Microfriction Model: a New Material Constitutive Model

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Abstract

A microfriction model is proposed to delineate the linear and nonlinear properties of engineering materials. The model may be served to disclose the physical meaning of some empirical formulas. Thus, the elasto-plastic behavior of engineering materials may be clarified with physical terms, furthermore, certain clues aboit isotropic and kinematic hardening might also be available through this channel.

The validity of this model was checked by working out the finite element analysis with some examples and found the results well conformel with the experimental data. Some common used models were compared and contrasted numerically and it had shown that the present model is able to be used to estimate the elasto-plastic behavior of structures and components.

1. Introduction

Nowadays more and more practical engineering problems require elasto-plastic analysis of structures or components. In nonlinear finite element method, in order to describe material stress-strain properties, the elasto-plastic model (ideal plasticity or plastisity with strain hardening), and the piecewise curve description model have been widely used. Other models, which have been adopted in the finite element method, are Kelvin-Voigt (visco-elastic) model[1], rheological (visco-plastic) model[2], overlay model[3], and Endochronic model[4].

The object of this paper is to constitute a physical model which can represent material elastic-plastic properties more closely to the real material. This proposed model will be implemented into the finite element method scheme. As the first step of this work, only the time independent problems have been considered in this paper.

2. The Microfriction Component

In order to simulate the rigid-plastic property of material, let us make a simple friction element consisting of a friction slider and an elastic spring in parallel (fig. 1). The friction slider is not "active" until the applied stress is equal to or bigger than its "active stress" σ_0.

Then consider a model of unit length consisting of a large number of simple friction elements in series (fig. 2), and let us call this model a microfriction component. The threshold stress σ_1, σ_2, ... σ_n are all different and randomly distributed in the unit length. However let us assume that the thresholds could be reranged in ascending order such that there is a relationship

$$g = f(x) \tag{1}$$

Suppose that the applied external stress equals σ. Thus all elements below this value yield while those above conduct stress but are rigid. Elogation of one element is

$$d\epsilon = \left\langle \frac{\sigma - \sigma(g)}{E(g)} \right\rangle dg \tag{2}$$

Thus elogation of the component is:

$$\epsilon = \int_0^G \left\langle \frac{\sigma - \sigma(g)}{E(g)} \right\rangle dg \tag{3}$$

equals the strain. In eq. (3), G is the "active length".

We will now consider the integration of some specific formes of functions $\sigma(g)$ and $E(g)$.

Case 1. Suppose that the microfriction component has a uniform $E(g)$, i.e. $E(g) = E$ and the function $\sigma(g)$ takes the form of $\sigma(g) = \sigma_0 + \alpha g^n$, where E, α, n and σ_0 are material constants, and α must be positive. Then it results

$$\epsilon = \frac{n}{n+1} \frac{(\sigma - \sigma_0)^{\frac{n+1}{n}}}{E\,\alpha^{\frac{1}{n}}} \tag{4}$$

The stress-strain relationship for different n are shown in fig. 3.

Case 2. Suppose that the function $E(g)$ takes the form of $E(g) = E_0 + kg$, and the function $\sigma(g)$ is still as case 1. Here E_0, k are the material constants too. When n equals 1, 2, and 3, explicit results are:

$$n = 1 \quad \epsilon = \frac{\alpha E_0}{k^2} + \frac{\sigma - \sigma_0}{k} + \left(\frac{\sigma - \sigma_0}{k} - \frac{\alpha E_0}{k^2} \right) \ln\left(1 - \frac{k(\sigma - \sigma_0)}{\alpha E_0} \right)$$

$$n = 2 \quad \epsilon = \frac{\alpha}{k^3} \left(\frac{1}{2} k^2 \frac{\sigma - \sigma_0}{\alpha} - 2E_0^2 - E_0 k \sqrt{\frac{\sigma - \sigma_0}{\alpha}} \right) + \left(\frac{\sigma - \sigma_0}{k} + \frac{\alpha E_0^2}{k^3} \right) \ln\left(1 + \frac{k}{E_0} \sqrt{\frac{\sigma - \sigma_0}{\alpha}} \right)$$

$$n = 3 \quad \epsilon = \frac{\sigma - \sigma_0}{3k} - \frac{\alpha E_0}{2k^2} \sqrt[3]{\left(\frac{\sigma - \sigma_0}{\alpha}\right)^2} + \frac{\alpha E_0^2}{k^2} \sqrt[3]{\frac{\sigma - \sigma_0}{\alpha}} + \left(\frac{\sigma - \sigma_0}{k} + \frac{\alpha E_0^3}{k^4} \right) \ln\left(1 + \frac{k}{E_0} \sqrt{\frac{\sigma - \sigma_0}{\alpha}} \right) \tag{5}$$

