THERMAL SHOCK ON A FINITE ORTHOTROPIC THIN SHELL

H. K. WOO

General Atomic Company, P.O. Box 81608, San Diego, California 92138, U.S.A.

P. G. KIRMSER, C. L. HUANG

Department of Mechanical Engineering, Kansas State University, Manhattan, Kansas 66506, U.S.A.

SUMMARY

The complex structure of a nuclear reactor must be designed and built to withstand not only the normal operating conditions, but also the conditions which could occur in unusual circumstances in which larger than normal or faster than normal temperature variations exist. Thermal stresses are always critical factors in the design of the components.

Because graphites have excellent thermal, nuclear, and thermal shock resistance, and actually get stronger as temperature increases to about 4000°F, they are frequently used as components of reactors. The crystal structure of graphite has orthotropic symmetry, and this property must be included in the analysis or design of components using this material. Ribbed steel containment vessels could also be analyzed as orthotropic structures.

In this paper, temperature variations, induced stresses and deformations are calculated for an orthotropic cylindrical shell of finite length (such as shells used for cladding fuel rods or those used for containment) when acted on by a thermal shock which is an instantaneous heat pulse acting in a small area of the thin shell structure. The quasi-linear theory of thermoelasticity is assumed. The material of the shells is taken to be homogeneous, and orthotropic in both mechanical and thermal properties. The material constants are assumed to be independent of temperature.

Using the methods of transform calculus, the Green's function for the variation of temperature is found from infinitely long shell. Then the variations of temperature in a shell of finite length with various given boundary conditions are obtained by the method of reflections, such as frequently used in potential field theory. Infinite series solutions are written in terms of theta functions. These are rapidly convergent for small values of time. Error bounds are found for the use of truncated series in calculation. Once the solutions are obtained for temperature distributions, the differential equations of thermoelasticity are solved by a method of double Fourier Transforms. Solutions for the thermal stresses and induced deformations in an orthotropic thin shell of finite length with simply supported edges are obtained.

The materials assumed for use in the numerical examples are grades ATJ and ZTA graphite. Curves for heat storage and temperature distribution in a thin cylindrical shell of finite length are obtained as functions of time. The radial displacements, membrane forces, and bending moments are also calculated as functions of the space coordinates. These functions fell off rapidly with distance in both the axial and circumferential directions from the instantaneous hot-spot source point. The functions for radial deflection and axial membrane forces are monotonically decreasing in the axial direction, but oscillatory in the circumferential direction. For the circumferential membrane forces and bending moments, the curves oscillate in both axial and circumferential directions, except the curves for circumferential bending moment in the axial direction. The membrane forces are always compressive in the neighborhood of the heat source.

1. Introduction

Takeuti and Noda [1] and Hsu [2] investigated the problem of thermal shock on a finite disk. DeSilva and Whitman [3] extended Nordgren's work [4] by employing Green's functions to study cylindrical elastic shells of infinite length. Recently, Shirakawa [5] used Fourier Transforms in a study of a finite cylindrical shell subjected to an instantaneous hot spot. All of these investigations were restricted to objects made of isotropic materials. Problems involving thermal shock on structural elements of anisotropic media have received only a little attention [6,7,8].

The purpose of this paper is to investigate the temperature variations, induced stresses, and deformations which occur in an orthotropic cylindrical shell of finite length when a thermal shock acts on a small area of the shell structure. The quasi-linear theory of thermo-elasticity is assumed. the material of the shells is assumed to be homogeneous, and orthotropic both in the mechanical and thermal properties. Also the material constants are taken to be independent of temperature.

Using the methods of transform calculus, the Green's function for the temperature in an infinitely long shell is found. Then, the variations of temperature in a shell of finite length with a given boundary condition are obtained by the method of reflections [7]. Solutions for the thermal stresses and induced deformations in an orthotropic thin shell of finite length with simply supported edges are obtained by use of double Fourier Transforms.

The material used in the numerical examples is graphite. Graphical presentations of temperature variations and elastic responses in a thin shell of finite length due to a thermal shock are given. Finally, the significant trends, and the effects of material orthotropy of the graphite on the temperature field and elastic responses are discussed.

2. Heat Conduction Equations

Consider an orthotropic cylindrical thin shell of average radius a, and thickness h. A cylindrical coordinate system, which is oriented in such a manner that the principal directions of heat conduction as well as elasticity coincide with the coordinate axes, x, ϕ and z, is shown in Fig. 1. Without loss of generality, the temperature at any point of the thin shell can be assumed as

$$T(x,\phi,z,t) = T_o(x,\phi,t) + zT_1(x,\phi,t) \ , \tag{1}$$

where T_o is the temperature of the middle surface, T_1 is the temperature gradient in the direction normal to the middle surface of the shell, t is the time variable.

The right side of the equation (1) states that a hot-spot on the surface of a thin shell can be treated as a sum of two equivalent hot-spots; the first temperature term $T_o(x,\phi,t)$ is induced by a "plane hot-spot", and the second $zT_1(x,\phi,t)$ is induced by another hot-spot called the "bending hot-spot." The resultant temperature T is the sum of the effects due to both hot-spots, and varies linearly along the normal to the middle surface of the shell.

