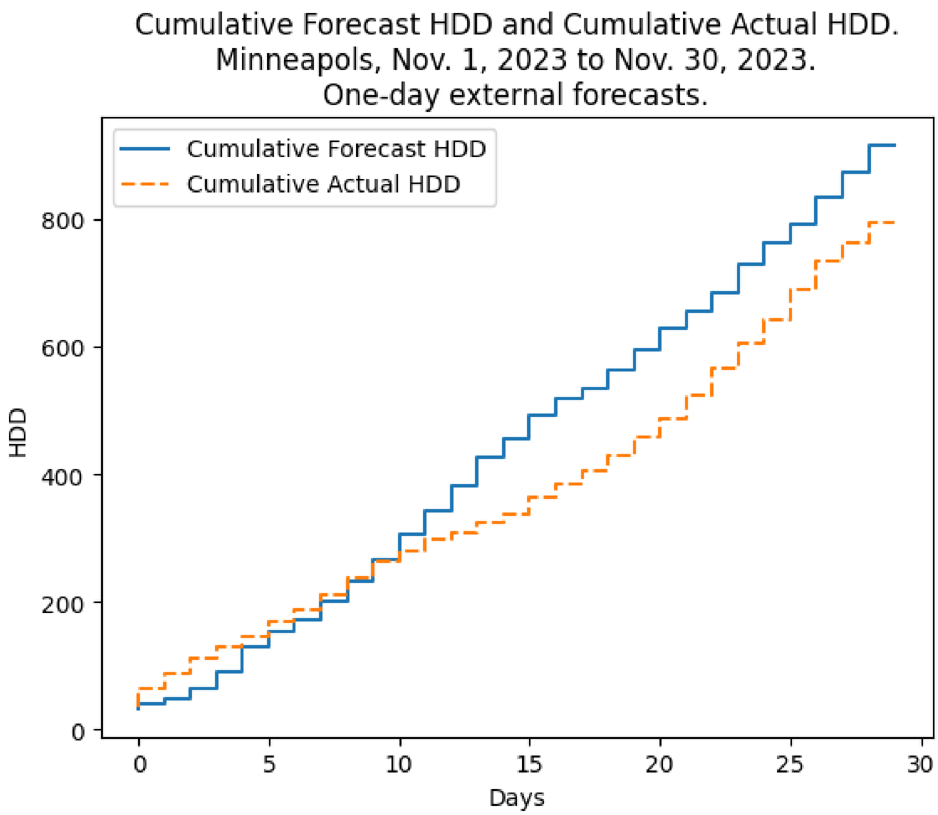

Figure 4.2: Cumulative Forecast HDD and Cumulative Actual HDD. Minneapolis, November 1, 2023 to November 30, 2023. One-day external forecasts.

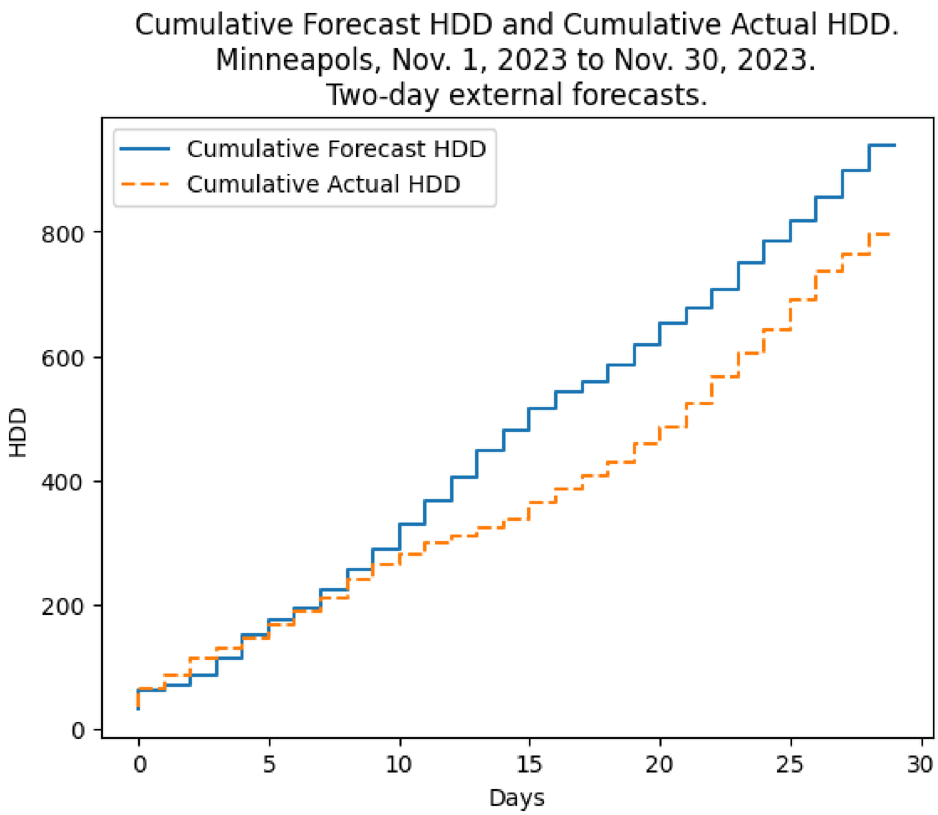

Figure 4.3: Cumulative Forecast HDD and Cumulative Actual HDD. Minneapolis, November 1, 2023 to November 30, 2023. Two-day external forecasts.

Table 4.3: Model forecasted daily maxima, minima, rounded maxima, rounded minima, average temperature and heating degree days for Minneapolis in degrees Fahrenheit. November 1, 2023 to November 30, 2023.

Date	Max	Min	Rounded Max	Rounded Min	Average	HDD
2023-11-01	56.4	47.8	56.0	48.0	52.0	13.0
2023-11-02	67.4	46.9	67.0	47.0	57.0	8.0
2023-11-03	66.6	47.8	67.0	48.0	57.5	7.5
2023-11-04	47.7	48.3	48.0	48.0	48.0	17.0
2023-11-05	51.6	25.9	52.0	26.0	39.0	26.0
2023-11-06	30.7	23.7	31.0	24.0	27.5	37.5
2023-11-07	47.6	31.7	48.0	32.0	40.0	25.0
2023-11-08	53.2	38.9	53.0	39.0	46.0	19.0
2023-11-09	43.1	29.1	43.0	29.0	36.0	29.0
2023-11-10	45.8	20.9	46.0	21.0	33.5	31.5
2023-11-11	45.2	19.7	45.0	20.0	32.5	32.5
2023-11-12	33.6	16.8	34.0	17.0	25.5	39.5
2023-11-13	34.2	19.2	34.0	19.0	26.5	38.5
2023-11-14	32.6	19.6	33.0	20.0	26.5	38.5
2023-11-15	24.0	18.3	24.0	18.0	21.0	44.0
2023-11-16	45.2	25.5	45.0	25.0	35.0	30.0
2023-11-17	42.9	17.2	43.0	17.0	30.0	35.0
2023-11-18	43.0	31.1	43.0	31.0	37.0	28.0
2023-11-19	59.1	37.7	59.0	38.0	48.5	16.5
2023-11-20	41.4	32.9	41.0	33.0	37.0	28.0
2023-11-21	38.0	30.0	38.0	30.0	34.0	31.0
2023-11-22	40.8	21.4	41.0	21.0	31.0	34.0
2023-11-23	49.0	28.0	49.0	28.0	38.5	26.5
2023-11-24	39.0	32.3	39.0	32.0	35.5	29.5
2023-11-25	27.4	16.6	27.0	17.0	22.0	43.0
2023-11-26	37.3	21.8	37.0	22.0	29.5	35.5
2023-11-27	41.6	27.9	42.0	28.0	35.0	30.0
2023-11-28	33.6	16.1	34.0	16.0	25.0	40.0
2023-11-29	33.5	15.4	33.0	15.0	24.0	41.0
2023-11-30	34.6	12.1	35.0	12.0	23.5	41.5

Table 4.4: Model forecasted heating degree days, actual heating degree days and differences for Minneapolis in degrees Fahrenheit. November 1, 2023 to November 30, 2023.

Date	Forecast HDD	Actual HDD	Diff
2023-11-01	13.0	34.5	-21.5
2023-11-02	8.0	30.0	-22.0
2023-11-03	7.5	22.5	-15.0
2023-11-04	17.0	25.5	-8.5
2023-11-05	26.0	17.5	8.5
2023-11-06	37.5	16.0	21.5
2023-11-07	25.0	22.5	2.5
2023-11-08	19.0	19.5	-0.5
2023-11-09	29.0	24.0	5.0
2023-11-10	31.5	27.0	4.5
2023-11-11	32.5	25.0	7.5
2023-11-12	39.5	17.0	22.5
2023-11-13	38.5	18.0	20.5
2023-11-14	38.5	10.5	28.0
2023-11-15	44.0	15.0	29.0
2023-11-16	30.0	12.5	17.5
2023-11-17	35.0	28.0	7.0
2023-11-18	28.0	20.5	7.5
2023-11-19	16.5	21.5	-5.0
2023-11-20	28.0	22.0	6.0
2023-11-21	31.0	29.0	2.0
2023-11-22	34.0	28.5	5.5
2023-11-23	26.5	36.5	-10.0
2023-11-24	29.5	44.0	-14.5
2023-11-25	43.0	37.5	5.5
2023-11-26	35.5	37.5	-2.0
2023-11-27	30.0	48.5	-18.5
2023-11-28	40.0	45.0	-5.0
2023-11-29	41.0	28.0	13.0
2023-11-30	41.5	32.0	9.5
Totals	**896.0**	**795.5**	**100.5**

Table 4.5: Model forecasted daily maxima, minima, rounded maxima, rounded minima, average temperature and heating degree days with external one-day temperature forecast for Minneapolis in degrees Fahrenheit. November 1, 2023 to November 30, 2023.

