Structure (NPP) responses to single/multiple rotational excitations

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1 ABSTRACT

In case of the single support structure, the general theory on seismic forces under the action of rocking ground motion is completed through some derivation, analysis as well as summarization.

A comparison of the response to base rocking with that to base (horizontal) translation is made for the high-rise structure.

The results from the computation for a Λ-typed piping show that the effect of the response to the multi-support rocking excitations can never be negligible.

2 RESPONSES OF CANTILEVER TO BASE ROCKING

A cantilever is shown in Fig.1. In the figure, θ_g is the angular displacement of rocking at the base (ground or support); H, the full height; h_i, the height of the lumped mass m_i. The relative displacement $u(i,t)$ can be expanded in series of the normal modes $\bar{u}_r(i)$ and the generalized coordinates $q_r(t)$

(1) $$u(i,t) = \sum_r \bar{u}_r(i)\, q_r(t)$$

where, r is the modal ordinal and t, the time. Thus, the absolute velocity is

(2) $$\dot{u}(i,t) + h_i\dot{\theta}_g(t) = \sum_r \bar{u}_r(i)\,\dot{q}_r(t) + h_i\,\dot{\theta}_g(t)$$

By the orthogonality condition, the kinetic energy can be determined as

(3) $$\begin{aligned} K &= \frac{1}{2}\sum_i m_i(\dot{u}_i + h_i\dot{\theta}_g)^2 \\ &= \frac{1}{2}\dot{\theta}_g^2\sum_i m_i h_i^2 + \dot{\theta}_g\sum_r m_i h_i \bar{u}_r(i) + \frac{1}{2}\sum_r \dot{q}_r^2 \sum_i m_i \bar{u}_r^2(i) \end{aligned}$$

Denoting the natural frequency by ω_r, the strain energy should be

(4) $$\begin{aligned} U &= \frac{1}{2}\sum_i m_i\Big[\sum_r \omega_r^2 \bar{u}_r(i) q_r \cdot \sum_s \bar{u}_s(i) q_s\Big] \\ &= \frac{1}{2}\sum_r \omega_r^2 q_r^2 \sum_i m_i \bar{u}_r^2(i) \end{aligned}$$

The energy dissipation function can be written as

$$D=\sum_i m_i\Big[\sum_r \mu_r \bar{u}_r(i)\dot{q}_r \cdot \sum_s \bar{u}_s(i)\dot{q}_s\Big]=\sum_r \mu_r \dot{q}_r^2 \sum_i m_i \bar{u}_r^2(i) \tag{5}$$

Substituting Eq.(3),(4) and (5) into Lagrange Equation

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_r}\right)+\frac{\partial U}{\partial q_r}+\frac{\partial D}{\partial \dot{q}_r}=0 \tag{6}$$

and making some rearrangement, we have

$$\ddot{q}_r+2\mu_r\dot{q}_r+\omega_r^2 q_r=-\ddot{\theta}_g\sum_i m_i h_i \bar{u}_r(i)\Big/\sum_i m_i \bar{u}_r^2(i) \tag{7}$$

where, μ_r is the damping coefficient.

Putting

$$q_r(t)=H\xi_r\delta_r(t) \tag{8}$$

$$h_i=H\bar{h}_i \tag{9}$$

$$\varepsilon_r=\mu_r/\omega_r \tag{10}$$

one may obtain from Eq.(7)

$$\ddot{\delta}_r+2\varepsilon_r\omega_r\dot{\delta}_r+\omega_r^2\delta_r=-\ddot{\theta}_g \tag{11}$$

and

$$\xi_r=\sum_i m_i\bar{h}_i\bar{u}_r(i)\Big/\sum_i m_i\bar{u}_r^2(i) \tag{12}$$

In Eq.(11), δ_r is the response of single-mass oscillator. In Eq.(12), ξ_r is the mode participation factor and has the following noted mathematical characteristic

$$\bar{h}_i=\sum_r \xi_r\bar{u}_r(i) \tag{13}$$

by which, the absolute acceleration is expressed as

$$\begin{aligned}\ddot{u}(i,t)+h_i\ddot{\theta}_g&=H\sum_r\xi_r\bar{u}_r(i)\ddot{\delta}_r(t)+H\sum_r\xi_r\bar{u}_r(i)\ddot{\theta}_g(t)\\&=H\sum_r\xi_r\bar{u}_r(i)\big[\ddot{\delta}_r(t)+\ddot{\theta}_g(t)\big]\end{aligned} \tag{14}$$

Considering the relationship $\omega_r^2\delta_r\doteq-(\ddot{\delta}_r+\ddot{\theta}_g)$, the seismic loads

$$\begin{aligned}S(i,t)&=-m_i\big[\ddot{u}(i,t)+h_i\ddot{\theta}_g(t)\big]\\&=H\sum_r m_i\omega_r^2\xi_r\bar{u}_r(i)\delta_r(t)\end{aligned} \tag{15}$$

can be determined.

