Continuous-time Continuous Stochastic Process Models of Pine Stumpage Prices and Plantation Returns in the Southeast US

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ABSTRACT

DIXON, ERNEST, IV. Continuous-time Continuous Stochastic Process Models of Pine Stumpage Prices and Plantation Returns in the Southeast US. (Under the direction of Robert C. Abt).

This work presents an overview of how continuous-time continuous stochastic process models can be used in forestry related analysis, using data relevant for the coastal plain of North Carolina. Testing for stationarity, model estimation, simulated projections, and interpretation are presented for geometric Brownian motion and simple mean reverting models of pine plantation returns for this region. Additional models are estimated for pine stumpage and plantation returns for regions across the southeast US coastal plain.

ACKNOWLEDGMENTS

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Introduction

Stochastic models are used to describe the random behavior of an asset's value through time. When the value is observed constantly and can take any positive real value, the model is considered a continuous-time continuous stochastic process model. Alternatively, there are discrete-time models in which the asset's value is observed over regular intervals and discrete process models in which the asset's value can only be a countable number of possible values (Tsay 2005). Continuous-time continuous stochastic process models are commonly used in the financial industry to simulate the price behavior of assets such as equities and are often incorporated into a larger analysis such as a real options analysis. For examples see: Song et al. (2011), Plantinga (1998), and Schatzki (2003). If it is assumed that the behavior of the process will not change from the historical time series on which the model is based, these models may also be used to generate simulated forecasts and confidence intervals (Dixit and Pindyck 1994).

The purpose of this analysis is to develop continuous-time, continuous stochastic process models of timber prices for pine sawtimber and pulpwood and also for the equivalent annual returns of pine plantation management in coastal regions of the southeast US. The equivalent annual return is the yearly payment throughout the rotation that, when discounted, would equal the lump sum of the net present value (Bettinger et al. 2009). Mei et al. (2010) conducted a similar analysis to this one. They fit continuous-time continuous stochastic process models, along with several types of discrete-time models, to quarterly pine sawtimber prices for regions across the southeast US for the years 1977-2008 and then tested the models for forecasting accuracy. They used quarterly price data and developed models only for pine sawtimber, so their models will not be directly comparable to those in this analysis, which are based on annual values. This work differs from theirs by using an assumed timber management regime to create estimates of annualized returns, an extension that shows how the models can be incorporated into other types of analyses.

Description of models - Two types of continuous-time continuous stochastic process models are used in this analysis, the geometric Brownian motion model and the simple mean reverting model. Following Dixit and Pindyck (1994) both geometric Brownian motion and simple mean reverting models are based on the Weiner process, a stochastic process with the following properties:

1) Future values of the process only depend on the most recent value (called the Markov property).
2) The probability distribution of any changes in the process over some time period is independently distributed from that of other time periods (called independent increments).
3) The changes in the process over some time period are normally distributed with a variance that increases linearly with time.

Mathematically, this process can be represented as a function of a single variable, time: $w(t)$ and the change, also called the increment, of the Wiener process is represented as the function

$$dw = \epsilon_t \sqrt{dt} \tag{1}$$

where ϵ_t is a normally distributed random variable with zero mean and a standard deviation of one.

Following Tsay (2005), if x_t is the price of an asset at time t, then the equation

$$dx_t = \mu x_t dt + \sigma x_t dw_t, \tag{2}$$

with μ and σ as constant values and dw_t as the increment of the Wiener process, describes a geometric Brownian motion (GBM) model of the asset's value. μ and σ are called the drift and variance parameters, respectively. With this model, changes in the natural logarithm of the asset's value are described by

$$dlnx_t = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dw_t \tag{3}$$

and it is this property that is used to fit these models empirically. Additionally, if $x(0) = x_0$, then the expected future value of the asset in time t is

$$\varepsilon[x(t)] = x_0 e^{\mu t} \tag{4}$$

and the variance is

$$V[x(t)] = x_0{}^2 e^{2\mu t}(e^{\sigma^2 t} - 1) \tag{5}$$

(Dixit and Pindyck, 1994).

The second model used in this analysis, the simple mean reverting (MR) model, is also called the Ornstein-Uhlenbeck model. Following Dixit and Pindyck (1994), if x is the value of an asset, then a simple MR model of x is described by the function

$$dx = \eta(\overline{x} - x)dt + \sigma dw \tag{6}$$

where $\overline{x}$ is the long-run average value of the asset, η is the speed with which the asset's value will return to the long-run average, and dw is again the increment of the Weiner process. If $x(0) = x_0$, then the expected value of the asset in time t is

$$\varepsilon[x_t] = \overline{x} + (x_0 - \overline{x})e^{-\eta t} \tag{7}$$

and the variance of $(x_t - \overline{x})$ is

$$V[x_t - \overline{x}] = \frac{\sigma^2}{2\eta}(1 - e^{-2\eta t}). \tag{8}$$

While both geometric Brownian motion and simple mean reverting models are from the same family of models, those based on the Wiener process, there is an important difference in the behavior between the two that must be considered when deciding how to model an asset. The geometric Brownian motion model will allow the value of an asset to grow without bound but never become negative, while the value from a mean reverting model, true to its name, tends to revert back to the long-run average. This means that, depending on the asset being modeled, it may be more appropriate to use one model over another and this decision may be made *a priori* based on economic reasoning. Two studies in applied topics similar to this analysis that addressed this issue are Mei et al. (2010) and Song et al. (2011).