Date	Max	Min	Rounded Max	Rounded Min	Average	HDD
2023-11-01	38.0	26.0	38.0	26.0	32.0	33.0
2023-11-02	67.4	46.9	67.0	47.0	57.0	8.0
2023-11-03	66.6	47.8	67.0	48.0	57.5	7.5
2023-11-04	47.7	48.3	48.0	48.0	48.0	17.0
2023-11-05	51.6	25.9	52.0	26.0	39.0	26.0
2023-11-06	30.7	23.7	31.0	24.0	27.5	37.5
2023-11-07	47.6	31.7	48.0	32.0	40.0	25.0
2023-11-08	53.2	38.9	53.0	39.0	46.0	19.0
2023-11-09	43.1	29.1	43.0	29.0	36.0	29.0
2023-11-10	45.8	20.9	46.0	21.0	33.5	31.5
2023-11-11	45.2	19.7	45.0	20.0	32.5	32.5
2023-11-12	33.6	16.8	34.0	17.0	25.5	39.5
2023-11-13	34.2	19.2	34.0	19.0	26.5	38.5
2023-11-14	32.6	19.6	33.0	20.0	26.5	38.5
2023-11-15	24.0	18.3	24.0	18.0	21.0	44.0
2023-11-16	45.2	25.5	45.0	25.0	35.0	30.0
2023-11-17	42.9	17.2	43.0	17.0	30.0	35.0
2023-11-18	43.0	31.1	43.0	31.0	37.0	28.0
2023-11-19	59.1	37.7	59.0	38.0	48.5	16.5
2023-11-20	41.4	32.9	41.0	33.0	37.0	28.0
2023-11-21	38.0	30.0	38.0	30.0	34.0	31.0
2023-11-22	40.8	21.4	41.0	21.0	31.0	34.0
2023-11-23	49.0	28.0	49.0	28.0	38.5	26.5
2023-11-24	39.0	32.3	39.0	32.0	35.5	29.5
2023-11-25	27.4	16.6	27.0	17.0	22.0	43.0
2023-11-26	37.3	21.8	37.0	22.0	29.5	35.5
2023-11-27	41.6	27.9	42.0	28.0	35.0	30.0
2023-11-28	33.6	16.1	34.0	16.0	25.0	40.0
2023-11-29	33.5	15.4	33.0	15.0	24.0	41.0
2023-11-30	34.6	12.1	35.0	12.0	23.5	41.5

Table 4.6: Model forecasted heating degree days, actual heating degree days and differences with external one-day temperature forecast for Minneapolis in degrees Fahrenheit. November 1, 2023 to November 30, 2023.

Date	Forecast HDD	Actual HDD	Diff
2023-11-01	33.0	34.5	-1.5
2023-11-02	8.0	30.0	-22.0
2023-11-03	7.5	22.5	-15.0
2023-11-04	17.0	25.5	-8.5
2023-11-05	26.0	17.5	8.5
2023-11-06	37.5	16.0	21.5
2023-11-07	25.0	22.5	2.5
2023-11-08	19.0	19.5	-0.5
2023-11-09	29.0	24.0	5.0
2023-11-10	31.5	27.0	4.5
2023-11-11	32.5	25.0	7.5
2023-11-12	39.5	17.0	22.5
2023-11-13	38.5	18.0	20.5
2023-11-14	38.5	10.5	28.0
2023-11-15	44.0	15.0	29.0
2023-11-16	30.0	12.5	17.5
2023-11-17	35.0	28.0	7.0
2023-11-18	28.0	20.5	7.5
2023-11-19	16.5	21.5	-5.0
2023-11-20	28.0	22.0	6.0
2023-11-21	31.0	29.0	2.0
2023-11-22	34.0	28.5	5.5
2023-11-23	26.5	36.5	-10.0
2023-11-24	29.5	44.0	-14.5
2023-11-25	43.0	37.5	5.5
2023-11-26	35.5	37.5	-2.0
2023-11-27	30.0	48.5	-18.5
2023-11-28	40.0	45.0	-5.0
2023-11-29	41.0	28.0	13.0
2023-11-30	41.5	32.0	9.5
Totals	**916.0**	**795.5**	**120.5**

Table 4.7: Model forecasted daily maxima, minima, rounded maxima, rounded minima, average temperature and heating degree days with external two-day temperature forecast for Minneapolis in degrees Fahrenheit. November 1, 2023 to November 30, 2023.

Date	Max	Min	Rounded Max	Rounded Min	Average	HDD
2023-11-01	38.0	26.0	38.0	26.0	32.0	33.0
2023-11-02	41.0	28.0	41.0	28.0	34.5	30.5
2023-11-03	66.6	47.8	67.0	48.0	57.5	7.5
2023-11-04	47.7	48.3	48.0	48.0	48.0	17.0
2023-11-05	51.6	25.9	52.0	26.0	39.0	26.0
2023-11-06	30.7	23.7	31.0	24.0	27.5	37.5
2023-11-07	47.6	31.7	48.0	32.0	40.0	25.0
2023-11-08	53.2	38.9	53.0	39.0	46.0	19.0
2023-11-09	43.1	29.1	43.0	29.0	36.0	29.0
2023-11-10	45.8	20.9	46.0	21.0	33.5	31.5
2023-11-11	45.2	19.7	45.0	20.0	32.5	32.5
2023-11-12	33.6	16.8	34.0	17.0	25.5	39.5
2023-11-13	34.2	19.2	34.0	19.0	26.5	38.5
2023-11-14	32.6	19.6	33.0	20.0	26.5	38.5
2023-11-15	24.0	18.3	24.0	18.0	21.0	44.0
2023-11-16	45.2	25.5	45.0	25.0	35.0	30.0
2023-11-17	42.9	17.2	43.0	17.0	30.0	35.0
2023-11-18	43.0	31.1	43.0	31.0	37.0	28.0
2023-11-19	59.1	37.7	59.0	38.0	48.5	16.5
2023-11-20	41.4	32.9	41.0	33.0	37.0	28.0
2023-11-21	38.0	30.0	38.0	30.0	34.0	31.0
2023-11-22	40.8	21.4	41.0	21.0	31.0	34.0
2023-11-23	49.0	28.0	49.0	28.0	38.5	26.5
2023-11-24	39.0	32.3	39.0	32.0	35.5	29.5
2023-11-25	27.4	16.6	27.0	17.0	22.0	43.0
2023-11-26	37.3	21.8	37.0	22.0	29.5	35.5
2023-11-27	41.6	27.9	42.0	28.0	35.0	30.0
2023-11-28	33.6	16.1	34.0	16.0	25.0	40.0
2023-11-29	33.5	15.4	33.0	15.0	24.0	41.0
2023-11-30	34.6	12.1	35.0	12.0	23.5	41.5

Table 4.8: Model forecasted heating degree days, actual heating degree days and differences with external two-day temperature forecast for Minneapolis in degrees Fahrenheit. November 1, 2023 to November 30, 2023.

Date	Forecast HDD	Actual HDD	Diff
2023-11-01	33.0	34.5	-1.5
2023-11-02	30.5	30.0	0.5
2023-11-03	7.5	22.5	-15.0
2023-11-04	17.0	25.5	-8.5
2023-11-05	26.0	17.5	8.5
2023-11-06	37.5	16.0	21.5
2023-11-07	25.0	22.5	2.5
2023-11-08	19.0	19.5	-0.5
2023-11-09	29.0	24.0	5.0
2023-11-10	31.5	27.0	4.5
2023-11-11	32.5	25.0	7.5
2023-11-12	39.5	17.0	22.5
2023-11-13	38.5	18.0	20.5
2023-11-14	38.5	10.5	28.0
2023-11-15	44.0	15.0	29.0
2023-11-16	30.0	12.5	17.5
2023-11-17	35.0	28.0	7.0
2023-11-18	28.0	20.5	7.5
2023-11-19	16.5	21.5	-5.0
2023-11-20	28.0	22.0	6.0
2023-11-21	31.0	29.0	2.0
2023-11-22	34.0	28.5	5.5
2023-11-23	26.5	36.5	-10.0
2023-11-24	29.5	44.0	-14.5
2023-11-25	43.0	37.5	5.5
2023-11-26	35.5	37.5	-2.0
2023-11-27	30.0	48.5	-18.5
2023-11-28	40.0	45.0	-5.0
2023-11-29	41.0	28.0	13.0
2023-11-30	41.5	32.0	9.5
Totals	**938.5**	**795.5**	**143.0**

Table 4.9: Summary of HDD forecasts from model with no external daily temperature forecast, one-day external daily temperature forecast and two-day external daily temperature forecast. November 15, 2023 to November 30, 2023. The error is the forecast HDD minus actual HDD. Actual HDD: 486.0.

Scenario	Forecast HDD	Error
No external forecast	532.0	46.0
One-day external forecast	501.5	15.5
Two-day external forecast	486.0	0

4.4 Summary

In this chapter we have shown how a practitioner may leverage the model to incorporate external temperature forecasts. We find that the model is responsive to the inclusion of external temperature forecasts. However, the performance over an entire month is lackluster. Unseasonably warm (for CDD) or cold (for HDD) months can lead to large differences between model forecasted cumulative degree days and actual degree days.

Forecasts generated later in a month benefit from the resultant shorter forecasting period and knowledge of temperature actuals to that point. As degree day forecasts earlier in the month lead to less impact on total degree day model forecasts, a party seeking to hedge temperature risk would need to update their position throughout the month of the contract in question. In Chapter 5, we use the model to guide the balancing of a weather derivative futures portfolio to hedge temperature risk.

Table 4.10: Model forecasted daily maxima, minima, rounded maxima, rounded minima, average temperature and heating degree days for Minneapolis in degrees Fahrenheit. November 15, 2023 to November 30, 2023.

