SELF-SUSTAINED VIBRATIONS OF RODS IN PARALLEL FLOW

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ABSTRACT

A correlation is reached between the maximum displacement in self substained vibration of a cylindrical rod of a bundle in parallel flow and the system's physical and geometrical parameters. Foreseen displacement values agree well with experimental data reported by several authors.

1. INTRODUCTION

A cylindrical straight rod belonging to a cluster is considered. In condition of parallel flow one may observe the appearing of self sustained vibrations with frequencies near the rod natural frequency and strongly dependent on fluid velocity amplitudes. In the present work the problem to predict the maximum value of displacement is considered from a theoretical point of view; a general correlation is reached and few parameters are fixed manipulating a great quantity of experimental data obtained essentially from reference [1]. Utilizing the correlation presented in this paper it is possible to foresee self substained vibration displacements of rods belonging to clusters taking into account different types of end fixity and the presence of a number of spacers or supports. To tackle the problem we start from the following hypothesis:

- The amount of energy necessary to produce the self substaining of vibration is subtracted from the energy released by the fluid because of friction.
- There is a viscous damping action on the vibrating rod because maximum displacements are contained in the boundary layer. (this supposition is confirmed by observation that rods tested by several authors show the same natural frequency either in stagnant or in moving fluid).

- The equation of motion, for each section of rod is:

$$y = y_o \varphi(z) \sin(\omega t) \tag{1}$$

where z is the value of section's abscissa normalized to rod lenght ($z = x/b$).

2. ANALYSIS OF PROBLEM

The amount of energy released by rod each 1/4 of cycle, under the above second hypothesis is:

$$E = K \int_o^b \int_o^{\pi/2\omega} \left(\frac{\partial y}{\partial t}\right)^2 dx\, dt \tag{2}$$

being K the viscous friction coefficient per unit lenght.

Energy, then, released by rod per unit time is:

$$P(\omega) = K \frac{2\omega}{\pi} \int_o^b \int_o^{\frac{\pi}{2\omega}} \left(\frac{\partial y}{\partial t}\right)^2 dx\, dt \tag{3}$$

We can assign to K the value:

$$K = C \eta \tag{4}$$

being η dinamic viscosity and C constant.

If we call α the fraction of the energy per unit time released by fluid because of friction which contributes to substain vibration, we reach the following equation:

$$(\alpha f) \frac{b}{D_{id}} \frac{\pi d D_{id}}{8} \gamma v^3 = C \eta b y_o^2 \omega^2 \int_o^1 \varphi^2(z)\, dz \tag{6}$$

- d = rod diameter
- f = friction factor
- D_{id} = hydraulic diameter
- γ = fluid density

It is easy to evaluate with a good accuracy the shape of rod bowing during its maximum deflection position normalized with respect to the maximum displacement.

In the simplest case of supported ends is:

$$\varphi(z) = \sin(\pi z) \tag{7}$$

$$\int_o^1 \varphi^2(z)\, dz = \frac{1}{2} \tag{8}$$

In case of clamped ends:

$$\varphi(z) = \frac{1}{2} \left[1 - \cos(2\pi z)\right] \tag{9}$$

$$- \int_0^1 \varphi^2(z)\, dz = \frac{3}{8} \qquad (10)$$

3. CONSIDERATIONS ON α FACTOR

The fully developed turbolent flow surrounding the rod is composed of a number of N vortexes each containing E_v energy. These vortexes produce the transmission of energy from fluid to rod. Because of great value of N we can suppose that statistically the amount of energy transmitted at a certain vibration frequency is constant at constant fluid speed and Reynolds number. Always depending on great value of N we can also consider that the number of vortexes intervening to produce the energy exchange at different frequencies is the same. Under these suppositions fluid is considered like a white noise source therefore the amount of energy absorbed by rods with identical geometry but different natural frequencies is the same. This supposition is confirmed analyzing some experimental data produced by Burgreen et al [1]. Fig. 1 shows the results of experiments on three geometrically identical rods in the same rod bundles, under conditions of self sustained vibration produced by parallel water flow at room temperature. Because of different materials used and different linear weights, the three rods vibrated respectively to about 10, 15 and 20 Hz in water. It is possible to observe, in the same figure, that the term $y_o f_o/v^{1.5}$ gives values very close each other at the same Reynolds number. The term $y_o f_o/v^{1.5}$ is a characteristic group obtained from eq. 6 and represents a value proportional to the square root of the amount of energy transmitted to the rod. y_o is maximum rod displacement, f_o is measured or calculated frequency of rod, v is fluid velocity. α factor contained in eq. 6 is proportional to the product $N \cdot E_v$ then we can express:

$$- \quad \alpha = \psi(Re) \cdot \phi(T) \qquad (11)$$

T = fluid temperature

Being friction factor f function of the Reynolds number alone we can write:

$$- \quad \frac{\alpha \cdot f}{32 \pi C} = \bar{\Psi}(Re) \cdot \phi(T) \qquad (12)$$

4. DISPLACEMENT DETERMINATION

The general expression of displacement is deduced from eq. 6 and eq. 12:

$$- \quad y_o = \left[\frac{d}{\int_0^1 \varphi^2(z)dz} \cdot \frac{\gamma}{\eta} \cdot \bar{\Psi}(Re)\, \phi(T) \right]^{0.5} \frac{v^{1.5}}{f_o} \qquad (13)$$

$\phi(T)$ function is proportional to the vortex energy. In Appendix I is treated the case of plane steady vortex and it is shown that:

$$- \quad E_v \equiv \phi(T) \equiv \frac{\eta}{\gamma} \qquad (14)$$

An empirical evaluation of data plotted in Fig. 1 allows to give the following expression to $\left[\bar{\Psi}(Re)\right]^{0.5}$ when fluid is liquid water:

$$- \quad \left[\bar{\Psi}(Re)\right]^{0.5} = 0.055 \exp\left[7,374\times10^{6}\ Re - 11.28\right] \qquad (15)$$

Eq. 15 has been obtained using C.G.S. unit and assuming $\phi(T)=1$ at room temperature. Under this assumption is:

$$- \quad \phi(T) = \frac{\eta \gamma_o}{\gamma \eta_o} \qquad (16)$$

In eq. 16 the subscript "o" means that values are calculated at room temperature. When fluid is liquid water, final expression for y_o, in C.G.S. units, is:

$$- \quad y_o = 0.55 \left[\frac{d}{\int_0^1 \varphi^2(z)\,dz} \right]^{0.5} \exp(7,374\times10^{-6} Re - 11.28)\ \frac{v^{1.5}}{f_o} \qquad (17)$$

In eq. 17 the only term affected by a temperature variation is Re. Eq. 17 agree with data of several authors except in the case of high temperatures. Data plotted in Fig. 5 and Fig. 9 show that amplitude of self substained vibration is pratically independent with respect to fluid temperature. It is necessary, then, to introduce in eq. 17 an empirical correction factor to take into account this fact. The most rapid way to do it is to use a reference Reynolds number evaluated at room (or reference) temperature.

$$- \quad Re_o = 100\ v\ D_{id} \qquad (18)$$

Proposed correlation is then:

$$- \quad y_o = \lambda(Re_o)\ \frac{v^{1.5}}{f_o} \left(\frac{d}{\int_0^1 \varphi^2(z)\,dz} \right)^{0.5} \qquad (19)$$

where:

$$- \quad \lambda(Re_o) = 0.55 \exp\left[(7.374\times10^{-6})Re_o - 11.28\right] \qquad (20)$$