Timber price data - Timber Mart South (TMS), a private company associated with the University of Georgia, has been reporting prices for major forest products since the late 1970s. During this time, TMS has changed both the frequency of the reports and the boundaries of their geographic regions, which affects the quality of these data as a time series. Prestemon and Pye (2000) address this problem directly and suggest a methodology to correct for these changes. The timber price data for pine pulpwood and sawtimber used in this analysis were created following this procedure and were obtained from Prestemon directly (Prestemon 2013). Figure 1 is a map of the coastal Timber Mart South regions used in this analysis. Figures 2 and 3 present product prices in dollars per ton for pine pulpwood and sawtimber respectively, deflated to base year 2005, for each of the coastal Timber Mart South regions. Due to data limitations, chip-n-saw prices are assumed to be twice that of pulpwood. See Appendix A for more information about the deflation procedure.

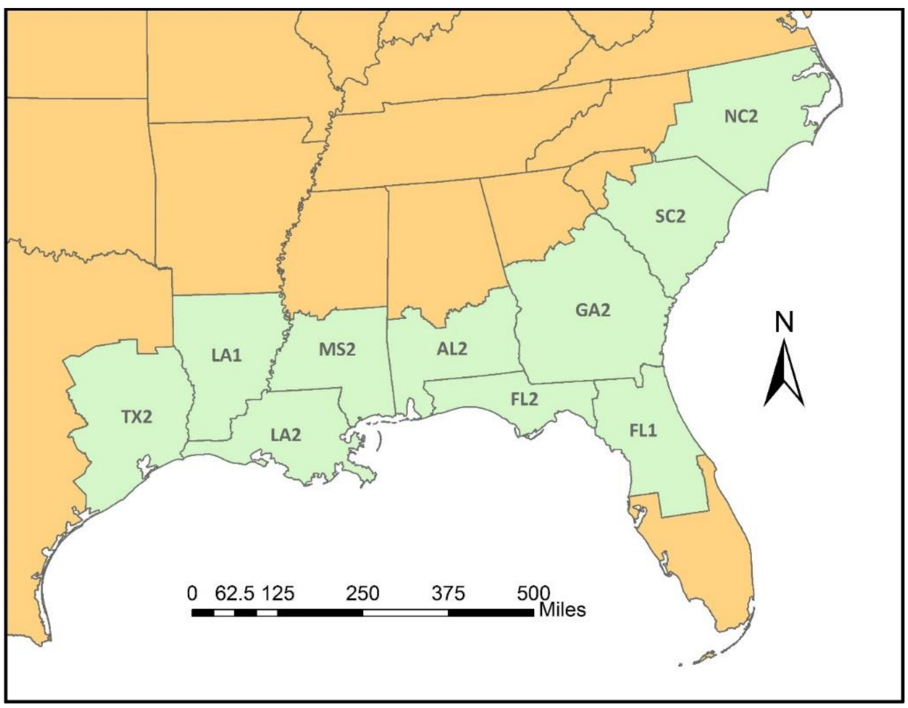

Figure 1 - Map of coastal Timber Mart South regions.

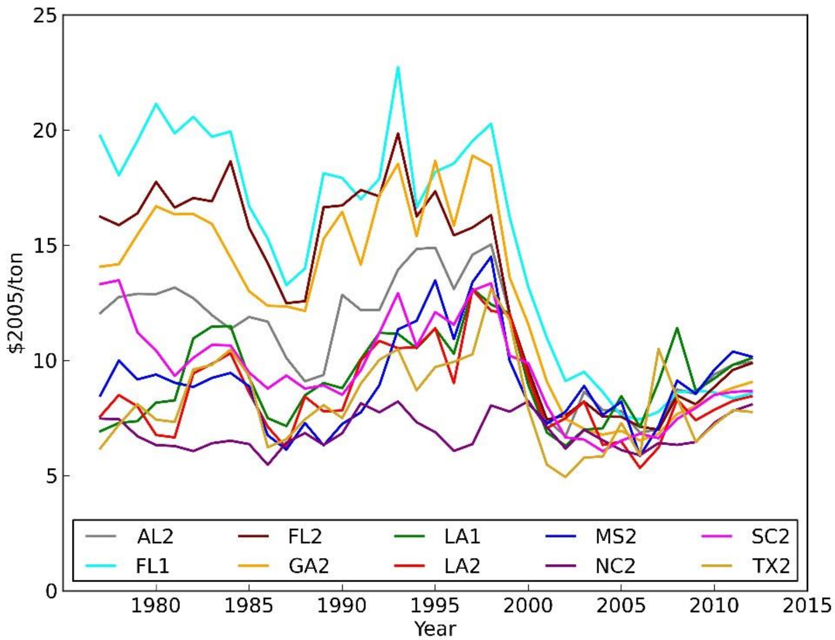

Figure 2 - Real pine pulpwood prices, $2005, for coastal Timber Mart South zones.