Date	Max	Min	Rounded Max	Rounded Min	Average	HDD
2023-11-15	24.1	18.4	24.0	18.0	21.0	44.0
2023-11-16	45.3	25.5	45.0	26.0	35.5	29.5
2023-11-17	43.0	17.3	43.0	17.0	30.0	35.0
2023-11-18	43.0	31.2	43.0	31.0	37.0	28.0
2023-11-19	59.2	37.8	59.0	38.0	48.5	16.5
2023-11-20	41.5	32.9	42.0	33.0	37.5	27.5
2023-11-21	38.1	30.1	38.0	30.0	34.0	31.0
2023-11-22	40.9	21.5	41.0	21.0	31.0	34.0
2023-11-23	49.1	28.0	49.0	28.0	38.5	26.5
2023-11-24	39.1	32.4	39.0	32.0	35.5	29.5
2023-11-25	27.4	16.7	27.0	17.0	22.0	43.0
2023-11-26	37.4	21.8	37.0	22.0	29.5	35.5
2023-11-27	41.7	27.9	42.0	28.0	35.0	30.0
2023-11-28	33.7	16.1	34.0	16.0	25.0	40.0
2023-11-29	33.6	15.4	34.0	15.0	24.5	40.5
2023-11-30	34.7	12.2	35.0	12.0	23.5	41.5

Table 4.11: Model forecasted heating degree days, actual heating degree days and differences for Minneapolis in degrees Fahrenheit. November 15, 2023 to November 30, 2023.

Date	Forecast HDD	Actual HDD	Diff
2023-11-15	44.0	15.0	29.0
2023-11-16	29.5	12.5	17.0
2023-11-17	35.0	28.0	7.0
2023-11-18	28.0	20.5	7.5
2023-11-19	16.5	21.5	-5.0
2023-11-20	27.5	22.0	5.5
2023-11-21	31.0	29.0	2.0
2023-11-22	34.0	28.5	5.5
2023-11-23	26.5	36.5	-10.0
2023-11-24	29.5	44.0	-14.5
2023-11-25	43.0	37.5	5.5
2023-11-26	35.5	37.5	-2.0
2023-11-27	30.0	48.5	-18.5
2023-11-28	40.0	45.0	-5.0
2023-11-29	40.5	28.0	12.5
2023-11-30	41.5	32.0	9.5
Totals	**532.0**	**486.0**	**46.0**

Table 4.12: Model forecasted daily maxima, minima, rounded maxima, rounded minima, average temperature and heating degree days with external one-day temperature forecast for Minneapolis in degrees Fahrenheit. November 15, 2023 to November 30, 2023.

Date	Max	Min	Rounded Max	Rounded Min	Average	HDD
2023-11-15	59.0	44.0	59.0	44.0	51.5	13.5
2023-11-16	45.3	25.5	45.0	26.0	35.5	29.5
2023-11-17	43.0	17.3	43.0	17.0	30.0	35.0
2023-11-18	43.0	31.2	43.0	31.0	37.0	28.0
2023-11-19	59.2	37.8	59.0	38.0	48.5	16.5
2023-11-20	41.5	32.9	42.0	33.0	37.5	27.5
2023-11-21	38.1	30.1	38.0	30.0	34.0	31.0
2023-11-22	40.9	21.5	41.0	21.0	31.0	34.0
2023-11-23	49.1	28.0	49.0	28.0	38.5	26.5
2023-11-24	39.1	32.4	39.0	32.0	35.5	29.5
2023-11-25	27.4	16.7	27.0	17.0	22.0	43.0
2023-11-26	37.4	21.8	37.0	22.0	29.5	35.5
2023-11-27	41.7	27.9	42.0	28.0	35.0	30.0
2023-11-28	33.7	16.1	34.0	16.0	25.0	40.0
2023-11-29	33.6	15.4	34.0	15.0	24.5	40.5
2023-11-30	34.7	12.2	35.0	12.0	23.5	41.5

Table 4.13: Model forecasted heating degree days, actual heating degree days and differences with external one-day temperature forecast for Minneapolis in degrees Fahrenheit. November 15, 2023 to November 30, 2023.

Date	Forecast HDD	Actual HDD	Diff
2023-11-15	13.5	15.0	-1.5
2023-11-16	29.5	12.5	17.0
2023-11-17	35.0	28.0	7.0
2023-11-18	28.0	20.5	7.5
2023-11-19	16.5	21.5	-5.0
2023-11-20	27.5	22.0	5.5
2023-11-21	31.0	29.0	2.0
2023-11-22	34.0	28.5	5.5
2023-11-23	26.5	36.5	-10.0
2023-11-24	29.5	44.0	-14.5
2023-11-25	43.0	37.5	5.5
2023-11-26	35.5	37.5	-2.0
2023-11-27	30.0	48.5	-18.5
2023-11-28	40.0	45.0	-5.0
2023-11-29	40.5	28.0	12.5
2023-11-30	41.5	32.0	9.5
Totals	**501.5**	**486.0**	**15.5**

Table 4.14: Model forecasted daily maxima, minima, rounded maxima, rounded minima, average temperature and heating degree days with external two-day temperature forecast for Minneapolis in degrees Fahrenheit. November 15, 2023 to November 30, 2023.

Date	Max	Min	Rounded Max	Rounded Min	Average	HDD
2023-11-15	59.0	44.0	59.0	44.0	51.5	13.5
2023-11-16	62.0	40.0	62.0	40.0	51.0	14.0
2023-11-17	43.0	17.3	43.0	17.0	30.0	35.0
2023-11-18	43.0	31.2	43.0	31.0	37.0	28.0
2023-11-19	59.2	37.8	59.0	38.0	48.5	16.5
2023-11-20	41.5	32.9	42.0	33.0	37.5	27.5
2023-11-21	38.1	30.1	38.0	30.0	34.0	31.0
2023-11-22	40.9	21.5	41.0	21.0	31.0	34.0
2023-11-23	49.1	28.0	49.0	28.0	38.5	26.5
2023-11-24	39.1	32.4	39.0	32.0	35.5	29.5
2023-11-25	27.4	16.7	27.0	17.0	22.0	43.0
2023-11-26	37.4	21.8	37.0	22.0	29.5	35.5
2023-11-27	41.7	27.9	42.0	28.0	35.0	30.0
2023-11-28	33.7	16.1	34.0	16.0	25.0	40.0
2023-11-29	33.6	15.4	34.0	15.0	24.5	40.5
2023-11-30	34.7	12.2	35.0	12.0	23.5	41.5

Table 4.15: Model forecasted heating degree days, actual heating degree days and differences with external two-day temperature forecast for Minneapolis in degrees Fahrenheit. November 15, 2023 to November 30, 2023.

Date	Forecast HDD	Actual HDD	Diff
2023-11-15	13.5	15.0	-1.5
2023-11-16	14.0	12.5	1.5
2023-11-17	35.0	28.0	7.0
2023-11-18	28.0	20.5	7.5
2023-11-19	16.5	21.5	-5.0
2023-11-20	27.5	22.0	5.5
2023-11-21	31.0	29.0	2.0
2023-11-22	34.0	28.5	5.5
2023-11-23	26.5	36.5	-10.0
2023-11-24	29.5	44.0	-14.5
2023-11-25	43.0	37.5	5.5
2023-11-26	35.5	37.5	-2.0
2023-11-27	30.0	48.5	-18.5
2023-11-28	40.0	45.0	-5.0
2023-11-29	40.5	28.0	12.5
2023-11-30	41.5	32.0	9.5
Totals	**486.0**	**486.0**	**0**

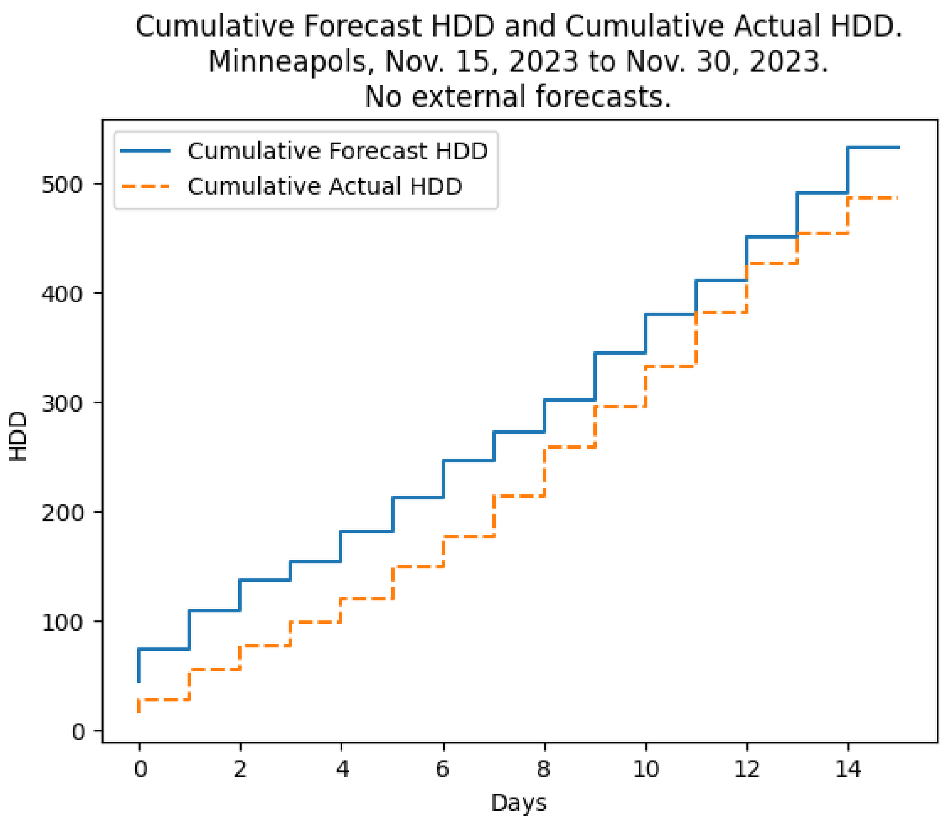

Figure 4.4: Cumulative Forecast HDD and Cumulative Actual HDD. Minneapolis, November 15, 2023 to November 30, 2023. No external forecasts.