Assuming that no heat source exists in the shell, and heat transfer on the shell surface follows Newton's law, the heat conduction equations for a thin cylindrical orthotropic shell can be written as follows [8];

$$k_x \frac{\partial^2 T^{(o)}}{\partial x^2} + k_\phi \frac{1}{z^2} \frac{\partial^2 T^{(o)}}{\partial \phi^2} - \eta_o T^{(0)} - c\rho' \frac{\partial T^{(o)}}{\partial t} = 0 \tag{2a}$$

$$k_x \frac{\partial^2 T_1}{\partial x^2} + k_\phi \frac{1}{a^2} \frac{\partial^2 T_1}{\partial \phi^2} - \eta_1 T_1 - c\rho' \frac{\partial T_1}{\partial t} = 0 \tag{2b}$$

where

$$\eta_o = \frac{2h'}{h} \quad , \quad \eta_1 = \frac{6h'}{h} + \frac{12k_z}{h^2} \ .$$

$$T^{(o)} = T_o - T_\infty = (\frac{1}{h} \int_{-h/2}^{h/2} Tdz) - T_\infty \quad , \quad T_1 = \frac{12}{h^3} \int_{-h/2}^{h/2} Tzdz$$

and T_∞ is the ambient temperature. In equations (2), c is the specific heat, ρ' is the density of the material of shell, h' is heat transfer coefficient on the surface of the shell, h is the thickness of shell, and k_x, k_ϕ and k_z are the thermal conductivities in the x, ϕ and z directions, respectively.

By methods of transform calculus, the general solutions of Eqs. (2) for a shell of infinite length heated instantaneously at the time t_o by a hot-spot at (x_o,ϕ), are

$$T^{(0)} = H_o G_o(x,\phi,t;x_o,\phi_o,t_o) \quad , \quad T_1 = H_1 G_1(x,\phi,t;x_o,\phi_o,t_o) \tag{3}$$

where H_o and H_1 are the amounts of heat released by the plane and bending hot-spot, respectively; functions $G_o(x,\phi,t)$ and $G_1(x,\phi,t)$

$$G_o(x,\phi,t) = \frac{e^{-\eta_o(t-t_o)/c\rho'}}{2\pi ahc\rho} \sqrt{\frac{c\rho'}{4\pi k_x(t-t_o)}}\ e^{-\frac{(x-x_o)^2 c\rho'}{4k_x(t-t_o)}}\ \Theta_3(\frac{\phi-\phi_o}{2\pi}, \tau_\phi) \tag{4a}$$

$$G_1(x,\phi,t) = \frac{6e^{-\eta_1(t-t_o)/c\rho'}}{\pi h^3 c\rho'} \sqrt{\frac{c\rho'}{4\pi k_x(t-t_o)}}\ e^{-\frac{(x-x_o)^2 c\rho'}{4k_x(t-t_o)}}\ \Theta_3(\frac{\phi-\phi_o}{2\pi}, \tau_\phi) \tag{4b}$$

are Green's functions, in which theta function, θ_3, is defined as [9]

$$\Theta_3(x,t) = \frac{1}{\sqrt{\pi t}} \sum_{n=-\infty}^{\infty} \exp[-(n+x)^2/t] \quad , \quad t > 0 ,$$

or

$$\Theta_3(x,t) = 1 + 2 \sum_{m=1}^{\infty} e^{-m^2\pi^2 t} \cos^2 m\pi x \ , \quad t > 0 ,$$

and

$$\tau_\phi = \frac{k_\phi(t-t_o)}{\pi^2 a^2 c\rho'}$$

Therefore, temperature variation in an orthotropic cylindrical thin shell of infinite length due to a thermal shock at (x_o,ϕ_o,t_o) is

$$T(x,\phi,z,t) = H_o G_o + z\, H_1 G_1 \tag{5}$$

By the method of reflections, the solutions of temperature field for an orthotropic thin shell of finite length L, with constant temperature at both ends are [8]

$$G_o(x,\phi,t) = \frac{e^{-\eta_o(t-t_o)/c\rho'}}{4\pi ahc\rho L} \left[\Theta_3(\frac{x-x_o}{2L}, \tau_x) - \Theta_3(\frac{x+x_o}{2L}, \tau_x)\right] \Theta_3(\frac{\phi-\phi_o}{2\pi}, \tau_\phi) , \tag{6a}$$

$$G_1(x,\phi,t) = \frac{3e^{-\eta_1(t-t_o)/c\rho'}}{\pi h^3 c\rho' L} [\Theta_3(\frac{x-x_o}{2L}, \tau_x) - \Theta_3(\frac{x+x_o}{2L}, \tau_x)]\ \Theta_3(\frac{\phi-\phi_o}{2\pi}, \tau_\phi) \tag{6b}$$

where $$\tau_x = \frac{k_x(t-t_o)}{L^2 c\rho'}$$

3. Thermal Stresses and Deformations

Based on Sander's first-approximation theory of thin shell [10], the differential equations for an orthotropic cylindrical shell subjected to non-uniform heating may be written as follows [8]:

$$\begin{aligned}
&A_1\Big[\frac{\partial^8}{\partial x^8} + A_2\frac{1}{a^2}\frac{\partial^8}{\partial x^6\partial\phi^2} + A_3\frac{1}{a^4}\frac{\partial^8}{\partial x^4\partial\phi^4} + A_4\frac{1}{a^6}\frac{\partial^8}{\partial x^2\partial\phi^6} + A_5\frac{1}{a^8}\frac{\partial^8}{\partial\phi^8} + A_6\frac{1}{a^2}\frac{\partial^4}{\partial x^6}\\
&\quad A_7 + \frac{1}{a^4}\frac{\partial^6}{\partial x^4\partial\phi^2} + A_8\frac{1}{a^6}\frac{\partial^6}{\partial x^2\partial\phi^4} + A_9\frac{1}{a^8}\frac{\partial^6}{\partial\phi^6} + A_{10}\frac{1}{a^4}\frac{\partial^4}{\partial x^4} + A_{11}\frac{1}{a^6}\frac{\partial^4}{\partial x^2\partial\phi^2}\\
&\quad + A_{12}\frac{1}{a^8}\frac{\partial^4}{\partial\phi^4}\Big]w = \frac{1}{a}\Big[B_1\frac{\partial^6}{\partial x^6} + B_2\frac{1}{a^2}\frac{\partial^6}{\partial x^4\partial\phi^2} + B_3\frac{1}{a^4}\frac{\partial^6}{\partial x^2\partial\phi^4} + B_4\frac{1}{a^6}\frac{\partial^6}{\partial\phi^6}\\
&\quad + B_5\frac{1}{a^2}\frac{\partial^4}{\partial x^4} + B_6\frac{1}{a^2}\frac{\partial^4}{\partial x^2\partial\phi^2} + B_7\frac{1}{a^4}\frac{\partial^4}{\partial\phi^4}\Big]T_o + \Big[B_1^*\frac{\partial^6}{\partial x^6} + B_2^*\frac{1}{a^2}\frac{\partial^6}{\partial x^4\partial\phi^2}\\
&\quad + B_3^*\frac{1}{a^4}\frac{\partial^6}{\partial x^2\partial\phi^4} + B_4^*\frac{1}{a^6}\frac{\partial^6}{\partial\phi^6} + B_5^*\frac{1}{a^2}\frac{\partial^4}{\partial x^4} + B_6^*\frac{1}{a^2}\frac{\partial^4}{\partial x^2\partial\phi^2} + B_7^*\frac{1}{a^4}\frac{\partial^4}{\partial\phi^4}\Big]T_1\ ,
\end{aligned} \tag{7a}$$

$$\begin{aligned}
&\Big[C_1\frac{\partial^4}{\partial x^4} + C_2\frac{1}{a^2}\frac{\partial^4}{\partial x^2\partial\phi^2} + C_3\frac{1}{a^4}\frac{\partial^4}{\partial\phi^4}\Big]u = \Big[C_4\frac{1}{a}\frac{\partial^5}{\partial x^3\partial\phi^2} + C_5\frac{1}{a^3}\frac{\partial^5}{\partial x\partial\phi^4} + C_6\frac{1}{a}\frac{\partial^3}{\partial x^3}\\
&\quad + C_7\frac{1}{a^3}\frac{\partial^3}{\partial x\partial\phi^2}\Big]w + \Big[D_1\frac{1}{a}\frac{\partial^3}{\partial x^3} + D_2\frac{1}{a^3}\frac{\partial^3}{\partial x\partial\phi^2}\Big]T_o + \Big[D_1^*\frac{\partial^3}{\partial x^3} + D_2^*\frac{1}{a^2}\frac{\partial^3}{\partial x\partial\phi^2}\Big]T_1\ ,
\end{aligned} \tag{7b}$$

$$\begin{aligned}
&\Big[C_1\frac{\partial^4}{\partial x^4} + C_2\frac{1}{a^2}\frac{\partial^4}{\partial x^2\partial\phi^2} + C_3\frac{1}{a^4}\frac{\partial^4}{\partial\phi^4}\Big]v = \Big[E_1\frac{\partial^5}{\partial x^4\partial\phi} + E_2\frac{1}{a^2}\frac{\partial^5}{\partial x^2\partial\phi^3} + E_3\frac{1}{a^4}\frac{\partial^5}{\partial\phi^5}\\
&\quad + E_4\frac{1}{a^2}\frac{\partial^3}{\partial x^2\partial\phi} + E_5\frac{1}{a^4}\frac{\partial^3}{\partial\phi^3}\Big]w + \Big[F_1\frac{1}{a}\frac{\partial^3}{\partial x^2\partial\phi} + F_2\frac{1}{a^3}\frac{\partial^3}{\partial\phi^3}\Big]T_o\\
&\quad + \Big[F_1^*\frac{\partial^3}{\partial x^2\partial\phi} + F_2^*\frac{1}{a^2}\frac{\partial^3}{\partial\phi^3}\Big]T_1\ ,
\end{aligned} \tag{7c}$$

where u, v, and w are the displacements of a point of the middle surface in x, ϕ, and z directions; E_x and E_ϕ are Young's moduli; G is the shear modulus; $\nu_{x\phi}$ and $\nu_{\phi x}$ are Poisson's ratios; and a is the average radius of shell. The coefficients are defined as follows:

$$\nu = \nu_\phi = \nu_{x\phi}\ ,\quad \nu_x = \nu_{\phi x}\ ,\quad \alpha_x' = \alpha_x + \nu_\phi\alpha_\phi\ ,\quad \alpha_\phi = \alpha_\phi + \nu_x\alpha_x$$

$$\lambda = E_\phi/E_x\ ,\quad \beta = G/E_x\ ,\quad A_1 = \beta\ ,\quad A_2 = \lambda-\nu(2\beta+\nu)\ ,$$