5. COMPARISON WITH EXPERIMENTS AND CONCLUSIONS

In figures 2, 3 and 4 a comparison between foreseen values and Burgreen data is shown. [1] In Figure 5 there is a comparison between Pavlica and Marshall experimental data [3] on rods of a cluster with different number of support at different temperatures and the present correlation. It shows that the slope of the correlation presented is more actual with respect to experimental data than the one obtained using Paidoussis [4] and Burgreen correlation. In the same work [3] authors had to determine experimentally the bundle stiffness to introduce it in the Paidoussis and Burgreen correlation. Eq. 19 has been applied to the same case using only rod characteristics as diameter and calculated natural frequency and no experimental support was necessary. Figures 6 and 9 show Quinn [2] data for cluster and single rods in single phase flow. Experiments are carried out at different temperatures. One can note good agreement between experimental data and foreseen value only at fluid speeds over 150÷200 cm/sec. Figures 7 and 8 show single rod tests made by Wambganns and Boers [5] and Basile ed al. [6]. In all these single rod tests eq. 19 has been used assuming $D_{id} = d$. To sum up eq. 19 agrees well with experimental data and does not need experimental supports to be used. After all parameters have been carefully chosen, it is possible to foresee with good approximation maximum displacements in self substained vibration.

6. APPENDIX I: Vortex energy

Considering a coordinates system with origin in the centre of vortex, the force acting on an element $2\pi r\, dr$ and unit lenght is:

- $$dF = 2\pi(\tau dr + r d\tau) \tag{21}$$

being τ shear stress per unit lenght.

We have further:

- $$\tau = \eta \frac{dv}{dr} \tag{22}$$

The total power lost by vortex is:

- $$P = \int_0^{\infty} 2\pi \eta \left(\frac{dv}{dr} dr + r\frac{d^2v}{dr^2} dr\right) v \tag{23}$$

If vortex is steady it must catch the same amount of energy per unit time from fluid. Expressing the amount of energy absorbed in form of kinetic energy:

- $$\Delta E_c = \frac{\rho}{2} \int_0^{\infty} 2\pi r \gamma\, v^2\, dr \tag{24}$$

where β is logarithmic increase in kinetic energy per unit time:

$$- \quad \beta = \frac{\Delta Ec}{Ec} \tag{25}$$

eq (21) and eq (20) combined give:

$$- \quad \frac{d^2 v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - \frac{\beta \gamma}{2\eta} v = 0 \tag{26}$$

this is a modified Bessel equation the general solution of which is:

$$- \quad v = C_1 I_o \left(\sqrt{\frac{\beta \gamma}{2\eta}}\, r \right) + C_2 K_o \left(\sqrt{\frac{\beta \gamma}{2}}\, r \right) \tag{27}$$

C_1 and C_2 constants.

v must be 0 at $r = \infty$ then

$$- \quad v = C_2 K_o \left(\sqrt{\frac{\beta \gamma}{2\eta}}\, r \right) \tag{28}$$

Vortex dimension is about:

$$- \quad r_o \simeq \sqrt{\frac{2\eta}{\beta \gamma}} \tag{29}$$

Vortex energy per unit lenght is:

$$- \quad E_v = \frac{1}{2} \gamma \pi \frac{2\eta}{\beta \gamma} \cdot \bar{u}^2 \tag{30}$$

where $\bar{u}$ is mean velocity and it is always the same varying η. Then E_v is proportional to η.

The density of liquid water is near constant varying T than we can assume:

$$- \quad E_v = \frac{\eta}{\gamma} \tag{31}$$

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[6] BASILE D., FAURE J., OHLMER E., "Experimental study on the vibrations of various fuel rods models in parallel flow", Nucl. Eng. Des. 7 (1968)

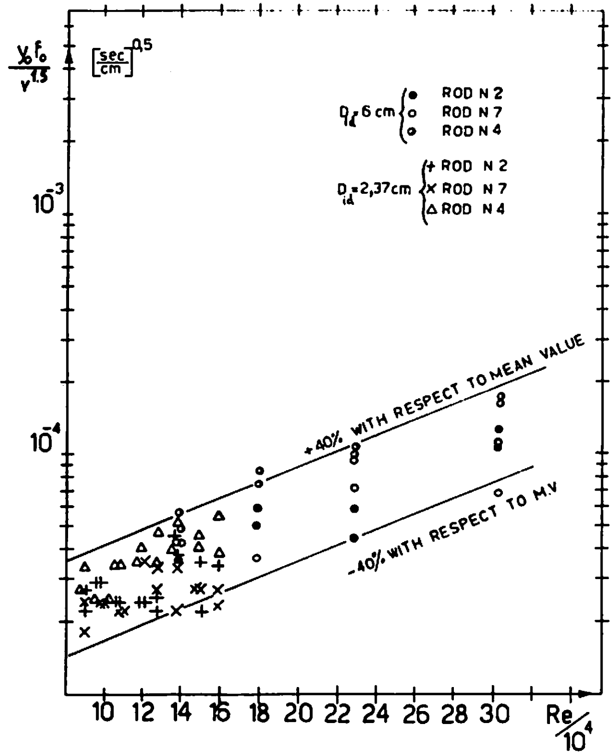

Fig. 1

Burgreen data for rods with different natural frequencies.

