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Dynamic Analysis of Elastic Shell Structures under Thermal Gradient

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Abstract : In this paper an extensive analysis has been presented for the nonlinear thermal vibrations of elastic shallow spherical shell under linear and parabolic temperature distributions. The formulation of the problem is based on the von Karman nonlinear equations and the mathematical analysis on Galerkin's method. Numerical results have been presented for different variations of thermal as well as other parameters considering different boundary conditions. Criteria for thermal buckling have also been discussed.

INTRODUCTION :

Thermal vibrations and thermal stresses play an important role in structural mechanics, aeronautics and high-speed space-crafts. Compared to linear analysis very few problems have been dealt with for thermally-stressed plates e.g. Nowinski (1963), Pal(1970, 1973), Nowinski and Ohnabe (1972), Satyamoorthy and Pandalai(1974), Banerjee and Datta (1979), Majumder et al. (1980), Biswas (1983), and Biswas and Kapoor (1984) and thin shallow shells e.g. Nash and Modeer (1960), Bhattacharya (1976), Ramachandran (1981) Sinharay and Banerjee (1985a,b), Banerjee et al (1993), and Ohnabe (1995) in the nonlinear cases.

In this paper basic governing equations have been derived in the von Karman sense in terms of displacement components and stress functions under thermal loading extended to thin shallow spherical shell. These equations have been solved satisfying prescribed boundary conditions by application of Galerkin's method considering linear and parabolic temperature distributions.

Numereical results have been presented for different variations of thermal as well as other parameters which occured in the analysis considering clamped movable and immovable edge conditions. Criterion for thermal buckling has also been discussed.

FORMULATION OF GOVERNING DIFFERENTIAL EQUATIONS

A thin shallow spherical shell of uniform thickness is considered. The geometry and coordinate system is shown in Fig.1. The shell is made of elastic, homogeneous and isotropic material. The vertical component of displacement of the middle surface of the shell is denoted by w(r,t) and the radial displacement of a point in the middle surface is denoted by u(r,t), measured horizontally. The elevation of the middle surface of the shell above the base plane is represented by

$$Z(r) = R^2/2R_0\,[(1-(r^2/R^2)] \tag{1}$$

The distribution of temperature is assumed to vary radially being independent of θ coordinate. It is considered as follows :

(i) For linear varition of temperature $T(r) = T_O [1 - (r/R)]$ (2a)

(ii) For parabolic variation of temperature $T(r) = T_O [1 - (r^2/R^2)]$ (2b)

where T_0 = temperature at the center of the shell .

Considering the equilibrium equations of nonlinear free vibrations for exisymmetric thin shallow spherical shell including temperature loading basic garving equations in the von Karman sense are obtained in the forms :

$$r^2u,_{rr} + ru,_r - u = ((\nu-1)/2)rw^2,_r + (\nu-1)rw,_rz,_r - r^2w,_rw,_{rr} - r^2(w,_rz,_{rr} + w,_{rr}z,_r) + r^2(1-\nu^2)(N_T),_r/EH \quad (3)$$

$D\nabla^4w + \rho Hw,_{tt} + \nabla^2M_T = (1/r)(_rN_{rr}w,_r)_r$ where M_T and N_T are thermal stress resultant and thermal stress couple respectively.

These two equations are to be used to study the static and dynamic nonlinear behavior of such shell structures at elevated temperature.

For free vibrations $M_T = 0$ is not exactly true; it is an assumption based on neglecting temperature variation along depth due to compression; nor it does follow from Majuder et al. (1980) who considered $M_T = 0$. For free thermal vibrations the temperature field is considered to depend on the polar coordinate r only.

BOUNDARY CONDITIONS

This method can be applied to axisymmetric spherical shell for any boundary conditions. In this paper the spherical shell with clamped edges, movable or immovable, is considered. So the boundary conditions at r = R are

(i) w = 0 (deflection along z-direction is zero) (ii) $w,_r = 0$ (slope is zero in r-z plane)

(iii) $N_{rr} = 0$ (radial stress is zero for movable edges)

or, u = 0 (radial displacement is zero for immovable edges)

SOLUTION

Since the distribution of temperature and mass is symmetrical over the shell, the form of symmetrical deflection of the clamped spherical shell is assumed as follows in the first approximation

$$w(r,t) = A [1+A_2(r^2/R^2) + A_4 (r^4/R^4) \quad (6)$$

where A is the maximum deflection at the center of the shell; A_2 and A_4 are constants to be determined from the boundary conditions. Applying boundary conditions (i) and (ii) in the above equation one obtains

$$w(r,t) = A [1-(r^2/R_2)]^2 \quad (7)$$

Substitution of eqns. (1) and (7) into (3) yields

$$r^2u,_{rr} + ru,_r - u = C_1r^3+C_2 r^5+C_3 r^7 + \psi(r) \quad (8)$$

in which C_1,C_2,C_3 and $\psi(r)$ are known.