$$A_3 = 2\{3\lambda\beta+\nu[\lambda-(2\beta+\nu)^2]\}\ ,\quad A_4 = 2\lambda[\lambda+4\beta^2-\nu^2]\ ,\quad A_5 = \lambda^2\beta\ ,$$

$$A_6 = 0\ ,\quad A_7 = A_3\ ,\quad A_8 = A_4\ ,\quad A_9 = 2\lambda^2\beta\ ,\quad A_{10} = \beta(\lambda-\nu^2)(\tfrac{9}{4} + \tfrac{1}{p^2})\ ,$$

$$A_{11} = \lambda(\lambda+4\beta^2-\nu^2)\ ,\quad A_{12} = A_5\ ,\quad B_1 = 0\ ,\quad B_2 = -(2\beta+\nu^2)\alpha_x' + \lambda(3\beta+\nu)\alpha_\phi'\ ,$$

$$B_3 = -\lambda(2\beta+\nu)\alpha_x' + \lambda[\lambda+2\beta(2\beta+\nu)]\alpha_\phi'\ ,\quad B_4 = \lambda^2\beta\alpha_\phi'\ ,$$

$$B_5 = (\lambda\beta\alpha'_\phi - \beta\nu\alpha'_x)\ (\frac{9}{4} + \frac{1}{p^2}) \quad ,$$

$$B_6 = (\lambda\beta\alpha'_x - \lambda\beta\nu\alpha'_\phi)/p^2 - \lambda[(\frac{3}{4}\ \beta+\nu)\alpha'_x - (\lambda+4\beta^2 + \frac{3}{4}\ \beta\nu)\alpha'_\phi] \quad , \quad B_7 = \lambda^2\beta\alpha'_\phi \quad ,$$

$$B_1^* = -\beta\alpha'_x \quad , \quad B_2^* = [\nu(2\beta+\nu)-\lambda]\alpha'_x - \lambda\beta\alpha'_\phi \quad , \quad B_3^* = -\lambda\beta\alpha'_x - \lambda[\lambda-\nu(2\beta+\nu)]\alpha'_\phi \quad ,$$

$$B_4^* = -\lambda\ \beta\alpha'_\phi \quad , \quad B_5^* = 0 \quad , \quad B_6^* = -\lambda[\lambda-\nu(\beta+\nu)]\alpha'_\phi \quad , \quad B_7^* = -\lambda^2\beta\alpha'_\phi \quad ,$$

$$C_1 = \beta \quad , \quad C_2 = \lambda-\nu(2\beta+\nu) \quad , \quad C_3 = \lambda\beta \quad , \quad C_4 = -p^2(2\beta+\nu)^2 \quad ,$$

$$C_5 = -p^2\lambda(2\beta+\nu) \quad , \quad C_6 = -\beta\nu(1 + \frac{9p^2}{4}) \quad , \quad C_7 = \lambda[\beta - (\frac{3}{4}\ \beta+\nu)] \quad ,$$

$$D_1 = \beta\alpha'_x \quad , \quad D_2 = \lambda[\alpha'_x - (\beta+\nu)\alpha'_\phi] \quad , \quad D_1^* = 0 \quad , \quad D_2^* = -p^2\lambda(\beta+\nu)\alpha'_\phi \quad ,$$

$$E_1 = p^2(3\beta+\nu) \quad , \quad E_2 = p^2[\lambda+2\beta(2\beta+\nu)] \quad , \quad E_3 = p^2\lambda\ \beta \quad ,$$

$$E_4 = -[\lambda-\nu(\beta+\nu)] \quad , \quad E_5 = -\lambda\beta \quad , \quad F_1 = -(\beta+\nu)\alpha'_x + \lambda\alpha'_\phi \quad , \quad F_2 = \lambda\beta\alpha'_\phi \quad ,$$

$$F_1^* = p^2\lambda\alpha'_\phi \quad , \quad F_2^* = p^2\lambda\beta\alpha'_\phi \quad , \quad \text{and} \quad p^2 = (\frac{h}{a})^2/12 \quad .$$

For a cylindrical orthotropic thin shell of length L with both ends hinged, the boundary conditions are:

$$v=0, \ w=0, \ N_x=0, \ M_x=0, \ \text{at}\ x=0,L \ .$$

3.1 The solutions for the displacements in the shell due to a plane hot-spot are obtained by a method of double Fourier transform. They are:

$$w(x,\phi) = \frac{H_o e^{-\eta_o(t-t_o)/c\rho'}}{\pi hLc\rho'} \sum_{m=1}^{\infty} e^{-m^2\pi^2\tau_x} \sin\frac{m\pi x_o}{L} \sin\frac{m\pi x}{L} \left\{ \frac{S_1(m,o)}{S_2(m,o)} \right.$$

$$\left. + 2\sum_{n=1}^{\infty} e^{-n^2\pi^2\tau_\phi} \frac{S_1(m,n)}{S_2(m,n)} \cos[n(\phi-\phi_o)] \right\} \quad . \tag{8a}$$

$$u = -\frac{H_o e^{-\eta_o(t-t_o)/c\rho'}}{\pi hLc\rho} \sum_{m=1}^{\infty} e^{-m^2\pi^2\tau_x} \sin\frac{m\pi x_o}{L} \cos\frac{m\pi x}{L} \left\{ \frac{S_1(m,o)S_4(m,o) + S_2(m,o)S_5(m,o)}{S_2(m,o)S_3(m,o)} \right.$$