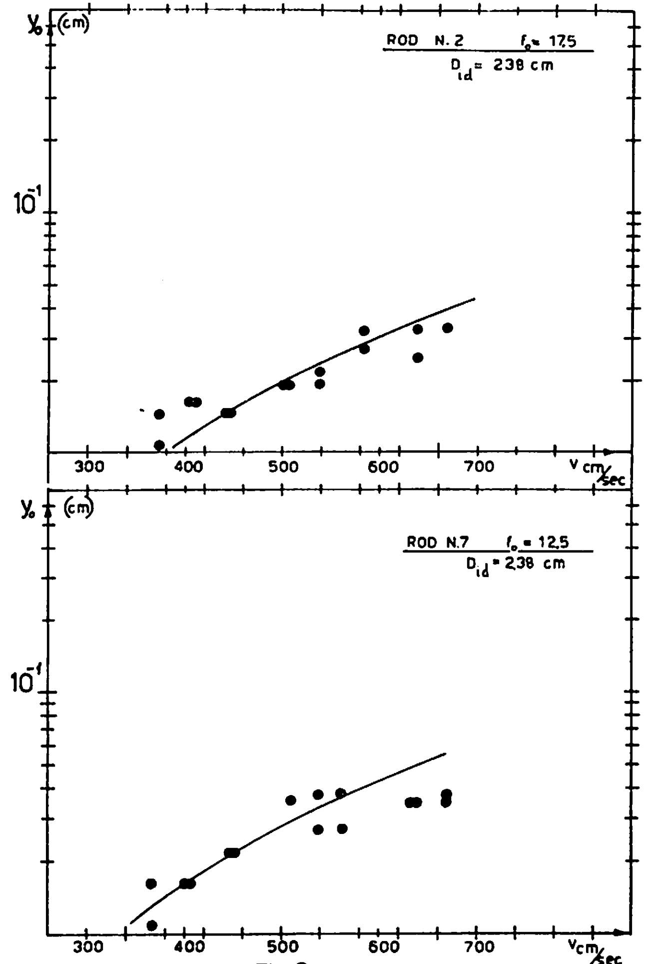

Fig. 2

Comparison between Burgreen results and present correlation.