The complete solution of eqn. (8) is obtained in the form

$$u = C\,r + (C_1/8)r^3 + (C_2/24)r^5 + (C_3/48)\,r^7 + \psi(r) \qquad (9)$$

where $\psi(r)$ is the particular solution for the term $\phi(r)$ of eqn.(8); the constant C can be found from boundary condition (iii) for movable or immovable edges.

Now, we turn to eqn.(6) with M_T being neglected for free vibration as discussed earlier. Combining eqns. (1),(2) (6) and (7) and applying Galerkin's error minimising technique one gets well known nonlinear time differential equation in the following form

$$F_{,tt}(t) + \lambda_1\,F(t) + \lambda_2\,F^2\,(t) + \lambda_3\,F^3(t) = 0 \qquad (10)$$

where the parameters λ's are known.

The solution of eqn. (10) with initial conditions $F(0) = 1$, $F_{,t}(0) = 0$ have been given by Bhattacharya (1976) and the ratio of nonlinear and linear frequencies takes the form

$$\omega_{NL}/\omega_L = [[\ 1+ (A^2/H^2)\ [\ (3\lambda_3/4\lambda_1)-(5/6))\ [(\lambda_2/\lambda_1)]^2]]^{1/2} \qquad (11)$$

where ω_{NL} is the nonlinear frequency and ω_L is the linear frequency of the structure.

The eqn. (11) assumes different forms depending on the values of λ's corresponding to different boundary conditions and temperature distributions.

NUMERICAL RESULTS, OBSERVATIONS AND DISCUSSIONS :

Numerical results are presented in this section to understand the effects of different parameters viz. (i) nondimensional curveture (R^2/R_0H), (ii) nondimensional amplitude A/H, (iii) R/H ratio, and (iv) the nature of temperature distribution and its magnitude etc. on dynamic behaviors of clamped thin shallow spherical shell with movable and immovable edges. The material properties of the plate considered in the numerical calculations are as follows :

Poisson's ratio $\nu = 0.3$; mass density $\rho = 7.833\text{x}10^{-6}$ kg/mm^3 , Young's modulus $E = 2\ \text{x}10^5$ N per mm^2; coefficient of thermal linear deformation $\alpha_t = 12\text{x}10^{-6}$ per ^{0}C.

Effects of nondimensional curvature [R^2/R_0H] :

Table 1 shows variations of nondimensional curvature for both movable and immovable edges with and without temperature loading. Both hardening and softening type of behaviors are noticed. The behavior is nonhardening-nonsoftening type i.e. $\omega_{NL}=\omega_L$ for the value, called transition value, of nondimensional curveture equal to 4.996 and 1.887 for movable and immovable edges respectively. The behavior changes from hardening type to softening type as the value of nondimensional curvature becomes greater than the transition value 4.996 and 1.887 for movable and immovable edges. It is also observed that the presence of temperature field makes the variation steeper and the transition value of nondimensional curvature become smaller.

Effects of nondimensional amlitude [A/H] :

Table 2 shows variations of relative nonlinear frequency with the variation of nondimensional amplitude for different values of nondimensional curvature and different edge conditions. Both hardening and softening types of behavior are observed for both clamped movable and immovable edges. Also, the effect is more pronounced i.e. relative nonlinear frequency increases much faster with increase of nondimensional amplitude in the latter case than the former. Similar behaviors are also observed by Sinharay and Banerjee (1985a) though the results presented in this paper deviate from them numerically which may be attributed to different approaches in formulation ofthe problem.

Effects of temperature distribution and its magnitude :

Numerical results showing varitions of relative nonlinear frequency with the variation of temperature T_0 for both linear and parabolic temperature field when nondimensional amplitude A/H is taken equal to unity considering a particular value of r/H ratio equal to 50 for some particular values of nondimensional curvature r^2/R_0H are presented in tables 3 and 4. It is observed that when the value of nondimensional curvature remains less than 4.996 and 1.887 for movable and immovable edges respectively, with increase of temperature the relative nonlinear frequency increases initially upto a maximum point and then decreases abruptly to zero; the behavior being mixed hardening and softening type. When the value of nondimensional curvature becomes greater than the transition value 4.996 and 1.887 for movable and immovable edges respectively, it is observed that the relative nonlinear frequency which is less than unity decreases with increase of temperature at relatively slower rate initially and after certain point it decreases abruptly to zero; the behavior being fully softening type. The effect of temperature is more pronounced in case of parabolic temperature field than the linear one. Since the strain softeining type of behavior leads to degradation of stiffness, such structures buckle owing to stiffness degradation due to temperature rise.

