Two Principles of the Hydroelasticity of Plate–Tubes Structures

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I. INTRODUCTION

1.1. A subject. Two circular plates are perforated following the nodes of any triangular grid. They are connected by a system of tubes clamped into the perforations. So the plate-tube structure shown on Fig. 1 is obtained. The multiconnected volume of inviscid, compressible and heavy liquid, bounded by absolutely rigid circular cylindrical shell and both aforementioned compliant plates (serving as a bottom and a seiling of the structure) is under consideration. The vibrating tubes are represented dually: (i) their real cross dimension (diameter) is taken into account when the liquid acoustical pressure to their surfaces is under consideration; (ii) as structural components they are taken as beam-like vibrating.

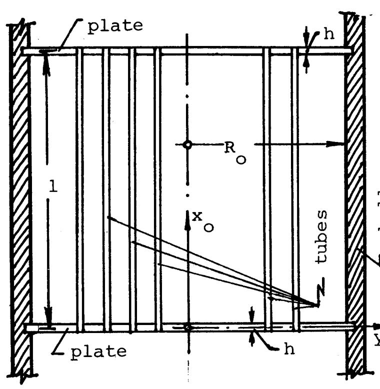

Fig. 1. The tube-plate geometry

The hydroelasticity of perforated plates is considered in a paper, published in the Proceedings of III Keswick Conference on Vibration of Nuclear Plants (England, 1982). All investigation is essentially based to the application of Bessel's function summation formulas and this presupposes the application of two special principles which were used but not rigorously formulated in that early work, [Dzhupanov, 1983]. Later [Golovkina & Dzhupanov, 1983] the first of both principles was formulated explicitly and used to prove a theorem on orthogonality of the modes of liquid vibration in multiconnected reactor volume. Both principles were reported (in realized and complete form) on XXVI Polish Solid Body Mechanics Conference (Sobieshevo 1986, Poland) but they were not published until now.

So the object of the present investigation is to formulate the two principles (PI.- of objectivity & PII.- of problem matching) harmonically supplementing them by a system of regular mechanical assumptions of the problem.

1.2. Technological information. The mechanical problems connected with the design of the structures under consideration are mainly dynamical ones [Dzhupanov 1983_2, 1987]. Really said, plate-tubes structures are used in heatexchangers, in chemical reactors, or in PWR (e.g. the Russian WWER-440) where the dynamical loads of different nature predominate [Dzhupanov 1981, 1983, 1985, Dzhupanov & Manoach 1983, 1987, etc.].

II. PREREQUISITES

2.1. General consideration. Obviously the practical hydroelastic problem has to be modelled by three systems of assumptions, namely: (i) connected with the solid body structure, (ii) connected with the liquid nature and its flow; (iii) connected with structure-liquid interaction.

2.2. Structural material. The classical assumptions modelling the Hookian elastic body are accepted here: (i) homogeneons material; (ii) isotropic material; (iii) small deformation (small displacements of rigid body); (iv) linear elasticily, etc.

2.3. Kirchoff-Love assumption. Considering thin plates the classical Kirchoff-Love assumption on the rectilinear normals to the mid plate surface is presumed as valid. In accordance with this assumption we shall accept that the plates under consideration are represented by their mid surfaces only [Dzhupanov 1983, Dzhupanov & Manoach 1983, 1987].

2.4. Bernoully-Navier assumption. The length of the tubes, taken between both plate mid planes is considered as quite greater than the dimension (diameter) of their cross section. This gives right to accept Bernoully-Navie assumption as valid and to consider the tube vibrations as beam like ones [Dzhupanov 1983_1].

2.5. Liquid. The heatcarrier liquid is presumed to be inviscid compressible and heavy one.

2.6. Co-ordinates. The analytical description of the plate vibration will be given in local co-ordinates connected with the axes of the perforated holes. All local co-ordinate systems are enumerated by the counting index m, where $m = 0,1,2,\ldots,\nu$. Zero number corresponds to the barrel vessel of the structure containing the plate-tube unit. Number of the holes corresponds to tube number and that is why the same current index m to indicate the tubes is used also. In both cases, if one would like to indicate a point considered in near vicinity of any tube or any hole current index s is introduced, having $s = 0,1,2,\ldots,\nu$ also. All local co-ordinate systems are cylindrical ones e.g. $0(r_m,\theta_m,x)$ being transformated in polar co-ordinate systems at $x = 0$ (bottom plate) and $x = h$ (ceiling plate) e.g. $0(r_m,\theta_m)$.