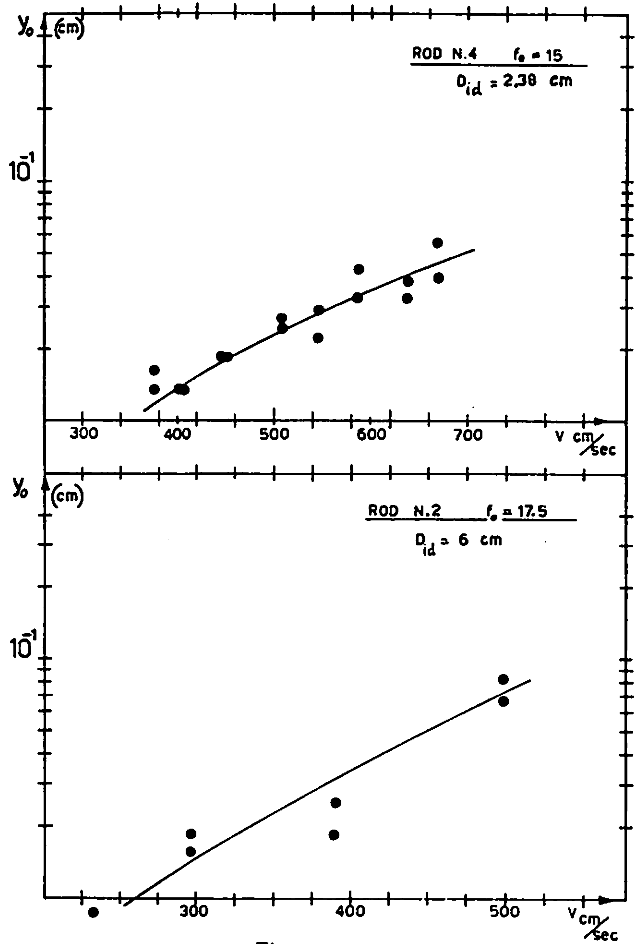

Fig. 3

Comparison between Burgreen results and present correlation.

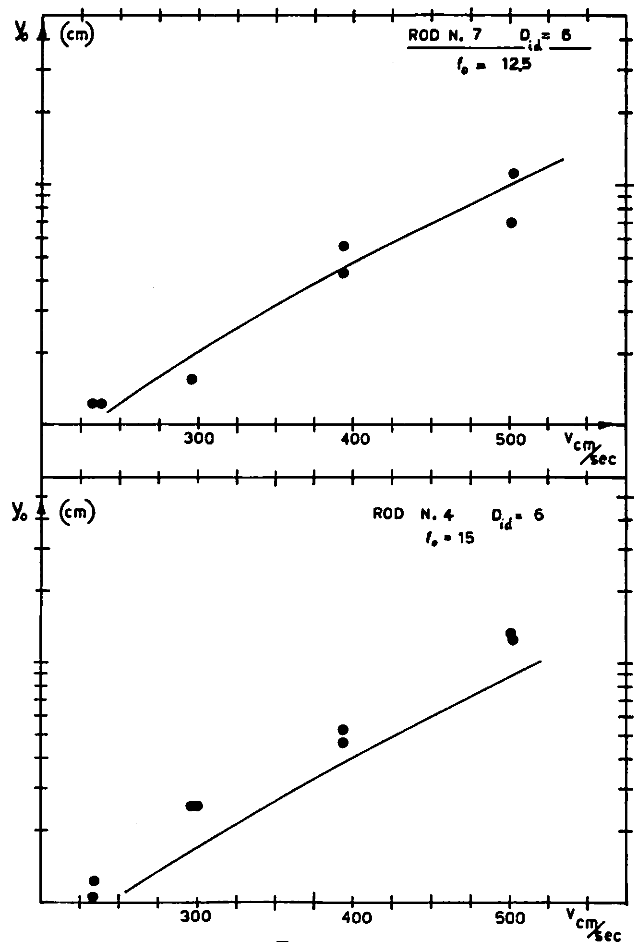

Fig. 4

Comparison between Burgreen results and present correlation.

