PREDICTION OF THE VIBRATORY BEHAVIOUR OF A MULTISTAGE GASEOUS DIFFUSION PLANT

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SUMMARY

A study has been made to predict the vibratory behaviour of the rotating machinery of a gaseous diffusion plant starting from the results obtained for a single machine.

TRICASTIN gaseous diffusion plant uses several hundred of enrichment stages but only three different sizes of machine are used. Each individual machine is a vertical assembly of a compressor heat exchanger and diffusion barriers, this column is supported on four lugs on a concrete slab. This slab must accomodate thermal expansion and is placed on neoprene pads.

Due to the compactness of the system the mass of concrete is relatively small. Typically the mass of a machine of the intermediate size is 84 T, the mass of associated concrete is 55 T. Furthermore this supporting slab is flexible, meaning that a dynamic analysis of the slab shows several frequencies below the compressor rotational speed.

Extensive dynamic tests have been conducted on a machine supported on a rigid foundation. These tests have shown that the main source of mechanical excitation was caused at 50 Hz by the unbalance of the electrical motor rotor. Then the problem remained to predict the behaviour of a group of twenty machines in the plant itself.

The study has been carried out in three steps, using a finite element procedure.

First, analysis of a single stage and comparaison with experiments.

Second, analysis of the concrete slab using a simplified model for the machines.

Third, synthesis of the two calculations to determine the mutual influence of the various machines.

Displacements has been measured on the concrete slab during start up tests, the results are in sactifactory agreement with the values calculated using the quadratic sum rule.

Combining the results of preliminary tests and of a multistep calculational approach, a method has been divised to correctly predict the vibratory behaviour of the TRICASTIN gaseous diffusion plant.

1. INTRODUCTION

TRICASTIN gaseous diffusion plant is built by EURODIF in the south of FRANCE. Construction started in 1974 and is due to be completed in 1981.

The plant uses several stages of enrichment and depletion stages with only three different machine sizes. Each machine is the assembly of a compressor, a heat-exchanger and diffusion barriers mounted vertically on four lugs, and is connected to the other stages by four pipes.

Machines are mounted by groups of 20 on independant concrete slabs placed on neoprene pads. Bellows are used in the piping to accomodate the thermal expansion.

Due to the compactness of the system, the concrete mass is relatively small and flexible, meaning that a dynamic analysis of the slab shows several frequencies below the compressor rotational speed.

Extensive dynamic tests have been conducted on machine supported on rigid foundation and have shown that the main source of mechanical excitation is the unbalance of electrical motor rotor.

In this paper we will expose the analytical method which has been devised to predict the behaviour of a group of twenty machines in the plant itself. Theoretical results will be compared to measurement made during start-up tests.

This study has been carried out by NOVATOME.

2. ANALYTICAL METHOD

The method is based on linear superposition of two calculations. In the first the machine is placed on a rigid foundation. The second is the calculation of the motion of the concrete slab under the action of the forces transmitted by the supporting lugs.

To simplify, damping is neglected in this study, therefore the response of a structure submitted to a harmonic loading is reduced to a simple static calculation.

The equation of motion of the complete system can be written under the form :

$$M \ddot{X} + K X = F$$

F is the vector of 20 independant set of elementary forces.

The solution can be derived from the superposition of the following two problems :

$$M \ddot{X}_E + K X_E = F - R$$

$$M \ddot{X}_A + K X_A = R$$

The vector R will be determined so that there is no motion of the slab in the first problem, then this problem is reduced to the study of a single machine on a rigid foundation and R is the force applied to base. The second problem will be solved in two steps :

In the first step, a simplified model of the column is used and the slab motion is calculated. In the second step, the effect of the coupling through the slab is obtained as the superposition on a given column of effect of this column itself and of the nineteen others

$$X = X_E + \sum_j X_{Aj} \qquad j = 1,20$$

displacements are harmonic functions of time :

$$X_{Aj} = \overline{X_{aj}} \quad \exp\,(j\,w\,t + \varphi_j)$$

The phase angles φ_j of the different machines are not correlated. Finite element program SAP IV has been used for the analysis.