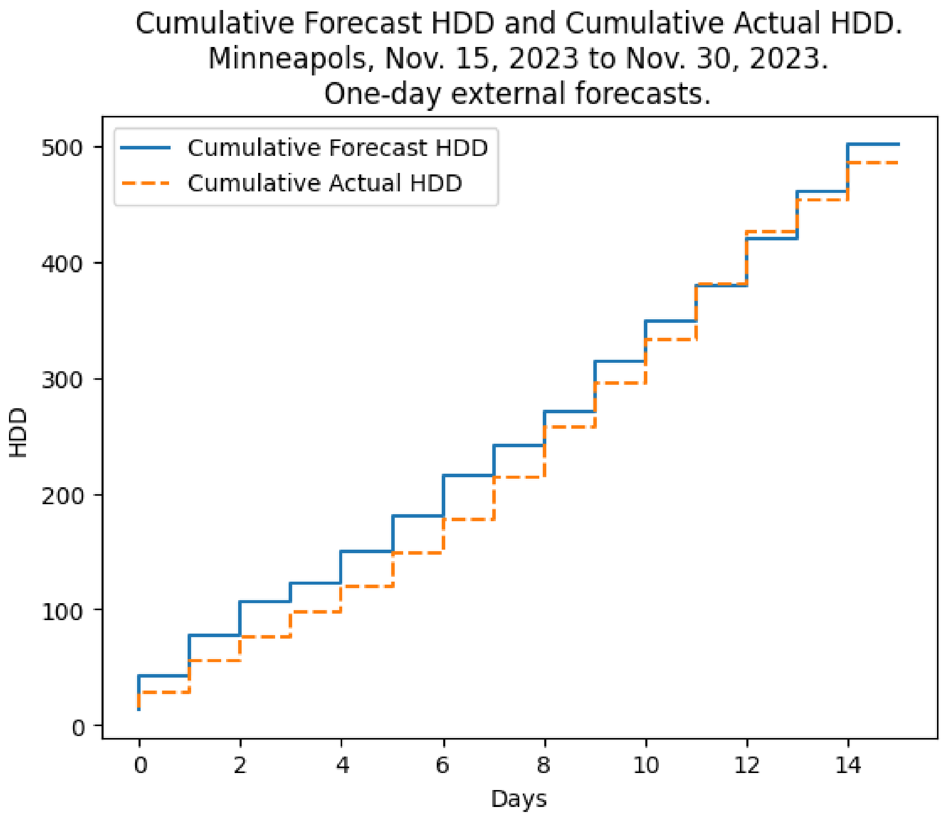

Figure 4.5: Cumulative Forecast HDD and Cumulative Actual HDD. Minneapolis, November 15, 2023 to November 30, 2023. One-day external forecasts.

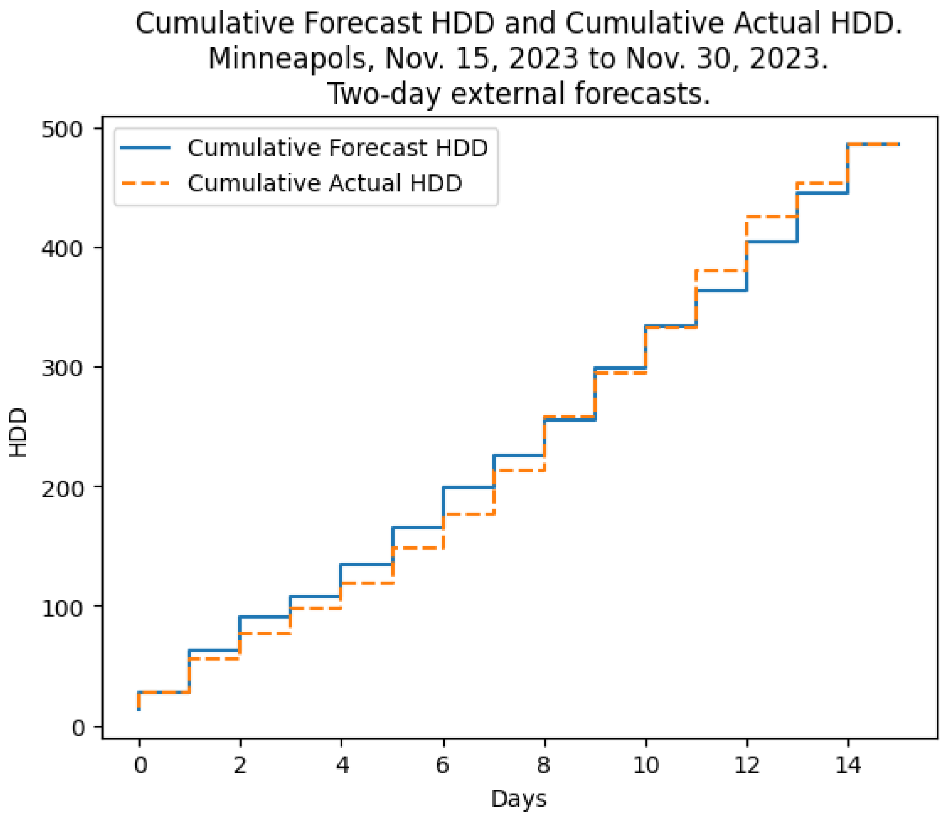

Figure 4.6: Cumulative Forecast HDD and Cumulative Actual HDD. Minneapolis, November 15, 2023 to November 30, 2023. Two-day external forecasts.

Chapter 5

Hedging example

In this chapter, we provide an example use of our model as part of a weather derivative hedge strategy using degree day future contracts. The hedge is updated at several days over a month using historical weather data and external forecasts. We give an example a trading strategy that uses both degree day future contracts and degree day option contracts.

5.1 Weather derivative market

As mentioned in Chapter 1, there are over-the-counter (OTC) and exchange-traded degree day weather derivatives. For the purpose of the hedging example in this chapter, we consider only exchange-traded degree day weather derivatives, as OTC are not standardized on tick amount or expiration date and data about the deals is not publicly available.

Figure 5.1 features information from the January 31, 2024 CME Daily Information Bulletin. The bottom third of the figure, under heading WEATHER INDEX FUTURES, contains information about exchange-traded heating degree day futures for Atlanta, Chicago and New York. From the first line of the weather index futures section, we see that for January 31, 2024 the settlement price for a February 2024 heating degree day weather future contract for Atlanta was 381.0, in heating degree days. The point change was -1200, or a decrease of twelve HDD from January 30, 2024. There were 500 open interest (i.e., open positions) on the February 2024 HDD future for Atlanta which was unchanged from the number on January 30, 2024. The highest bid for the contract was 395.0 heating degree days.

Later in this chapter, we use information from the CME daily information bulletins for price information on February 2024 HDD contracts.

WEATHER INDEX FUTURES										
	GLOBEX OPEN	GLOBEX HIGH	GLOBEX LOW	SETT. & PT.	PRICE CHGE	VOLUME TRADES CLEARED	OPEN INTEREST		--CONTRACT-- HIGH	LOW
HDD ATLANTA F										
FEB24	----	----	----	381.0	- 1200	----	500	UNCH	395.0B	----
JAN24	----	----	----	645.0	- 300	----	350	UNCH	675.0B	750.0A
TOTAL HDD ATLANTA F						0	850			
HDD CHICAGO F										
FEB24	----	----	----	880.0	+ NEW	700	700	+ 700	1040.0B	----
JAN24	----	----	----	1201.0	- 300	----	400	UNCH	1300.0B	1180.0A
TOTAL HDD CHICAGO F						700	1100	+ 700		
HDD NEW YORK F										
FEB24	----	----	----	731.0	- 1900	----	500	UNCH	800.0B	800.0A
JAN24	----	----	----	869.0	- 400	----	500	UNCH	905.0B	860.0A
TOTAL HDD NEW YORK F						0	1000			

Figure 5.1: Segment of CME Daily Bulletin, page 24. January 31, 2024 [31]. The blue box to the left shows that the associated row is for February 2024 heating degree day future contracts for New York. The red box to the right shows the highest degree day bid (800 HDD) and lowest degree day ask (also 800 HDD) for the contract on January 31, 2024.

5.2 Temperature risk hedge with futures contracts: February 2024

The parameters in the following hypothetical hedging scenario have been chosen for the purpose of providing a numerical example of a hedge. In practice, the values and functions involved would have to be based on historical data and knowledge of the firm's operations.

Suppose we are presented with a fictitious firm in the New York area that wants to hedge temperature risk in February 2024. For our scenario, the firm wants to protect its projected revenue for the month against being below \$4,000,000; that is, the firm is exposed to the risk of weather over the month being warmer which, in turn, would lead to fewer heating degree days. Furthermore, suppose that the firm has found from an analysis of its historical revenue data for the month of February that there is a nonlinear relationship between heating degree days and revenue.

For the purpose of this hypothetical hedging example we use the function $f(x)$ to represent the firm's revenue with

$$f(x) = 100(x - 490)^{1/4}, \qquad x \geq 500, \tag{5.1}$$

where x represents the number of heating degree days in a given February and $f(x)$ is in tens of thousands of dollars for the given February. As mentioned above, for a real hedging scenario the revenue function would be based on historical data and knowledge of the firm's operations.