$$\left. + 2\sum_{n=1}^{\infty} \frac{S_1(m,n)S_4(m,n) + S_2(m,n)S_5(m,n)}{S_2(m,n)S_3(m,n)} \times e^{-n^2\pi^2\tau_\phi} \cos[n(\phi-\phi_o)] \right\} \quad . \tag{8b}$$

$$v = \frac{H_o e^{-\eta_o(t-t_o)/c\rho'}}{\pi hLc\rho'}\ 2\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left\{ \frac{S_1(m,n)S_6(m,n) + S_2(m,n)S_7(m,n)}{S_2(m,n)S_3(m,n)} \right.$$

$$\left. \times\ e^{[-m^2\pi^2\tau_x - n^2\pi^2\tau_\phi]} \sin\frac{m\pi x_o}{L} \sin\frac{m\pi x}{L} \sin[n(\phi-\phi_o)] \right\} \quad . \tag{8c}$$

3.2 The solutions for the displacements due to a bending hot-spot are:

$$u = -\frac{12H_1 e^{-\eta_o(t-t_o)/c\rho'}}{\pi h^3 L c\rho'/a^2} \sum_{m=1}^{\infty} e^{-m^2\pi^2\tau_x} \sin\frac{m\pi x_o}{L} \cos\frac{m\pi x}{L} \left\{ \frac{S_1^*(m,o)S_4(m,o) + S_2(m,o)S_2^*(m,o)}{S_2(m,o)S_3(m,o)} \right.$$

$$\left. + 2\sum_{n=1}^{\infty} \frac{S_1^*(m,n)S_4(m,n) + S_2(m,n)S_2^*(m,n)}{S_2(m,n)S_3(m,n)} \times e^{-n^2\pi^2\tau_\phi} \cos[n(\phi-\phi_o)] \right\} , \qquad (8a)$$

$$v = \frac{24H_1 e^{-\eta_1(t-t_o)/c\rho'}}{\pi h^3 L c\rho'/a^2} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} e^{-m^2\pi^2\tau_x - n^2\pi^2\tau_\phi}$$

$$\times \frac{S_1^*(m,n)S_6(m,n) + S_2(m,n)S_3^*(m,n)}{S_2(m,n)S_3(m,n)} \sin\frac{m\pi x_o}{L} \sin\frac{m\pi x}{L} \sin[n(\phi-\phi_o)] , \qquad (9b)$$

$$w = \frac{12H_1 e^{-\eta_1(t-t_o)/c\rho'}}{\pi h^3 L c\rho'/a^2} \sum_{m=1}^{\infty} e^{-m^2\pi^2\tau_x} \sin\frac{m\pi x}{L} \sin\frac{m\pi x_o}{L}$$

$$\times \left\{ \frac{S_1^*(m,o)}{S_2(m,o)} + 2\sum_{n=1}^{\infty} e^{-n^2\pi^2\tau_\phi} \frac{S_1^*(m,n)}{S_2(m,n)} \cos[n(\phi-\phi_o)] \right\} . \qquad (9c)$$

where

$$S_1(m,n) = -B_1\rho^6 - B_2\rho^4 n^2 - B_3\rho^2 n^4 - B_4 n^6 + B_5\rho^4 + B_6\rho^2 n^2 + B_7 n^4 ,$$

$$S_2(m,n) = A_1\rho^8 + A_2\rho^6 a^2 + A_3\rho^4 n^4 + A_4\rho^2 n^6 + A_5 n^8 - A_6\rho^5 - A_7\rho^4 n^2$$

$$- A_8\rho^2 n^4 - A_9 n^6 + A_{10}\rho^4 + A_{11}\rho^2 n^2 + A_{12} n^4 ,$$

$$S_3(m,n) = C_1\rho^4 + C_2\rho^2 n^2 + C_3 n^4 , \quad S_4(m,n) = C_4\rho^3 n^2 + C_5\rho n^4 - C_6\rho^3 - C_7\rho n^2 ,$$

$$S_5(min) = -D_1\rho^3 - D_2\rho n^2 , \quad S_6(m,n) = -E_1\rho^4 n - E_2\rho^2 n^3 - E_3 n^5 + E_4\rho^2 n + E_5 n^3 ,$$

$$S_7(m,n) = F_1\rho^2 n + F_2 n^3 , \quad \text{and} \quad \rho = m\pi/L .$$

$$S_1^* = - B_1^*\rho^6 - B_2^*\rho^4 n^2 - B_3^*\rho^2 n^4 - B_4^* n^6 + B_5^*\rho^4 + B_6^*\rho^2 n^2 + B_7^* n^4 ,$$

$$S_2^* = - D_1^*\rho^3 + D_2^*\rho n^2 , \quad S_3^* = F_1^*\rho^2 n + F_2^* n^3 .$$