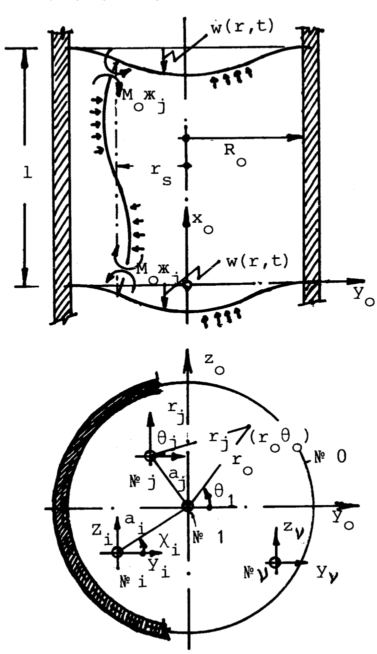

Fig. 2. Antisymmetric forms of vibration and additional geometrical data.

2.7. Coupling. The small structural displacement generate acoustical motion in the liquid with potential function $\varphi(r,\theta,x,t)$.

III. ADDITIONAL PRELIMINARIES

3.1. Designations: $w(r,\theta,t)$ - displacements of the plate mid surface; D - flexural rigidity; $p(r,\theta,h;t)$ & $p(r,\theta,0,t)$ hydrodynamical pressurs to the plates; ρ - liquid density; c - sound velocity of the liquid; $\Delta = (\partial^2/\partial r^2)+(1/r)(\partial/\partial r)+ + (1/r^2)(\partial^2/\partial\theta^2) + (\partial^2/\partial x^2)$; $(\dot{\ }) \equiv (\partial/\partial t)$; $q(r,\theta,t)$ - dynamical plate loading; $v_m(x,\theta_m,t)$ - displacement function of tube No m surface.

3.2. Preliminaries on the displacement functions. The displacement functions of the plates must be subordinated to some preliminary conditions and designations. We shall introduce the designations:

$w_m = w_m(r_m,\theta_m;t)$ - local displacement function (in near vicinity of hole, having No m);

$\sum_{m=1}^{\nu} w_m(r_m,\theta_m;t)$ - is impossible without co-ordinate transforms of all co-ordinates to one and same system;

$w_I, w_{II}, \dots, w_{\underline{s}}, \dots, w_{\underline{\nu}}$ are general expressions of one and same displacement function obtained following the rule

$$(1) \quad w_{\underline{s}} = w_s + \sum_{j=1}^{\nu} {}^{*} w_m \,(m \to s),$$

which has to be read: "The general displacement function of the plate in near vicinity of co-ordinates No s (i.e. $w_{\underline{s}}$) is equal to the sum of local displacement function No s (i.e. w_s) and all other local functions w_m but rewritten down in co-ordinate system No s i.e. $\Sigma^*_m w_m(m \to s)$ ", [Dzhupanov 1983, 1985]. The symbol $w_m(m \to s)$ introduces the co-ordinate transforme formulas.

The general displacement function w has to be an integral of differential equation

$$(2) \quad D\Delta\Delta w + \bar{m}\,\ddot{w} = q(r,\theta;t) - p(r,\theta,t)$$

and to satisfy the boundary conditions along the supporting contours of the plates and coresponding conditions in the points of plate-tubes couplings [Dzhupanov & Manoach 1983, 1987].

