Transactions **of the SMiRT 27**
Yokohama, Japan, March 3-8, 2024
Division V

EMPIRICAL FORMULAS OF NEW EQUIVALENT DAMPING FOR VARIOUS NONLINEAR CONSTITUTIVE LAWS AS USED IN ATC40 PROCEDURE

Thuong Anh Nguyen[1,*], Pierre Labbé[3], Guillaume Hervé-Secourgeon[2]

[1] Engineer-Researcher, EDF Lab, Palaiseau, France
[2] Senior Scientist, EDF Lab, Palaiseau, France
[3] Professor, Ecole Speciale des Travaux Publics, Cachan, France

ABSTRACT

The Direct Displacement-Based Design (DDBD) and Capacity Spectrum Method (CSM) enable earthquake engineers to design structures within acceptable deformation beyond elastic regime. These approaches are based on equivalent linear characteristics of nonlinear structure, usually linked to the secant stiffness and the dissipated hysteretic energy. However, the discrepancy of predicted nonlinear responses using different equivalent linear concepts remains non-negligeable with respect to the non-linear responses as reference. Two issues of these equivalent linear concepts are identified: (1) the equivalent linear criteria are not exclusively linked to maximal displacement even equivalent linear structure is supposed to give the same displacement of nonlinear one (e.g., it is usually linked also to the quantity of dissipated energy) and (2) the dynamic content of nonlinear system, expressed by means of f_0/f_c where f_c is the central frequency of the input signal and f_0 is elastic frequency of structures, is not considered even that many studies have shown the crucial importance of this parameter. In this paper, a new equivalent linear concept is investigated, in which the maximal displacement is exclusively the equivalent linearization criterion, and the influence of f_0/f_c is analysed. The conventional secant stiffness is adopted but the new proposed damping values are derived from systematic analysis of responses of a series of nonlinear single degree of freedom (SDOF) oscillators to seismic type input motions. Different nonlinear behaviors including elastoplastic and damage ones are assigned to oscillators with a hardening slope (α) equal to 10% or 20% of the initial stiffness. The dimensionless parameter f_0/f_c varies between 0.1 to 2.0 and the damping in elastic regime remains at 5%. For deriving new equivalent damping values, seismic type input motion is a set of one thousand synthetic filtered white noise signals. As results, new equivalent damping values exhibit strong dependence on the f_0/f_c ratio, even more than hardening parameter. In addition to that, this equivalent behavior for elastoplastic structures is considerably different from damage ones. Then, empiric formulas of equivalent damping coefficients are carried out for all configurations, then, such equivalent systems are expressed as function of f_0/f_c, α and ductility demand μ. In order to assets the robustness of the new equivalent damping values, predicted responses are compared to reference ones determined by transient nonlinear analyses for various oscillators undergoing to one hundred recorded signals. The small discrepancy of the proposed equivalent linear concept in comparison with reference solution shows the robustness of proposed approach.

Keywords: *Equivalent linear concept, dimensionless frequency dependence, Secant stiffness, Maximum displacement, different nonlinear oscillators.*

INTRODUCTION

In a large extent, when designing structures, the favourite approach of engineers is the force-based approach, which calculates the response based on elasticity assumption, takes non-linear responses into account through behavior factors, and uses force-based or stress-based acceptance criteria. Face to the increasing seismic hazard that is currently considered in nuclear power plant seismic safety revaluation, this approach results frequently in internal forces or stresses that either are not sustainable by the structure or require substantial reinforcements.

To overcome this issue, it is necessary that the post-elastic response of structures is accounted for. However, some decades ago, running non-linear time-history response analyses was very demanding in computer resource, and nowadays it is still very demanding in engineering resources, so that simplified cost-effective methods are necessary. In addition to that difficulty, the definition of nonlinear material in standard context is not well established because of variance in material data set. These are reasons why, based on the analysis of non-linear Single Degree Of Freedom (SDOF) systems, many attempts were made to introduce linear equivalent models that could conveniently replace non-linear ones for earthquake engineering purpose.

