ESTIMATION OF ELASTOPLASTIC DAMAGE PARAMETERS USING GUIDED WAVE SCATTERING COEFFICIENT MEASUREMENT

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ABSTRACT

In this paper a detailed study is reported on the damage severity estimation using ultrasonic guided wave interaction in a thin structure undergoing elastoplastic deformation due to uniaxial loading. The analytical understanding of the interaction of guided waves with elastoplastic boundaries near a damaged zone is developed by employing an elastoplastic material model. The structure with elastoplastic damage is modeled considering three regions. The damaged region which is at the middle is modeled as an elastoplastic process zone. Guided wave propagation in the stressed specimen and its interaction with the plastic zone are analyzed using the developed wave propagation model. The two-dimensional wave propagation problem is reduced to a one-dimensional model by considering various different possible wave travel paths passing through the damage in the structure. The developed model relates the guided wave reflection and transmission coefficients to the damage severity. Experimental study of the reflected and transmitted ultrasonic guided waves is carried out using piezoelectric wafers bonded to a uniaxially loaded specimen having a crack. The effect of the uniaxal loading on the performance of the piezoelectric actuator and the sensor in a pitch catch mode is investigated. It is found that the change in the sensor signal due to stressed condition is negligible. Increase in the plastic zone size around a stable crack with loading is correlated to the strength of the reflected and transmitted guided wave experimentally. Increase in the crack size with loading is also correlated. With this approach it is possible to detect and monitor elastoplastic damages in a structure. The developed technique can be used in estimating the elastoplastic damage parameters.

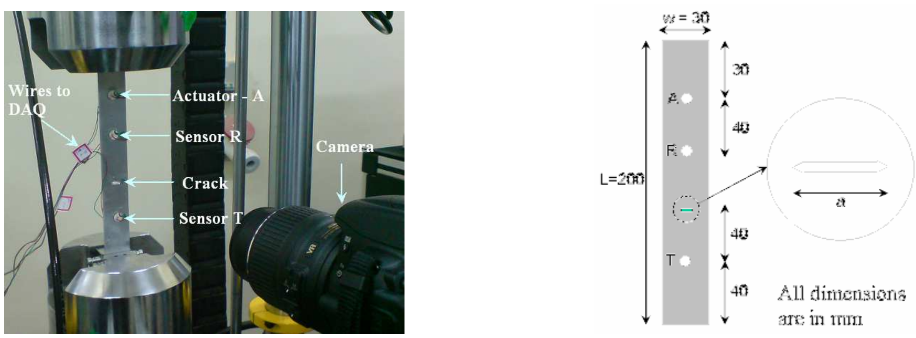
INTRODUCTION

Nondestructive assessment of the components for defects can be done using many available inspection techniques. Selection of a technique for inspection depends on the location of the component, type of damage to be monitored, time available for inspection and cost of the techniques. Development of the in-service inspection techniques such as ultrasonic testing, acoustic emission testing and eddy current testing and their judicious choice for inspection of nuclear power plants has been discussed in [1]. Work on digital signal analysis used for enhancing sensitivity and better defect characterization in stainless steel weldments using ultrasonic testing has been discussed in ref. [2]. The acoustic emission technique is found to be effective in detection, localization and monitoring of the defects due to fatigue [3]. Guided waves are the diagnostic waves used widely for health monitoring of plate like structures to detect possible damages. Estimation of damage growth in terms of damage parameters using ultrasonic guided waves has attracted attention. Work on guided wave based identification and estimation of damage parameter in metallic plate structure using circular Piezoelectric Wafer Active Sensors (PWAS) array has been described in refs. [4-6]. Monitoring of a crack under mixed-mode fatigue loading with PWAS using guided wave transmission method is reported in ref. [7]. Propagation of these waves involves the motion of particles along the thickness cut plane of the plate. Thus it is suited for the interrogation of one sided damages such as corrosion and dents, delamination in composites [4,8] which produce asymmetry in the particle motion in the thickness cut plane. The techniques that have been reported so far in literature, do not address the problem of detecting plastic deformation surrounding a crack like damage and the characteristics of the guided wave interaction with a plastic zone. Failure due to plastic deformation by overloading (unlike fatigue loading) and their effect on the bonding of the sensors and actuators used for interrogation are also not addressed in literature. In this work we focus on damage severity estimation using ultrasonic guided wave interaction in a thin structure undergoing elastoplastic deformation under uniaxial loading. Effect of sensitivity of PWAS under stressed condition is also investigated.

