Stochastic response of WWER Core Barrel to seismic ground motion

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1 INTRODUCTION

Earthquake resisting design of nuclear power plants ranks among the important problems of nuclear safety. There are two possible methods of analysis: the deterministic approach and the stochastic approach. In the former method the time histories of the motion of structure floors are calculated by the numerous available analytical or numerical techniques, some adequate strong motion records being considered as the input earthquake ground motion.

In this study the stochastic response of thin cylindrical shell (core barrel - CB) to random earthquake support motion is considered. The formal general solution in time domain is at first derived. The statistically stationary model of earthground acceleration in the form of filtered white noise is briefly reviewed and the general expression for the mean-square response is derived.

2 EQUATION OF MOTION

The equation of motion of thin cylindrical shell is given in the form

$$\mathbf{A}\mathbf{u} = \mathbf{B}\mathbf{u} + \mathbf{f}_g + \mathbf{f}_i \qquad (1)$$

$\mathbf{A}$ ... matrix of Flügge differential operators

$\mathbf{B} = -\dfrac{\varrho_s R_s (1-\nu^2)}{Eh} \dfrac{\partial^2}{\partial t^2} \mathbf{E}$ $\qquad$ $\mathbf{E}$ ... unit matrix

$$\mathbf{f}_g = \left[0; 0; -\frac{\varrho_s R_s (1-\nu^2)}{Eh} \ddot{u}_g(t)\right]^T$$

$\ddot{u}_g(t)$ ... horizontal earthquake acceleration

$\mathbf{f}_i = [0; 0; p_w]^T \dfrac{1-\nu^2}{Eh} R_s$ $\qquad$ ... vector of fluid - CB interaction

p_w ... radial pressure generated by fluid - CB interaction

Using the method derived in /1/, we obtain the differential equation as follows

$$\ddot{q}_\alpha + 2D_\alpha \omega_\alpha \dot{q}_\alpha + \omega_\alpha^2 q_\alpha = -\frac{1}{1+\delta_\alpha} \frac{1}{\int_0^L F_m^2 dz} \int_0^L \int_0^{2\pi} \ddot{u}_g(t) F_m(z) \cos m\varphi \, d\varphi dz \tag{2}$$

$F_m(z)$ - beam function for given boundary condition

δ_α - added mass coefficient

The radial displacement is supposed as

$$w(z,\varphi,t) = q(t) F_m(z) \cos m\varphi \tag{3}$$

Because of only radial component of $\ddot{u}_g(t)$, e.g. $\ddot{u}_g(t) \cos\varphi$ takes the place in (2), it will be received as a result

$$\ddot{q}_\alpha + 2D_\alpha^* \omega_\alpha \dot{q}_\alpha + \omega_\alpha^2 q_\alpha = -\frac{\pi}{1+\delta_\alpha} \frac{\int_0^L F_m(z) dz}{\int_0^L F_m^2(z) dz} \ddot{u}_g(t) \tag{4}$$

$$\alpha = (1, m)$$

The general solution in the time domain has the following form

$$q_\alpha(t) = -\frac{\pi}{1+\delta_\alpha} \frac{1}{\omega_\alpha} \frac{\int_0^L F_m(z) dz}{\int_0^L F_m^2(z) dz} \int_0^t e^{-D_\alpha^* \omega_\alpha (t-\tau)} \times$$

$$\times \sin \omega_\alpha (t-\tau) \ddot{u}_g(\tau) \, d\tau \tag{5}$$

3 RESPONSE TO STATIONARY GROUND MOTION

If the ground motion $\ddot{u}_g(t)$ is given in the form of power spectral density $S_{\ddot{u}_g}$, then the mean-square of $q_{1,m}(t)$ is defined by

$$\langle q_{1,m}^2 \rangle = \int_0^{+\infty} S_q(\omega) \, d\omega \tag{6}$$

with

$$S_q(\omega) = \lim_{T \to \infty} \frac{1}{2T} q(\omega) \hat{q}(\omega) \tag{7}$$

$$\hat{q}(\omega) = \mathcal{F}\{q(t)\} = C \frac{\hat{\ddot{u}}_g(\omega)}{\omega_{1,m}^2 - \omega^2 + 2iD_{1,m}^* \omega_{1,m}\omega} \tag{8}$$

Substituting (8) and (7) into (6) we obtain after some calculations

$$\langle q_{1,m}^2 \rangle = \sum_{m=1}^{M} C_m \int_0^{+\infty} \frac{S_{\ddot{u}_g}(\omega)}{(\omega_{1,m}^2 - \omega^2)^2 + 4D_{1,m}^{*2}\omega_{1,m}^2\omega^2} d\omega \tag{9}$$

$$C_m = -\frac{\pi}{1+\delta_{1,m}} \frac{1}{\omega_{1,m}} \frac{\int_0^L F_m(z)\,dz}{\int_0^L F_m^2(z)\,dz}$$