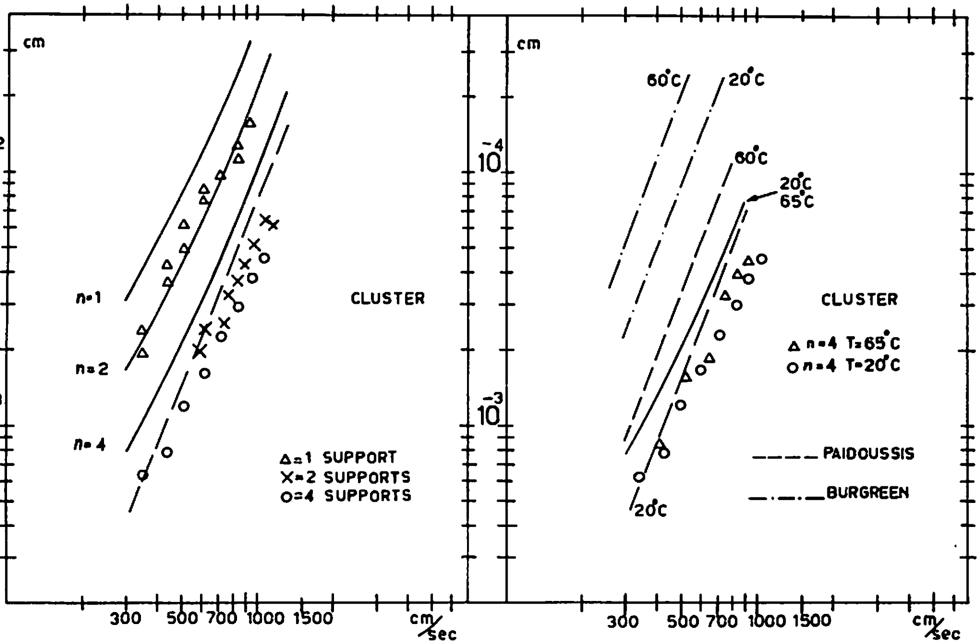

Fig.5

Comparison among Pavlica and Marshall (1966) data, present correlation, Paidoussis and Burgreen correlations.

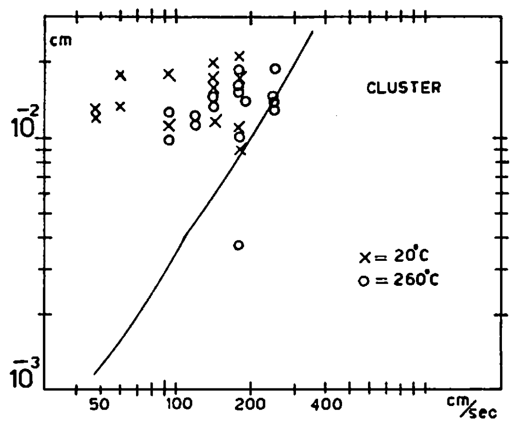

Fig.6

Queen data (1962) at low velocities and different temperatures.

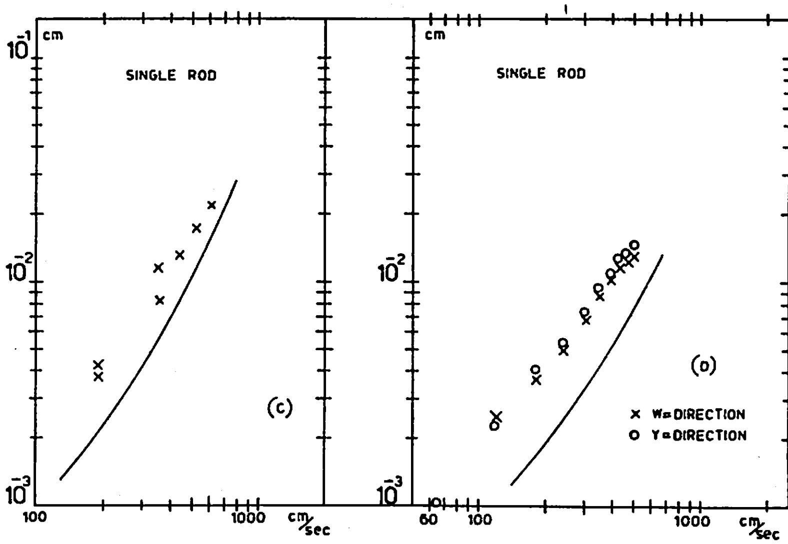

Fig.7

Wambsgnss and Boers single rod data (1968) in comparison with present correlation.

