Criterion of stability and implementation issues of hybrid frequency-time-domain procedure for nonlinear dynamic analysis

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1. INTRODUCTION

In the hybrid-frequency-time domain procedure (HFTD procedure), a nonlinear system is modeled by a linear one in which the nonlinear effects are represented by exciting pseudo forces (Kawamoto 1983). The latter compensate for the difference in internal forces as obtained from the pseudo-linear system and from the true nonlinear one. This corresponds to the initial-stress method used in static nonlinear analyses (Bathe 1982). The HFTD procedure is thus similar to the pseudo-force methods used in transient nonlinear analyses (Molnar et al. 1976), from which it differs only to the extent that the equations of motion are solved in the frequency domain.

Here, the pseudo forces are estimated for a limited number of time steps at a time and then transformed to the frequency domain where the equations of motion are solved (segmenting approach). The resulting response quantities are transformed back to the time domain and the pseudo forces are updated. The procedure is repeated iteratively until convergence is reached.

The condition which needs to be satisfied in order to guarantee stability of the HFTD procedure is presented in Section 2 and the implementation of the segmenting approach is discussed in Section 3. The application of the HFTD procedure to a nonlinear soil-structure-interaction analysis is presented in Section 4, where an uplifting rigid block is considered. This example is also used to illustrate the relationships between the number of time segments used in the calculation and the number of iterations and the CPU time needed to perform it. Final remarks conclude the paper.

2. CRITERION OF STABILITY

The criterion of stability applying to the HFTD procedure, derived in (Darbre & Wolf 1986), states that the spectral radius $\rho(\omega)$ evaluated at $\omega=-i\Omega$ must be less than unity. $\rho(\omega)$ is defined as the maximum of all the absolute values of the eigenvalues $\lambda_i(\omega)$ of the matrix $[I]-[S_o(\omega)]^{-1}[S(\omega)]$, where $[S_o(\omega)]$ is the dynamic-stiffness matrix of the pseudo-linear system and $[S(\omega)]$ is the dynamic-stiffness matrix of the system at the present level of nonlinearities. The matrix $[I]$ is the identity matrix and $\Omega=\pi/\Delta t$ is the Nyquist frequency associated with the discrete Fourier transform.

The validity of this criterion is illustrated here by the analysis of a SDOF system of mass m, stiffness k and damping c subjected to a force excitation P(t). The response of the system is obtained by use of the HFTD procedure, whereby the pseudo-linear system is of mass m_o, stiffness k_o and damping c_o. The equation of motion to be solved is thus

$$(-\omega^2 m_o + i\omega c_o + k_o)\, u_j(\omega) = P(\omega) + Q_j(\omega) \qquad (1)$$

where $u_j(\omega)$, $P(\omega)$ and $Q_j(\omega)$ are the Fourier transforms of the displacement $u_j(t)$, of the exciting force P(t) and of the pseudo force $Q_j(t)$. The latter is obtained in the time domain using the equation

$$Q_j(t) = (m_o - m)\ddot{u}_{j-1}(t) + (c_o - c)\dot{u}_{j-1}(t) + (k_o - k)u_{j-1}(t) \qquad (2)$$

In the equations above, the subscript j refers to the jth iteration.

The properties of the original SDOF system are selected, in consistent units, as m=9, k=5'685 (natural period of .25) and c=18.1 (critical damping ratio of .04) with the force excitation P(t) given by $P(t) = 100[-3\sin(8\pi t) + 9\sin(24\pi t)]$ for $0<t<2$ and by zero otherwise. The properties of the pseudo-linear system are selected as k_o=1'895 and c_o=18.1 while m_o is varied as indicated in Table 1 to achieve a wide variation of $\rho(\omega=-i\Omega)$. Unless indicated otherwise, the time period used in the discrete Fourier transform is 10.24 (1'024 time steps of length Δt=.01) and the time span for which the response is calculated is 3.2 (320 time steps). The time span of 3.2 is divided into segments of equal lengths. The minimum total number of iterations valid in the particular calculation performed is determined by varying the number of segments from one calculation to another.

The spectral radius $\rho(\omega=-i\Omega)$ is evaluated from the following equation in which $\omega=-i\Omega$ is substituted ($\Omega=100\pi$)

$$\rho(\omega) = \left| 1 - \frac{-\omega^2 m + i\omega c + k}{-\omega^2 m_o + i\omega c_o + k_o} \right| \qquad (3)$$

The minimum total number of iterations and the corresponding number of segments are presented in the table.