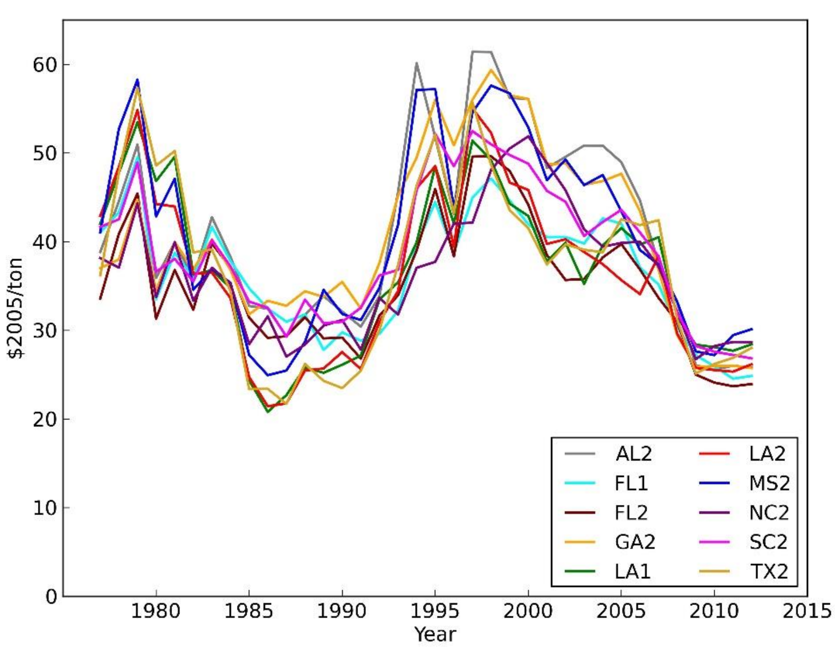

Figure 3 - Real pine sawtimber prices, $2005, for coastal Timber Mart South zones.

Timber management assumptions –To generate annualized returns from the timber prices series, it is necessary to assume some past management regime. For this analysis, treatment timing, costs (excluding yearly management), and harvest yields are based on those currently used for strategic planning for the Hofmann Forest (Catts 2013) and are assumed constant throughout the analysis. The Hofmann Forest is an 80,000 acre property in the coastal plain of North Carolina owned by NC State, a large portion of which is intensively managed pine plantation. Table 1 presents management assumptions and Table 2 presents harvest yields by product class. Establishment costs include chemical and mechanical site preparation, fertilization, seedlings, and planting for the establishment of a bedded stand on a 20’ by 5’ spacing. Yearly management cost estimates are from Barlow (2011) and represent a southeast US coastal plain average. The volume of stumpage sold, listed in Table 2, is the only source of revenue during the rotation. Rotation timing includes a one year delay between the harvest of the previous stand and establishment of the current, accounting for the lag in tree age within the timeline.

Table 1 – Coastal pine plantation management assumptions.

Description	Value	Unit	Year
Establishment costs	383	$/ac	1
Management costs	9.98	$/ac	1-25
Tree age at thinning	11	years	12
Tree age at final harvest	25	years	26
Discount rate	0.06	-	-

Table 2 – Coastal pine plantation assumed yields, tons/acre.		
Harvest Event	**Product**	**Yield**
Thinning	Pulpwood	23.1
Thinning	CNS	1.2
Final	Pulpwood	40.6
Final	CNS	14.7
Final	Sawtimber	67.6

<u>Methods</u>

Prices to return function – Using the above management assumptions, a function was created that calculates the annual equivalent return for a rotation of plantation management, with timber prices by product as the independent variables. This function was then used to calculate the returns for an array of possible price combinations, within a reasonable range for each product class. A multiple linear regression model of returns to prices was then fit to this data set using traditional OLS, with the form

$$annual\, return = \beta_0 + \beta_1 p_{pulp} + \beta_2 p_{CNS} + \beta_3 p_{st}. \quad (9)$$

For this analysis, the estimated model was

$$annual\, return = -37.60 + 1.57\, p_{pulp} + 0.29\, p_{CNS} + 1.14\, p_{st}. \quad (10)$$

Since this model is purposefully over-fitted, standard errors for the estimated parameters and fit statistics for the model are not reported. This regression equation was then used to calculate the yearly return for pine plantation management in each Timber Mart South zone from the historically observed price series. To extend the usefulness of this technique, price elasticities of annual returns are calculated for pine pulpwood and sawtimber; see Appendix D.

Unit root test for stationarity – The decision of which type of model is more appropriate for some time series, GBM or MR, is an important one given the difference in behavior between the two models. Dixit and Pindyck (1994) mention the use of a statistical test for unit root to decide whether a GBM or MR model would be most appropriate for a given time series. In the statistics and economics literature, the term used to describe the mean-reverting tendency of some time series is stationarity. A mean reverting series would be called stationary and a series that isn't is called non-stationary. Kennedy (2008) summarizes the difference between stationary and non-stationary series, stating that a stationary series appears erratic, has a finite variance, and shocks that are temporary, whereas a non-stationary series appears smooth, has a variance that grows with time, and shocks that are permanent.

One popular test for unit root and the one used by Insley, 2002, is the Dickey-Fuller test. Following Wooldridge (2009), the model is assumed to be

$$\Delta y_t = \alpha + \theta y_{t-1} + e_t \tag{11}$$

where y_0 is the initial observed value. θ is defined as $\rho - 1$, with ρ being the value to test for unity. The null hypothesis of the test is that $\theta = 0$, implying that the series has a unit root and is therefore non-stationary. This is tested against the alternative that $\theta < 0$ using a simple t-test with special critical values from the Dickey-Fuller distribution. The null hypothesis is rejected if $t_{\hat{\theta}} < c$, where c is the appropriate critical value. A rejection of the null hypothesis would give favor to the modeling of the series as a mean reverting process. Unfortunately, unit root tests often have low statistical power and require very long data sets, so the results from these tests must be considered carefully in empirical work (Dixit and Pindyck 1994).