3.3. Preliminaries on the potential function. Let $\varphi_m(r_m,\theta_m;t)$ is space local liquid velocity potential function in near viciniti of tube No m. Let $\varphi_I, \varphi_{II}, \dots, \varphi_{\underline{\nu}}$, are different designations of one and same general velocity potential function, describing all liquid motion in near vicinity of corresponding co-ordinates (e.g. No s). By using the same formulas this general function can be presented in the form

$$(3) \quad \varphi_{\underline{s}} = \varphi_s + \Sigma^*_m\, \varphi_m(m \to s).$$

It has to be an integral of the acoustical differential equation

$$(4) \quad \Delta\varphi_{\underline{s}} - (2/c^2)\ddot{\varphi}_{\underline{s}} = 0 \to \Delta\varphi_m - (1/c^2)\ddot{\varphi}_m = 0$$

and to satisfy boundary conditions Dzhupanov 1983_1 :

$$(5) \quad \left.\frac{\partial\varphi_{\underline{s}}}{\partial x}\right|_{x=0,h} = \dot{w} \text{ and } \left.\frac{\partial\varphi_{\underline{s}}}{\partial r_s}\right|_{r_s=R_s} = \dot{v}_s .$$

If (3) is known, acoustical pressure p can be determined, e.g.

$$(6) \quad p_s = -\rho\left[\partial\varphi_{\underline{s}}/\partial t\right]_{x=0} \text{ (or } x = h),$$

and can be put into Eq. (2).

3.4. Problem matching. Obviously both problems (for the plate-tubes vibration from one side and for the liquid mothion from another side) are matched dynamically by Eqs. (2) & (6), and kinematically by Eqs. (5).

IV. PRINCIPLES SOUGHT

4.1. Objectivity principle (P.I.). If in a multiconnected domain any process (or phenomenon) is described by selfconjugated linear differential equation with constant coefficients, its analytical solution can be represented by one and same function for all domain in form (1) - for the two dimensional case, or in form (3) - for the three-dimensional case [Golovkina & Dzhupanov 1983].

4.2. Matching principle (P.II.). The functions of two linear variables $w_s(r_s,\theta_s;t)$ and the functions of three linear variables $\varphi_s(r_s,\theta_s,x;t)$ (but at $x=0$, or $x=h$) must be matched in such way, that to every function w, with given number s (e.g. $w_{\underline{s}}$) only function φ with the same number s (e.g. $\varphi_{\underline{s}}$), must correspond in dynamical Eq. (2).

4.3. Comments. (i) P.I. is a necessary and enough condition to prove the orthogonality of eigenfunctions φ in 3D case and eigenfunctions w in 2D case. For the velocity potential functions describing free liquid vibrations in a multiconnected cylindrical volume (conditionelly called "Reactor volume") the orthogonality theorem was proved by Dzhupanov & Golovkina, see Ref. [10] (1983). For the displacement functions w describing free vibrations of multiconnected circular plate the orthogonality theorem was proved (using essentially the idea of [10]) by E.S. Manoach in his "Candidat of Sciences" - degree thesis and published later [Dzhupanov & Manoach 1983]. However, by applying systematically P.I., E.S. Manoach has proved some aditional theorems on the orthogonality of displacement eigen functions w. The roots of some of these theorems (ideology and first formulations) can be discovered in Ref. [12] = [Kito 1970].

(ii). P.II. states the correspondence $w_{\underline{s}} \to \varphi_{\underline{s}}$, which seems to be evident, as the other matchings (e.g. $w_{\underline{s}} \leftrightarrow \varphi_{\underline{\nu}}$) have not physical sense. The usefulness of this principle is seen better in detailed investigation of plate-tube (structures) hydroelasticity.

V. CONCLUSION

5.1. The hydroelastic problem of the plate-tubes structures can be treated analytically (theoretically) only after a formulation of a system of principles. Two of them were formulated above. Here other of them will be mentioned.

5.2. Symmetry principle (P.III). Stating on the Betti's symmetry principle (for the lineary elastic bodies) and on the symmetry prinsiple of the hydroacoustics one can prove the symmetry of so called interaction matrix [Golovkina, Dzhupanov & Yanatchkov 1984]. It is remarcable how in the nature of (cylindrical functions) summation formulas the symmetry principle is hidden. In general terms P.III. states the correspondence:

(7) $$\varphi_m(m \to s) \Leftrightarrow \varphi_s(s \to m) \; \& \; w_m(m \to s) \Leftrightarrow w_s(s \to m).$$

5.3. Restriction principle (P.IV). Usually summation formulas $\Sigma^* w_m(m \to s)$ introduce infinite sums on any summation index k, e.g.

