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A STUDY ON THE LOAD DISTRIBUTION FACTOR IN THE PERFORATED SQUARE PLATE WITH ELASTIC SUPPORT

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ABSTRACT

The load distribution factor in the perforated square plate supporting by angle shape legs under concentrated load acting at arbitrary points through elastic media is calculated. For the calculation the perforated plate was converted into an orthotropic plate using the method suggested by J.B.Mahoney. The deflection for the calculation of the load distribution factor was obtained from the auxiliary plate which was extended both sides of the plate and it was compared with the results from ANSYS calculation. With this deflection, the calculation of the load distribution factor was performed. The result shows that the load distribution factor at the periphery of the plate is larger than that of in the central location. This load distribution factor could be used for re-distribution of the applied load in more accurate analysis of the plate as well as in the analysis of the elastic media as the load factor.

1. INTRODUCTION

The perforated plates which have regular hole array are frequently found in heat exchanger or structual components in fluid engineering field.

For the analysis of the perforated plate ,Mahoney[1,2] suggested the theoretical method how to convert the perforated plate into an orthotropic plate with effective stiffness coefficients.

Recently the analysis of the plate can be performed by numerical method using computer application.

Figure 1 shows the bottom nozzle of a typical PWR fuel assembly with square penetration hole array with angle shaped elastic supports at four corners. Generally the strength analysis of the bottom nozzle are performed by FEM with most applicable load during its life time. In this case the load is assumed to be uniformly distributed through the guide thimble and applied at the guide thimble locations. Since the guide thimble and the support legs as well as the plate are not completely rigid, the load will not actually be uniformly distributed over the guide thimble locations.

This paper deals with the load distribution factor on the perforated plate such as the bottom nozzle under load applied via elastic media.

To get the load distribution factor at each loading point of the perforated plate, the plate was converted to an equivalent orthotropic plate and then the deflection of the plate was calculated. Now that the boundary conditions of the bottom nozzle is not simple to apply analytic solution for the deflection, the plate was extended to make an auxiliary plate with four edge simple supports. But the deflection of the auxiliary plate depend on the side length of the plate, the length of the plate was determined based on the comparison of the deflection with that of ANSYS. With this deflection of the plate the reaction forces at the elastic supprts are applied on the auxiliary plate to get the reaction forces at loading points of which the ratios is the load ditribution factor.

2. EQUIVALENT ORTHOTROPIC PLATE

2.1 Elastic Constants

The perforated plate with minimum ligament H′ as in Fig.2 can be idealized as in Fig.3 with averaged ligament length H and then it can be converted to an equivalent orthotropic plate as were in ref.[1] and [2].

$$2R/H = \int_0^{\pi/2} \frac{\cos\phi}{[\,(A/2R)-\cos\phi\,]}\, d\phi \qquad \text{------------------------------} \ (1)$$

The elastic strain energy of the orthotropic plate with AxB is

$$V = (t^3/24)(A\cdot B)[C_{11}W^2{}_{xx} + 2C_{12}W_{xx}W_{yy} + C_{22}W^2{}_{yy} + 4C_{44}W^2{}_{xy}] \qquad \text{----------} \ (2)$$

The strain energy of the crucial element comprises the strain energy of the central plate and the beam tips :

$$\begin{aligned} V_1 = {} & (D/2)H^2[(B/H)W^2{}_{xx} + \nu(B/H + A/H)W_{xx}W_{yy} + (A/H)W^2{}_{yy} \\ & + (1-\nu)(B/H + A/H)W^2{}_{xy}] \qquad \text{--------------------} \ (3) \end{aligned}$$

for the central element, and

$$V_2 = (1/2)[EI_yR\cdot H\cdot W^2{}_{xx}/B + EI_xR\cdot H\cdot W^2{}_{yy}/A + \{GC_1R\cdot H/B + GC_2R\cdot H/A\}W^2{}_{xy}] \ \text{----} \ (4)$$

for the beam elements.
Thus the total strain energy of the crucial element is

$$\begin{aligned} V = {} & (Et^3/24)[\{B\cdot H/(1-\nu^2) + R\cdot H^2/B\}W^2{}_{xx} + \{A\cdot H/(1-\nu^2) \\ & + R\cdot H^2/A\}W^2{}_{yy} + \nu/(1-\nu^2)\{B\cdot H + A\cdot H\}W_{xx}W_{yy} \\ & + 1/(1+\nu)\{B\cdot H + A\cdot H + 6C_1R\cdot H/(t^3B) \\ & + 6C_2R\cdot H(t^3A)\}W^2{}_{xy}] \qquad \text{------------------------------} \ (5) \end{aligned}$$