Effects of the parameter R/H ratio :

Numerical results showing variations of relative nonlinear frequency with the variation of the parameter R/H ratio for both linear and parabolic temperature field when nondimensional amplitude A/H is taken equal to unity considering some particular values of temperature paramete T_0 and some particular values of nondimensional curvature R^2/R_0H are presented in tables 5 and 6 . It is observed that the effects of the parameter R/H ratio on the relative nonlinear frequency is simillar to those of temperature distribution and its magnitude discussed in the previous section.

BUCKLING CRITERION AND CRITICAL BUCKLING TEMPERATURES :

Bukcling is characterised as the phenomenon of instability of stable equilibrium form and subsequently indefinitely large deflection of structure. From this fact one can conclude that buckling occurs when relative nonlinear frequency becomes infinite or zero for structures exhibiting strain-hardening type or strain-softening type of behaviors respectively. Preceding sections reveal that shallow spherical shell strain-softens finally as the temperature T_0 increases. Such structures buckle owing to stiffness degradation due to temperature rise; critical buckling temperature being dependent on nondimensional amplitude, nondimensional curveture and R/H ratio besides temperature distributions and edge conditions. Accordingly, expressions of critical buckling temperature $(T_0)_{cr}$ for different edge conditions and temperature distributions can be obtained.

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Table 1 : Relative nonlinear frequencies for different edge conditions, temperature fields and R^2/R_oH values; when A/H ratio = 1.0 and R/H ratio = 50.0

R^2/R_oH	Relative nonlinear frequencies ω_{NL}/ω_L					
	Clamped movable edge			Clamped movable edge		
	No Temp. field	Linear Temp. field $T_0 = 500°C$	Parabolic Temp. field $T_0 = 500°C$	No temp. field	Linear Temp. field $T_0 = 50°C$	Parabolic field $T_0 = 50°C$
0.25	1.013399	1.022737	1.034788	1.089104	1.136013	1.185005
0.75	1.013299	1.021964	1.032969	1.076821	1.106512	1.128331
1.25	1.012597	1.020416	1.029321	1.051827	1.045013	1.005445
1.75	1.011794	1.018091	1.023826	1.013179	0.945294	0.785878
2.25	1.010722	1.014982	1.016452	0.959230	0.793064	0.305260
2.75	1.009381	1.011082	1.007159	0.887192	0.546043	
3.25	1.007769	1.006382	0.995892	0.792146		
3.75	1.005886	1.000871	0.982585	0.664287		
4.25	1.003729	0.994536	0.967152	0.477973		
4.75	1.001297	0.987359	0.949491			
5.25	0.998588	0.979324	0.929473			
5.75	0.995599	0.970409	0.906944			

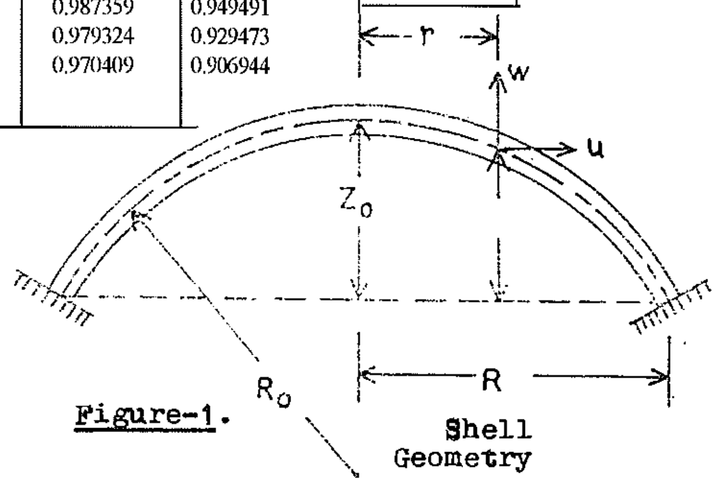

Figure-1. Shell Geometry

Table 2 : Relative nonlinear frequencies for different edge conditions, R^2/R_oH and A/H ratio.