Inner force and deformation responses $R(i,t)$ are derived from the relative displacement

$$R(i,t)=H\sum_r\xi_r\bar{R}_r(i)\delta_r(t) \tag{16}$$

In Eq.(16), the modal response $\bar{R}_r(i)$ should be determined from the influence function $I(i,j)$

(17) $$\bar{R}_r(i)=\sum_j I(i,j)m_j\omega_r^2\,\bar{u}_r(j)$$

Substitution of Eq.(17) into Eq.(16) gives

(18) $$\begin{aligned} R(i,t) &= H\sum_r \xi_r\Big[\sum_j I(i,j)m_j\omega_r^2\,\bar{u}_r(j)\Big]\delta_r(t)\\ &= H\sum_j I(i,j)\sum_r m_j\omega_r^2\xi_r\bar{u}_r(j)\delta_r(t)\\ &= \sum_j I(i,j)\cdot S(j,t)\end{aligned}$$

which shows that the inner force and deformation responses determined either from the relative displacement [Eq.(1)] or from the seismic load [Eq.(15)] are identical.
Substituting Eq.(8) into Eq.(4), we have

(19) $$U=\frac{H^2}{2}\sum_r m_r\big[\omega_r\delta_r\big]^2$$

where,

(20) $$\begin{aligned} m_r &= \xi_r\sum_i m_i\bar{h}_i\,\bar{u}_r(i)\\ &= \Big[\sum_i m_i\bar{h}_i\,\bar{u}_r(i)\Big]^2\Big/\sum_i m_i\bar{u}_r^2(i)\end{aligned}$$

is defined as the equivalent mass which is usually positive and decreases generally as r increases.
Now, the summation of equivalent masses is

(21) $$\begin{aligned} m^* &= \sum_r m_r = \sum_r \xi_r \sum_i m_i\bar{h}_i\,\bar{u}_r(i)\\ &= \sum_i m_i\bar{h}_i\sum_r \xi_r\bar{u}_r(i)\\ &= \sum_i m_i\bar{h}_i^2 = J_0/H^2\end{aligned}$$

where, J_0 is the moment of inertia of the cantilever about the bottom. It is noted that $\sum_r m_r$ is usually smaller than $\sum_i m_i$ for the multi-lumped-mass system.
The overturning moment about the bottom is

(22) $$\begin{aligned} M_0 &= \sum_i S(i,t)h_i = -\sum_i m_i H\sum_r \xi_r\bar{u}_r(i)\big[\ddot{\delta}_r(t)+\ddot{\theta}_g(t)\big]h_i\\ &= -H^2\sum_r\xi_r\sum_i m_i\bar{h}_i\bar{u}_r(i)\big[\ddot{\delta}_r(t)+\ddot{\theta}_g(t)\big] = -H^2\sum_r m_r\big[\ddot{\delta}_r(t)+\ddot{\theta}_g(t)\big]\end{aligned}$$

For the absolute rigid case, $\delta_r=0$, we have

(23) $$M_0=-H^2\,\ddot{\theta}_g(t)\sum_r m_r=-J_0\,\ddot{\theta}_g(t)$$

For the absolute flexible case, $\delta_r=-\theta_g$, it is evident that

(24) $$M_0=0$$

If the normal modes are taken as the following standardized form

(25) $$X_r(i)=\bar{u}_r(i)\,m^*\Big/\sum_i m_i\bar{h}_i\,\bar{u}_r(i)$$

then the corresponding mode participation factor will be

(26) $$\xi_r^* = \sum_i m_i \bar{h}_i X_r(i) / \sum_i m_i X_r^2(i)$$
$$= [\sum_i m_i \bar{h}_i \bar{u}_r(i)]^2 / m^* \sum_i m_i \bar{u}_r^2(i) = m_r/m^*$$

which expresses that ξ_r^* is the ratio of the modal equivalent mass to the summation of equivalent masses.

3 A COMPARISON OF RESPONSE TO BASE ROCKING WITH THAT TO BASE TRANSLATION

The related formulas may be summarized and listed in Table 1.