Estimating the GBM model – Tsay (2005) describes the estimation of the mean and variance parameters of the GBM model. Using a series of $n + 1$ annual observations of an asset's value, x_t, define $r_t = \ln(x_t) - \ln(x_{t-1})$. Then, if $\bar{r}$ is the sample mean and s_r is the sample standard deviation of r_t, μ and σ are estimated by

$$\hat{\mu} = \bar{r} + \frac{1}{2} {s_r}^2 \tag{12}$$

$$\hat{\sigma} = s_r. \tag{13}$$

Forecasting and confidence intervals of the GBM model – Dixit and Pindyck (1994) give examples of calculating optimal forecasts and the 66% confidence interval boundaries for the

GBM model. Given the known value of an asset x at time t, the unknown value of x at some future time T is forecasted as

$$\hat{x}_T = (1 + \mu)^{T-t} x_t \tag{14}$$

and the upper and lower bounds of the 66% confidence interval, respectively, are

$$(1 + \mu)^{T-t}(1 + \sigma)^{\sqrt{T-t}} x_t \text{ and } (1 + \mu)^{T-t}(1 + \sigma)^{-\sqrt{T-t}} x_t. \tag{15}$$

Estimating the MR model – Dixit and Pindyck (1994) describe the steps for estimating the parameters for the simple mean reverting model from discrete data. It begins by estimating the regression

$$x_t - x_{t-1} = a + b x_{t-1} + \epsilon_t. \tag{16}$$

Then parameter estimates for the MR model are

$$\hat{\bar{x}} = -\frac{\hat{a}}{\hat{b}}, \tag{17}$$

$$\hat{\eta} = -\ln(1 + \hat{b}), \text{ and} \tag{18}$$

$$\hat{\sigma} = \hat{\sigma}_\epsilon \sqrt{\frac{\ln(1 + \hat{b})}{(1 + \hat{b})^2 - 1}}. \tag{19}$$

<u>Results</u>

Most of the results presented will focus on the annual returns series for a single region, NC2, the coastal plain of North Carolina. The exception is Figure 4, below, which shows the real annual returns for all of the coastal TMS zones under the assumed plantation management regime. For a given year, the returns are calculated using the prices to return function described previously. Note the wide range in returns from the beginning of the data set in the late seventies through the late nineties. This appears to be followed by a convergence in returns between the regions since, with the tightest range during the most recent recession. Figure 5 shows the historical annual returns for NC2 along with the value of the 75th percentile of these returns, which was $22.50/ac/year.

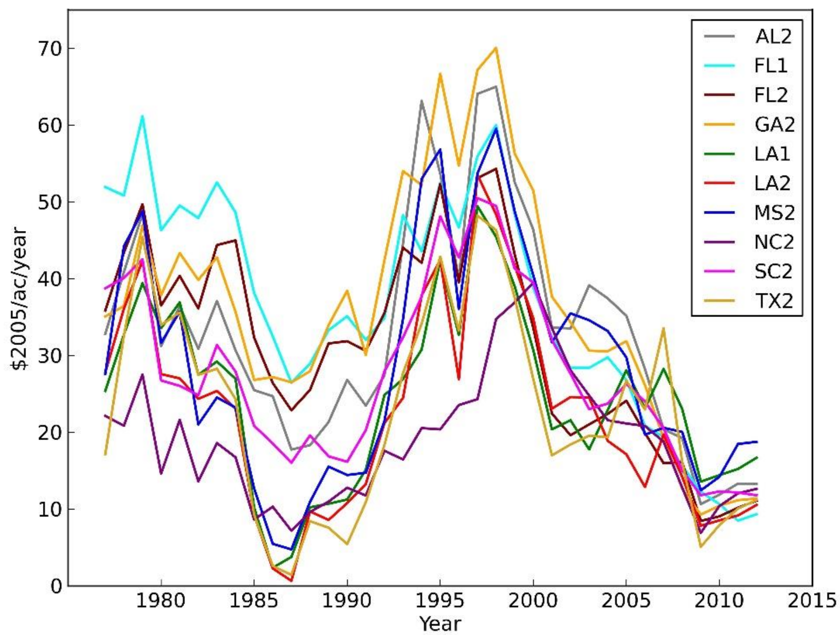

Figure 4 - Real annual returns for TMS zones under assumed plantation management regime.

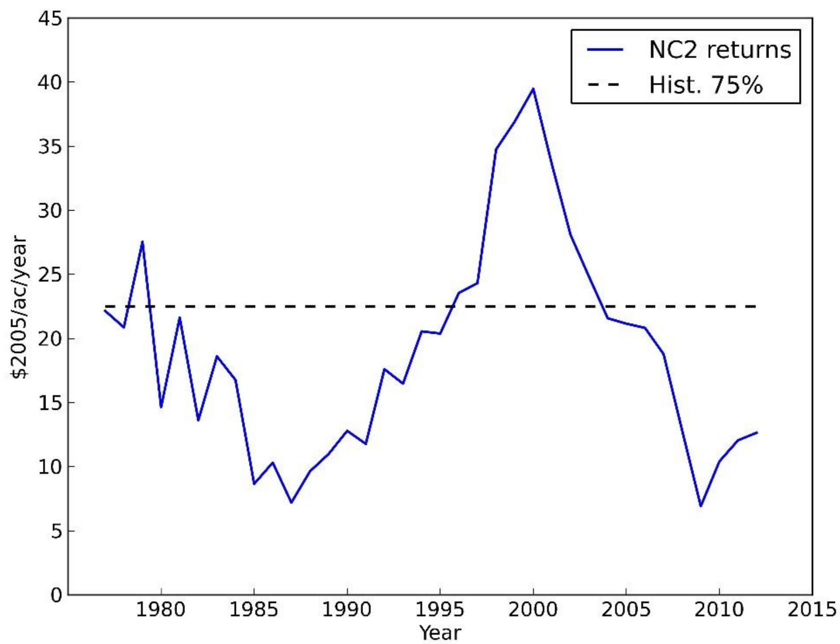

Figure 5 - Real annual returns and 75% boundary for TMS region NC2 under assumed plantation management regime.