3. ANALYSIS OF A COLUMN

3.1. Mode shape and frequencies

The finite element model has 612 nodes and 3 573 degrees of freedom. Shell and beam elements are used. 26 eigenvalues have been found between 0 and 40 Hz, 6 between 40 Hz and 60 Hz. The following table gives the 12 eigenfrequencies closest to 50 Hz.

NB	Frequency	NB	Frequency	NB	Frequency
23	30,26	27	42,30	31	53,44
24	30,52	28	44,24	32	55,97
25	34,59	29	51,51	33	64,02
26	39,91	30	53,41	34	75,04

Figure 1 gives the mode shape of eigenvalues 25 and 29.

3.2. Response to rotor unbalance

A rotating force corresponding to a motor unbalance of 4 500 g. cm has been applied to the electrical motor center of gravity. This force is divided into two components :

Fo	cos w t	along OX	axis
Fo	sin w t	along OZ	axis

The equation of motion under the influence of the first component is :

$$m\ddot{q} + kq = Fo \quad \cos wt$$

The response is : $q = q_o \cos wt$ $\quad q_o = \left[k - mw^2\right]^{-1} F_o$

Damping being neglected, frequency w must not be close to a resonant frequency ω_N of the system.

Response of a damped mode is proportional to the factor

$$A = \frac{1}{\sqrt{\left(\frac{w^2}{w_N^2} - 1\right)^2 + \left(2\varepsilon \frac{w}{w_N}\right)^2}}$$

at resonance $A \simeq \frac{1}{2\varepsilon}$ and to have the same result with an undamped system one must choose $\frac{w - wN}{wN} = \varepsilon$

Structural damping measured during test is $2\varepsilon =$ 1,2 % around 50 Hz, then $w - w_N = 0{,}3$ Hz

Two quantities are calculated :

- Forces applied to the base reduced into a system of one vertical force F_y and two horizontal moments M_x and M_z.
- Stresses and displacements in the structure.

3.3. Response to the motion of the base

To build a simplified model of a column two calculations have been made at 49,5 and 47,5 Hz.

Horizontal forces are neglected and the motion is reduced to a vertical displacement δ_y and two rotations θ_x and θ_z.

Unit displacements are applied to the model through stiff boundary elements.

The results of the calculation are stresses, displacements and internal forces. From the calculation of the forces applied at the base, an equivalent one degree of freedom oscillator can be determined for each component of the input motion.

For example in the vertical direction, the force applied by this oscillator of mass m_y and stiffness $k_y = m_y \ w_y^2$ to the support moving with an amplitude unity at frequency w is :

$$F_y = \frac{my \quad w^2}{1 - \frac{w2}{w_y^2}}$$

then using the two values of w $\quad my = 21\,343$ kg $\quad wy = 33{,}42$ Hz between 47,5 and 49,5 Hz, this oscillator has the same mechanical impedance as the column itself.

4. CALCULATION OF THE SLAB MOTION

4.1. Finite element model

The slab has a logitudinal plane of symetry and it will be assumed that a satisfactory result can be obtained with a quarter size slab and 5 columns.

The model 628 nodes and 1 614 degrees of freedom. 213 eight nodes brick elements are used.

Two differents boundary conditions can be imposed along the axis OX and OZ, corresponding to a symetry or antisymetry of the full size structure deformation (see figure 2).

A modal analysis has been performed on the four models giving the following frequencies :

Mode Number	A	B	C	D
20	33,19		33,42	35,20
21	35,19	35,36	34,16	39,76
22	38,42	36,09	40,19	42,61
23	40,21	43,96	42,26	46,85
24	46,44	48,86	52,37	56,03
25	52,83	54,01	59,54	
26	54,54	58,08		

Figure 3 is a typical mode shape.