The stress resultants are:

$$N_\phi = D_\phi \left[\frac{1}{a}\left(\frac{\partial v}{\partial\phi} + w\right) + \nu_x \frac{\partial u}{\partial x}\right] - D_\phi\alpha_\phi' T_o , \quad N_x = D_x \left[\frac{\partial u}{\partial x} + \frac{\nu_\phi}{a}\left(\frac{\partial v}{\partial\phi} + w\right)\right] - D_x\alpha_x' T_o ,$$

$$M_\phi = K_\phi \left[\frac{1}{a^2}\left(\frac{\partial v}{\partial\phi} - \frac{\partial^2 w}{\partial\phi^2}\right) - \nu_x \frac{\partial^2 w}{\partial x^2}\right] - K_\phi\alpha_\phi' T_1 , \quad M_x = K_x \left[-\frac{\partial^2 w}{\partial x^2} + \frac{\nu_\phi}{a^2}\left(\frac{\partial v}{\partial\phi} - \frac{\partial^2 w}{\partial\phi^2}\right)\right] - K_x\alpha_x' T_1 .$$

$$\bar{N}_{x\phi} = D_{x\phi}\left(\frac{\partial v}{\partial x} + \frac{1}{a}\frac{\partial u}{\partial\phi}\right) , \quad \bar{M}_{x\phi} = K_{x\phi}\left(-\frac{1}{a}\frac{\partial^2 w}{\partial x\partial\phi} + \frac{3}{4a}\frac{\partial v}{\partial x} - \frac{1}{4a^2}\frac{\partial u}{\partial\phi}\right) . \qquad (10)$$

Substitution of displacement functions u, v, and w into equation (10), force resultants and moments can be calculated.

4. Numerical Examples

The examples for an orthotropic cylindrical shell subjected to an instantaneous point source located at the point (L/2,0) are presented. Temperatures at both edges are assumed to be the same as the ambient temperature. One short cylindrical shell is selected. Its dimensions are $L/a = 2$, $h/a = 0.05$ and $a = 2$ ft. ZTA Graphite is used in the numerical examples. Its thermophysical properties are listed [11]. Note that the thermophysical contents of the D-type ZTA graphite are obtained from those of the C-type by interchanging the subscripts x and ϕ.

Properties	Type of material C	D	Properties	Type of material C	D
E_x/E_o	2.85	0.7	α_ϕ/α_o	14.96	1.26
E_ϕ/E_o	0.7	2.85	k_x/k_o	104.02	38.7
G/E_o	0.5	0.5	k_ϕ/k_o	38.7	104.02
α_ϕ/α_o	1.26	14.96	ν_x	0.204	0.05
			ν_ϕ	0.05	0.204

$E_o = 10^6$ psi , $\alpha_o = 1$ in/(in.°F) , $k_o = 1$ BTU/(hr.ft.°F) .

For convenience, the following nondimensional parameters are used in the figures which illustrate the results.

$$B_i = \frac{h'h}{k_o} \quad , \quad \tau_o = \frac{k_o(t-t_o)}{a^2 c\rho'} \quad , \quad T_o' = T^{(o)}/(H_o/\pi h a^2 c\rho') \quad ,$$

$$T_1' = T_1/(H_1/\pi h a^3 c\rho') \quad , \quad w' = w/(H_o/\pi a h c\rho') \quad ,$$

$$(N_x', N_\phi') = (N_x, N_\phi)/[H_o \alpha_o E_o/\pi a^2 c\rho'(1-\nu_x\nu_\phi)] \quad ,$$

$$(M_x', M_\phi') = (M_x, M_\phi)/[H_o \alpha_o E_o/\pi a c\rho' \xi^2 (1-\nu_x\nu_\phi)] \quad \text{and} \quad \xi = \frac{h}{a} \ .$$

In general, the numerical results indicate that the reversal of the thermal and mechanical properties of two principal directions of the ZTA graphite material affects the temperature fields, stresses and deformations considerably. From a careful examination of the results obtained one concludes that the cylindrical shell undergoes smaller deformations and stresses if the grain direction of graphite is placed in the circumferential direction of the shell.

References

[1] TAKEUTI, Y. and NODA, N., "Transient Thermal Stress on a Hollow Circular Disk under an Instantaneous Point Heat Source," Nuclear Engineering and Design, Vol. 24, No. 3, 1973, pp. 440-450.

[2] HSU, T. R., "Thermal Shock on a Finite Disk due to an Instantaneous Point Heat Source," Journal of Applied Mechanics, Vol. 36, 1969, pp. 113-120.

[3] DeSILVA, C. N. and WHITMAN, A. B., "Green's Functions in the Non-stationary Theory of Elastic Shell," ZAMM, Heft 6, 1967, pp. 369-378.

[4] NORDGREN, R. P., "On the Method of Green's Function in the Thermoelastic Theory of Shallow Shells," International Journal of Engineering Science, Vol. 1, 1963, pp. 279-307.

[5] SHIRAKAWA, K., "Thermoelastic Behavior of a Cylindrical Shell Subjected to an Instantaneous Hot Spot," Bull. of Japanese Society of Mechanical Engineers, Vol. 17, No. 104, 1974, pp. 187-193.

[6] PARIDA, J. and DAS, A. K., "Thermal Stresses in a Thin Circular Disk of Orthotropic Material due to an Instantaneous Point Heat Source," Acta Mechanica, Vol. 13, 1972, pp. 205-214.

[7] HUANG, C. L. D., and WOO, H. K., "The Fundamental Solution for a Thermal Shock on a Finite Orthotropic Thin Shell," submitted for publication.

[8] WOO, H. K., "Thermal Shock on an Orthotropic Cylindrical Shell of Finite Length due to an Instantaneous Point Heat Source," Ph.D. Dissertation, Dept. of Applied Mechanics, Kansas State University, Manhattan, Kansas, 1975.