A pioneering example is the equivalent linear concept introduced by Iwan (1980), reported in Formula (1) where T_0 and ξ_0 are the period and damping of a SDOF system in elastic regime, while T_{eq} and ξ_{eq} are the equivalent period and damping associated to a given ductility demand μ. This formula was established based on different nonlinear behaviors including elastoplastic, degradation and damage (Iwan, 1974). Nowadays, trend is not to encapsulate so different non-linear behaviors in a single model. For instance, examining the seismic response of reinforced concrete shear walls, (Brun, et al., 2003) show that the stiffness degradation model is suitable, while interpreting the experimental results of the piping systems test campaign of EPRI (1994) (Labbé, 2013) recommends linearization based on elastic-plastic relationship.

$$T_{eq} = T_0(1 + 0.121(\mu - 1)^{0.939}) \quad \text{(1-a)}$$

$$\xi_{eq} = \xi_0 + 0.0587(\mu - 1)^{0.371} \quad \text{(1-b)}$$

The displacement-based design (DBD) concept was introduced in the 90's. As opposed to force-based, this approach requires a description of the post-elastic structural behavior and uses strain (rotation, inter-stories drift…) based acceptance criteria. An example of an early paper mentioning the DBD approach is (Moehle, 1992).

In this spirit, the direct displacement-based design (DDBD) was introduced by (Kowalsky, et al., 1995) for applications to bridge piles. In the same decade, the ATC-40 approach was developed (ATC, 1996), and some improved procedures of it were presented by Chopra & Goel (1999). This concept was then investigated for other types of structure such as hybrid prestressed concrete bridges, reinforced concrete frame, steel frame, elastoplastic frame in (Priestley, 2000) and Priestley & Grant (2005). The influence of higher modes is also examined in (Priestley, 2003) for cantilever structures.

The DDBD approach and its developments are based on the concept of linearization applied to SDOF systems. A non-linear SDOF is replaced by an equivalent one, characterized by an equivalent stiffness, namely the secant stiffness at maximum displacement, and an equivalent viscous damping, representative of the combined elastic damping and the hysteretic energy dissipated during nonlinear response. An example of this approach is summarized by formula (2), introduced by Chopra & Goel (2001). This formula was established for an elastic-plastic SDOF system with a bilinear tensile curve, α being the ratio of the hardening slope to the elasticity one. (Other notations are same as in Formula (1)).

$$T_{eq} = T_0 \sqrt{\frac{\mu}{1 + \alpha\mu - \alpha}} \tag{2-a}$$

$$\xi_{eq} = \xi_0 + \frac{2}{\pi} \frac{(\mu - 1)(1 - \alpha)}{\mu(1 + \alpha\mu - \alpha)} \tag{2-b}$$

At this stage, a note should be made that the frequency content of the input motion, or the flexibility/stiffness of the structure, do not play a role in the above introduced approaches. However, in a series of publications (Miranda, et al., 2002), (Miranda, et al., 2002) and (Akkar, et al., 2005)about the performance of different linear equivalent concepts, authors mention that their discrepancy depends not only on the amplitude of seismic excitation but on the natural period of structures. Concurrently, in the 90's, a focus was made by the SAFE research program on the frequency dependence of shear walls seismic margins (Labbé & al., 2016). Similarly, when eliciting the respective damaging capacity of near-field versus far-field input motions, Labbé & Altinyollar (2011) pointed out that the frequency content of the input motion plays a crucial role in the post elastic response of structures.

Recently, Labbé & Nguyen (2019) introduced the non-dimensional parameter $\varphi = f_0/f_c$ where f_0 is elastic frequency of the SDOF system and f_c the central frequency of the input motion. They provided evidence that nonlinear responses depend considerably on f_0/f_c regardless the type of constitutive relationship.

In this study, the secant stiffness is adopted, as equivalent stiffness. New equivalent damping curves, matched from systematic analyses of equivalent linearization, are proposed. Various constitutive relationships, ranging from damaged (stiffness degradation) to elastoplastic, controlled by a hybridity factor, χ, are considered. The frequency dependence of the equivalent damping, ξ_{eq}, is examined versus the non-dimensional frequency ratio, $\varphi = f_0/f_c$, which varies from 0.1 to 2.0. Formulas that establish ξ_{eq} as function of φ are obtained through a systematic analysis of the response of a series of non-linear SDOF systems to 1000 samples of stochastic process. These formulas are then validated against responses to 100 natural accelerograms.