Figure 6 presents the results of the Dickey-Fuller test for unit root for the NC2 annual return series. The test statistic value is reported as $Z(t) = -1.654$. At each of the statistical levels, this value is greater than the critical value, so we fail to reject the null hypothesis of a unit root. This means that according to this test the NC2 annual return series appears to be best modeled as a GBM rather than a mean reverting model.

```
. dfuller Return, regress

Dickey-Fuller test for unit root                   Number of obs   =        35

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -1.654            -3.682            -2.972            -2.618
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.4549

------------------------------------------------------------------------------
    D.Return |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      Return |
         L1. |  -.1594707   .0964072    -1.65   0.108    -.3556126    .0366712
             |
       _cons |   2.791913   2.017237     1.38   0.176    -1.312186    6.896012
------------------------------------------------------------------------------
```

Figure 6 - Dickey-Fuller test for unit root for the NC2 annual returns time series.

Geometric Brownian motion model - The estimated parameters of the geometric Brownian motion model of annual returns for TMS region NC2 are $\mu = 0.02$ and $\sigma = 0.29$. This yields the model

$$dx = 0.02xdt + 0.29xdw. \tag{20}$$

This implies that projections of future values from this model can be made using the formula

$$x_t = x_{t-1} + 0.02x_{t-1}dt + 0.29x_{t-1}dw. \tag{21}$$

Model parameters for GBM models of pine pulpwood price, sawtimber price, and annual plantation returns for all TMS regions are in Appendix B. Figure 7 shows the annual returns series for region NC2, along with a ten-year optimal forecast and a 66% confidence interval for the forecast. Figure 8 shows twenty simulations of ten years of projected returns using the estimated GBM model. Figure 9 shows the probability density functions of future returns for one, five, and ten years in the future for ten-thousand simulations of the estimated GBM model along with the 75th percentile value boundary. Using this boundary and the distributions, and assuming that returns will continue to evolve by this process, we can interpret that the likelihood of returns greater than the historical 75th percentile are expected to occur 0.4%, 17%, and 23% of the time for one, five, and ten years in the future, respectively. Following expected behavior, the distribution of returns becomes more skewed through time, with the majority of returns occurring in the lower range of possible values, but with an increasing likelihood of extreme values through time.

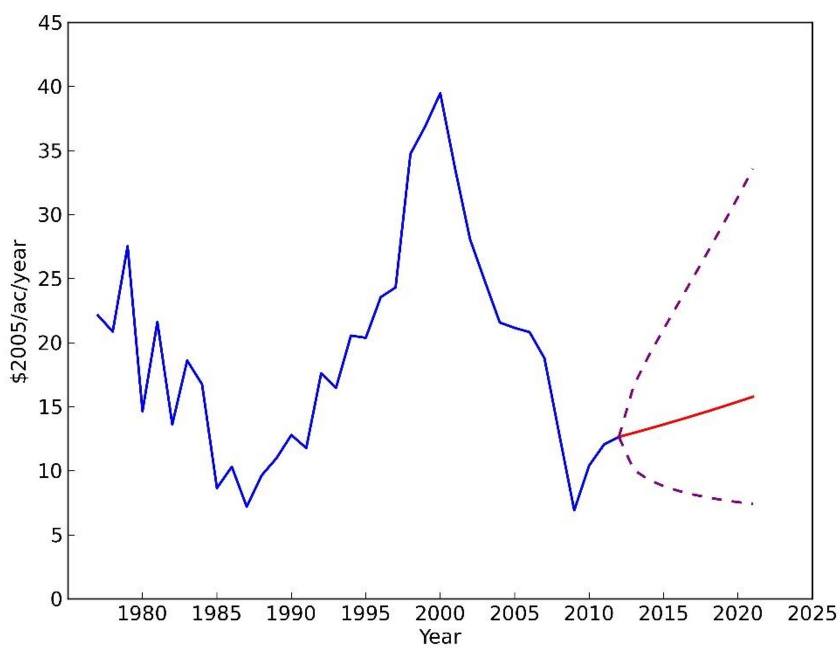

Figure 7 - Ten year forecast (red) and 66% confidence interval of annual returns, modeled as a GBM process for TMS region NC2.

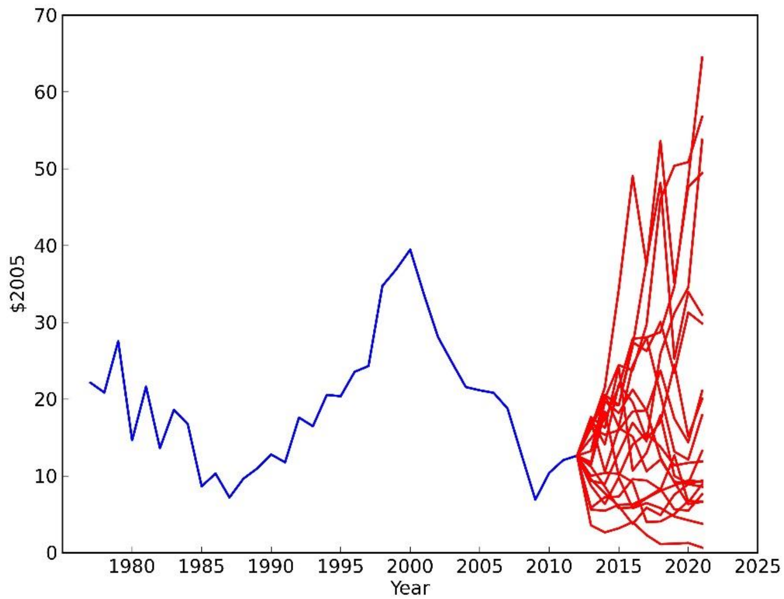

Figure 8 – Twenty simulations of annual returns for ten years in region NC2, with returns modeled as a GBM process.