An effective online remote health monitoring system needs to estimate the damage parameters with minimal error under operating conditions. Varying operating conditions are due to the environmental conditions and loading. Fatigue loads that are within the operating range of the component are under elastic limit. Such loading does not alter the sensitivity of the actuators and sensors mounted on the structure. Whereas fatigue loading above elastic limit of the material alters the sensitivity of the actuators and sensors mounted on the component and may lead to spurious results during the damage detection process. Unlike fatigue overloading, overloading such as ramp and step loads may also cause error in damage detection due to varying stress levels on the sensor and actuators. Effect of environmental conditions such as humidity and temperature is not addressed in the present work and need to be investigated in future work. A deeper understanding of the interaction of guided waves with elastoplastic boundaries near damaged zone is achieved by employing elastoplastic material model to model a damage zone in the analytical model. In the present work we investigate on the effect of stress level on the performance of PWAS for ramp type of loading on the component. The developed theoretical model relates the guided wave reflection coefficient to the severity of the damage including the plastic zone. Damage detection and damage severity estimation in terms of a damage parameter have been addressed experimentally using guided wave technique. In the next section analytical model is discussed followed by discussions on experimental investigation conducted on a thin beam specimen.

ONE-DIMENSIONAL WAVE PROPAGATION MODEL FOR ElASTO-PLASTIC MATERIAL

Monitoring a component undergoing damage growth process using guided wave technique requires the study of guided wave propagation in the stressed specimen and its interaction with elasto-plastic zone of the damage. In the present work we have considered the problem of monitoring a crack growth in a thin plate specimen loaded in uniaxial tension as shown in Fig. 1(a). The crack is detected and monitored using guided wave which is launched using PWAS and sensed using PWAS. First the interaction of guided wave with damage and crack boundaries is studied using a wave propagation model.



(a) (b)

Fig.1: (a) Experimental setup showing the locations of sensors, an actuator and a crack on a plate specimen (b) Schematic of a specimen with a crack and the crack configuration.

Approximation of Plastic Zone

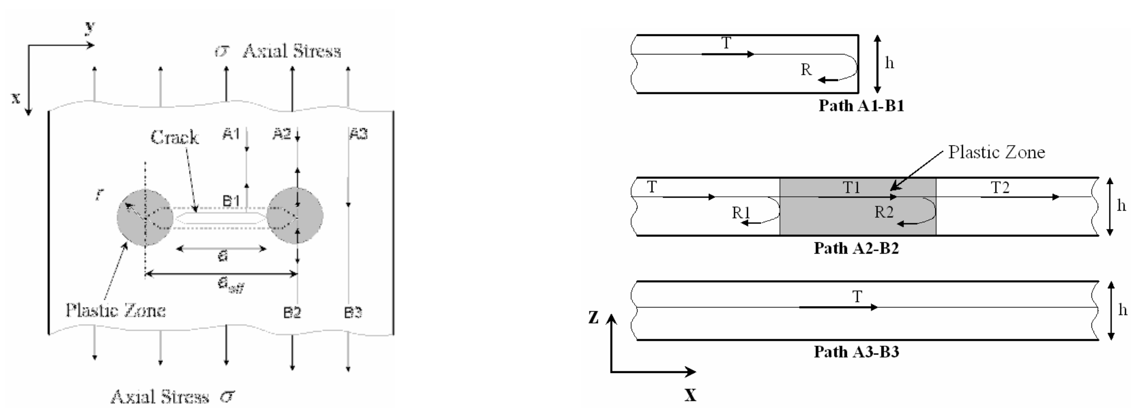
In the present study we consider a thin plate structure loaded uniaxially as shown in Fig. 1(a). The dimensions of the specimen and the crack configuration are shown in Fig. 1(b). Near the crack tip, a localized elastoplastic state is formed when the component is loaded. For a given specimen, the stress σ required to propagate the crack of size a is related to the stress intensity factor K [9] by the relation