[9] BELLMAN, R. E., A Brief Introduction to Theta Functions, Holt, Rinehart and Winston, New York, 1961.

[10] SANDERS, J. L., Jr., "An Improved First-Approximation Theory for Thin Shells," NASA TR-R24, 1957.

[11] The Industrial Graphite Engineering Handbook, Union Carbide Corporation, New York, 1970.

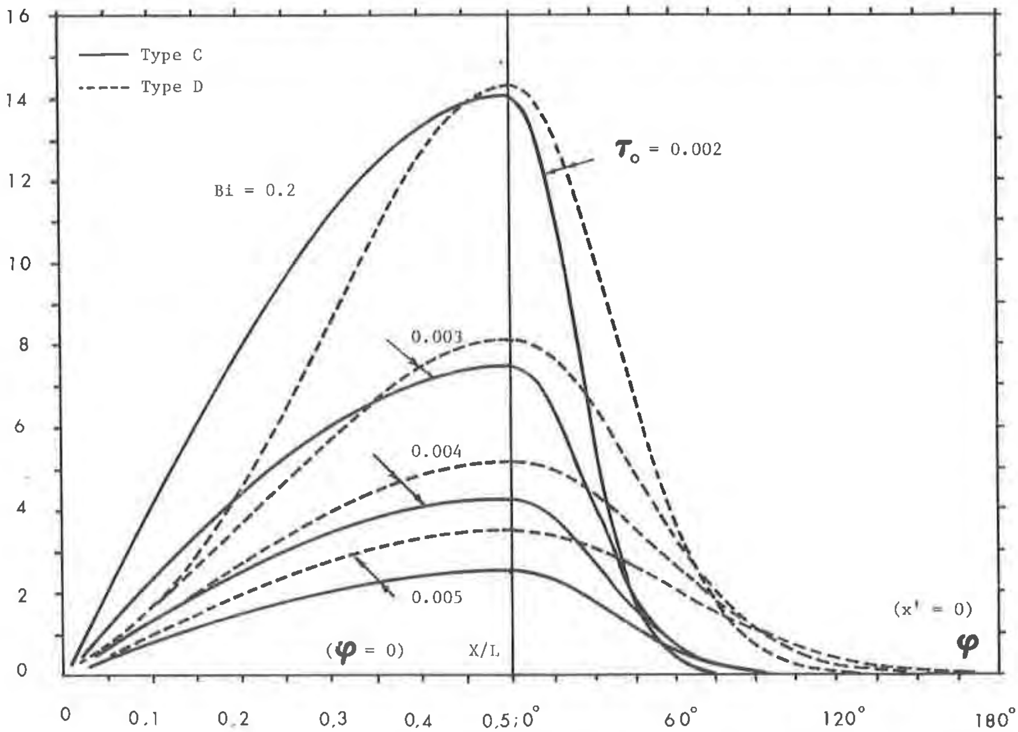

Fig. 1. Variation of Temperature.

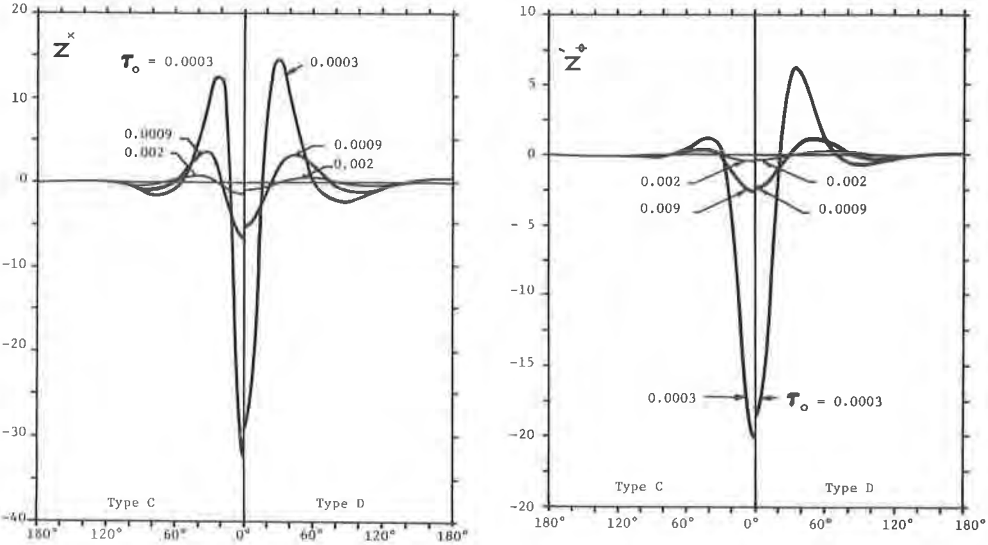

Figure 2. Membrane Forces due to a Plane Hot Spot (Bi = 0.2, x' = 0.5).