4.2. Response to base excitation

For each direction of the excitation, the forces applied to the base by one of the machines are equivalent to a set of three forces F_y, M_x, M_z

Then to summarize there are :

- 1 excitation frequency
- 4 boundary conditions
- 30 load cases, product of :
  - . 2 directions for the excitation
  - . 5 machine positions
  - . 3 forces

Calculation of the effect of one force on a quarter size model is equivalent to the effect of four forces on the full size structure (see figure 4). For example, the forces F_y in the quadrant q, α, β, γ corresponding to the models A, B, C, D, are given in the following table.

Quadrant	q	α	β	γ
Model A	+	+	+	+
B	+	+	-	-
C	+	-	-	+
D	+	-	+	-

Effect of a single force on the full size structure is obtained by superposition of the results given by four models.

Therefore using four different superposition, the displacements induced at a given point by a machine at any one of the twenty locations is obtained.

These vertical displacements are combined to give a vertical motion δ_y and two rotations θ_x and θ_z.

5. SUPERPOSITION OF THE EFFECT OF 20 MACHINES

5.1. Summation rule

For a column i a general expression for the rotating force is :

$$Fx = \overline{F} \cos (w t + \varphi i) \qquad \text{along OX}$$
$$Fz = \overline{F} \sin (w t + \varphi i) \qquad \text{along OZ}$$

w is practically the same for all the machines but the phase angles φ_i are not correlated. Under the action of component OX the response at any point k has the following expression :

$$R_{xk} = \sum_i \Psi_{ki} \cos (w t + \varphi i) \qquad i = 1,20$$

Then can be defined :

for the maximum effect an absolute sum $\overline{R_k} = \sum \left| \Psi_{ki} \right|$

for the average effect a quadratic sum $\tilde{R}_{xk} = \left[\sum \Psi_{ki}^2 \right]^{\frac{1}{2}}$

The same is done for the effect of component OZ.

Then the maximum effect resulting from the two directions of the excitation is :

$$R_k = \left[R_{xk}^2 + R_{zk}^2 \right]^{\frac{1}{2}}$$

5.2. Response of the columns

Displacements and stresses have been calculated in several points of the structure by superposition of the direct effect of the unbalance forces and the coupling through the concrete slab. In this last calculation, the worst combination for phase angle and the result of response calculation under unit displacement of the column base. The two directions of forces are added separately and then summed up quadratically.

The following table gives maximum displacements along axis OX and OZ (peak value in micron) at the top of the diffuser or at the electrical motor level of a column at location 1, 3 or 5 (see figure 2).

Location		Fixed base	Machine on slab location 1	3	5
Top	δx	19,36	19,60	19,44	19,46
	δz	17,98	18,14	18,23	18,21
Base	δx	27,34	32,47	29,61	29,81
	δz	27,64	29,37	29,27	29,15

5.3. Base slab motion

The two summation rules : absolute sum and quadratic sum have been used giving the following results at the concrete slab edge (peak value in micron) (see figure 3).

		Maximum displacement			Average displacement		
		X	Z	X+Z	X	Z	X+Z
Point	1	3,8	9,8	10,5	1,1	2,7	2,9
	57	28,6	13,4	31,5	7,8	4,0	8,8
	167	26,1	11,5	28,5	7,2	3,7	8,1
	277	24,6	10,3	26,7	6,9	3,4	7,7
	387	24,1	10,2	26,2	6,8	3,6	7,7
	497	21,3	8,3	22,9	6,7	2,7	7,3
	613	47,8	16,5	50,5	13,5	5,2	14,5

6. EXPERIMENTAL RESULTS

Two series of tests have been made, before start-up on columns with electrodynamic shakers and during start-up on a group of twenty machines.

6.1. Test on columns

Rotating forces have been simulated with shakers allowing to vary the rotating speed between 10 and 60 Hz. Testing columns with three different piping layout only small differences have been noticed.

The following table gives the most significant resonances measured and calculated in Hz.