Table 1

Base motion	Rocking	Horizontal translation
Participation factor	$\xi_r = \sum_i m_i \bar{h}_i \bar{u}_r(i) / \sum_i m_i \bar{u}_r^2(i)$	$\eta_r = \sum_i m_i \bar{u}_r(i) / \sum_i m_i \bar{u}_r^2(i)$
Seismic load	$S(i,t) = -H m_i \sum_r \xi_r \bar{u}_r(i)[\ddot{\delta}_r(t) + \ddot{\theta}_g(t)]$ $= H \sum_r m_i \omega_r^2 \xi_r \bar{u}_r(i) \delta_r(t)$ $\ddot{\delta}_r + 2\varepsilon_r \omega_r \dot{\delta}_r + \omega_r^2 \delta_r = -\ddot{\theta}_g$	$S(i,t) = -m_i \sum_r \eta_r \bar{u}_r(i)[\ddot{\delta}_r(t) + \ddot{u}_g(t)]$ $= \sum_r m_i \omega_r^2 \eta_r \bar{u}_r(i) \delta_r(t)$ $\ddot{\delta}_r + 2\varepsilon_r \omega_r \dot{\delta}_r + \omega_r^2 \delta_r = -\ddot{u}_g$
Equivalent mass	$m_r = [\sum_i m_i \bar{h}_i \bar{u}_r(i)]^2 / \sum_i m_i \bar{u}_r^2(i)$	$m_r = [\sum_i m_i \bar{u}_r(i)]^2 / \sum_i m_i \bar{u}_r^2(i)$
Summation of equivalent masses	$m^* = \sum_r m_r = \sum_i m_i \bar{h}_i^2 = \frac{J_o}{H^2} \leq \sum_i m_i$	$m^* = \sum_r m_r = \sum_i m_i$
Base overturning moment or shear	$M_o = -H^2 \sum_r m_r [\ddot{\delta}_r(t) + \ddot{\theta}_g(t)]$	$S_o = -\sum_r m_r [\ddot{\delta}_r(t) + \ddot{u}_g(t)]$
Standardized mode	$X_r(i) = m^* \bar{u}_r(i) / \sum_i m_i \bar{h}_i \bar{u}_r(i)$	$X_r(i) = m^* \bar{u}_r(i) / \sum_i m_i \bar{u}_r(i)$
Participation factor of standardized mode	$\xi_r^* = \frac{m_r}{m^*} \quad (m^* \leq \sum_i m_i)$	$\eta_r^* = \frac{m_r}{m^*} \quad (m^* = \sum_i m_i)$

For the auxiliary structure, the base is a point on the surface of main structure of which the translation and rocking motion can be predetermined in the analysis of main structure. The total response of the auxiliary structure is the summation of those to both base translation and base rocking.

For the structure supported on the ground, the base is, of course, the ground. There have been a large amount of records of the translational ground motion, which can be used to compute the structure responses. Only the computation of responses to the translational ground motion is needed in general in the conventional design. Now, in order to make a comparison of the response to the rocking with that to the translation, we calculate the base overturning moment (maximum) for the fundamental mode. For the rocking

(27) $$M_0 = \sum_i H\, m_i\, \xi^{(1)} \bar{u}^{(1)}(i) \beta_\theta^{(1)} |\ddot{\theta}_g|_{max} \cdot H\bar{h}_i$$

and for the horizontal translation

(28) $$M_0 = \sum_i m_i\, \eta^{(1)} \bar{u}^{(1)}(i) \beta_u^{(1)} |\ddot{u}_g|_{max} \cdot H\bar{h}_i$$

in Eq.(27) and (28), β's are the response spectra (relative value).
Division of Eq.(27) by Eq.(28) leads to

(29) $$e = \frac{\xi^{(1)}}{\eta^{(1)}} \cdot \frac{\beta_\theta^{(1)}}{\beta_u^{(1)}} \cdot \frac{|\ddot{\theta}_g|_{max} H}{|\ddot{u}_g|_{max}} = e_1 e_2 e_3$$

where, u_g is the horizontal displacement of ground motion.
According to Table 1, it is seen that

(30) $$e_1 = \sum_i m_i \bar{h}_i \bar{u}^{(1)}(i) \Big/ \sum_i m_i \bar{u}^{(1)}(i) \leq 1$$

Now, let's express roughly the fundamental mode of the cantilever (high -rise building) by the following power function

(31) $$\bar{u} = (\bar{h})^b$$

For the bending-typed structure, the exponential $b > 1$ and for the shear-typed-structure, $0 < b < 1$. Substituting Eq.(31) into Eq.(30), we obtain

(32) $$e_1 = \int_0^1 m\bar{h}(\bar{h})^b d\bar{h} \Big/ \int_0^1 m(\bar{h})^b d\bar{h} = (b+1)/(b+2)$$

under the uniform-mass-distribution assumption. Eq.(32) shows that e_1 is larger for the bending-typed structure than the shear-typed structure. In the extreme case, the former is twice as large as the latter.