$$K = \sigma \sqrt{\pi a \sec\left(\frac{\pi a}{w}\right)} \tag{1}$$

where w is the width of the specimen. The value of K can be estimated for a specimen for any crack length a and stress σ recorded during the experiment. The plastic zone shape and size for any crack size can be approximated using Irwin's approach [10] or the strip yield model [11]. Strip yield model bears little resemblance to actual plastic zone in metals and is much suited to polymers. Thus we consider Irwin's approach to estimate approximate plastic zone shape and size. The plastic zone size is approximated as a circle of radius r at the crack tip as shown in Fig. 2 and is given by

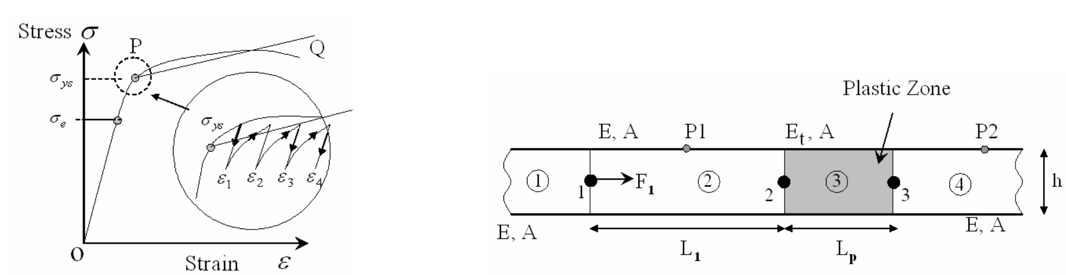
$$r = \frac{1}{\pi}\left(\frac{K}{\sigma_{ys}}\right)^2 \tag{2}$$

where σ_{ys} is the yield stress of the material. The crack tip plastic zone given by Eq. (2) is based on the corrections to Linear Elastic Fracture Mechanics (LEFM) model. The stress singularity is truncated at the crack tip by yielding and redistribution of the stresses. Thus the stress in the Irwin's plastic zone cannot exceed the yield stress. Because of the redistributed stress, the behavior of the structure is similar to the structure with crack with its tip at the center of the plastic zone. The crack with its tip at the center of the plastic zone is called effective crack and has a size of $a_{eff} = a + 2r$ as shown in Fig. 2(a). The effective cross-sectional dimension (w) also decreases as the crack propagates. The variation of crack size and the plastic zone size indicates the severity of the damage. These damage parameters need to be monitored during the crack propagation by an effective online health monitoring system. Diagnosis of damage growth can be done by guided wave interaction with crack tip plastic zone and the crack edge/surfaces.



(a) (b)

Fig.2: (a) Schematic of a specimen loaded under uniaxial state of stress showing plastic zones around a crack. (b) Three possible wave propagation paths A1-B1, A2-B2 and A3-B3.



(a) (b)

Fig.3: (a) Stress-strain diagram showing the strain response of a material at yield stress subjected to sinusoidal stress wave. (b) Wave propagation through plastic zone in 1D rod modeled using spectral finite element.