Table 1. Overview of HFTD calculation: k_o=1'895, c_o=18.1

m_o	$\rho(\omega=-i\Omega)$	nbr. segmnts.	min. nbr. iterations	
182	.95	320	30'528	*
45.5	.80	20	944	**
15	.39	10	184	
9	.00	10	120	
6.4	.42	10	327	
5	.82	20	1'671	
4.6	.97	320	31'378	***
4.4	1.04	diverges		

* : extrapolated from 5 converged time steps; time period of 81.92
** : time period of 30.72
***: extrapolated from 18 converged time steps

The results obtained indicate that the optimum number of time segments depends on the value of the spectral radius $\rho(\omega=-i\Omega)$. The more the latter tends towards unity, the larger this number is. The dependence of the minimum total number of iterations on the value of the spectral radius $\rho(\omega=-i\Omega)$ is similar to that of the optimum number of segments. Typically, it becomes more difficult to reach convergence as $\rho(\omega=-i\Omega)$ tends towards unity. The procedure diverges for the value of m_o leading to $\rho(\omega=-i\Omega)$ larger than 1. For values of m_o resulting in $\rho(\omega=-i\Omega)$ slightly below unity, time segments consisting of only one time step must be used to obtain convergence (for m_o=182 and m_o=4.6 in this application). These results are in total agreement with the criterion of stability.

It must be noted that the criterion will be satisfied in most structural applications due to the fact that it depends on the properties of the pseudo-linear system and of the nonlinear system at the Nyquist frequency only. The Nyquist frequency is generally high enough so that $[S_o(\omega)]$ and $[S(\omega)]$ are dominated by the mass, which does not generally change from the original system to the pseudo-linear one so that $[S_o(\omega=-i\Omega)]\approx[S(\omega=-i\Omega)]$ and thus $\rho(\omega=-i\Omega)\approx 0$.

3. SEGMENTING APPROACH

The numerical application of the HFTD procedure requires the introduction of time segments which must be implemented in the following way.

The entire time span for which the calculation is to be performed is divided into time segments consisting of one or several time steps. Starting with the first segment, the pseudo forces of the entire time period used in the discrete Fourier transform are set to zero and are transformed to the frequency domain where the equations of motion are solved. After the transformation back to the time domain, the pseudo forces are recalculated for the first segment only and the equations of motion are again solved in the frequency domain. Returning then from the frequency domain typically results in convergence of the response for all times up to a specific time t_c. New pseudo forces are then calculated for times larger than t_c only within this particular segment and a new iteration is started. This procedure is repeated until t_c coincides with the last time value of this segment, after which the next segment is investigated.

It is of the utmost importance not to update any response quantity or pseudo force on that part of the time span which has previously converged, i.e. for times less than t_c. Should such updates be performed, some very small changes in response would occur from one iteration to the next for times $t<t_c$. These small changes affect the response of that part of the investigated segment which has not yet converged. This can cause divergence, especially when a true/false situation of the type encountered in contact problems occurs.

4. APPLICATION TO AN UPLIFTING RIGID BLOCK

The HFTD procedure is particularly attractive when applied to nonlinear problems which cannot be analyzed easily using other formulations. Typical of such situations are systems in which some of the properties are originally defined in the frequency domain. This is, for example, the case of nonlinear soil-structure interaction systems, as discussed in (Kobori et al. 1984). In the following application, the soil-structure interaction of an uplifting rigid block is considered.

The system of Figure 1 is used. It is a rigid block of height 2h, width 2b, mass m and mass moment of inertia I referred to the center of mass M. The block rests on 4 circular foundations of radius a placed at each corner of the block. The system is subjected to a horizontal earthquake acceleration $\ddot{u}_g(t)$. Only the vertical and rocking motions w(t) and $\theta(t)$ of the bottom center of the block are considered in the calculation, with the horizontal motion being assumed to be identical to the ground motion. The numerical values used are as follows: h=9 m, b=6 m, $m=12.7\cdot10^6$ kg and $I=6.4\cdot10^7$ kg·m^2.