β_θ can be determined only from the estimated rocking ground motion based on the theory of wave propagation. A representative example of $\beta_\theta(T)$ (response spectrum, $\varepsilon=0.01$) and its associated $\ddot{\theta}_g(t)$ (time-history) are given in Reference [2] (Wolf et al. 1979).

In a series of research work in the recent years, it has been an attempt to estimate the rocking from the vertical ground motion based on the wave propagation theory in seismology. Referring to these studies, e_3 can be written as in the following symbolical expression.

(33) $$e_3 \propto |\ddot{v}_g|_{max} \bar{P} H / |\ddot{u}_g|_{max} C$$

where, $\bar{P}$, the average or the mediate frequency of the ground motion and C, the apparent velocity of propagation of the seismic wave along the ground surface. It is obvious that the higher the structure is and the softer the ground soil is, the larger the value of e_3 will be.

Evidently, it is more necessary to consider the influence of rocking of the ground motion for the higher bending-typed structure on the soft soil than the lower shear-typed one on the hard soil.

4 PIPING RESPONSE TO MULTI - SUPPORT ROCKING EXCITATIONS

A Λ-typed piping is shown in Fig.2. The inner moment responses to mul-

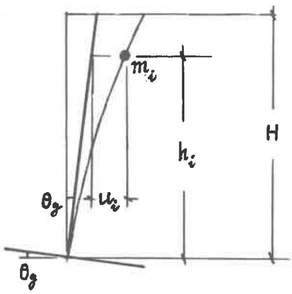

Fig.1

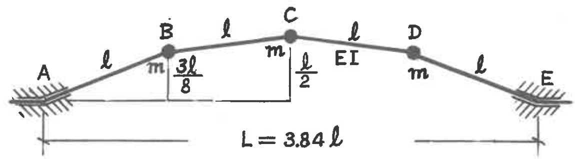

Fig.2

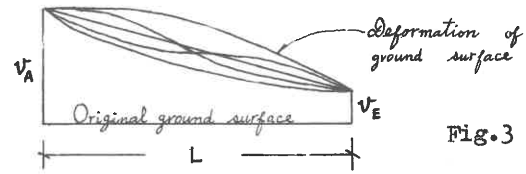

Fig.3

ti-support vertical translational and rocking excitations consist of two parts—pseudo—statical response and dynamic response. The results are written as follows:

$$(34)\qquad \frac{\ell^2}{EI}\begin{Bmatrix}M_{AB}\\M_{BC}\\M_{CD}\\M_{DE}\\M_{ED}\end{Bmatrix}=\begin{bmatrix}-0.38 & 0.38 & 0.54 & -0.16\\-0.20 & 0.20 & 0.11 & 0.09\\0.00 & 0.00 & -0.11 & 0.11\\0.20 & -0.20 & -0.09 & -0.11\\0.38 & -0.38 & 0.16 & -0.54\end{bmatrix}\begin{Bmatrix}v_A\\v_E\\\theta_A L\\\theta_E L\end{Bmatrix}$$

$$+\begin{bmatrix}1.43 & -1.43 & -0.26 & -0.26\\-1.17 & 1.17 & 0.21 & 0.21\\0.00 & 0.00 & 0.00 & 0.00\\1.17 & -1.17 & -0.21 & -0.21\\-1.43 & 1.43 & 0.26 & 0.26\end{bmatrix}\begin{Bmatrix}\delta_{vA}\\\delta_{vE}\\\delta_{\theta A}\\\delta_{\theta E}\end{Bmatrix}_{skew}+\begin{bmatrix}-0.14 & -0.14 & -0.10 & 0.10\\0.19 & 0.19 & 0.14 & -0.14\\-0.24 & -0.24 & -0.17 & 0.17\\0.19 & 0.19 & 0.14 & -0.14\\-0.14 & -0.14 & -0.10 & 0.10\end{bmatrix}\begin{Bmatrix}\delta_{vA}\\\delta_{vE}\\\delta_{\theta A}\\\delta_{\theta E}\end{Bmatrix}_{sym.}$$