Analytical Study of Interaction of Diagnostic Waves with Crack Tip Plastic Zone

Analytical solution for the propagation of guided waves in isotropic plates using three-dimensional elasticity is available in literature [12]. Solutions using elementary and higher order rod and beam models [13] are also available which are computationally faster than the 3D model. It is found that when the higher order displacement terms are considered in the Taylor series expansion of the displacement field the approximation of the solution increases at higher frequencies [14]. Due to the absence of higher order wave modes at lower frequencies, only a first few terms or a single term in Taylor series in sufficient to reasonably approximate the solution. We simplify the 2-D wave propagation phenomenon in plate by 1-D wave propagation using elementary rod theory by considering the wave paths at different locations as shown in Fig. 2(a) (A1-B1, A2-B2 and A3-B3). Consider a wave propagating from a far field in the direction from A1 to B1. Three different paths are possible in a plate structure undergoing damage depending upon location of the wave path. One possible wave travel path is the one along which the wave is reflected from a crack edge as shown in Fig. 2(b). The crack edge considered here is the actual crack edge since the guided wave interacts only with the actual crack. Whereas the effective crack with the plastic zone alters the global behavior of the structure. Second possible wave travel path is the the one along which transmission of the wave through the plastic zone takes place. The third possible wave travel path is the one along which the wave propagates without any obstacle. The longitudinal displacement u is obtained using the governing differential equation of the rod with negligible damping which is given by

$$EA\frac{\partial^2 u(x,t)}{\partial x^2} = \rho A\frac{\partial^2 u(x,t)}{\partial t^2} \tag{3}$$

where A is the cross sectional area and ρ is the density of the material and E is the Young's modulus to model a healthy structure within elastic limit. In a damaged zone undergoing plastic deformation with stress σ_{ys} (as explained in the previous section), the stress-strain curve follows the path as shown in Fig. 3(a) when subjected to an ultrasonic guided wave. In the present study we make a simple approximation of the material property in the plastic zone using the tangent modulus E_t as shown in Fig. 3(a) by the slope of the line PQ. More detailed study on the interaction of guided waves with material undergoing damage growth process can be found in [15]. We use spectral method to obtain the solution to the above partial differential equation which suits the wave propagation problems. The spectral representation of the solution is assumed as $u(x,t) = u(x,\omega)e^{i\omega_n t} = Ce^{ikx}e^{i\omega_n t}$. By substitution this form in Eq. (3) we get the roots of above equation as $k = \pm\omega_n / c$. The complete solution in case of longitudinal wave is given by

$$\hat{u} = A_1 e^{ikx} + A_2 e^{-ikx} \tag{4}$$

where A_1 and A_2 are the coefficients to be determined with the help of the boundary conditions. Interfaces due to different materials can be dealt with same approach. Let $\hat{u}_1$ and $\hat{u}_2$ be the displacements at coordinates x_1 and x_2, respectively. Eq. (4) in can be written in matrix form as

$$\begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix} = \begin{bmatrix} e^{ikx_1} & e^{-ikx_1} \\ e^{ikx_2} & e^{-ikx_2} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} \tag{5}$$

The natural boundary condition for rod is given by $F(x,t) = EA\partial u(x,t)/\partial x$ where A is the cross sectional area. If $\hat{F}(x,\omega)$ is the force amplitude in frequency domain, then the natural boundary condition in frequency domain becomes $\hat{F}(x,\omega) = EAd\hat{u}(x,\omega)/dx$. Using Eq. (5) the force evaluated at coordinates x_1 and x_2 can be written in matrix form as

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{F}_2 \end{Bmatrix} = AEik \begin{bmatrix} -e^{ikx_1} & e^{-ikx_1} \\ e^{ikx_2} & -e^{-ikx_2} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} \tag{6}$$

Using using Eq. (5) and the above equation the forces can be related to the displacements as

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{F}_2 \end{Bmatrix} = AEik \begin{bmatrix} -e^{ikx_1} & e^{-ikx_1} \\ e^{ikx_2} & -e^{-ikx_2} \end{bmatrix} \begin{bmatrix} e^{ikx_1} & e^{-ikx_1} \\ e^{ikx_2} & e^{-ikx_2} \end{bmatrix}^{-1} \begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix} = [\hat{K}]\{\hat{u}\}_e \tag{7}$$