Predictive performances using new equivalent damping values are compared to damping formula reported in Formulas (1) and (2), as proposed by Iwan (1980) and by Chopra & Goel (2001).

NEW EQUIVALENT CONCEPT SSMD

New concept of SSMD

The equivalent linearization criteria that we now introduce is exclusively based on maximal displacement, and the new concept is called by Secant Stiffness Maximum Displacement (SSMD) concept. In this SSMD concept, we adopt this definition of equivalent stiffness. Consequently, equivalent frequency is expressed by Formula (3), which is equivalent to Formula (2-a), where f_0 is the natural frequency of the oscillator.

$$\frac{f_{eq}}{f_0} = \sqrt{\frac{1 + \alpha\mu - \alpha}{\mu}} \tag{3}$$

The SSMD equivalent damping, ξ_{eq}, is derived from a systematic numerical investigation of the responses of a series of nonlinear oscillators, including different nonlinear behaviors and dimensionless frequency f_0/f_c, to a set of 1000 synthetic seismic type input signals. The determined equivalent damping

values are such that the maximum responses of equivalent oscillators are the same as the maximum responses of nonlinear oscillators.

The novelty of this numerical investigation is that, in addition to the dependence on α, we systematically analyze the dependence of ξ_{eq} on the dimensionless frequency of the oscillator, introduced as $\varphi = \frac{f_0}{f_c}$.

ESTABLISHMENT OF SSMD EQUIVALENT DAMPING

Synthetic input signals

As used in (Nguyen, 2017), one thousand samples of a filtered white noise stochastic process are generated, designated by $\{g_i(t), i = 1 \ldots 1000\}$.

As mentioned, it is important to describe the frequency content of the generated input signals by means of their preponderant frequencies (other called central frequency) f_c. The central frequencies are determined by using the (Rice, 1944).

Hybrid model of nonlinear oscillator SDOF

To consider both plasticity and damage in the structure, we introduce a Hybrid model that consists in an assemblage in parallel of the two models described previously, as schemed in Figure 1, with respective weights χ for the elastoplastic model and $1-\chi$ for the damage model. In addition, we assign the same elastic stiffness (k_e), post-elastic stiffness (k_p) and yielding displacement (X_y) values to both models. It implies that the elastic response of the Hybrid model, and consequently the natural frequency of an oscillator equipped with such a Hybrid model, does not depend on χ. The Hybrid mode is identical to elastoplastic one with $\chi = 1$ and damage one with $\chi = 0$.

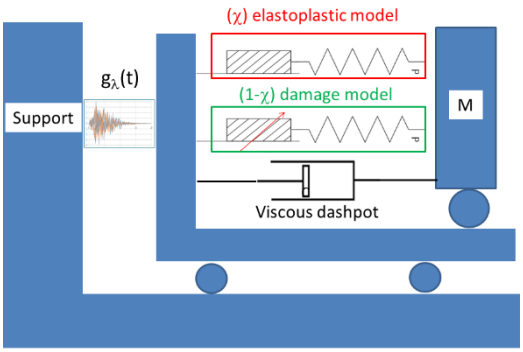


Figure 1: Hybrid SDOF oscillator

Beyond the elastic regime, the force exerted by the Hybrid model reads:

$$f(t) = \chi f_{EP}(t) + (1-\chi) f_{DM}(t) \tag{4}$$

where $f_{EP}(t)$ and $f_{DM}(t)$ are calculated as a function of the state variables of the elastoplastic model and of the damage model.

We designate as a Hybrid oscillator an oscillator equipped with such a Hybrid model. Its dynamic equilibrium equation reads:

$$\ddot{x}(t) + 2\xi_0\omega_0\dot{x}(t) + \frac{f(t)}{m} = -g(t) \quad (5)$$

where f(t) is calculated according to Equation (4).