A/H	Relative nonlinear frequencies ω_{NL}/ω_L					
	Clamped movable edge			Clamped movable edge		
	$R^2/R_oH = 0.5$	$R^2/R_oH=3.0$	$R^2/R_oH=6.0$	$R^2/R_oH=0.5$	$R^2/R_oH=1.0$	$R^2R_oH=2.0$
0.1	1.000134	1.000087	0.999940	1.000991	1.000681	0.999883
0.2	1.000535	1.000346	0.999761	1.003517	1.002722	0.999533
0.3	1.001204	1.000778	0.999461	1.007897	1.006113	0.998949
0.4	1.002140	1.001382	0.999042	1.013996	1.010843	0.998130
0.5	1.003341	1.002159	0.998503	1.021784	1.016891	0.997077
0.6	1.004808	1.003108	0.997844	1.031223	1.024234	0.995789
0.7	1.006538	1.004228	0.997064	1.042269	1.032846	0.994263
0.8	1.008531	1.005518	0.996964	1.054894	1.042694	0.992501
0.9	1.010785	1.006980	0.995142	1.068971	1.053744	0.990499
1.0	1.013299	1.008609	0.994000	1.084514	1.065958	0.988250

Table 3 : Relative nonlinear frequencies for movable edge conditions,different temperature fields and T_0 values; when A/H ratio = 1.0; R/H ratio =50.0 R^2/R_0H = 0.5 & 0.6

$R^2/R_0H = 0.5$				$R^2/R_0H = 6.0$			
Linear temperature field		Parabolic temperature field		Linear temperature field		Parabolic temperature field	
T_0	ω_{NL}/ω_L	T_0	ω_{NL}/ω_L	T_0	ω_{NL}/ω_L	T_0	ω_{NL}/ω_L
00.0	1.013299	00.0	1.013299	00.0	0.993999	00.0	0.993999
100.0	1.014479	100.0	1.015147	100.0	0.991518	100.0	0.989964
200.0	1.015889	200.0	1.017592	200.0	0.988111	200.0	0.983339
300.0	1.017603	300.0	1.020978	300.0	0.983305	300.0	0.971639
400.0	1.019732	400.0	1.025978	400.0	0.976294	400.0	0.948631
500.0	1.022447	500.0	1.034106	500.0	0.965613	500.0	0.894679
600.0	1.026027	600.0	1.049628	600.0	0.948371	600.0	0.712788
700.0	1.030966	700.0	1.090954	700.0	0.918169	650.0	0.311385
800.0	1.038217	750.0	1.154892	800.0	0.858216	655.0	0.176982
900.0	1.049896	760.0	1.179653	900.0	0.708351	657.0	0.056134
1000.0	1.071827	770.0	1.212825	950.0	0.513514		
1100.0	1.127733	780.0	1.256859	960.0	0.444241		
1120.0	1.150982	785.0	1.280810	970.0	0.349537		
1140.0	1.184004	790.0	1.294128	980.0	0.191205		
1160.0	1.232847	795.0	1.230970	983.0	0.092471		
1170.0	1.264368	796.0	1.183632				
1180.0	1.292320	797.0	1.104105				
1190.0	1.234721	798.0	0.962120				
1191.0	1.207279	799.0	0.662933				
1192.0	1.168862	799.5	0.280367				
1193.0	1.114289	799.6	0.019174				

Table 4 : Relative nonlinear frequencies for immovable edge condition, different temperature fields and T_0 values; when A/H ratio = 1; R/H ratio = 50 and R^2/R_0H = 0.5 & 0.2.

$R^2/R_0H = 0.5$				$R^2/R_0H = 2.0$			
Linear temperature field		Parabolic temperature field		Linear temperature field		Parabolic temperature field	
$T_0{}^0C$	ω_{NL}/ω_L	T_0	ω_{NL}/ω_L	$T_0{}^0C$	ω_{NL}/ω_L	$T_0{}^0C$	ω_{NL}/ω_L
0.0	1.084514	0.0	1.084514	0.0	0.988257	0.0	0.988257
25.0	1.100895	10.0	1.093654	10.0	0.978100	5.0	0.980918
50.0	1.125041	20.0	1.104990	20.0	0.964317	10.0	0.971723
75.0	1.163875	30.0	1.119407	30.0	0.945191	15.0	0.960071
100.0	1.233488	40.0	1.138316	40.0	0.917862	20.0	0.945102
105.0	1.253015	50.0	1.164075	50.0	0.877232	25.0	0.925540
110.0	1.273483	60.0	1.200701	60.0	0.813278	30.0	0.899429
115.0	1.290724	70.0	1.253580	70.0	0.702807	35.0	0.863613
120.0	1.288089	80.0	1.287176	80.0	0.467833	40.0	0.812631
125.0	1.180326	81.0	1.273597	81.0	0.426834	45.0	0.736026
126.0	1.114642	82.0	1.246859	82.0	0.378577	50.0	0.609740
127.0	1.008750	83.0	1.196609	83.0	0.319528	55.0	0.344247
128.0	0.822413	84.0	1.004530	84.0	0.241520	56.0	0.230883
129.0	0.380343	85.0	0.897790	85.0	0.108708	56.7	0.063146
129.2	0.070964	86.0	0.162692	85.2	0.046528		