TEST	CALCULATION	
36 - 37,3 40,2 - 41,9	34,59 39,91	Flexion - 2 nodes
45,8	42,3 44,24	
53,1 56 - 58	51,51 53,41 - 53,44 55,97	Flexion - 3 nodes

At the nominal speed of 49,5 Hz and a rotor unbalance of 1 000 g. cm the following displacements (micron peak to peak) have been measured and calculated.

POSITION	TEST	CALCULATION
Top of column	1,2 - 2,6	8,6
Diffuseur shell	0,5 - 1,4	2,45
Heat eachanger	3,5	5,07
Motor	7,3	12,15

Only a small coupling effect have been measured during tests, for the above amplitude of 7,3 on the motor, a value of 0,4 has been measured on the rotor of the next machine.

6.2. Test on a group of 20 machines

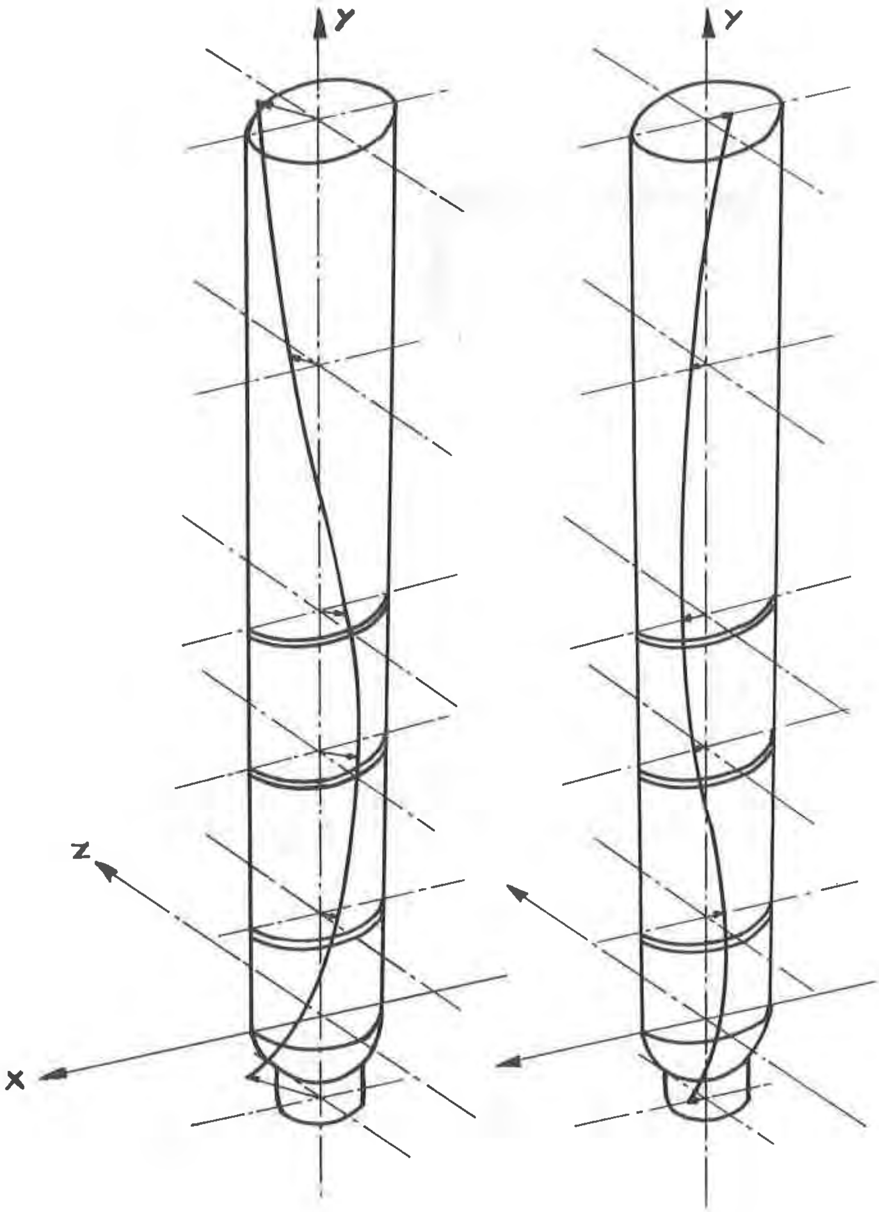
A frequency analysis of accelerometers located on the concrete slab gave frequencies which can be compared to calculated values.
In the same way vibration amplitude (o-peak in micron) has been compared in five points of the slab with calculated average values.