Systematic numerical analyses of nonlinear responses to obtain equivalent damping ratio SSDM

In the performed numerical experiments, we first calculate the central frequency $f_{c,i}$ of every input signal $g_i(t)$, and for every of them, nonlinear oscillators are considered such that:

- $f_{0,i,j} = \varphi_j f_{c,i}$ with φ_j, where $j = 1\ldots5$, takes values 0.1; 0.5; 1.0; 1.5; 2.0;
- ξ_0 is fixed to 5%;
- α_p, where $p = 1$ and 2, takes values 10% and 20%;
- χ_n take 0.0, 0.5 and 1.0 where $n = 1, 2, 3$

There are in total 30 configurations of $[\varphi_j; \alpha_p; \chi_n]$ values. Every configuration encompasses 1000 oscillators because the natural frequency $f_{0,i,j}$ of every of them is equal to φ_j multiplied by central frequency $f_{c,i}$ of $g_i(t)$, i=1…1000.

From here, the procedure of identification of equivalent damping for each configuration is described in 5 steps, and these indices j, n, p are dropped for simplification.

Step 1: Elastic responses

For every oscillator, we first calculate its response in elasticity, $x_{e,i}(t)$, to the input signal $g_i(t)$. Then we assign its maximum response to the yield displacement:

$$X_{y,i} = \max\left|x_{e,i}(t)\right| \ , \ i = 1....1000 \quad (6)$$

Step 2: Amplitude of input signals

We determine the maximal level of input signal, noted by λ_{max}, such that the mean value of ductility demands of nonlinear oscillators submitted to 1000 signals $g_{i,max}(t) = \lambda_{max}\, g_i(t), i = 1 \ldots 1000$ is equal to 20. Once λ_{max} is established, ten intermediate values of λ_m where $m = 1 \ldots 10$ are considered, evenly distributed between 1 and λ_{max}. By definition, λ_1=1 and $\lambda_{10} = \lambda_{max}$. Then, the amplified input signal is determined $g_{m,i}(t) = \lambda_m g_i(t), m = 1 \ldots 10$. It means that λ_m is a dimensionless level of seismic input motion, λ_1 equal to 1 corresponding to the level which just leads the oscillator to its elastic limit.

Step 3: Nonlinear responses

Then, for each amplified input $g_{m,i}(t)$, we calculate the response $x_{m,j}(t)$ of the nonlinear oscillator by using β - Newmark scheme, with β= ¼ et γ= ½. Eventually, for every configuration, we get 10,000 time-responses by combination of 10 input levels and 1000 input signals: $\{x_{m,i}(t), m$=1… 10 and i = 1…1000$\}$.

For every configuration noted by $[\varphi_j; \alpha_p; \chi_n]$, the corresponding 10,000 ductility demand values are calculated by:

$$\mu_{m,i} = \frac{\max\left|x_{m,i}(t)\right|}{X_{y,i}} \quad (7)$$

where, m =1…10 and i = 1…1000.

Then, for each of λ_m value we determine the associated mean value of ductility demand $\mu_m = \text{mean}\,(\mu_{m,i},\ i = 1 \ldots 1000)$. In total, we obtain 10 μ_m versus 10 λ_m values for every configuration of $[\varphi_j; \alpha_p; \chi_n]$.

Step 4: Raw curves of SSMD equivalent damping

For each input level λ_m, an equivalent damping value $\xi_{eq,m}$ is calibrated. First, we determine mean value of the yielding displacement $X_y = sd(f_0, \xi_0)$, where sd is the mean response spectrum in relative displacement of the 1000 $g_i(t)$, natural frequency $f_0 = \varphi f_c$, f_c being the mean central frequency of 1000 $g_i(t)$ and damping in elastic regime $\xi_0 = 5\%$.

Finally, for every of the 10 μ_m values, the equivalent frequency $f_{eq,m}$ is calculated with formula (3). Then, the equivalent damping $\xi_{eq,m}$ is calibrated such that:

$$\lambda_m sd\left(f_{eq,m}, \xi_{eq,m}\right) = X_y\, \mu_m \tag{8}$$

We obtain 10 $\xi_{eq,m}$ values for every configuration of $[\varphi_j; \alpha_p; \chi_n]$.

Empiric SSMD equivalent damping curves

Empiric SSMD equivalent damping curves are displayed as function of $\xi_{eq,m}$ *vs* μ_m in Figure 2 for 10 configurations $[\varphi_j; \chi_n]$. with $\alpha = 10\%$. Other configurations are presented in the appendix 2.