To inform our hedging activity, we compute hedging ratios. A hedge ratio is the ratio of the size of a position in a hedging instrument to the size of the position being hedged [33, 34]. Our hedge ratios is of the form

$$\Delta\text{Revenues} \div \Delta\text{Value of Futures}, \tag{5.2}$$

where ΔRevenues is the size of our revenue that we are protecting multiplied by the derivative of the revenue function, $f'(x)$, evaluated at our model forecast degree days per the month. As the units of $f(x)$ for this exercise are tens of thousands of dollars, we use tens of thousands of dollars as the unit for ΔRevenues which gives us

$$\Delta\text{Revenues} = 400 \times f'(x). \tag{5.3}$$

For ΔValue of Futures we use the value of a single degree day future contract at the market price with the same units as ΔRevenues. As an example, if HDD futures were trading at 950, we would have

$$\Delta\text{Value of Futures} = \text{Market Price} \times \text{Tick} \times 0.0001 = 950 \times 20 \times 0.0001. \tag{5.4}$$

5.2.1 January 31, 2024 hedging activity

On January 31, 2024 we run our forecasting model with two-day external forecasts and the HDD forecast for February 2024 is 822. From Figure 5.1 we see that our forecasted number of HDD is close to the January 31, 2024 highest bid and lowest ask offers of 800 HDD for February 2024 New York HDD futures. We form the hedge ratio

$$\frac{400 \times f'(822)}{800 \times 20 \times 0.0001} = \frac{400 \times 0.321}{800 \times 20 \times 0.0001} = 80.25, \tag{5.5}$$

where the numerator is the revenue that the firm wants to protect (in tens of thousands of dollars) multiplied by f^0 evaluated at the forecasted HDD value of 822, and the denominator is the value of one HDD contract at the trading HDD price (also in tens of thousands of dollars). To protect the $4,000,000 in revenue, the firm could open their hedging position by selling 80 New York HDD contracts for February 2024 on January 31, 2024 and receive $800   20   80, or $1,280,000.

5.2.2 February 14, 2024 hedging activity

On February 14, 2024 we run our forecasting model with two-day external forecasts and the HDD forecast for February 15, 2024 to February 29, 2024 is 442.5. The actual HDD value from February 1, 2024 to February 14, 2024 was 333, for a total HDD forecast for February 2024 of 775.5. From the February 14, 2024 CME Daily Information Bulletin we find a highest bid of 800 and a lowest ask of 780 for February 2024 New York HDD futures. As the forecasted number of HDD has reduced, the firm would look to buy additional futures. Using the bid of 800 and our updated forecast of 775.5 HDD to form our hedge ratio, we have

$$\frac{400 \quad f^0(775{:}5)}{800 \quad 20 \quad 0{:}0001} = \frac{400 \quad 0{:}360}{800 \quad 20 \quad 0{:}0001} = 90{:} \tag{5.6}$$

Thus, to update the hedging position the firm could short (that is, sell) an additional ten New York HDD contracts for February 2024 on February 14, 2024 and receive $800   20   10, or $160;000.

5.2.3 February 21, 2024 hedging activity

On February 14, 2024 we run our forecasting model with two-day external forecasts and the HDD forecast for February 22, 2024 to February 29, 2024 is 253. The actual HDD value from February 1, 2024 to February 21, 2024 was 545.5, for a total HDD forecast for February 2024 of 798.5. The February 21, 2024 CME Daily Information Bulletin lists a highest bid of 800 and a lowest ask of 780 for February 2024 New York HDD futures. The forecasted number of HDD has increased, so the firm would look to buy futures. Using the ask of 780 and our updated forecast of 798.5 HDD to form our hedge ratio, we have

$$\frac{400 \quad f^0(798{:}5)}{780 \quad 20 \quad 0{:}0001} = \frac{400 \quad 0{:}340}{780 \quad 20 \quad 0{:}0001} = 87{:}18{:} \tag{5.7}$$

To update the hedging position, the firm could buy three New York HDD contracts for February 2024 on February 21, 2024 at a cost of $\$780 \quad 20 \quad 3$, or $\$46;800$.

5.2.4 Hedging cost

The total actual HDD for New York in February 2024 was 736. The firm would need to pay to settle the 87 outstanding shorted future contracts at a total of $\$736 \quad 20 \quad 87$, or $\$1;280;640$. Thus, the total cost for the hedge is

$$\$1;280;640 + \$46;800 \quad \$160;000 \quad \$1;280;000 = \quad \$112;560: \tag{5.8}$$

That is, the firm gained $112,560 from the hedging activity. Note that at 736 HDD, the firm expects the monthly revenue to be $f(736)$ which is roughly $3,960,000. The hedge brought in $72,560 more than the expected shortfall in revenue.

5.3 Position hedge with option contract

In the previous section, our objective was to protect revenues by using degree day future contracts. We now explore a scenario in which an investor enters into a short (that is, sell) position on a degree day future and wishes to hedge that position.

Consider the scenario for New York on February 14, 2024 from the previous section. Our forecast for the cumulative HDD for the month was 775.5. The market price to sell a HDD future on that day was $800. Based on our forecast for HDD, our view on February 14, 2024 is that the HDD is overpriced. To take advantage of the potential opportunity, an investor could sell (short) a HDD future contract at 800 degree days and purchase a call option on the same future contract. The call option would provide a hedge against losses incurred if the number of HDD were greater than 800 for the month. As stated in the previous section, the total actual HDD for New York in February 2024 was less than 800. Thus, the investor would not exercise the option to purchase a HDD future at 800. The investor's profit would be

$$\$800 \quad 20 \quad \$736 \quad 20 \quad \text{cost to buy the option} = \$1;280 \quad \text{cost to buy the option}: \tag{5.9}$$

That is, if the option were to cost less than $1,280 to purchase then the investor would realize a gain. If, instead, the number of HDD for the month were to have been higher than 800, the investor could have exercised the call option to cover the obligation of the short future position.

In Chapter 6 we detail ideas about how our degree day forecasting approach could be enhanced to allow for option pricing.

5.4 Summary

In this chapter, we built upon the work from Chapter 3 and Chapter 4 to show how we may leverage two-day external temperature forecasts with our degree day forecasting framework to perform a weather risk hedge using exchange-traded heating degree day future contracts. We have also given a brief glimpse into how we could use our degree day forecasting model to answer questions about the price of a degree day option contract.

In Chapter 6 we propose refinements to the model that may allow for improved heating degree day forecasts.

Chapter 6

Summary and future work

This chapter is a brief summary of the work produced in this dissertation along with several ideas for future work and extensions.

6.1 Summary of dissertation

In Chapter 3 of this dissertation, we have shown that the series of daily maximum temperatures and the series of daily minimum temperatures for cities in the US that have degree day weather derivative products feature the long-memory property. We have also shown that that long-memory property may be modeled by an ARFIMA process and that by combining linear and sinusoidal trend factors with the ARFIMA component to model we may model and forecast daily maximum temperatures and daily minimum temperatures. In Chapter 4 we have shown that we may incorporate external temperature forecasts to improve the degree day forecasts of our model. In Chapter 5 we used the model and external temperature forecasts to illustrate how the model could be used to hedge temperature risk.

6.2 Ideas for future work

To this point, we have used point estimates of daily maximum temperatures and daily minimum temperatures. Given probability distributions of temperature casts, we could construct a lattice of temperature forecasts. Each path through the lattice could represent a realization of temperatures over the external forecast horizon. A simplified example is depicted in Figure 6.1, in which given a particular reference day (t_n) the temperature forecasts for the subsequent day (t_{n+1}) may be +2°, +1°, the same, -1° or -2° from the reference day. The red sample path represents the case where the the temperature on day t_1 is two degrees warmer than the temperature on day t_0 and the temperature on day t_2 is one degree cooler than the temperature on day t_1. The probability associated with the sample path would be given by the external temperature forecast distribution. For each path, the daily temperatures would be fed into the model as if they were actuals to produce temperature forecasts beyond the horizon of the external temperature forecast source. Through exhaustion of all of the sample paths or through Monte Carlo methods we could price options on the degree day futures and estimate values such as *delta*, the price sensitivity of the option contract to changes in the price of the underlying degree day future, and *gamma*, the price sensitivity of delta to changes in the price of the underlying degree day future. By using the delta and gamma, an option trader can adjust their position to maintain delta neutrality, a scenario in which the trader's portfolio is insensitive to changes in small changes in the price of the underlying asset, and gamma neutrality, a scenario in which the portfolio is insensitive to larger changes in the price of the underlying asset [33, 34].

We could modify our approach by employing a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. Note that from Equation 3.17 and Equation 3.18, the l-step ahead forecast for our model is a function of d and l. The GARCH family of models would allow for us to model temporal variability of the error terms [15, 14]. Furthermore, the Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) model could allow us to model long-term dependence between the variance of the error terms if such dependence were detected in the daily temperature data [6].

Together, the external forecast distribution and GARCH approaches could provide an extension to our degree day future pricing framework that would allow for further temperature risk mitigation via hedge strategies that extend to degree day option contracts and Seasonal Strip contracts.

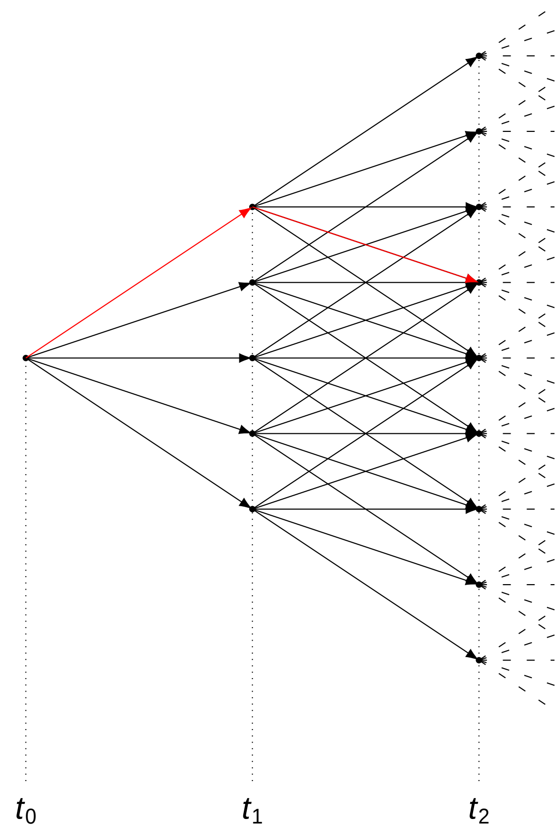

Figure 6.1: Simplified lattice representation of external temperature forecasts with a distribution. Sample path depicted in red.