Only the vertical stiffness $S_z(\omega)$ of the individual foundations defined by the closed-form solution for a rigid circular massless foundation on the surface of a halfspace is considered here (Veletsos & Verbic 1973). The radius a equals 4 m, the shear wave velocity c_s 750 m/sec, the shear modulus G $1.3\cdot10^6$ kN/m^2 and the Poisson's ratio ν 1/3.

The idealized earthquake ground acceleration $\ddot{u}_g(t)$ acting during 2 seconds is taken as $g/10\cdot[-3\sin(8\pi t) + 9\sin(24\pi t^9)]$, where g=9.81 m/sec^2. Nonlinear effects are present in the system due to the fact that the excitation is large enough to produce uplift of the block. The pseudo-linear system is logically selected as the one in which no uplift occurs. It is noted that 4 degrees of freedom must be introduced: 2 to describe the motion of the block and 2 to describe that of the footings.

The response time histories obtained by application of the HFTD procedure to this example problem are depicted in Figs. 2 to 5. The high level of nonlinearities occurring in this application may be appreciated by referring to Fig. 4, in which the contact conditions are shown. In spite of this, no difficulties are encountered in the application of the HFTD procedure and convergence is reached without any difficulty. The power of the HFTD procedure is clearly demonstrated by this application which is typical of nonlinear soil-structure-interaction problems.

This example also illustrates how the total number of iterations and the corresponding CPU time depend on the number of segments introduced in the calculation. This is shown in Fig. 6. It is first observed that the CPU time is essentially proportional to the total number of iterations. Second, it is seen that using many segments, each containing only a few time steps, wastes computer time. It must be recalled that using too few segments can cause instability. In this application, instability occurs when using a single segment of 320 time steps. Finally, it is noted that the sensitivity of the total number of iterations to the number of segments is not dramatic when one operates in a range relatively close to the optimum number of segments (equal to 4 in this application).

5. SUMMARY AND CONCLUSIONS

A criterion of stability pertaining to the hybrid frequency-time-domain procedure is presented. The criterion depends on the dynamic stiffnesses of both the original nonlinear system and of the pseudo-linear system evaluated at the Nyquist frequency only. At this frequency, both stiffnesses are usually controlled by the mass and the criterion of stability is thus satisfied for most structural systems. The time span of interest must, however, generally be divided into time segments to which the procedure is applied sequentially. The number of segments does not strongly affect the total number of iterations and the necessary CPU time, except when using too many of them. Selecting too few time segments can cause divergence.

The hybrid-frequency-time domain procedure allows stringent nonlinear dynamic problems to be solved accurately. This is demonstrated trough the

analysis of a nonlinear soil-structure interaction system in which the soil's stiffness coefficients are directly defined in the frequency domain.

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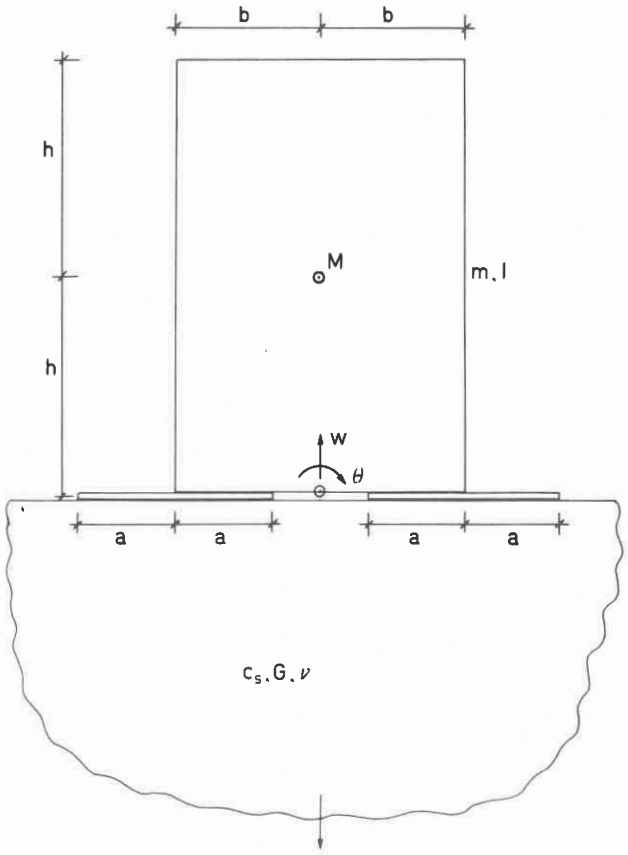