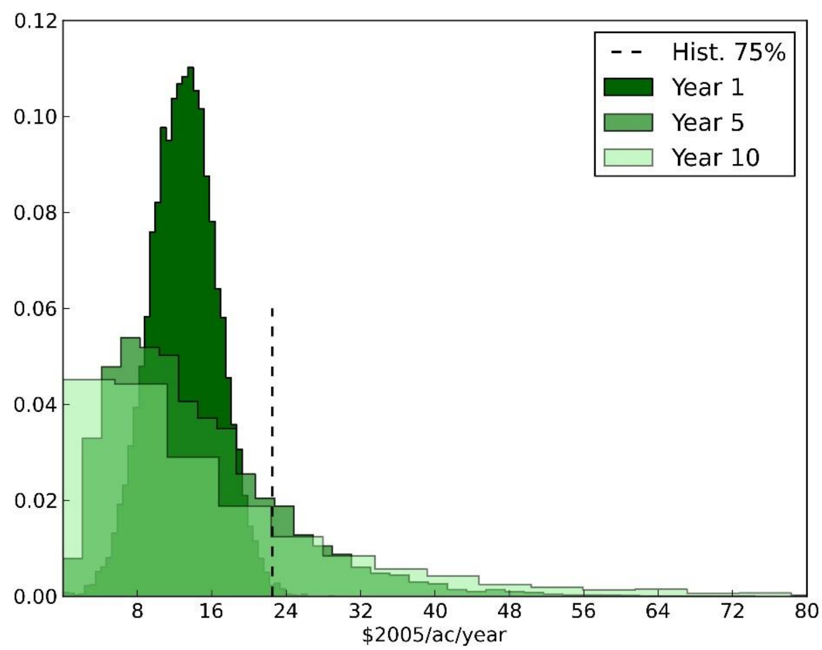

Figure 9 - Probability density function of simulated annual returns in years 1, 5, and 10 for region NC2, with returns modeled as a GBM process, along with the historical 75% value boundary.

Mean reverting model - The estimated parameters of the simple mean reverting model of annual returns for TMS region NC2 are $\overline{x} = 17.51$, $\eta = 0.17$, and $\sigma = 3.64$. This yields the model

$$dx = 0.17(17.51 - x)dt + 3.64dw. \quad (22)$$

This implies that projections of future values from this model can be made using the formula

$$x_t = x_{t-1} + 0.17(17.51 - x_{t-1})dt + 3.64dw. \quad (23)$$

Model parameters for MR models of pine pulpwood price, sawtimber price, and annual plantation returns for all TMS regions are in Appendix C. Figure 12 shows twenty simulations of ten years of projected returns using the estimated MR model. Figure 13 shows the probability density function of future returns for one, five, and ten years in the future for ten-thousand simulations of the estimated MR model along with the 75th percentile value boundary. Using this boundary and the distributions, and assuming that returns will continue to evolve by this process, we can interpret that the likelihood of returns greater than the historical 75th percentile are expected to occur 0.7%, 12%, and 18% of the time for one, five, and ten years in the future, respectively. Following expected behavior, the distribution of returns becomes more normal through time as returns tend toward the long run average value of $17.51/ac/year.

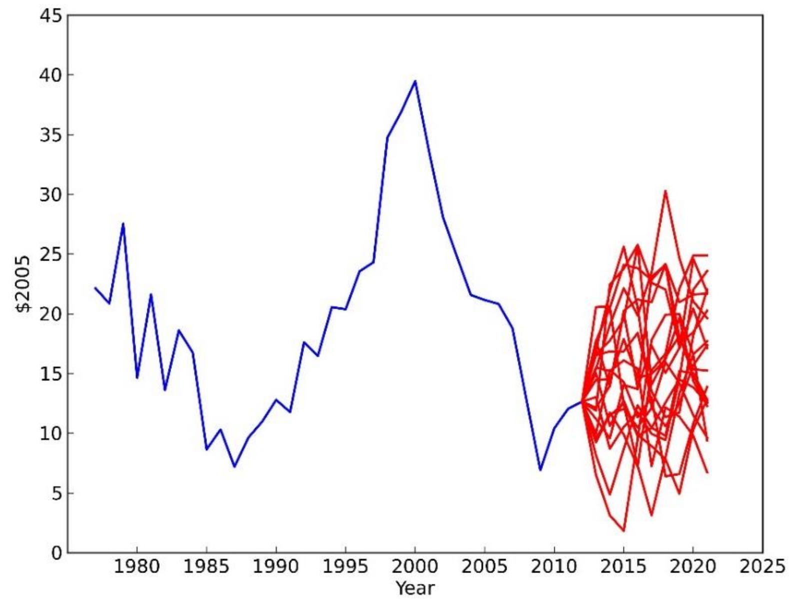

Figure 10 - Twenty simulations of annual returns for ten years in region NC2, with returns modeled as a mean reverting process.