where $[\hat{K}]$ is the dynamic stiffness matrix. In the first possible wave travel path shown in Fig. 2(b) the entire wave packet incident on the crack surface gets reflected. We assume that there is no conversion incident wave mode after reflection, thus the size and the shape of the wave packet remains unaltered after reflection. In the third possible wave travel path A3-B3, the wave packet propagates unaltered towards the other end of the rod. But in the second possible wave travel path A2-B2, the incident wave undergoes double reflection due to mismatch in the material property due to the presence of a plastic zone. Thus we consider path A2-B2, where the reflected and transmitted wave packet shapes and sizes depend upon the plastic zone size and the associated material property at that particular state of stress and strain. The three segment problem with different tangent modulus is modeled using spectral elements as shown in Fig. 3(b). The 1D bar is considered to be extending to infinity at both the ends. Semi-infinite elements are used to model a situation where the wave energy needs to be removed out of the system or in other words the boundary reflections need to be eliminated artificially. The ultimate result of this process is often equivalent to what can be obtained also by using appropriate damping and other losses that are present in the actual structure. Semi-infinite elements are derived by considering any one of the forward or backward moving wave in the displacement given by Eq. (4). Elements 1 and 2 represent the semi-infinite elements in Fig. 3(b). Element 2 is considered to introduce a source or excitation point at node 1. This in turn can be treated as a wave launched form a far field (since only one-dimensional setting is used instead of circular plan wave). Element 3 represents the plastic zone lying in the wave travel path A2-B2.

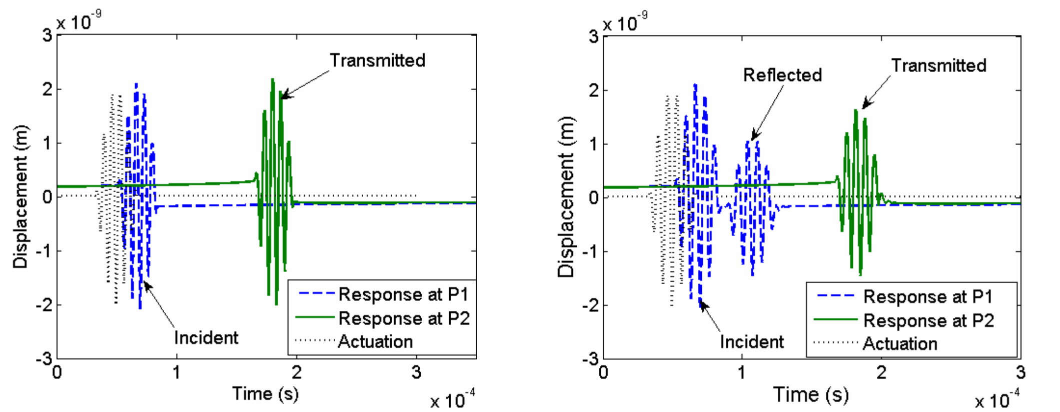
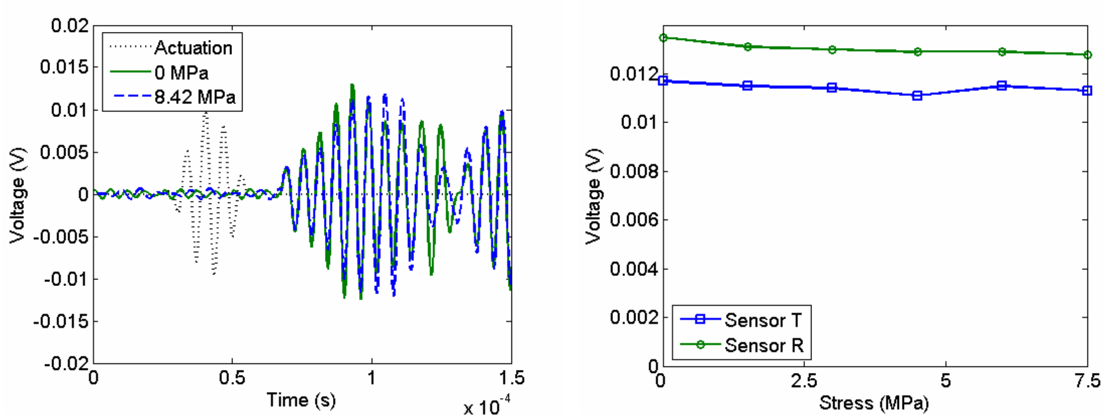


Fig.4: Displacement vs. time response obtained from of wave propagation model discretized by spectral finite elements. Excitation of modulated sinusoidal packet with a central frequency of 150 kHz is used. (a) response without plastic zone (b) response with plastic zone.