(8) $$J_\lambda(\Psi r_m)\cos\lambda\theta_m = \sum_{k=-\infty}^{\infty} J_{\lambda+k}(\Psi a) J_k(\Psi r_s)\cos k\theta_s, \; (r_s < a),$$

where Ψ is wave number, a - distance between the origines of the co-ordinate systems $O_m(r_m,\theta_m)$ and $O_s(r_s,\theta_s)$. The restriction of these sums can not be arbitrary, and has to be in correspondence with the basical idea of Bubnov-Galerkin method on the solution representation by restricted sums.

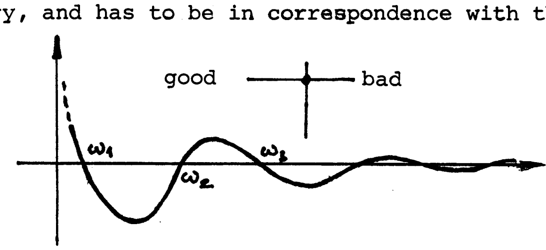

Fig. 3.

5.4. Eigen value importance principles. Usually the multitude of the lowest natural frequences is "well" marked and as consequence - well determinable. They are called "important" or "good" frequenses - Fig. 3. The higher part of frequenses is not well marked as the solution curve is crossing the frequence - abscissa under small angles. That is why they are called "bad stipulated", or "insignificant" frequenses. (These terms are introduced by us and are not obligible).

REFERENCES

1. Dzhupanov, V.A. Theoretical study on the interference of the circular cylindrical shells through the fluid in case of mixed hydroelastic problem. Proc. of Intern. Symp. on Vibration Problems in Industry. 10-12 April 1973, Keswick, England.
2. Dzhupanov, V.A. On the dynamical pressure cumulation in any tipes of PWR. SMiRT-6 Conference, Paris, France 1981. P. No J 1/10.

3. Dzhupanov, V.A. Hydroelasticity of the perforated plates applicable in nuclear technology. Proc. of III Keswick Symp. on "Vibration in Nuclear Plant". Vol. 2; 1062-1084. BNES - London, 1983.
4. Dzhupanov, V.A. Currents and Heatexchange in Power Apparatuses of Nuclear Plants (General lecture on Session Cd). XX IAHR Congress, Sept. 1983, Moscow, USSR. Proceedings, Vol. VIII, 257-288.
5. Dzhupanov, V.A. Hydroelasticity of the fuel rods for the Nuclear Power Reactors. XX IAHR Congress; Sept. 1983; Moscow, USSR; Proceedings, Vol. I, Part II; 103-105.
6. Dzhupanov, V.A. Hydroelasticity of Energetical Structures. Proc. of V-th Bulgarian Nat. Congress on TAM. Vol. I, 228-242. BUL. AC. SCI., 1985.
7. Dzhupanov, V.A. Hydroelasticity in Mechanics of Nuclear Energetical Reactors. BUL. AC. SCI. Center of Sci. Information, "Actual problems in Science" 1987, No 3; 72 p.
8. Dzhupanov, V.A., E.S. Manoach. Theoretical modelling of the plate-tube coupling. SMiRT-7 Conference, Chicago, USA; 1983. P No B 3/11.
9. Dzhupanov, V.A., E.S. Manoach. Consideration on the plate-tube dynamics. SMiRT-9 Conference, Lausanne, Switzerland, 1987. Proceedings, Vol. E, 419-424.
10. Golovkina, A.B., V.A. Dzhupanov. Liquid free vibration modes in a multiconnected reactor volume. SMiRT-7 Conference; Chicago, USA, 1983; P. No B 7/13.
11. Golovkina, A.B., V.A. Dzhupanov, O.P. Yanatchkov. On the symmetry of the interaction matrix in case of hydroelastic problem of cylindrical structural components... Proc. of ASME Winter Annual M-eting, 9-14 Dec., 1984. New Orleans, USA.
12. Kito, F. Principles of Hydroelasticity. Keio Univ. Press, 1970.