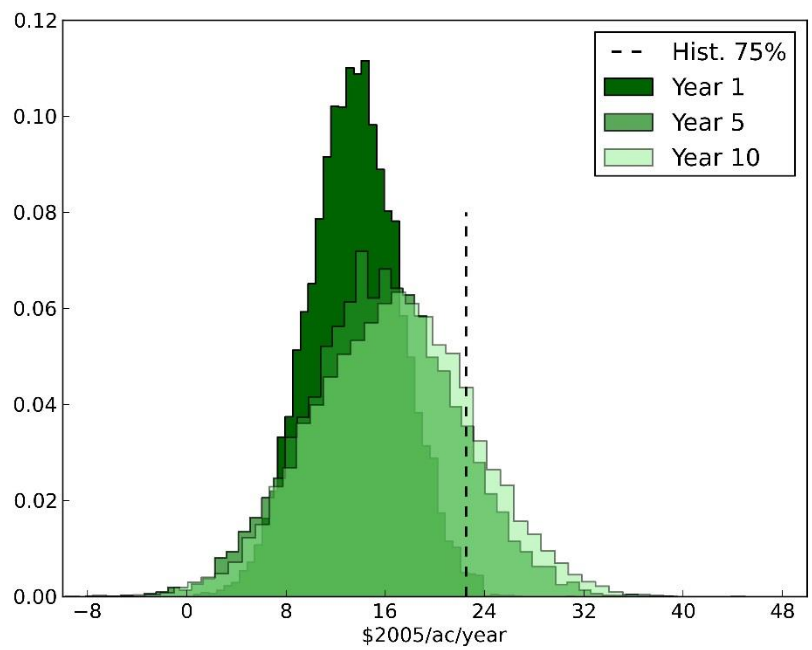

Figure 11 - Probability density function of simulated annual returns in years 1, 5, and 10 for region NC2, with returns modeled as a MR process, along with the historical 75% value boundary.

Discussion

This work presented an overview of how continuous-time continuous stochastic process models can be used in forestry related analysis. It covered estimating the models, using them to simulate future realizations, showed how the distribution of these realizations change through time for each type of model, and showed how to calculate optimal forecasts and confidence intervals for the GBM model. The use of a prices-to-returns function to change the focus of the modeling from timber prices to annual returns is presented as an example of how this sort of analysis can be extended to include management decisions. Finally, a statistical test was presented to help decide on the appropriateness of one model over another for a given time series.

There are a few simplifying assumptions that were made and data limitations that may adversely affect the results of this analysis. Plantation management variables, including costs and yields, were assumed to be constant through time, which is certainly not true. This work presents an incomplete accounting of all of the revenues and costs of managing a forest; for example cost-share and hunting lease revenues are excluded, as are property taxes. In order to use the full time series, timber prices were not allowed to evolve realistically; the same timber prices were assumed to occur for both the thinning and the harvest rather than using prices from fifteen years apart. Finally, the price data used covers a relatively short time period and annualizing the data certainly caused a loss of detail, and perhaps biased the results. Mei et al (2010) discuss this problem in the context of their own work as well.

There are many types of models that are used in time series analysis and there is a broad literature in both the statistics and economics fields covering this topic. Depending on the goals of the analysis and the underlying data itself, there may be other types of models that would be more appropriate than those presented here. For example, Mei et al. (2010) found VAR models, a type of discrete time model, to have the best out of sample performance in their study modeling quarterly pine sawtimber prices. The models presented covered in this work are commonly used in real options analysis and were selected *a priori* for this reason.

There is a rich literature on the usefulness of real options to solve forestry problems; some examples are Plantinga (1998), Schatzki (2003), Duku-Kaakyire and Nanang (2003), and Insley (2002).

REFERENCES

Barlow, R. J. and M. R. Dubois. 2011. Cost and cost trends for forestry practices in the South. *Forest Landowner* 70(6):14-24.

Bettinger, Peter, K. Boston, J.P. Siry, and D.L. Grebner. 2009. *Forest Management and Planning*. Elsevier, Inc., Burlington, MA. 331 p.

Catts, Glenn. 2013. Personal communication – Hofmann Forest data request. North Carolina State University, Raleigh, NC.

Dixit, Avinash K., and R.S. Pindyck. 1994. *Investment Under Uncertainty.* Princeton University Press, Princeton, NJ. 468 p.

Duku-Kaakyire, Armstrong, and David M. Nanang. 2004. Application of real options theory to forestry investment analysis. *Forest Policy and Economics* 6(6):539-552.

Federal Reserve Bank of Dallas. 2013. Deflating nominal values to real values. Available online at http://www.dallasfed.org/research/basics/nominal.cfm; last accessed 04/16/2013.

Insley, Margaret. 2002. A real options approach to the valuation of a forestry investment. *Journal of Environmental Economics and Management* 44(3):471-492.

Kennedy, Peter. 2008. *A Guide to Econometrics*. Blackwell Publishing, Malden, MA. 585 p.

Mei, Bin, Michael Clutter, and Thomas Harris. 2010. Modeling and forecasting pine sawtimber stumpage prices in the US South by various time series models. *Canadian Journal of Forest Research* 40(8):1506-1516.

OECD. 2010. "Main Economic Indicators - Complete Database". Main Economic Indicators (GDP Implicit Price Deflator in United States). http://dx.doi.org/10.1787/data-00052-en. Available online at http://research.stlouisfed.org/fred2/series/USAGDPDEFAISMEI; last accessed 04/16/2013.

Plantinga, Andrew J. 1998. The optimal timber rotation: An option value approach. *Forest Science* 44(2):192-202.

Prestemon, Jeffrey P., and John M. Pye. 2000. A technique for merging areas in Timber Mart-South data. *Southern Journal of Applied Forestry* 24(4):219-229.

Prestemon, Jeff. 2013. Personal communication – Timber Mart South data request. USDA Forest Service, Research Triangle Park, NC.