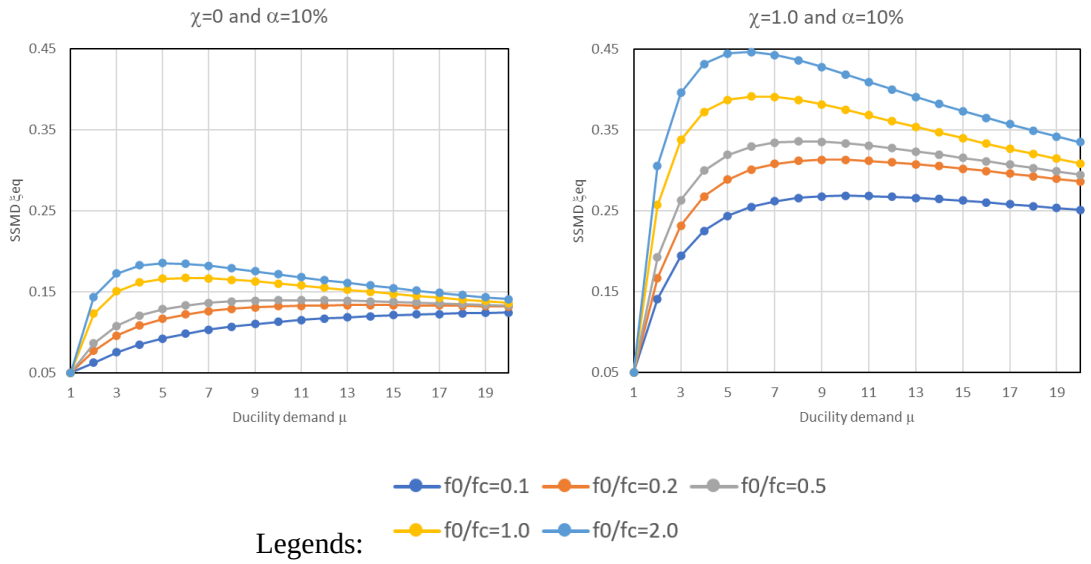


Figure 2: SSMD equivalent damping for the 5 considered configurations with α= 10 % for (a) $\chi = 0$ and (b) $\chi = 1$

According to empiric curves of SSMD equivalent damping, three dependencies are observed:

- The strong dependency of SSMD ξ_{eq} on the dimensionless frequency $\varphi = \frac{f_0}{f_c}$. For example, for $\mu = 5$, SSMD ξ_{eq} varies from 25% for φ =0.1 to 45% for $\varphi = 2.0$. The bigger φ, the bigger ξ_{eq}.

- The SSMD ξ_{eq} depend on type of nonlinear behavior (different values of χ). The ξ_{eq} for elastoplastic is bigger than damage one. This trendy is also observed in the recommended curves of (Priestley, et al., 2007).

- Dependence on hardening parameter α is more significantly observed for elastoplastic behavior than damage one. The bigger α, the lower ξ_{eq}.

SSMD ROBUTNESS

Investigation of nonlinear analyses for reference results

The Resorce database gathers seismic ground motion signals in European, Middle-Eastern, and neighboring countries (Akkar et al. 2013). To the present paper a set of 100 accelerograms was extracted from the database, designated as $\{\mathrm{G_k(t)}, k = 1 \ldots 100\}$. Details of these signals is presented in (Nguyen, et al., 2021).

Nonlinear responses with real signals

For each configuration $[\varphi_j; \alpha_p; \chi_n]$ of nonlinear oscillators, we determine the ductility demand μ as functions of amplification factor λ as reference results as following (index j, p, n are dropped for simplification).