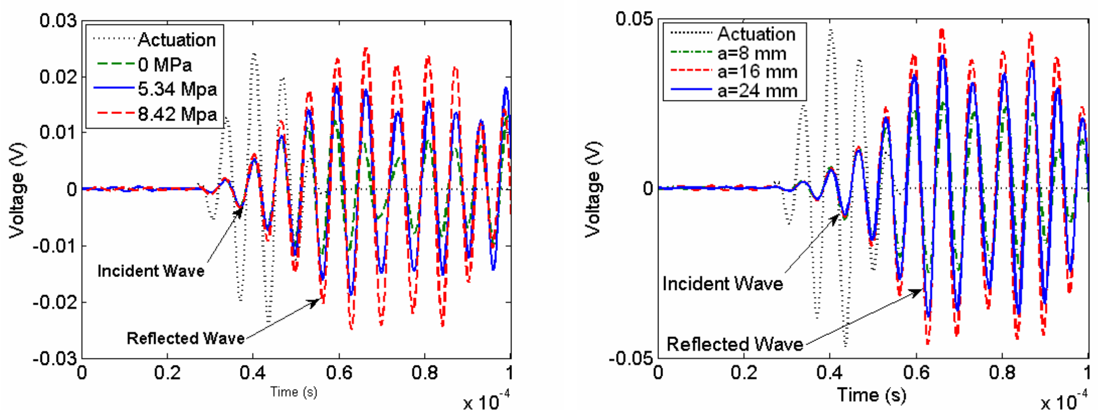
Since there are three nodes in the spectral finite element model, the assembled dynamic stiffness matrix size is 3x3. A dynamic force with single frequency sinusoidal packet enveloped by a hanning window containing 5 cycles is applied at node 1. The system of equations (7) is solved at each discrete frequency to obtain the nodal displacements. Using Eqs. (4) and (5) response at any other location can be found in terms of evaluated nodal displacements at each discrete frequency. Using inverse Fourier transform the response in time domain is computed. The length of element 1 is $L_1 = 0.2m$ and length of plastic zone is $L_p = 0.005m$. Density of the material is taken as $\rho = 2700\,\mathrm{kg/m^3}$. Thickness of the bar $\mathrm{h} = 0.005\,\mathrm{m}$ and the width is $0.03\,\mathrm{m}$. Young's modulus for the elements 1, 2 and 4 is $\mathrm{E} = 70\,\mathrm{GPa}$. We consider two cases where the element 3 is assigned the tangent modulus $\mathrm{E} = 70\,\mathrm{GPa}$ to represent a healthy path (path A3-B3). In another case, the element 3 is assigned the tangent modulus $\mathrm{E_t} = 70\,\mathrm{GPa}$ representing the plastic zone (path A2-B2). The response obtained at locations P1 and P2 for the case without plastic zone is shown in Fig. 4(a). The distance of P2 is higher than the distance of P1 from node 1 where the excitation is given. Thus, the time of arrival of the incident wave packet at location P1 is less than the time of arrival of the wave packet at P2. The amplitude of the wave packet obtained at locations P1 and P2 is same since there is no obstacle in the wave travel path A3-B3. In presence of a plastic zone between node 2 and 3, the response is shown in Fig. 4(b). At location P1, the response has two wave packets. The first wave packet corresponds to the incident wave packet and the second wave packet is due to the reflection of the wave packet from the plastic zone. The magnitude of the reflected wave packets is decreased which can be attributed to the partial transmission of wave energy through the plastic zone. The response at location P2 shows the transmitted wave through the plastic zone whose amplitude is less as compared to the incident wave packet amplitude at location P1. The reason of reduction in amplitude is due to the partial reflection of the wave energy during transmission through the plastic zone.