Schatzki, Todd. 2003. Options, uncertainty and sunk costs: An empirical analysis of land use change. *Journal of Environmental Economics and Management* 46(1):86-105.

Song, Feng, Jinhua Zhao, and Scott M. Swinton. 2011. Switching to perennial energy crops under uncertainty and costly reversibility. *American Journal of Agricultural Economics* 93(3):768-783.

Tsay, Ruey S. 2005. *Analysis of Financial Time Series.* John Wiley & Sons, Inc., Hoboken, NJ. 605 p.

Wooldridge, Jeffrey M. 2009. *Introductory Econometrics: A Modern Approach.* South-Western Cengage Learning, Mason, OH. 865 p.

APPENDIX A – Deflation procedure

The Federal Reserve Bank of Dallas has published instructions for deflating nominal values to real values on their website (Federal Reserve Bank of Dallas 2013) and the St. Louis Federal Reserve has published GDP deflators with a base year of 2005 for the United States on their website (OECD 2010). These tools were used to deflate timber prices and plantation returns for this analysis using this formula

$$real\ value = \left(\frac{nominal\ value}{GDP\ deflator}\right) * 100. \tag{24}$$

APPENDIX B - Estimated GBM model parameters

Table 3 - GBM model parameters for pine pulpwood price.		
Region	Drift, μ	Variance, σ
AL2	0.002	0.122
FL1	-0.017	0.121
FL2	-0.007	0.121
GA2	-0.005	0.124
LA1	0.022	0.148
LA2	0.018	0.170
MS2	0.018	0.158
NC2	0.007	0.094
SC2	-0.006	0.108
TX2	0.026	0.199

Table 4 - GBM model parameters for pine sawtimber price.		
Region	Drift, μ	Variance, σ
AL2	-0.001	0.149
FL1	-0.008	0.112
FL2	-0.001	0.132
GA2	-0.005	0.107
LA1	-0.002	0.134
LA2	-0.004	0.141
MS2	0.002	0.149
NC2	-0.002	0.115
SC2	-0.008	0.099
TX2	0.005	0.157

Table 5 - GBM model parameters for pine plantation annual return.		
Region	Drift, μ	Variance, σ
AL2	0.004	0.242
FL1	-0.037	0.154
FL2	-0.012	0.209
GA2	-0.014	0.189
LA1	0.074	0.415
LA2	0.173	0.633
MS2	0.049	0.348
NC2	0.025	0.286
SC2	-0.019	0.175
TX2	0.143	0.556

APPENDIX C - Estimated MR model parameters

Table 6 - MR model parameters for pine pulpwood price.

Region	Mean value, $\bar{x}$	Reversion speed, η	Variance, σ
AL2	10.56	0.14	0.96
FL1	11.00	0.08	1.51
FL2	11.05	0.08	1.23
GA2	11.25	0.09	1.33
LA1	9.57	0.37	1.10
LA2	8.68	0.38	1.17
MS2	9.24	0.32	1.17
NC2	6.89	0.47	0.53
SC2	8.91	0.20	0.84
TX2	8.35	0.49	1.34

Table 7 - MR model parameters for pine sawtimber price.

Region	Mean value, $\bar{x}$	Reversion speed, η	Variance, σ
AL2	39.56	0.19	5.08
FL1	34.34	0.21	3.37
FL2	34.34	0.24	3.85
GA2	36.62	0.08	3.24
LA1	34.57	0.17	3.86
LA2	33.62	0.17	4.18
MS2	39.30	0.20	4.97
NC2	35.07	0.17	3.14
SC2	35.29	0.11	2.98
TX2	36.11	0.19	4.69

Table 8 - MR model parameters for pine plantation annual return.

Region	Mean value, $\bar{x}$	Reversion speed, η	Variance, σ
AL2	30.58	0.18	6.83
FL1	16.81	0.06	4.59
FL2	25.47	0.11	5.12
GA2	27.91	0.08	5.44
LA1	23.42	0.21	5.41
LA2	20.22	0.21	6.26
MS2	27.48	0.20	6.95
NC2	17.51	0.17	3.64
SC2	19.91	0.10	3.86
TX2	22.09	0.22	6.47

APPENDIX D – Price Elasticities of Annual Returns

Elasticities are common values calculated for economic analysis that are useful to summarize the sensitivity of change in one variable to the change in another. In this case the price elasticity of annual returns in defined as:

$$\varepsilon_{annual\ return} = \frac{\partial annual\ return}{\partial\ price_{product}} * \frac{price_{product}}{annual\ return}$$

where the price of the product is the mean for the region, the annual return is the value for the mean of all products, and the partial derivative of annual return with respect to product price is the appropriate slope parameter from the estimated prices to returns function. Table 9 and 10 report the mean prices by product and calculated elasticities for the coastal TMS regions.

Table 9 - Mean prices, $2005/ton

	AL2	FL1	FL2	GA2	LA1	LA2	MS2	NC2	SC2	TX2
Sawtimber	41.30	36.45	35.29	40.59	36.82	36.38	40.76	36.56	38.98	37.18
Pulpwood	10.99	14.95	13.29	12.79	9.29	8.60	9.09	6.88	9.62	8.22

Table 10 - Price elasticity of annual returns by product

	AL2	FL1	FL2	GA2	LA1	LA2	MS2	NC2	SC2	TX2
Sawtimber	1.42	1.15	1.29	1.28	1.72	1.85	1.64	2.21	1.62	1.89
Pulpwood	0.52	0.65	0.67	0.56	0.60	0.60	0.50	0.57	0.55	0.57