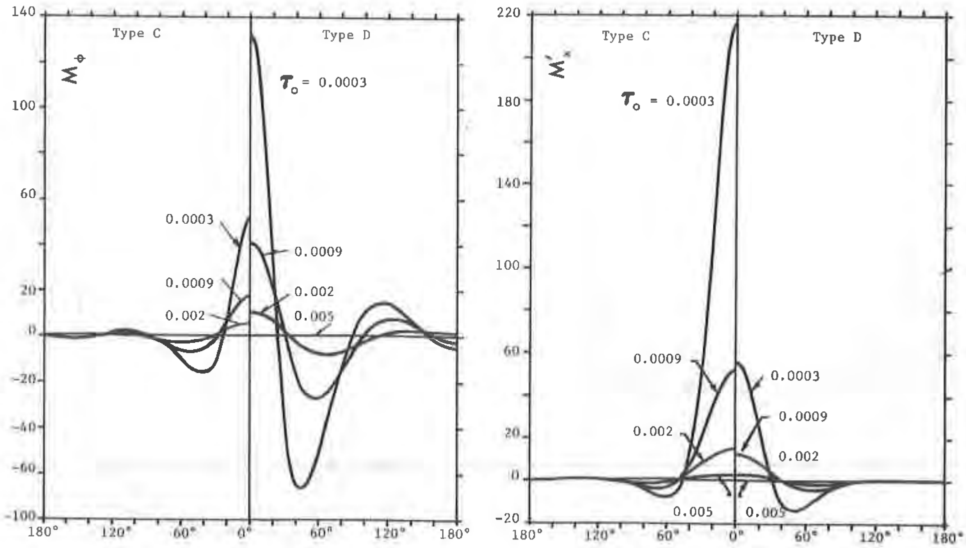

Figure 3. Bending Moments due to a Plane Hot Spot (Bi = 0.2, x' = 0.5).

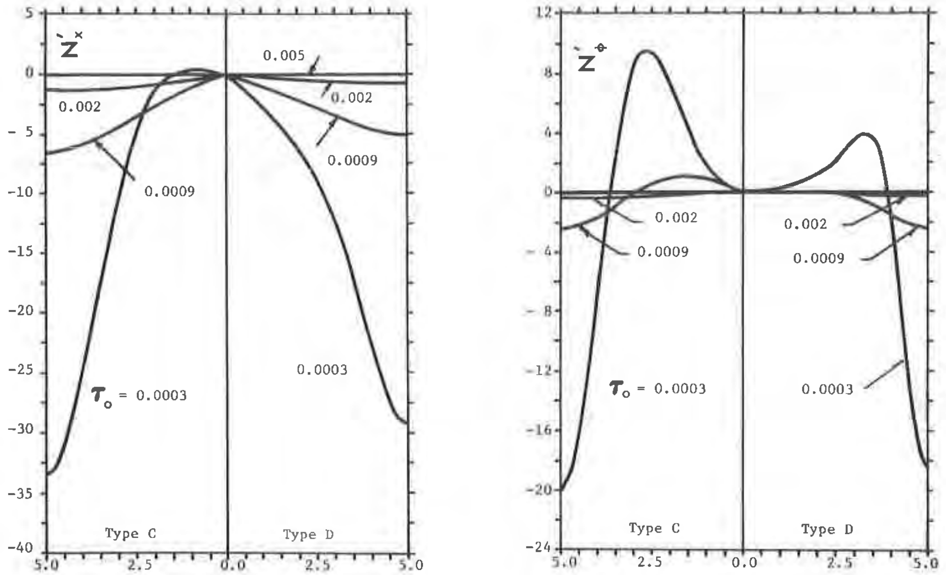

Figure 4. Membrane Force due to a Plane Hot Spot (Bi = 0.2, ϕ = 0).

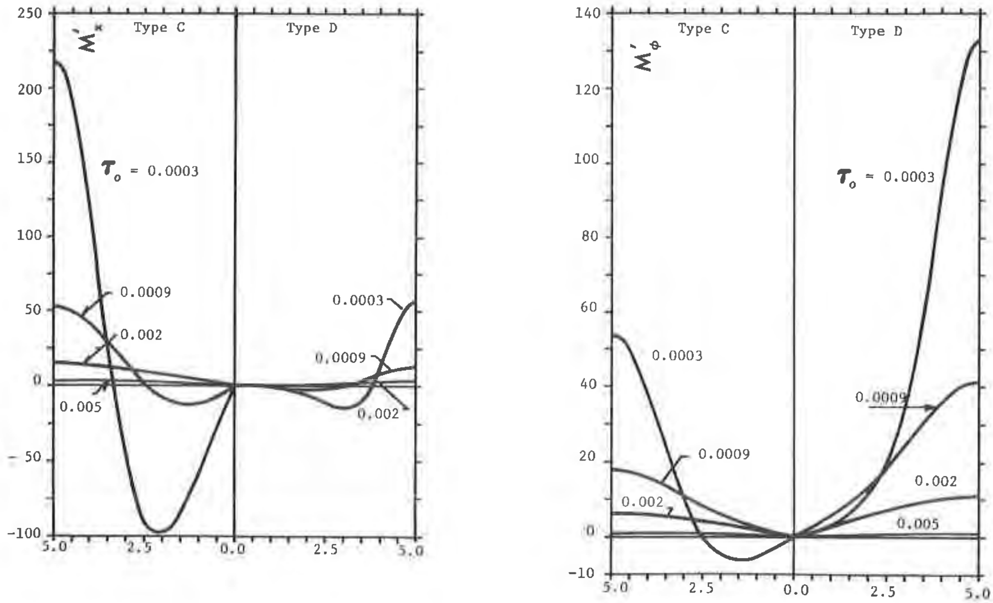

Figure 5. Bending Moment due to a Plane Hot Spot (Bi = 0.2, ϕ = 0).