Comparing eq.(2) with eq.(5),the elastic constants become

$$C_{11} = [E/(A\cdot B)][(B\cdot H/(1-\nu^2) + R\cdot H^2/B] \tag{6.a}$$

$$C_{21} = C_{12} = \nu[E/(2A\cdot B)][(B\cdot H + A\cdot H)/(1-\nu^2)] \tag{6.b}$$

$$C_{22} = [E/(A\cdot B)]\cdot[A\cdot H/(1-\nu^2) + R\cdot H^2/A] \tag{6.c}$$

$$C_{44} = E\cdot[B\cdot H + A\cdot H + \{(6C_1R\cdot H)/(t^3B) + (6C_2R\cdot H)/(t^3A)\}]/\{4A\cdot B(1+\nu)\} \tag{6.d}$$

2.2 Flexural Rigidities

The governing equation of orthotropic plate is[3]

$$D_1W_{xxxx} + 2(D_2 + D_4)W_{xxyy} + D_3W_{yyyy} = q(x,y) \tag{7}$$

where $D_1=(t^3/12)C_{11}$ ---- (8.a)

$D_2=(t^3/12)C_{12}$ ---- (8.b)

$D_3=(t^3/12)C_{22}$ ---- (8.c)

$D_4=(t^3/6)C_{44}$ ---- (8.d)

from eq.(6) and eq.(8), the flexural rigidities for the equivalent orthotropic plate are

$$D_1=[Et^3/(12A\cdot B)][A\cdot H/(1-\nu^2) + R\cdot H^2/B] \tag{9.a}$$

$$D_2=[\nu Et^3/(24A\cdot B)][\{B\cdot H + A\cdot H\}/(1-\nu^2\}] \tag{9.b}$$

$$D_3=[Et^3/(12A\cdot B)][A\cdot H/(1-\nu^3) + R\cdot H^2/A] \tag{9.c}$$

$$D_4=[Et^3/\{24A\cdot B(1+\nu)\}][B\cdot H + A\cdot H + (6C_1R\cdot H)/(t^3B) + (6C_2R\cdot H)/(t^3A)] \tag{9.d}$$

2.3. Reaction Forces at Supports

Elastic support at four corners of the perforated plate is simplified as a point support with equivalent elastic stiffness.

$$S_{eq} = EA_s/L_s \tag{10}$$

where A_s and L_s are area and length of the support.
The reaction forces at supports are obtained by applying the compatibility condition and flexibility influence coefficients [4].

$$[A_{ij}]\{F_j\} = \{w_j\} \tag{11}$$

where

$$A_{ij} = a_{ij} + 1/K\cdot\delta_{ij}$$

Here a_{ij} is flexibility influence coefficients, K and δ are stiffness of the support and Kronecker delta, respectively.

2.4 Deflection

The deflection of the orthotropic plate which satisfying the boundary condition due to the applied load and reaction forces can be calculated using Navier method[5].

$$W(x,y) = \sum_{m}\sum_{n} \frac{4P\cdot[\mathrm{Sin}(m\pi x/A)\cdot\mathrm{Sin}(n\pi y/B)]}{(A\cdot B)[D_1(m\pi/A)^4 + D^*(m\pi/A)^2\cdot(n\pi/A)^2 + D_3(n\pi/B)^4]} \tag{12}$$

2.5 Load Distribution Factor

The load distribution factor which is the ratio of the reaction forces at each loaing media can be obtained with the aid of the equ.(11) by applying the reaction forces at elastic support as loads.

3. RESULTS

For the numerical examples typical 14x14 and 17x17 type bottom nozzle of Korean fuel assmbly are taken.