ACRONYMS

AR autoregressive. 24

ARFIMA Autoregressive Fractionally Integrated Moving Average. 23–25, 59

ARIMA autoregressive integrated moving average. 15

ARMA Autoregressive Moving Average. 13

CDD cooling degree day. 3–6, 12–14, 29, 43, 71

CME Chicago Mercantile Exchange. 1, 3, 4, 6, 53, 54, 56, 73

FIGARCH Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity. 15, 60

GARCH Generalized Autoregressive Conditional Heteroskedasticity. 15, 60

HDD heating degree day. vii, viii, x, 3–6, 12–14, 29–31, 33, 43, 53–58, 71

NOAA National Oceanic and Atmospheric Administration. 31, 33

OTC over-the-counter. 53

PDE partial differential equation. 12

SDE stochastic differential equation. 12–14

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APPENDICES

.1 Forwards and Futures

.1.1 Definitions

A forward contract (referred hereafter as ***forward***) is an agreement to buy an asset in a certain date in the future for a certain price. The date in the contract is called the expiration date. A future contract (referred hereafter as ***future***) is an agreement to buy an asset at a future date that is traded on an exchange. A future is a standardized forward. [33, 34]

In general, the payoff curve for a future with price F and underlying asset price at expiration of the future $S(T)$ is as shown in Figure 2. Note that the case of $S(T) = K$ is a break-even point.

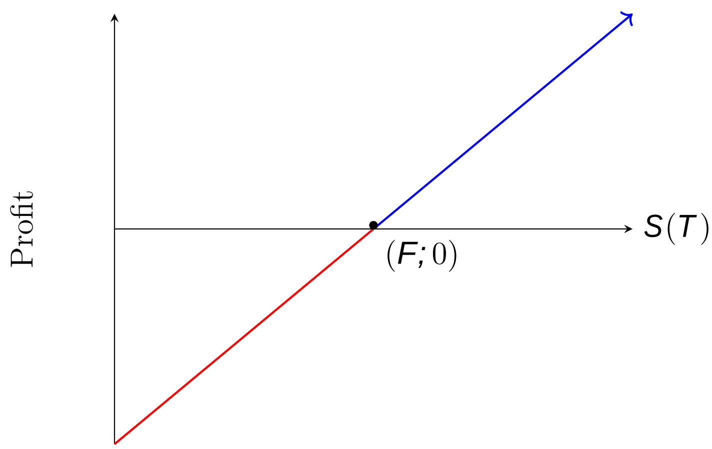

Figure 2: Profit curve for a future with price F and underlying asset with price at expiration $S(T)$. The red portion of the curve represents a net loss and the blue portion of the curve represents a net gain. The point $(F; 0)$ represents the break-even point.

.1.2 Options

An option is a contract which gives the holder the right, but not the obligation, to buy (in the case of a call option) or sell (in the case of a put option) the underlying asset by a certain date in the future for a certain price. The price in the contract is referred to as the strike price and the date in the contract is referred to as the expiration date. A

European option is an option that can be exercised only on the expiration date.

An example of a payoff curve for a European call option is given in Figure 3 and example of a payoff curve for a European put option is given in Figure 4

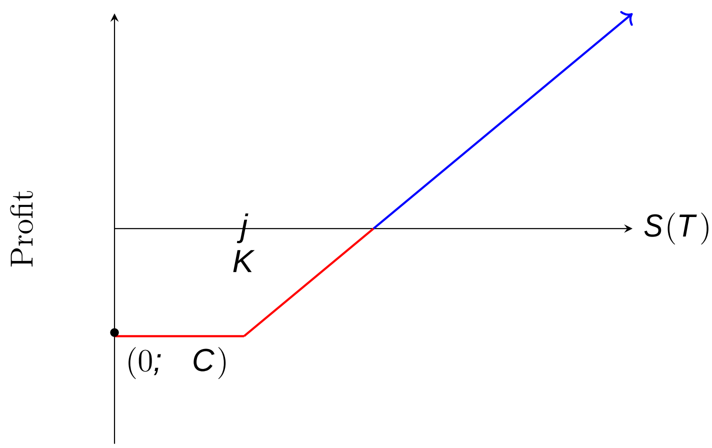

Figure 3: Profit curve for a European call option with price C, strike price K and underlying asset with price at expiration $S(T)$. The red portion of the curve represents a net loss and the blue portion of the curve represents a net gain.

.1.3 Weather derivative futures and options

A degree day future is a future contract that features a payoff of $20 (the tick price of the future) per heating degree day (HDD) or cooling degree day (CDD) over the period specified in the contract.

A degree day option is an option contract that features a "strike price" given as a number of degree days over the time period of the contract and an option premium which is the number of degree day future ticks (again, at $20 per tick) per contract.

Firms buy and sell degree day futures and options to hedge their temperature risk. Speculators buy and sell degree day futures and options as part of their portfolio management and strategy. [30, 36]

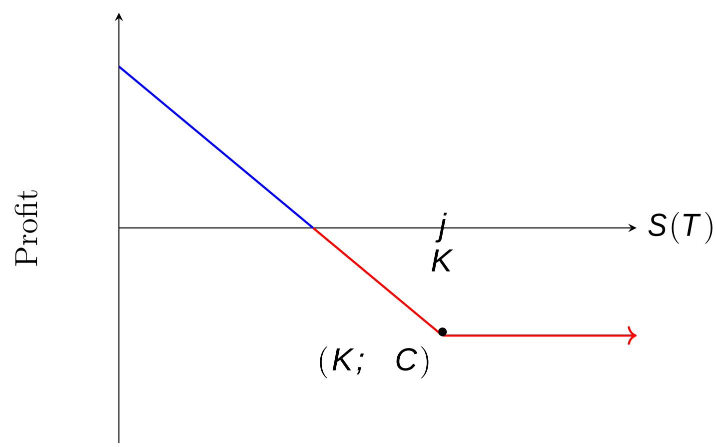

Figure 4: Profit curve for a European put option with price C, strike price K and underlying asset with price at expiration $S(T)$. The red portion of the curve represents a net loss and the blue portion of the curve represents a net gain.

Table 1: Estimates of trend and seasonal parameters for Atlanta daily maxima and daily minima. January 1, 1999 to December 31, 2018.

Parameter	Maxima estimate	Minima estimate
$\hat{c_0}$	71:55859	52:58343
$\hat{c_1}$	$3{:}36801 \quad 10^{4}$	$3{:}77202 \quad 10^{4}$
$\hat{A}$	17:96657	18:68711
ˆ	4:44068	4:40408

.2 Parameter estimates

The following parameter estimates are for the cities for which degree day weather derivatives are available from CME. The dates considered are from January 1, 1999 to December 31, 2019 with leap days removed. The number of observations for each series is $365 \quad 20 = 7700$.

Table 2: Parameter d estimation for Atlanta daily maxima. $\hat{d} = \quad 0{:}17$.

d	$S(d)$	d	$S(d)$
-0.50	783.8041	-0.25	48.2228
-0.49	698.2145	-0.24	44.9636
-0.48	621.7467	-0.23	42.3132
-0.47	553.4445	-0.22	40.2096
-0.46	492.4537	-0.21	38.5969
-0.45	438.0115	-0.20	37.4248
-0.44	389.4360	-0.19	36.6481
-0.43	346.1179	-0.18	36.2260
-0.42	307.5124	**-0.17**	**36.1218**
-0.41	273.1322	-0.16	36.3025
-0.40	242.5413	-0.15	36.7382
-0.39	215.3494	-0.14	37.4020
-0.38	191.2071	-0.13	38.2697
-0.37	169.8017	-0.12	39.3195
-0.36	150.8527	-0.11	40.5317
-0.35	134.1092	-0.10	41.8885
-0.34	119.3457	-0.09	43.3739
-0.33	106.3606	-0.08	44.9734
-0.32	94.9727	-0.07	46.6739
-0.31	85.0194	-0.06	48.4637
-0.30	76.3551	-0.05	50.3322
-0.29	68.8486	-0.04	52.2698
-0.28	62.3826	-0.03	54.2677
-0.27	56.8513	-0.02	56.3183
-0.26	52.1598	-0.01	58.4144

Table 3: Parameterd estimation for Atlanta daily minima. $\hat{d} = \ 0.18$.

d	S(d)	d	S(d)
-0.50	524.925550	-0.25	38.091007
-0.49	467.414232	-0.24	36.094686
-0.48	416.195251	-0.23	34.508786
-0.47	370.579180	-0.22	33.292991
-0.46	329.955183	-0.21	32.411012
-0.45	293.781745	-0.20	31.830152
-0.44	261.578525	-0.19	31.520940
-0.43	232.919192	-0.18	31.456790
-0.42	207.425129	-0.17	31.613706
-0.41	184.759890	-0.16	31.970014
-0.40	164.624315	-0.15	32.506127
-0.39	146.752241	-0.14	33.204330
-0.38	130.906706	-0.13	34.048595
-0.37	116.876621	-0.12	35.024409
-0.36	104.473820	-0.11	36.118623
-0.35	93.530463	-0.10	37.319318
-0.34	83.896748	-0.09	38.615685
-0.33	75.438882	-0.08	39.997912
-0.32	68.037291	-0.07	41.457091
-0.31	61.585043	-0.06	42.985132
-0.30	55.986443	-0.05	44.574677
-0.29	51.155801	-0.04	46.219040
-0.28	47.016328	-0.03	47.912135
-0.27	43.499174	-0.02	49.648424
-0.26	40.542557	-0.01	51.422867

Table 4: Estimates of trend and seasonal parameters for Chicago daily maxima and daily minima. January 1, 1999 to December 31, 2018.