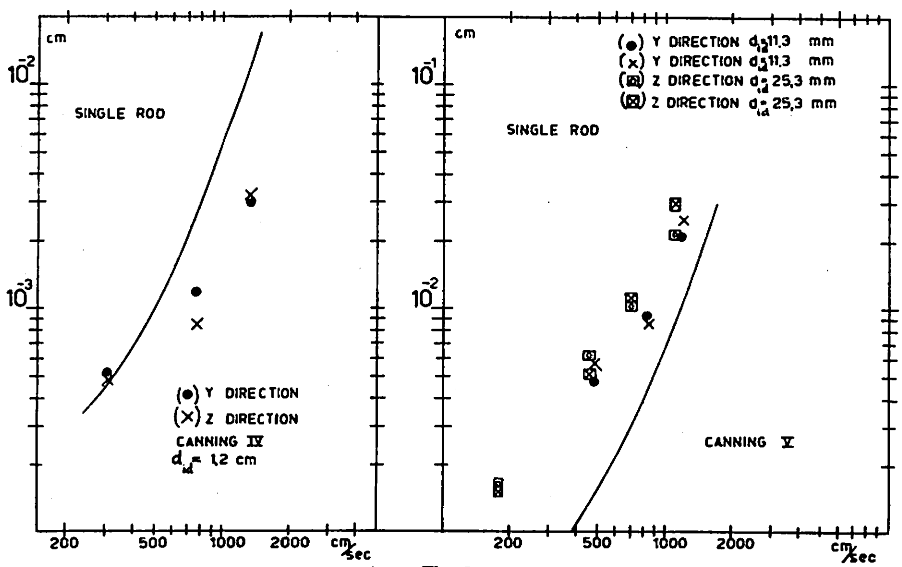

Fig.8

Basile single rod data (1968) compared with present correlation.

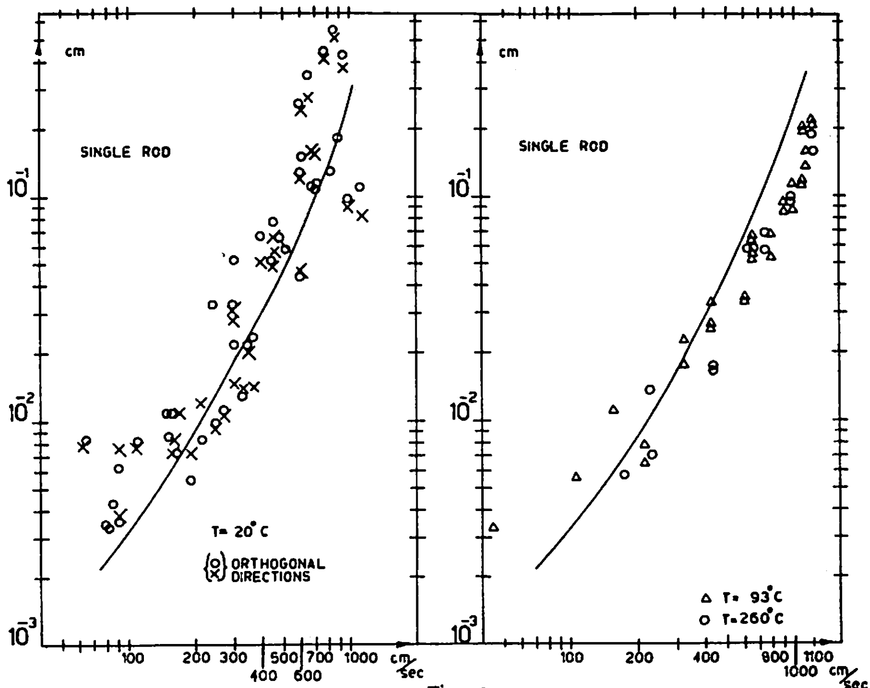

Fig. 9

Quinn single rod, different temperatures data (1962) compared
with present correlation.

DISCUSSION

Q E. OHLMER, JRC Ispra, Italy

Permit me some remarks as far as concerns the confrontation of our analytical model with the mentioned experimental results: The referred tests were made without taking account of the actual pressure fluctuations level and structure in the test loops. As we have seen before, this parameter may influence very much the measured rod displacement. So, the confrontation between these results and your calculations in which you exclude formally such disturbances perhaps will not agree so well ?

A P. G. AVANZINI, Italy

It is very difficult to avoid pressure fluctuations in a test loop. All experimental data are affected by this defect. Nevertheless the proposed correlation is composed of two parts: the first one is reached theoretically and the second one has been adjusted using experimental data. If we shall find better experimental data, we can modify in the best way this second part.

Q L. CEDOLIN, Italy

Has the correlation presented been employed in the prediction of vibration in two-phase flow ?
For example, the experimental data of paper E 4/5 could be analysed ?

A P. G. AVANZINI, Italy

I never tried to apply this correlation to a two-phase flow, but correlation is obtained in such a way that it is easy and possible to extend it to two-phase flow.