The deflection of the auxiliary plate and that of ANSYS are shown in Fig.4 and 5. From these Figures, half the length of the auxiliary plate which produce the same deflection were determined as 300 mm for 14x14, 290 mm for 17x17. With this side length of the auxiliary plate the calculated load distribution factor as a functions of stiffness of the elastic media where loading is applied are as in Table 1. and Table 2.

4. CONCLUSION

The load distribution factor in the perforated square plate under concentrated load acting at arbitrary points through elastic media are calculated. Numerical results are presented as an example for 14x14 and 17x17 bottom nozzles of typical PWR fuel assembly. The result shows that the load distribution factor of the guide thimbles is larger values at the peripheral regions than the central location which depend on the stiffness of the loading media. This factors can be used for the analysis of the loading media and the plate.

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Table 1. Load Distribution Factor on 14x14 Bottom Nozzle

Node No.	X (mm)	Y (mm)	LDF 1	LDF 2	LDF 3
1	-63.54	-63.54	1.02046	1.04357	1.08345
2	63.54	-63.54	1.02046	1.04357	1.08345
3	63.54	63.54	1.02046	1.04357	1.08345
4	63.54	63.54	1.02046	1.04357	1.08345
5	35.30	-35.30	1.00000	1.00000	1.00000
6	35.30	-35.30	1.00000	1.00000	1.00000
7	35.30	35.30	1.00000	1.00000	1.00000
8	35.30	35.30	1.00000	1.00000	1.00000
9	21.18	-63.54	1.00706	1.01651	1.02876
10	21.18	-63.54	1.00706	1.01651	1.02876
11	21.18	63.54	1.00706	1.01651	1.02876
12	21.18	63.54	1.00706	1.01651	1.02876
13	63.54	21.16	1.00706	1.01651	1.02876
14	63.54	-21.16	1.00706	1.01651	1.02876
15	63.54	21.16	1.00706	1.01651	1.02876
16	63.54	-21.16	1.00706	1.01651	1.02876

* LDF : Load Distribution Factor
1) LDF 1 : Guide Thimble Stiffness 562.92 N/mm
2) LDF 2 : Guide Thimble Stiffness 1125.84 N/mm
3) LDF 3 : Guide Thimble Stiffness 2251.68 N/mm

Table 2. Load Distribution Factor on 17x17 Bottom Nozzle

Node No.	X (mm)	Y (mm)	LDF 1	LDF 2	LDF 3
1	-62.87	62.87	1.02958	1.05870	1.12079
2	62.87	-62.87	1.02958	1.05870	1.12079
3	62.87	62.87	1.02958	1.05870	1.12079
4	-62.87	62.87	1.02958	1.05870	1.12079
5	-37.72	-75.44	1.02567	1.05091	1.10460
6	0.00	-75.44	1.01914	1.03746	1.07805
7	-37.72	0.00	1.00000	1.00000	1.00000
8	37.72	-75.44	1.02567	1.05091	1.10460
9	75.44	0.00	1.01914	1.03795	1.07805
10	75.44	37.72	1.02567	1.05091	1.10460
11	37.72	75.44	1.02567	1.05091	1.10460
12	0.00	75.44	1.01914	1.03795	1.07805
13	-37.72	75.44	1.02567	1.05091	1.10460
14	-75.44	37.72	1.02567	1.05095	1.10460
15	-75.44	0.00	1.01914	1.03795	1.07805
16	-75.44	-37.72	1.02567	1.05091	1.10460
17	-37.72	-37.72	1.00690	1.01380	1.02840
18	37.72	-37.72	1.00690	1.01380	1.02840
19	37.72	37.72	1.00690	1.01380	1.02840
20	-37.72	37.72	1.00690	1.01380	1.02840
21	0.00	-37.72	1.00000	1.00000	1.00000
22	37.72	0.00	1.00000	1.00000	1.00000
23	0.00	37.72	1.00000	1.00000	1.00000
24	75.44	-37.72	1.02567	1.05091	1.10460

* LDF : load Distribution Factor
1) LDF 1 : Guide Thimble Stiffness 483.80 N/mm
2) LDF 2 : Guide Thimble Stiffness 967.60 N/mm
3) LDF 3 : Guide Thimble Stiffness 1935.22 N/mm