If $S_{\ddot{u}_g}(\omega)$ is suggested by Zeman /2/ as

$$S_{\ddot{u}_g}(\omega) = \sum_{i=1}^{I} S_{og,i} \frac{\omega_{g,i}^2 + 4D_{g,i}^{*2}\omega^2}{(\omega_{g,i}^2 - \omega^2) + 4D_{g,i}^{*2}\omega_{g,i}^2\omega^2}$$

the general form of $\langle q_{1,m}^2 \rangle$ is given by

$$\langle q_{1,m}^2 \rangle = \sum_{i=1}^{I} C_{1,m} \int_{-\infty}^{+\infty} \frac{S_{og,i}}{(\omega_{1,m}^2 - \omega^2)^2 + 4D_{1,m}^{*2}\omega_{1,m}^2\omega^2} \; *$$

$$* \frac{\omega_{g,i}^2 + 4D_{g,i}^{*2}\omega^2}{(\omega_{g,i}^2 - \omega^2)^2 + 4D_{g,i}^{*2}\omega_{g,i}^2\omega^2} d\omega \tag{10}$$

The computation of $\int_{-\infty}^{+\infty} \dots d\omega$ will be carried out using the rezidue theorem. The zero points of denominator of (10) are

$$z_{1,2,3,4} = \pm\omega_{1,m}\sqrt{1-D_{1,m}^{*2}} \pm iD_{1,m}^*\omega_{1,m} \tag{11a}$$

$$z_{5,6,7,8} = \pm\omega_{g,i}\sqrt{1-D_{g,i}^2} \pm iD_{g,i}^*\omega_{g,i} \tag{11b}$$

Because of

$$\mathrm{rez}\, f(z_1) = \frac{\omega_{g,i}^2 + 4D_{g,i}^{*2}z_1^2}{\prod_{j=2}^{8}(z_1 - z_j)} \tag{12a}$$

$$rez\, f(z_2) = \frac{\omega_{g,i}^2 + 4D_{g,i}^{*2} z_2^2}{\prod_{\substack{j=1 \\ j\neq 5}}^{8} (z_2 - z_j)} \tag{12b}$$

$$rez\, f(z_6) = \frac{\omega_{g,i}^2 + 4D_{g,i}^{*2} z_6^2}{\prod_{j=1, j\neq 6}^{8} (z_6 - z_j)} \tag{12c}$$

$$rez\, f(z_5) = \frac{\omega_{g,i}^2 + 4D_{g,i}^{*2} z_5^2}{\prod_{j=1, j\neq 5}^{8} (z_5 - z_j)} \tag{12d}$$

the resulting formula of $\int_{-\infty}^{+\infty} \ldots d\omega$ takes the form of

$$\int_{-\infty}^{+\infty} f(z)\,dz = 2\pi i \left[rez\, f(z_1) + rez\, f(z_2) + \right.$$
$$\left. + rez\, f(z_5) + rez\, f(z_6) \right] \tag{13}$$

Substituting (11) and (12) into (13) and rearranging, we obtain the mean-square of $\langle q_{1,m}^2 \rangle$ as

$$\langle q_{1,m}^2 \rangle = -4\pi C^2 S_{og,i}\, \omega_{g,i}^2 \left[\frac{H_1 E_1 - F_1 G_1}{E_1^2 + F_1^2} + \frac{H_3 E_3 - F_3 G_3}{E_3^2 + F_3^2} \right] \tag{14}$$

with

$E_1 = A_1 C_1 - B_1 D_1$ $\qquad$ $F_1 = B_1 C_1 + A_1 D_1$

$A_1 = -8\omega_1^3 D_1^{*2}$ $\qquad$ $B_1 = 8\omega_1^3 D_1^*$

$$C_1 = (\omega_{g,i}^2 - \omega_1^2)^2 - 4\omega_1^2 (D_1^{*2}\omega_1^2 - D_{g,i}^{*2}\omega_1^2 -$$
$$- D_{g,i}^{*2}\, \omega_{g,i}^2)$$

$$D_1 = 4D_1^* \omega_1^2 (\omega_1^2 - \omega_{g,i}^2) + D_{g,i}^{*2} \omega_{g,i}^2$$

$$G_1 = \omega_{g,i}^2 + 4D_{gi}^* \omega_1^2$$

$$H_1 = 8 D_1^* D_{g,i}^* \omega_1^2$$

$$E_3 = A_3 C_3 - B_3 D_3 \qquad F_3 = B_3 C_3 + A_3 D_3$$

$$A_3 = - 8 D_{g,i}^{*2} \omega_{gi}^3 \qquad B_3 = 8 D_{gi}^* \omega_{gi}^3$$

$$C_3 = (\omega_1^2 - \omega_{gi}^2)^2 - 4\omega_{gi}^2 (D_{gi}^{*2} \omega_{gi}^2 - D_1^{*2} \omega_1^2)$$

$$D_3 = 4 D_{gi}^* \omega_{gi}^2 (\omega_{gi}^2 - \omega_1^2) + D_{gi}^{*2} \omega_{gi}^2$$

$$G_3 = \omega_{gi}^2 (1 + 4 D_{gi}^*)$$

$$H_3 = 8 D_{gi}^{*2} \omega_{gi}^2$$

CONCLUSIONS

In our calculations, the 3σ level of radial deflection, e.g.

$$3\left[\langle W_{1,m}^2 (z,\varphi)\rangle\right]^{0.5} = 3\left[\langle q_{1,m}^2\rangle F_m^2(z)\cos^2\varphi\right]^{0.5}$$

has been, for the given accelerogram, compared with deterministic approach $W_{1,m}(z,\varphi,t) = q_{1,m}(t)\, F_m(z) \cos\varphi$ obtained by direct integration of equation (4). It has been determined that the 3σ level is a conservative estimate of the maximum response. The $1{,}5\sigma$ level is more realistic. This fact is in a good agreement with /3/.

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