Parameter	Maxima estimate	Minima estimate
$\hat{c}_0$	59.69222	41.36056
$\hat{c}_1$	$2.84153\ \ 10^{5}$	$1.24955\ \ 10^{4}$
$\hat{A}$	26.41717	23.65341
$\hat{\ }$	1.01674	$9.65975\ \ 10^{1}$

Table 5: Parameter d estimation for Chicago daily maxima. $\hat{d} =$ 0:17.

d	S(d)	d	S(d)
-0.50	1187.1117	-0.25	77.2536
-0.49	1058.3929	-0.24	72.3584
-0.48	943.3350	-0.23	68.3970
-0.47	840.5095	-0.22	65.2754
-0.46	748.6410	-0.21	62.9093
-0.45	666.5902	-0.20	61.2226
-0.44	593.3393	-0.19	60.1465
-0.43	527.9784	-0.18	59.6193
-0.42	469.6938	-0.17	59.5850
-0.41	417.7578	-0.16	59.9933
-0.40	371.5191	-0.15	60.7987
-0.39	330.3948	-0.14	61.9603
-0.38	293.8628	-0.13	63.4408
-0.37	261.4554	-0.12	65.2070
-0.36	232.7535	-0.11	67.2284
-0.35	207.3814	-0.10	69.4778
-0.34	185.0020	-0.09	71.9304
-0.33	165.3132	-0.08	74.5640
-0.32	148.0435	-0.07	77.3583
-0.31	132.9493	-0.06	80.2950
-0.30	119.8121	-0.05	83.3577
-0.29	108.4351	-0.04	86.5313
-0.28	98.6417	-0.03	89.8025
-0.27	90.2732	-0.02	93.1588
-0.26	83.1867	-0.01	96.5894

Table 6: Parameter d estimation for Chicago daily minima. $\hat{d} = 0.17$.

d	S(d)	d	S(d)
-0.50	1109.590002	-0.25	63.193636
-0.49	987.905877	-0.24	58.494047
-0.48	879.174774	-0.23	54.652899
-0.47	782.043621	-0.22	51.581859
-0.46	695.302993	-0.21	49.201478
-0.45	617.871579	-0.20	47.440285
-0.44	548.782358	-0.19	46.233969
-0.43	487.170297	-0.18	45.524652
-0.42	432.261388	-0.17	45.260236
-0.41	383.362895	-0.16	45.393817
-0.40	339.854657	-0.15	45.883159
-0.39	301.181338	-0.14	46.690224
-0.38	266.845530	-0.13	47.780750
-0.37	236.401587	-0.12	49.123871
-0.36	209.450140	-0.11	50.691777
-0.35	185.633201	-0.10	52.459410
-0.34	164.629790	-0.09	54.404188
-0.33	146.152038	-0.08	56.505761
-0.32	129.941705	-0.07	58.745787
-0.31	115.767076	-0.06	61.107736
-0.30	103.420178	-0.05	63.576707
-0.29	92.714306	-0.04	66.139272
-0.28	83.481808	-0.03	68.783330
-0.27	75.572100	-0.02	71.497975
-0.26	68.849898	-0.01	74.273381

Table 7: Estimates of trend and seasonal parameters for Cincinatti daily maxima and daily minima. January 1, 1999 to December 31, 2018.

Parameter	Maxima estimate	Minima estimate
$\hat{c}_0$	63.91646	44.29116
$\hat{c}_1$	$1.57150 \quad 10^{4}$	$2.59659 \quad 10^{4}$
$\hat{A}$	23.68181	21.64799
^	4.45638	4.43285

Table 8: Parameterd estimation for Cincinatti daily maxima. $\hat{d} =$ 0:18.

d	S(d)	d	S(d)
-0.50	969.8450	-0.25	70.1716
-0.49	867.5707	-0.24	66.1131
-0.48	775.8308	-0.23	62.8507
-0.47	693.5617	-0.22	60.3074
-0.46	619.8100	-0.21	58.4135
-0.45	553.7206	-0.20	57.1060
-0.44	494.5266	-0.19	56.3277
-0.43	441.5397	-0.18	56.0271
-0.42	394.1421	-0.17	56.1573
-0.41	351.7790	-0.16	56.6758
-0.40	313.9519	-0.15	57.5442
-0.39	280.2130	-0.14	58.7278
-0.38	250.1596	-0.13	60.1948
-0.37	223.4297	-0.12	61.9165
-0.36	199.6975	-0.11	63.8671
-0.35	178.6698	-0.10	66.0227
-0.34	160.0827	-0.09	68.3621
-0.33	143.6985	-0.08	70.8657
-0.32	129.3031	-0.07	73.5158
-0.31	116.7033	-0.06	76.2964
-0.30	105.7253	-0.05	79.1929
-0.29	96.2119	-0.04	82.1921
-0.28	88.0217	-0.03	85.2818
-0.27	81.0266	-0.02	88.4511
-0.26	75.1114	-0.01	91.6901

Table 9: Parameterd estimation for Cincinatti daily minima. $\hat{d} =\ 0.19$.

d	S(d)	d	S(d)
-0.50	662.208308	-0.25	55.224411
-0.49	592.214273	-0.24	52.700033
-0.48	529.580154	-0.23	50.716340
-0.47	473.540709	-0.22	49.221942
-0.46	423.413048	-0.21	48.170394
-0.45	378.587497	-0.20	47.519695
-0.44	338.519512	-0.19	47.231844
-0.43	302.722499	-0.18	47.272438
-0.42	270.761449	-0.17	47.610315
-0.41	242.247289	-0.16	48.217232
-0.40	216.831869	-0.15	49.067581
-0.39	194.203508	-0.14	50.138126
-0.38	174.083035	-0.13	51.407772
-0.37	156.220283	-0.12	52.857363
-0.36	140.390959	-0.11	54.469489
-0.35	126.393868	-0.10	56.228319
-0.34	114.048443	-0.09	58.119453
-0.33	103.192546	-0.08	60.129786
-0.32	93.680515	-0.07	62.247383
-0.31	85.381416	-0.06	64.461370
-0.30	78.177493	-0.05	66.761839
-0.29	71.962790	-0.04	69.139752
-0.28	66.641909	-0.03	71.586865
-0.27	62.128918	-0.02	74.095654
-0.26	58.346366	-0.01	76.659249

Table 10: Estimates of trend and seasonal parameters for Las Vegas daily maxima and daily minima. January 1, 1999 to December 31, 2018.

Parameter	Maxima estimate	Minima estimate
$\hat{c}_0$	80.00043	57.82075
$\hat{c}_1$	$2.97683\ 10^{4}$	$5.01021\ 10^{4}$
$\hat{A}$	23.54329	21.27679
^	4.48415	4.45358

Table 11: Parameter d estimation for Las Vegas daily maxima $\hat{d} = 0.18$.

d	S(d)	d	S(d)
-0.50	544.3416	-0.25	37.4869
-0.49	485.0254	-0.24	35.3065
-0.48	432.1743	-0.23	33.5677
-0.47	385.0719	-0.22	32.2286
-0.46	343.0856	-0.21	31.2516
-0.45	305.6568	-0.20	30.6024
-0.44	272.2915	-0.19	30.2502
-0.43	242.5526	-0.18	30.1670
-0.42	216.0531	-0.17	30.3274
-0.41	192.4497	-0.16	30.7083
-0.40	171.4381	-0.15	31.2887
-0.39	152.7478	-0.14	32.0495
-0.38	136.1385	-0.13	32.9735
-0.37	121.3965	-0.12	34.0448
-0.36	108.3314	-0.11	35.2490
-0.35	96.7733	-0.10	36.5730
-0.34	86.5708	-0.09	38.0047
-0.33	77.5886	-0.08	39.5334
-0.32	69.7058	-0.07	41.1489
-0.31	62.8140	-0.06	42.8421
-0.30	56.8164	-0.05	44.6046
-0.29	51.6259	-0.04	46.4289
-0.28	47.1644	-0.03	48.3079
-0.27	43.3617	-0.02	50.2352
-0.26	40.1548	-0.01	52.2049

Table 12: Parameter d estimation for Las Vegas daily minima. $\hat{d} =$ 0.18.

d	S(d)	d	S(d)
-0.50	373.084197	-0.25	24.313319
-0.49	333.134285	-0.24	22.714551
-0.48	297.397611	-0.23	21.425711
-0.47	265.427023	-0.22	20.417572
-0.46	236.825345	-0.21	19.663653
-0.45	211.239545	-0.20	19.139943
-0.44	188.355599	-0.19	18.824656
-0.43	167.893975	-0.18	18.698008
-0.42	149.605661	-0.17	18.742027
-0.41	133.268660	-0.16	18.940368
-0.40	118.684908	-0.15	19.278164
-0.39	105.677553	-0.14	19.741881
-0.38	94.088557	-0.13	20.319192
-0.37	83.776583	-0.12	20.998864
-0.36	74.615122	-0.11	21.770655
-0.35	66.490850	-0.10	22.625222
-0.34	59.302166	-0.09	23.554041
-0.33	52.957904	-0.08	24.549328
-0.32	47.376193	-0.07	25.603977
-0.31	42.483450	-0.06	26.711494
-0.30	38.213482	-0.05	27.865949
-0.29	34.506696	-0.04	29.061919
-0.28	31.309397	-0.03	30.294450
-0.27	28.573164	-0.02	31.559010
-0.26	26.254296	-0.01	32.851459

Table 13: Estimates of trend and seasonal parameters for Minneapolis daily maxima and daily minima. January 1, 1999 to December 31, 2018.