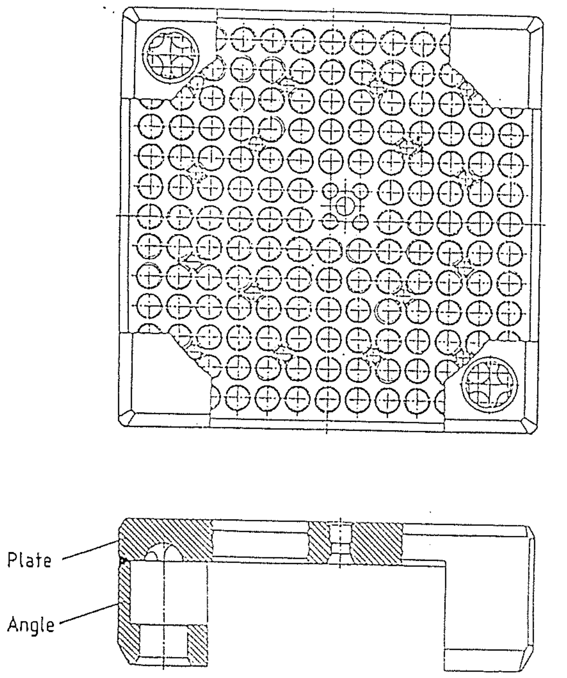

Fig.1 Bottom Nozzle for Typical PWR Fuel Assembly

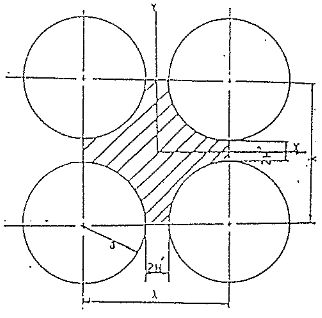

Fig.2 Perforated Plate Element

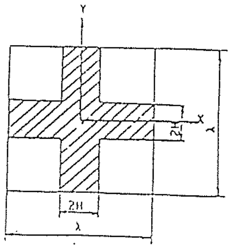

Fig.3 Equivalent Crucial Element

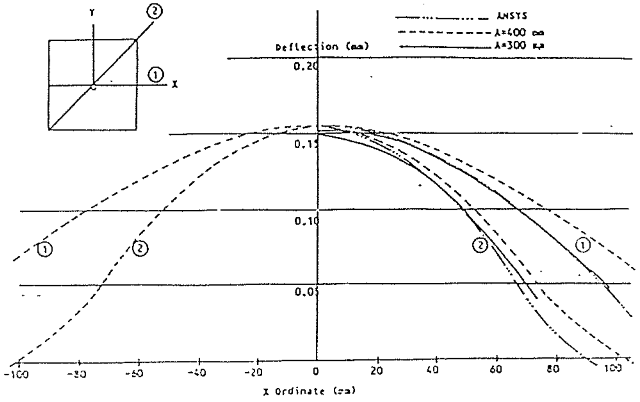

Fig.4 Comparison of the Calculated Deflection with ANSYS Results for 14x14 Bottom Nozzle

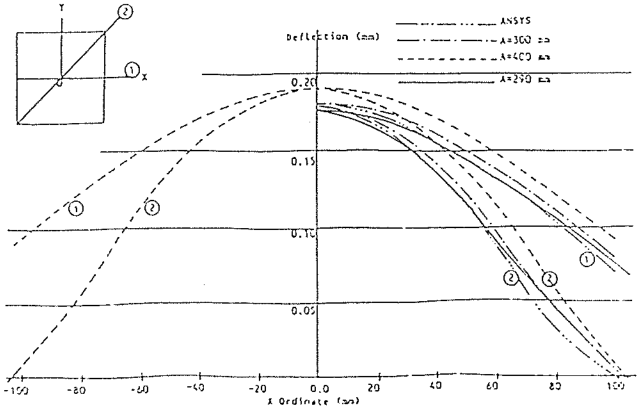

Fig.5 Comparison of the Calculated Deflection with ANSYS Results for 17x17 Bottom Nozzle