Case 3. If the function $E(g)$ takes the form of $E(g) = E_0 + \beta g^m$, and the function $\sigma(g)$ is same as before. Where E_0, β, m are material constants, and β can be positive or nagetive. Now the strain will be the sum of integrations I1 and I2. Some explicit results of these two integrations are listed in table 1.

Some literatures [5-9] mentioned several empirical formulas in the form of exponent functions or logarithm functions, which these empirical formulas have almost the same forms as those we derived above, it seems very likely that the microfriction component might be capable of providing a reasonable explanation for these empirical formulas. And, if we let the function $\sigma(g)$ and $E(g)$ take other forms, stress-strain relationships of new form are expected to be derived. For simplicity we will discuss the case 1 metioned above only from now on.

3. The Microfriction Model (MFM)

Connecting two microfriction components and one elastic spring in series, we call this assembly microfriction model (MFM). Within this model, it is suggested that each slider of simple friction element of the two microfriction components has the "ratchet" structure, so that it can slide in one direction only. All the sliders of one microfriction component can slide in the tension direction while the sliders of another microfriction component can slide in compression direction. When some load is applied to this model, the elastic spring will give the elastic response of displasment. And if the applied load makes part of simple friction elements of this model become "active", since the slide is irreversible, the slide provides the plastic part of the displasment.

4. Strain Hardening

In the MFM, we suggest that the function of $\sigma(g)$ is antisymmatric with respect to the centre, i.e. $\sigma(g) = -\sigma(-g)$, while the function of $E(g)$ is symmatric, $E(g) = E(-g)$. If this model is applied some load, and along with one direction (say the direction of tension), some amount of plastic deformation has taken place, further plastic deformation will occure only at a higher stress level (call current active point). Therefore, the MFM has reflected the isotropic hardening already.

Following the treatment of W. Prager [10] on kinematic hardening, we assume that in the MFM if plastic deformation of one direction has taken place, the active point of the same direction is "pushed forward" by "distance" of $c \cdot \epsilon_p$, while the active point in the opposite direction is "draw back" by the "distance" of $c \cdot \epsilon_p$, where c is a material constant or more generally is some material property function, while ϵ_p is a parameter to represent the total "history" of irreversible deformation of the material, which is something like the "intrinsic time" in the Endochronic theory of K.C. Valanis [4]. It can be seen, since the introduction of the translation of current active point due to the plastic deformation, the MFM can represent the kinematic hardening of material, so that this model can be used to describe the Baushinger effect.

5. Multidimension Case

When MFM is used to multidimension case, the Von Mises plastic yielding function and normality flow law are adopted, and only the associated plasticity problems are considered here. Following the method introduced by O.C. Zienkiewicz [11], we can get the elasto-plastic matrix is:

$$D_{ep} = D - \frac{9G^2 \{\sigma - c\epsilon_p\}\{\sigma - c\epsilon_p\}^T}{\left(\frac{H'}{1 - c\frac{\partial \bar{\epsilon}_p}{\partial F}} + 3G \right) F^2} \qquad (6)$$

6. Examples

Using the microfriction model in the finite element method scheme, some examples have been analysed. Two of the examples have experimental results, and others have the results calculated according to another constitutive model or using another program. All these results are used to make comparison with the results from MFM.

(1) Thick wall cylinder

An infinite length thick wall cylinder with internal diameter of 100 mm and outer diameter of 200 mm is subjected to internal pressure P. This problem was solved using Von Mises (ideal plasticity) and MFM (with kinematic hardening parameter c=0 and c=1000 respectively). Fig. 4 shows the displacements at inner and outer surface of the cylinder, and the stress distribution along the different radius when the internal pressure P = 300N/mm . It can be seen that before the initial yield at the inner surface the results from different constitutive models and different kinematic hardening parameters of microfriction model are the same. But, after the inner surface yields, due to the isotropic hardening, the stresses in MFM are higher than those in Von Mises results, and because kinematic hardening the stresses from c=1000 is higher than that from c=0. Therefore MFM can consider isotropic and kinematic hardening of material respectively in calculation.