The three possible wave travel paths that were considered for analysis, can be used to analyze the reflected and transmitted waves along any other path. To obtain the effective response at any location, a finite set of paths are considered at equal distances along the width of the plate w. For higher accuracy, the distance between the paths can be kept low. The average of the reflected wave amplitude of all the equidistant paths along the width gives the effective reflected wave amplitude. In a healthy structure, the only wave travel path is of type A3-B3. The wave travel path A3-B3 does not contribute to any reflection of the wave, thus the reflected wave amplitude measured at location P1 for healthy structure would be zero. The effective transmitted wave amplitude measured at P2 would be same as the incident wave amplitude measured at P1. In presence of crack and plastic zone the wave travel paths A1-B1 and A2-B2 contribute to the reflected wave amplitude, respectively. As the crack grows, the number of wave travel paths contributing to the reflected wave packet increases. Thus the effective reflected wave amplitude at location P1 increases. The energy of the reflected wave packet for a wave travel path can be estimated from the response at location P1 using wavelet transforms as elaborated in [5,6]. The average energy measured using all the wave travel paths is used to calculate the reflection coefficient. The estimated reflection coefficient directly correlates the crack size accompanied with its plastic zone which is the damage parameter in the present case.



(a) **(b)**

Fig.5: (a) Signal from sensor T (Fig 1) for an excitation of modulated sine with a central frequency of 150 kHz. (b) Variation of peak amplitude of the transmitted wave for different background quasi-static stress level in the plate.



(a) **(b)**

Fig.6: Signals from sensor R (Fig. 1) for an excitation of modulated sine with a central frequency of 150 kHz (a) effect of plastic zone in presence of a constant crack of size 8 mm at different levels of background quasi-static stress (b) effect of crack propagation.

EXPERIMENTAL RESULTS

Online structural health monitoring system must possess the capability to detect the occurrence of damage and its growth during the operation of the structure. The structure might be subjected to normal operating conditions or even overloading. Thus the sensing and actuation system mounted on the structure must be immune to such stresses. Occurrence of damage in a metallic structure may be in the form of a crack, local yielding or corrosion. Many features in the plate structures such as notches, holes and slots may serve as crack initiators. Local yielding forms a plastic zone that is accompanied by the formation of a crack which progresses if the loading is unchecked. Online structural health monitoring system capable of detecting such yielding may help in prevention of component failure and reduce risk and maintenance cost. In this section, the effect of external stress on the actuator and sensor performance in pitch-catch mode of operation is investigated experimentally, followed by damage severity correlation. The experimental setup is shown in Fig. 1(a) with a plate structure in uniaxial tension mounted in a universal testing machine. The plate dimension with a crack along with the location of actuator A and two sensors R and T is shown in Fig. 1(b). The thickness of the plate is 3 mm and material used is aluminium alloy of grade 6001. Sensor R is used to monitor the reflected signals form the crack and sensor T is used to monitor the transmitted signals through the crack. The sensors and the actuator are connected to the Data Acquisition System (DAQ) with control cards. A camera is used to observe the crack extension with time. For the sensitivity studies of sensors and actuator under background quasi-static stress, a plate with same dimensions without the crack is considered. The plate is loaded at different stress levels using universal testing machine (UTM) and a voltage signal of 5 cycle tone burst in a Hanning window with central frequency of 150 kHz is applied to the actuator. The response from the sensors is measured simultaneously using the DAQ. The response of the sensor signals under different stress levels σ in the plate is shown in the Fig. 5. There is a negligible change in the waveform even when the plate is loaded to a stress level of 8.42 MPa as seen in Fig. 5(a). Peak amplitude of first wave packet for different stress levels on the plate varies negligibly as seen in Fig. 5(b). Thus even on a stressed component, the performance of the sensors do not vary significantly if they are properly bonded and measurements can be made with fairly good accuracy.