Parameter	Maxima estimate	Minima estimate
$\hat{c}_0$	56.02538	38.38137
$\hat{c}_1$	$8.30258 \quad 10^{6}$	$1.23084 \quad 10^{4}$
$\hat{A}$	30.13174	27.98318
^	4.45645	4.42288

Table 14: Parameter d estimation for Minneapolis daily maxima. $\hat{d} = 0.17$.

d	S(d)	d	S(d)
-0.50	1362.9505	-0.25	84.0526
-0.49	1217.6420	-0.24	78.0805
-0.48	1087.2858	-0.23	73.2138
-0.47	970.3810	-0.22	69.3427
-0.46	865.5799	-0.21	66.3683
-0.45	771.6719	-0.20	64.2010
-0.44	687.5694	-0.19	62.7601
-0.43	612.2951	-0.18	61.9723
-0.42	544.9704	-0.17	61.7716
-0.41	484.8056	-0.16	62.0981
-0.40	431.0905	-0.15	62.8978
-0.39	383.1862	-0.14	64.1217
-0.38	340.5180	-0.13	65.7255
-0.37	302.5686	-0.12	67.6692
-0.36	268.8724	-0.11	69.9165
-0.35	239.0101	-0.10	72.4345
-0.34	212.6042	-0.09	75.1936
-0.33	189.3146	-0.08	78.1668
-0.32	168.8347	-0.07	81.3298
-0.31	150.8884	-0.06	84.6604
-0.30	135.2266	-0.05	88.1388
-0.29	121.6249	-0.04	91.7468
-0.28	109.8806	-0.03	95.4681
-0.27	99.8110	-0.02	99.2879
-0.26	91.2513	-0.01	103.1926

Table 15: Parameter d estimation for Minneapolis daily minima. $\hat{d} = -0.16$.

d	S(d)	d	S(d)
-0.50	1405.130898	-0.25	73.244543
-0.49	1251.151291	-0.24	67.085279
-0.48	1113.437130	-0.23	62.020287
-0.47	990.305502	-0.22	57.936775
-0.46	880.250088	-0.21	54.733227
-0.45	781.922315	-0.20	52.318262
-0.44	694.114550	-0.19	50.609610
-0.43	615.745123	-0.18	49.533192
-0.42	545.844955	-0.17	49.022293
-0.41	483.545641	-0.16	49.016823
-0.40	428.068804	-0.15	49.462655
-0.39	378.716603	-0.14	50.311023
-0.38	334.863252	-0.13	51.517991
-0.37	295.947452	-0.12	53.043970
-0.36	261.465628	-0.11	54.853286
-0.35	230.965889	-0.10	56.913791
-0.34	204.042630	-0.09	59.196514
-0.33	180.331711	-0.08	61.675350
-0.32	159.506140	-0.07	64.326771
-0.31	141.272223	-0.06	67.129579
-0.30	125.366110	-0.05	70.064674
-0.29	111.550716	-0.04	73.114848
-0.28	99.612954	-0.03	76.264600
-0.27	89.361270	-0.02	79.499973
-0.26	80.623432	-0.01	82.808398

Table 16: Estimates of trend and seasonal parameters for New York daily maxima and daily minima. January 1, 1999 to December 31, 2018.

Parameter	Maxima estimate	Minima estimate
$\hat{c}_0$	62.71707	49.21049
$\hat{c}_1$	1.94245×10^{-4}	1.82653×10^{-4}
$\hat{A}$	22.94492	21.81716
^	4.37020	4.28597

Table 17: Parameter d estimation for New York daily maxima. $\hat{d} = 0.17$.

d	S(d)	d	S(d)
-0.50	819.7226	-0.25	55.5676
-0.49	730.8733	-0.24	52.2465
-0.48	651.4452	-0.23	49.5591
-0.47	580.4624	-0.22	47.4404
-0.46	517.0513	-0.21	45.8321
-0.45	460.4298	-0.20	44.6818
-0.44	409.8978	-0.19	43.9421
-0.43	364.8279	-0.18	43.5705
-0.42	324.6586	-0.17	43.5289
-0.41	288.8863	-0.16	43.7828
-0.40	257.0602	-0.15	44.3012
-0.39	228.7757	-0.14	45.0561
-0.38	203.6705	-0.13	46.0224
-0.37	181.4196	-0.12	47.1776
-0.36	161.7314	-0.11	48.5011
-0.35	144.3443	-0.10	49.9747
-0.34	129.0238	-0.09	51.5818
-0.33	115.5592	-0.08	53.3075
-0.32	103.7616	-0.07	55.1384
-0.31	93.4612	-0.06	57.0624
-0.30	84.5056	-0.05	59.0686
-0.29	76.7580	-0.04	61.1473
-0.28	70.0954	-0.03	63.2895
-0.27	64.4073	-0.02	65.4873
-0.26	59.5945	-0.01	67.7336

Table 18: Parameter d estimation for New York daily minima. $\hat{d} = -0.17$.

d	S(d)	d	S(d)
-0.50	684.501408	-0.25	37.832485
-0.49	609.037553	-0.24	34.934783
-0.48	541.635840	-0.23	32.559187
-0.47	481.453337	-0.22	30.651120
-0.46	427.736241	-0.21	29.161559
-0.45	379.810315	-0.20	28.046461
-0.44	337.072379	-0.19	27.266252
-0.43	298.982714	-0.18	26.785374
-0.42	265.058295	-0.17	26.571868
-0.41	234.866748	-0.16	26.597011
-0.40	208.020966	-0.15	26.834986
-0.39	184.174303	-0.14	27.262584
-0.38	163.016286	-0.13	27.858944
-0.37	144.268784	-0.12	28.605309
-0.36	127.682597	-0.11	29.484819
-0.35	113.034397	-0.10	30.482315
-0.34	100.124011	-0.09	31.584169
-0.33	88.771982	-0.08	32.778131
-0.32	78.817399	-0.07	34.053188
-0.31	70.115953	-0.06	35.399444
-0.30	62.538198	-0.05	36.808001
-0.29	55.968003	-0.04	38.270867
-0.28	50.301165	-0.03	39.780860
-0.27	45.444164	-0.02	41.331530
-0.26	41.313057	-0.01	42.917083

Table 19: Estimates of trend and seasonal parameters for Portland daily maxima and daily minima. January 1, 1999 to December 31, 2018.

Parameter	Maxima estimate	Minima estimate
$\hat{c}_0$	62.42804	45.48851
$\hat{c}_1$	2.47380×10^{-4}	2.01131×10^{-4}
$\hat{A}$	17.54780	11.78945
$\hat{\ }$	4.41511	4.34362

Table 20: Parameter d estimation for Portland daily maxima. $\hat{d} = -0.17$.

d	S(d)	d	S(d)
-0.50	1059.8645	-0.25	50.3633
-0.49	934.1720	-0.24	46.3886
-0.48	823.2607	-0.23	43.1411
-0.47	725.3878	-0.22	40.5408
-0.46	639.0210	-0.21	38.5166
-0.45	562.8125	-0.20	37.0053
-0.44	495.5760	-0.19	35.9504
-0.43	436.2676	-0.18	35.3016
-0.42	383.9675	-0.17	35.0142
-0.41	337.8657	-0.16	35.0481
-0.40	297.2481	-0.15	35.3677
-0.39	261.4849	-0.14	35.9409
-0.38	230.0207	-0.13	36.7394
-0.37	202.3652	-0.12	37.7377
-0.36	178.0855	-0.11	38.9129
-0.35	156.7992	-0.10	40.2447
-0.34	138.1683	-0.09	41.7149
-0.33	121.8939	-0.08	43.3069
-0.32	107.7117	-0.07	45.0063
-0.31	95.3876	-0.06	46.7999
-0.30	84.7143	-0.05	48.6759
-0.29	75.5082	-0.04	50.6237
-0.28	67.6065	-0.03	52.6339
-0.27	60.8649	-0.02	54.6979
-0.26	55.1551	-0.01	56.8082

Table 21: Parameter d estimation for Portland daily minima. $\hat{d} = -0.17$.

d	S(d)	d	S(d)
-0.50	433.895155	-0.25	21.805516
-0.49	383.200460	-0.24	20.155214
-0.48	338.365935	-0.23	18.810454
-0.47	298.714370	-0.22	17.738243
-0.46	263.648617	-0.21	16.909162
-0.45	232.641944	-0.20	16.296963
-0.44	205.229567	-0.19	15.878220
-0.43	181.001209	-0.18	15.632010
-0.42	159.594580	-0.17	15.539635
-0.41	140.689637	-0.16	15.584381
-0.40	124.003549	-0.15	15.751297
-0.39	109.286270	-0.14	16.027000
-0.38	96.316649	-0.13	16.399508
-0.37	84.899010	-0.12	16.858087
-0.36	74.860141	-0.11	17.393115
-0.35	66.046654	-0.10	17.995964
-0.34	58.322649	-0.09	18.658892
-0.33	51.567672	-0.08	19.374951
-0.32	45.674907	-0.07	20.137902
-0.31	40.549589	-0.06	20.942139
-0.30	36.107603	-0.05	21.782625
-0.29	32.274254	-0.04	22.654832
-0.28	28.983174	-0.03	23.554685
-0.27	26.175372	-0.02	24.478522
-0.26	23.798381	-0.01	25.423046

.3 CME Weather Futures And Options Bulletins

The following pages contain the CME Weather Futures and Options Bulletins from January 31, 2024, February 14, 2024 and February 21, 2024.[31] Information from the bulletins about market prices for exchange-traded weather derivatives was used for the hedging example in Chapter 5.

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