Table 5 : Relative nonlinear frequencies for movable edge conditions,different temperature fields and R/H values; when A/H ratio = 1.0; T_0 =500°C, R^2/R_0H = 0.5 & 0.6

$R^2/R_0H = 0.5$				$R^2/R_0H = 6.0$			
Linear temperature field		Parabolic temperature field		Linear temperature field		Parabolic temperature field	
R_H	ω_{NL}/ω_L	R_H	ω_{NL}/ω_L	R_H	ω_{NL}/ω_L	R_H	ω_{NL}/ω_L
30.00	1.015585	30.00	1.017042	30.00	0.988886	30.00	0.984960
35.00	1.016617	35.00	1.018970	35.00	0.986159	35.00	0.978939
40.00	1.017991	40.00	1.021818	40.00	0.982114	40.00	0.968265
45.00	1.019852	45.00	1.026292	45.00	0.975864	45.00	0.946948
50.00	1.022447	50.00	1.034106	50.00	0.965613	50.00	0.894679
55.00	1.026236	55.00	1.050783	55.00	0.947247	55.00	0.693232
60.00	1.032188	60.00	1.109040	60.00	0.909452	56.00	0.572880
65.00	1.042717	61.00	1.143321	65.00	0.809693	57.00	0.315363
70.00	1.066026	62.00	1.208297	66.00	0.769452	57.10	0.265377
75.00	1.158128	62.30	1.239139	67.00	0.713857	57.20	0.199970
75.50	1.184403	62.50	1.262972	68.00	0.632409	57.30	0.089728
76.00	1.219315	62.60	1.275089	69.00	0.499787		
76.50	1.265839	62.70	1.286067	70.00	0.191205		
77.00	1.286452	62.80	1.293407	70.10	0.102154		
77.10	1.256347	62.90	1.291202	70.30	0.048927		

Table 6 : Relative nonlinear frequencies for movable edge conditions,different temperature fields and T_0 values; when A/H ratio = 1.0; T_0 =500°C R^2/R_0H = 0.5 & 2.0

$R^2/R_0H = 0.5$				$R^2/R_0H = 2.0$			
Linear temperature field		Parabolic temperature field		Linear temperature field		Parabolic temperature field	
R_H	ω_{NL}/ω_L	R_H	ω_{NL}/ω_L	R_H	ω_{NL}/ω_L	R_H	ω_{NL}/ω_L
30.00	1.095706	30.00	1.102511	30.00	0.967434	30.00	0.951558
35.00	1.100506	35.00	1.111028	35.00	0.956508	35.00	0.927751
40.00	1.106674	40.00	1.122771	40.00	0.940503	40.00	0.886504
45.00	1.114634	45.00	1.139416	45.00	0.916203	45.00	0.806367
50.00	1.125041	50.00	1.164075	50.00	0.877232	50.00	0.609740
55.00	1.138930	55.00	1.202920	55.00	0.809153	51.00	0.531056
60.00	1.158034	58.00	1.237453	60.00	0.670640	52.00	0.416598
65.00	1.185380	60.00	1.265918	61.00	0.624466	53.00	0.202306
70.00	1.226206	61.00	1.280399	62.00	0.566471	53.10	0.161457
72.00	1.247700	62.00	1.291799	63.00	0.490490	53.20	0.104327
74.00	1.271549	63.00	1.291465	64.00	0.382772	53.27	0.007618
76.00	1.291898	64.00	1.249678	65.00	0.188233		
77.00	1.293135	65.00	1.019700	65.10	0.152428		
78.00	1.273350	65.10	0.963015	65.28	0.104016		
79.00	1.189356	65.20	0.891399				
80.00	0.822415	65.30	0.798113				
80.10	0.729203	65.40	0.670116				
80.20	0.603608	65.50	0.472980				
80.30	0.412100	65.58	0.101991				
80.40	0.024657						