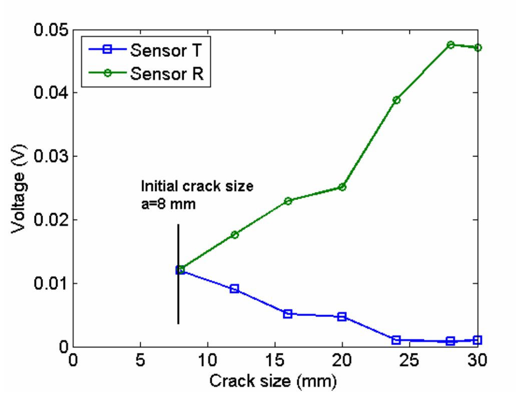


Fig.7: Peak amplitude response of wave packet for an excitation of modulated sine with a frequency 150 kHz.

Detection of the crack is accomplished by mounting the plate specimen with a crack in the UTM and loading it in the force controlled mode till complete failure. During loading, initially a crack tip plastic zone develops and propagates only when the magnitude of the applied stress reaches the critical stress σ_c that satisfies Eq. (1). σ_c at which the crack propagates was found to be 8.42 MPa for which the critical stress intensity factor K_c is estimated to be 1.63 Mpa-m$^{1/2}$. The radius of crack tip plastic zone size for K_c is as estimated from Eq. (2) as 0.0021 m. The ratio of crack tip plastic zone and crack size is 0.52 which is considerable. Thus, the two plastic zones at two tips of the crack contribute to considerable reflection of the incident wave packet. Fig. 6 shows the effect of reflection of wave from the plastic zones on the sensor signals due to initial loading without crack propagation. The radius of plastic zone increases as the stress level increases with constant crack size (See Eq. (2)), thus the reflected energy from the plastic zone also increases as discussed in the earlier section on the wave propagation model. This trend is observed in Fig. 6(a) from the signals of the sensor R where it can be seen that the amplitude of the reflected wave packet increases as the applied stress level increases. Above σ_c the crack grows which in turn decreases K_c. Thus the crack tip plastic zone size decreases with propagating crack and the main change in the reflected signal amplitude is due to the increase in the crack size. Fig. 6(b) shows an increase in the reflected wave amplitude with the increasing crack size. As the reflected wave energy increases with the crack size, the transmitted energy decreases which can be verified from Fig. 7. The peak amplitude of the reflected wave increases with crack size whereas the peak amplitude of the transmitted wave decreases. The transmitted wave amplitude finally tends to zero due to absence of wave travel path at complete failure. Whereas, in the reflected wave response the amplitude does not vary significantly during the last stages of failure, which is due to the absence of the wave transmission path through the crack. Variation of voltage responses obtained from experiments and the displacement responses from the numerically simulated wave packet gives the same predictions. The reflection coefficient representing the severity of the damage can be estimated using wavelet analysis as done in [5,6]. Thus the wave propagation model along with guided wave technique can be used to design an online structural health monitoring system for the structures in service.

CONCLUSION

In this work a technique is proposed to detect the plastic deformations in the loaded structure using guided wave. A wave propagation model is used to understand the interaction of the guided wave with the plastic zone in a structure undergoing damage growth. Displacement responses from the wave propagation model aids in understanding the energy distribution in the structure in the presence of damage. Wave propagation model suggests the possible detection of the crack with the plastic zone present in the wave travel path using reflected and transmitted waves. Analytical estimation of the reflection coefficient relating crack length with its plastic zone as a damage parameter is proposed. Experimental evaluation of the sensitivity of PWAS used as sensor and actuator in pitch-catch mode under stress is studied. Negligible variation in the sensor response was found under stressed condition up to 8.42 MPa. Experimentally determined reflected and transmitted ultrasonic guided wave strength varies monotonically with plastic zone size and the crack size. Thus the guided wave technique using the PWAS in the pitch catch mode can be used to detect the plastic zones formed in the structure due to loading. The initiation of crack and its propagation can also be effectively detected and quantified by estimation of the reflection coefficient. A good correlation was found between the displacement response of the wave propagation model and the voltage response obtained using experiments. Thus it is possible to design an active structural health monitoring system to monitor damage involving plastic deformation in a structure using the guided waves.

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