Fig. 1: Uplifting block investigated

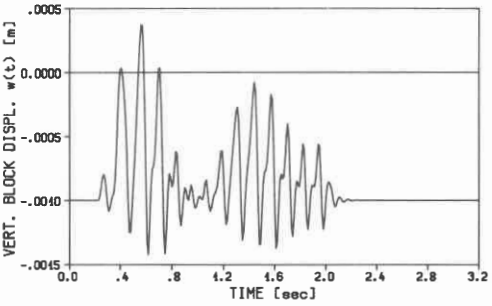

Fig. 2: Vert. block displ.

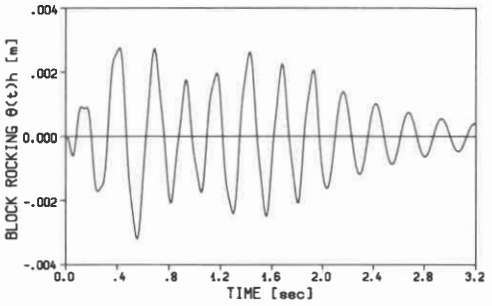

Fig. 3: Block rocking

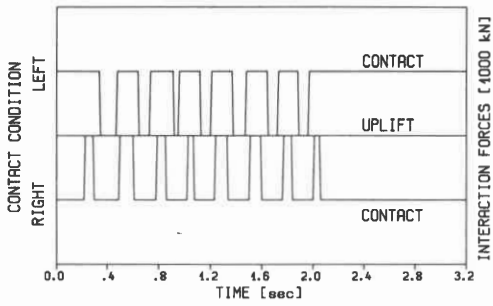

Fig. 4: Contact conditions

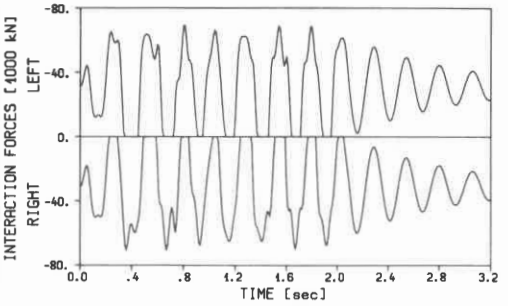

Fig. 5: Interaction forces

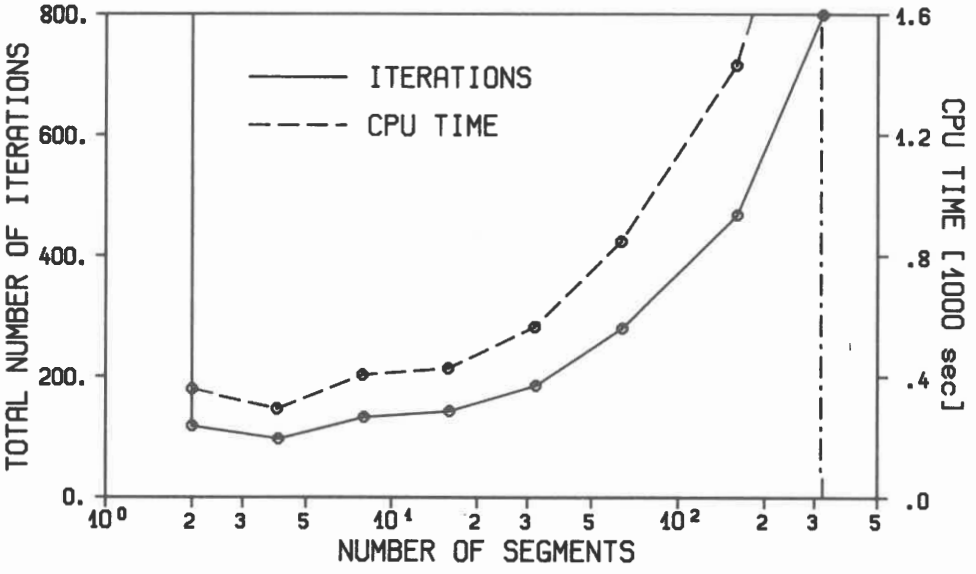

Fig. 6: Total nbr. of iterations and CPU time versus nbr. of segments