- For each real signal $G_k(\mathrm{t}), \mathrm{k} = 1 \ldots 100$, the central frequency $f_{c,k}$ is calculated.
- Elastic frequency of nonlinear oscillator $\mathrm{f_{0,k}}$ is equal to $\varphi \mathrm{f_{c,k}}$.
- Yielding displacement $X_{y,k}$ of nonlinear oscillator is equal to maximal displacement of elastic oscillator of $[\mathrm{f_{0,k}}, \xi_0 = 5\%]$ undergoing the real signal $G_k(\mathrm{t})$.
- We increase the level of the input signal by applying on it a factor λ_m, such that it becomes $G_{m,k}(\mathrm{t}) = \lambda_m G_k(\mathrm{t})$. Ten values of λ_m between 1 and $\lambda_{\max}$ are used. The $\lambda_{\max}$ value correspond to the case with mean value over 100 input signals of ductility demand equal to 20.
- For each input motion, $G_{\mathrm{m},k}(\mathrm{t})$, we calculate the response $\mathrm{x_{m,k}(t)}$ of the nonlinear oscillator under consideration (β - Newmark scheme, with $\beta = ¼$ et $\gamma = ½$).
- Eventually, we get 1000 time-responses $\mathrm{x_{m,k}(t)}$ where $m = 1 \ldots 10$ and $k = 1 \ldots 100$. The ducility demand is obtained by $\mu_{m,n} = \frac{max|x_{m,k}(t)|}{X_{y,k}}$ where $m = 1 \ldots 10$, $k = 1 \ldots 100$. Then, mean value is calculated as reference of ductility: $\mu_m = \mathrm{mean}(\mu_{m,k})$.

Finally, for each configuration $[\varphi_j; \alpha_p; \chi_n]$, we obtain 10 values of μ_m as function of λ_m as reference results with real signals.

Robustness of SSMD equivalent damping empiric curves

To assess the robustness of SSMD equivalent damping, we calculate now predictive values of ductility demand for each value of λ_m of every configuration $[\varphi_j; \alpha_p; \chi_n]$.

The seismic load is the response spectrum $(\lambda_m SD)$ where SD being mean response spectrum of the 100 real signals $G_k(\mathrm{t})$ with $k = 1 \ldots 100$. Natural frequency of nonlinear oscillator is $\mathrm{f_0} = \varphi \cdot f_c$, with f_c = 6.60 Hz (mean value of central frequencies of the 100 reals signals). The yielding displacement $X_{y,m}$ of nonlinear oscillator is equal to $SD(\mathrm{f_0}; \xi_0 = 5\%)$. The classical iterative method is used to find predictive ductility demand of nonlinear oscillator of $[\mathrm{f_0}, \xi_0 = 5\%; X_{y,m}]$. And, three options of equivalent linear concept are tested: SSMD, Chopra and Goel (2001), Iwan (1980). The respective resulting predictive ductility demands are denoted: $\mu_{m,\mathrm{pre,SSMD}}$, $\mu_{m,\mathrm{pre,Chopra\_Goel}}$, $\mu_{m,\mathrm{pre,Iwan}}$. Eventually, for every configuration $[\varphi_j; \alpha_p; \chi_n]$ and for each equivalent linear concept, we obtain ten $(\lambda_m, \mu_{\mathrm{m,pre}})$ couples.

Eventually, the reference μ_m and predictive $\mu_{m,pre}$ values are plotted versus λ_m, as presented in the next section. In addition, for each λ_m value, the error is calculated as indicated in Formula (9) and plotted versus μ_m.

$$\text{error}(\mu_m) = \frac{\mu_{m,\text{pre}} - \mu_m}{\mu_m} \tag{9}$$

For each configuration $[\varphi_j; \alpha_p; \chi_n]$, 10 values of λ_m are chosen, equally spaced from 1 to 95%λ_{max} where λ_{max} correspond to the case of reference result equal to 20. However, the predicted results with $\lambda_m = 1$ is excluded in the statistic calculation on robustness of different concepts because by definition $\lambda_m = 1$ corresponds to elastic case.

Finally, for each configuration $[\varphi_j; \alpha_p; \chi_n]$, we obtain 9 $(\mu_m, \text{error}(\mu_m))$ couples for each of equivalent linear concept.