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Table 1 Some Explicit Forms of Integration I1 and I2

		$m=1$	$m=2$	$m=3$
I1		$\frac{A}{C}\ln\left(\frac{B+CG}{B}\right)$	$\frac{A}{\sqrt{BC}}\,\mathrm{arctg}\left(\sqrt{\frac{C}{B}}\,G\right)$	$\frac{A\sqrt[3]{\frac{B}{C}}}{3B}\left\{\frac{1}{2}\ln\left[\frac{\left(\sqrt[3]{\frac{B}{C}}+G\right)^2}{\sqrt[3]{\left(\frac{B}{C}\right)^2}-\sqrt[3]{\frac{B}{C}}\,G+G^2}\right]+\sqrt{3}\,\mathrm{arctg}\left(\frac{2G-\sqrt[3]{\frac{B}{C}}}{\sqrt{3}\sqrt[3]{\frac{B}{C}}}\right)+\frac{\sqrt{3}\pi}{6}\right\}$
I2	$n=1$	$\frac{D}{C}\left[CG-B\ln\left(\frac{B+CG}{B}\right)\right]$	$\frac{D}{2C}\ln\left(\frac{B+CG^2}{B}\right)$	$\frac{D}{3C\sqrt[3]{\frac{B}{C}}}\left\{\frac{1}{2}\ln\left[\frac{\sqrt[3]{\left(\frac{B}{C}\right)^2}-\sqrt[3]{\frac{B}{C}}\,G+G^2}{\left(\sqrt[3]{\frac{B}{C}}+G\right)^2}\right]+\sqrt{3}\,\mathrm{arctg}\left(\frac{2G-\sqrt[3]{\frac{B}{C}}}{\sqrt{3}\sqrt[3]{\frac{B}{C}}}\right)+\frac{\sqrt{3}\pi}{6}\right\}$
	$n=2$	$\frac{D}{C^3}\left[\frac{1}{2}C^2G^2-BCG+B^2\ln\left(\frac{B+CG}{B}\right)\right]$	$D\left[\frac{G}{C}-\frac{B}{C\sqrt{BC}}\,\mathrm{arctg}\left(\sqrt{\frac{C}{B}}\,G\right)\right]$	—
	n 3	$D\left[\frac{G^3}{3C}-\frac{BG^2}{2C^2}+\frac{B^2G}{C^3}-\frac{B^3}{C^4}\ln\left(\frac{B+CG}{B}\right)\right]$	—	—

Note: in table 1, $A = \sigma - \sigma_0$, $B = E_0$, $C = \beta$, $D = \alpha$.

Table 2 The Comparison of MFM Results With Experimental Data and BERSAFE Results

Specimen	Gauge Mark Displacement (mm)		
	Experimental Values	MFM Results	BERSAFE Results
5 / 10 N	0.0139	0.01477	0.0148
5 / 10 S	0.0135	0.01388	0.0141

(2) The stress concentration in the notched bar

Two notched bars have been calculated using MFM. Both bars have the net and gross diameters of 5 and 10 mm respectively. The first specimen (5/10N) has the notch root radius r=0.8 mm, whilst the second (5/10S) has the r=0.08 mm. On both specimens the gauge points is symmetric to the notch and the distance between the gauge points is 12mm. Both specimens were under the constant load which gave the same net section stress of 375 N/mm . Table 2 lists the calculated and experimental values of initial elastic-plastic elongation at gauge points. It can be seen that the MFM solutions agree with the experimental data very well. In this table, the results of another well known program BERSAFE are listed too. It is clear the results of two approaches are nearly same. Fig. 5 is the stress distribution in the net section of these two notched bars. From these figures it can be seen the results of MFM and BERSAFE are close to each other, while the MFM solution gives more sharp stress concentration at the area near the notch root, eventhough both results were based on the same meshes.

7. Conclusion

(1) The microfriction model can be used to describe material elasto-plastic stress-strain relationship very well, especially for those materials which do not have clear yield point, and those which have the obvious tendency of strain hardening. The MFM provides physical explanation to some empirical formulas which were originated and formed on pure empirical data basis. Since use the same formula to describe the elastic and plastic stages in MFM, nonlinear problems can be dealt with like linear one.