FREQUENCIES (HZ)	
Test	Calculation
42,5	42,26 43,96
47,0	46,44 46,85
48,1	48,86
50,6	52,37
51,9 53,0	52,83 54,01

AMPLITUDE		
Point	Test	Calculation
627	1,4	9,4
617	2,9	8,2
613	3,6	14,5
497	1,4	7,3
387	1,4	7,6

7 - CONCLUSION

Calculation has correctly predicted that the column behaviour was not affected by the supporting slab.



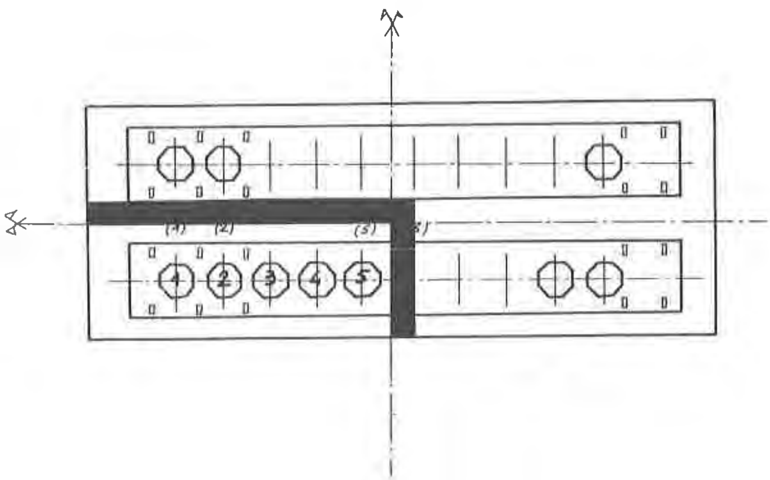
Mode n° 25

f = 34,59 Hertz

Mode n° 29

f = 51,51 Hertz

FIGURE 1 Mode shape of the column



: FIGURE 2 Geometry of the concrete slab

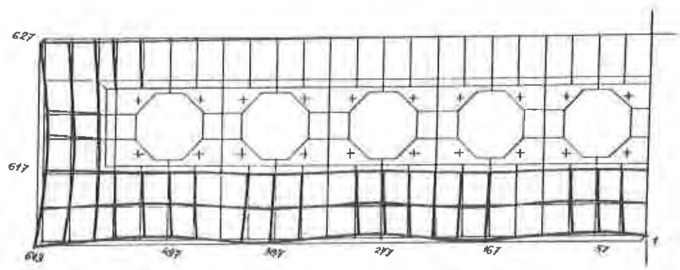


FIGURE 3 Mode shape of the slab

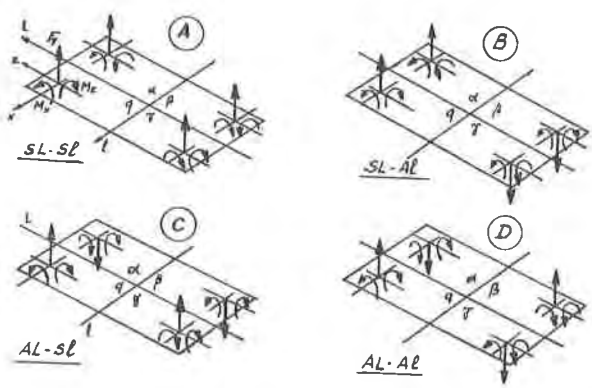


FIGURE 4 Effect of boundary condition : loading of the total slab