Outputs of robustness of SSMD equivalent linear empiric curves

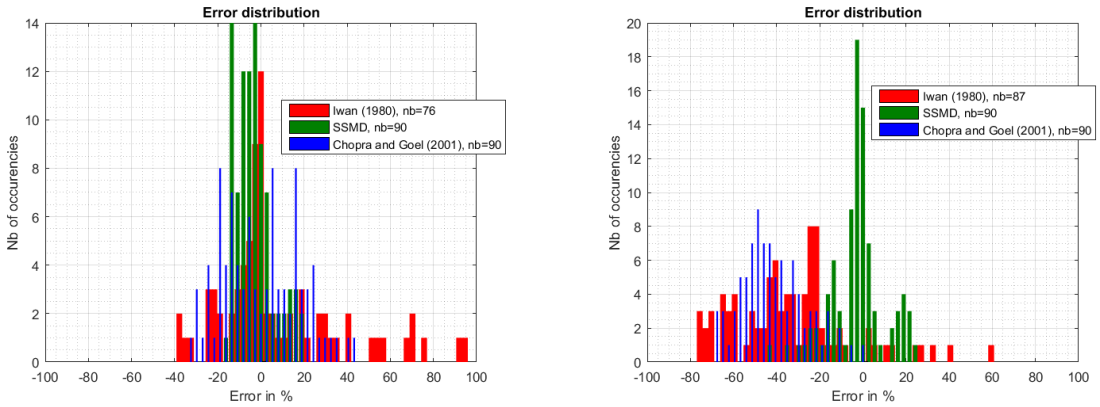
According to results of validation of new equivalent damping, some remarkable observations are highlighted as following:

- The SSMD predictive results are in very good agreement with the reference results for the range of oscillators. Unlikely to SSMD equivalent linear concept, the predictive using (Iwan, 1980) equivalent concept is exposed to non-convergence problem and has important discrepancy. Important discrepancy is also observed for the equivalent concept of Chopra and Goel (2001).

- The SSMD error increases with the level of input motion. It should however be observed that it remains very small (nearly null) for oscillators at "resonance" (φ=1) and for φ=1.5, regardless the level of input motion.

The error of SSMD is smaller than 5% for this configuration. This excellent coherence with reference result shows the robustness of this new equivalent concept.

Complementary to these observations, we can also note that the discrepancy of the results is much more important for Iwan (1980) and Chopra and Goel (2001) than for the SSMD concept. The results have been summarized in Figure 3 through a set of three histograms exhibiting the relative distribution of errors of the three concepts. data few errors larger than 100% are not reported.



(a) $\chi = 1$ (b) $\chi = 0$

Figure 3: Relative distribution of errors between predictive and reference results for the set of considered configuration for each concept. (a) for $\chi = 1$, (b) for $\chi = 0$.

"nb" is number of errors less than 100% (in absolute), with 90 maximum values corresponding to:

- 9 values of λ (λ=1 is excluded from the comparison)
- 2 values of α, respectively equal to 10% and 20%
- and 5 value of φ.

Only errors between -100% and 100% is considered in descriptive statistics of errors on predicted results. It is noted that predicted results by using linear equivalent concept by (Iwan, 1980) give errors more than 100% in absolute value (3 cases for $\chi = 0$ and 14 cases for $\chi = 1$). These cases are often linked to the non-convergence issue.

CONCLUSIONS

In this paper, a new concept of equivalent linear oscillator, designated as SSMD concept, is introduced, based on the exclusive equivalent linearization criterion: maximum displacement. The equivalent frequency is conventionally controlled by the secant stiffness linked to maximal displacement. The new equivalent damping is derived from a systematic numerical analysis of elastoplastic oscillators undergoing seismic type input motion.

To asset many nonlinear behaviors, the hybrid SDOF oscillator is introduced which is controlled by hybridity index χ. While $\chi = 1$, it is elastoplastic behavior and $\chi = 0$ it is damage one.

An analytical formula that fits the equivalent damping curves is identified for different nonlinear behaviors.

The SSMD novelty is that, in addition to the dependence on the hardening parameter α and nonlinear behaviors χ, the dependence of the equivalent damping, ξ_{eq}, on the oscillator dimensionless frequency, φ f_0/f_c. Eventually it appears that ξ_{eq} is much more sensitive to φ than it is to α.

The robustness of SSMD equivalent damping is asset by comparing predictive results to reference results for all considered configurations of oscillators undergoing 100 natural signals collected from the RESORCE database. The comparisons show a very good agreement of SSMD predictive results with respect to nonlinear time-response analysis reference results. In the vast majority of examined configurations, less satisfactory outputs are obtained when using other equivalent linear oscillator concepts.

In addition, the concept will be tested on multi degree of freedom real structures.

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