(2) In MFM apart the isotropic hardening the effect of kinematic hardening has been considered, this model can describe the Bauschinger effect of materials therefore.

(3) Using MFM within the finite element method scheme some two-dimensional examples have been analysed. The results of these analyses are promising.

(4) So far, only the simplest case, i.e. $E(g)$=const. and $\sigma(g)=\sigma_0+\alpha g^n$, has been programmed. It might be worth to investigate other possible cases of $E(g)$ and $\sigma(g)$ functions further.

(5) It seems straightforward to develop MFM so that it can deal with the time dependent problems (such as creep problems) or the crack growth problems.

8. References

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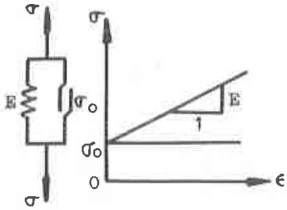

Fig. 1 Simple friction element

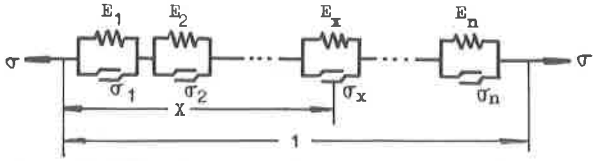

Fig. 2 The microfriction component

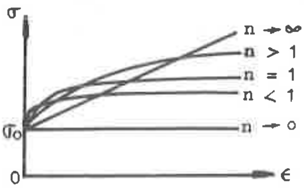

Fig. 3 Stress-strain curve for case 1

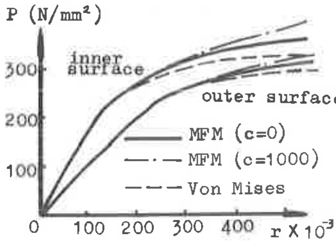

Fig. 4(a) Displacement at inner and outer surface of the thick wall cylinder

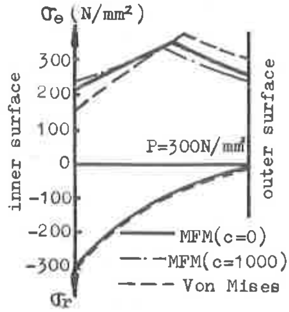

Fig. 4(b) stress in the thick wall cylinder

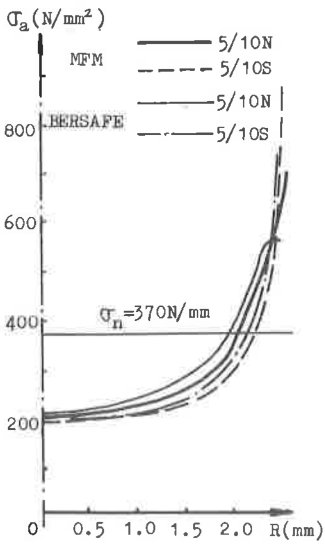

Fig. 5(a) Axial stress on the net section

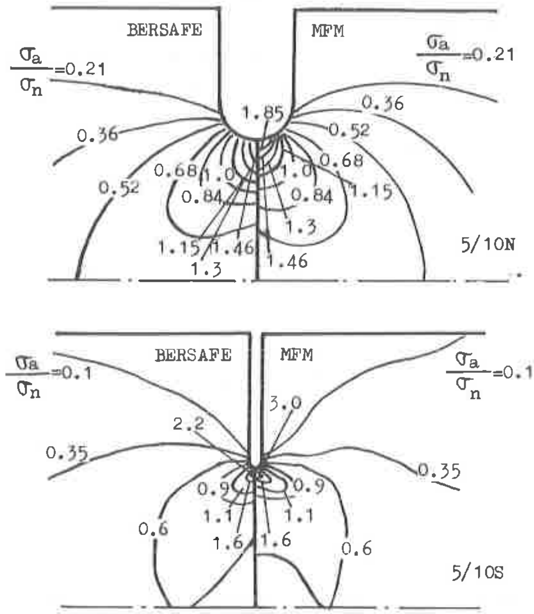

Fig. 5(b) The distribution of normalized stress (σ_n=370N/mm²)