In Eq.(34), v is the vertical displacement; δ_{vA} , $\delta_{\theta A}$,··· are the responses of the single-mass oscillator with the natural frequencies

$$(35)\qquad \omega_{skew}=3.57\sqrt{EI/m\ell^3} \qquad \text{and} \qquad \omega_{sym}=5.74\sqrt{EI/m\ell^3}$$

associated respectively with the skew-symmetrical and symmetrical modes to the ground excitations v_A , $\theta_A L$,··· . In the analysis, the bending deformation is principally considered geometrically.

The records of rocking ground motion during earthquake are seldom and may be even not available so far. Now, referring to Fig.3, let's consider $\theta=(v_A-v_E)/L$ to be a probable mean value of θ_A and θ_E. It is seen from Eq.(34) that the influence of rocking component is not less important than that of vertical translational component of the ground motion for the piping under study.

5 RESPONSES OF SOME STRUCTURES OTHER THAN NPP TO MULTI-SUPPORT ROTATIONAL EXCITATIONS

Recently, the authors of the present paper made some earthquake-resistant analysis for several different kinds of structures under the action of rotational ground motion which is estimated in virtue of the propagation principle of the seismic waves. Results are listed in Table 2-4.

Table 2 : Responses of Chimneys

Height, Material \ Record of ground motion		El Centro, 1940 3 components (translation)	El Centro, 1940 6 components (transl. + rotat.)	Tianjin Hospital (Tangshan, China, 1976) 3 components (translation)	Tianjin Hospital (Tangshan, China, 1976) 6 components (transl. + rotat.)
60 m brick	τ_x	1.55	1.66	1.97	2.15
	τ_y	1.08	1.58	4.19	4.17
	σ_x	4.61	5.91	15.67	16.46
	σ_y	6.87	7.36	8.13	8.91
100 m R.C.	τ_x	1.55	1.93	3.66	5.17
	τ_y	1.12	1.27	5.21	7.38
	σ_x	7.11	8.95	33.62	49.68
	σ_y	11.56	13.87	22.27	35.07
180 m R.C.	τ_x	3.77	4.82	5.14	7.54
	τ_y	2.25	4.46	10.09	9.81
	σ_x	17.63	29.93	37.48	38.29
	σ_y	*30.37	38.46	27.75	38.94

*Relative values of the time-history maxima of stresses.

Table 3 : Responses of Arch.

Time lag (sec)		El Centro 3 components	El Centro 6 components	Tianjin Hospital 3 components	Tianjin Hospital 6 components
0	τ_x	4.49	14.93	7.37	13.98
	τ_y	6.00	6.43	5.39	6.18
	σ_x	10.18	28.99	4.63	9.14
	σ_y	23.17	24.07	14.54	11.62
0.5	τ_x	5.64	10.33	5.75	13.38
	τ_y	13.56	13.34	11.34	12.05
	σ_x	12.68	23.38	4.51	11.16
	σ_y	29.20	28.36	14.77	17.95
1.0	τ_x	6.56	12.77	4.43	25.54
	τ_y	15.69	15.95	13.96	14.24
	σ_x	12.18	19.20	4.71	11.29
	σ_y	33.04	32.66	14.90	18.00

Table 4 : Responses of Cable-Stayed Bridge Tower.

Time lag (sec)		El Centro 3 components	El Centro 6 components	Tianjin Hospital 3 components	Tianjin Hospital 6 components
0	τ_x	3.61	5.84	3.69	6.70
	τ_y	24.12	22.45	22.04	17.33
	σ_x	8.56	24.04	8.86	16.04
	σ_y	18.44	17.55	16.36	13.13
0.5	τ_x	3.66	5.61	3.69	6.51
	τ_y	16.72	17.00	14.09	14.05
	σ_x	8.78	13.46	8.85	15.60
	σ_y	12.65	12.86	10.24	8.84
1.0	τ_x	3.66	5.75	3.75	6.76
	τ_y	14.36	15.98	14.20	13.88
	σ_x	8.78	13.78	8.89	16.18
	σ_y	10.67	11.60	11.36	11.12

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2 J. P. Wolf, P. Obernhüber 1979. Effects of Horizontally Travelling Waves in Soil-Structure Interaction, Nuclear Engineering And Design, Vol.57, No.2.