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A Consistent Response Spectrum Analysis Including the Resonance Range

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Summary

The present report, a supplement to the preceeding SMIRT-papers [1], [2], provides a complete consistent Response Spectrum Analysis for any component. The effect of supports with different excitation is taken into consideration, as is the description of the resonance ranges.

This report includes information which was omitted in [2], explaining how the contributions of the eigenforms with higher eigenfrequencies are to be considered.

The case in which the building and component vibrate in resonance was not considered in [2]. This case is treated in accordance with the considerations in [1].

Stocking of floor response spectra is also possible using the method described here. However, modified floor response spectra must now be calculated for each building mode. Once these have been prepared, the calculation of the dynamic component values is practically no more complicated than with the conventional, non-consistent methods.

The consistent Response Spectrum Analysis can supply smaller and larger values than the conventional theory, a fact which can be demonstrated using simple examples.

The present report contains a consistent Response Spectrum Analysis (RSA), which, as far as we know, has been formulated in this way for the first time. A consistent RSA is so important because today this method is preferentially applied as an important tool for the earthquake proof of components in nuclear power plants.

1. Introduction

The component design of a nuclear power plant against earthquakes is generally realised using the Response Spectrum Analysis (RSA). The application of the RSA permits the calculation of deformations, forces and stresses of a component in two ways:

1. With the RSA only for the component, whereby floor response spectra are used (method 1)

2. With the RSA for the system building and component, whereby ground response spectra are applied (method 2).

The logical demand that the results for the component should be the same in both cases cannot be fulfilled with present methods.

In principle, method 2 can always be applied. It does, however, have two essential disadvantages:

- The building must always be explicitly described too

- The mass and stiffness of building and component generally differ to such an extent that numerical difficulties can result.

Our task is therefore to calculate the dynamic values for the component in such a way that they are in agreement with the values corresponding to method 2, but without having to use the overall system explicitly as model. Once they have been calculated, building data may only be implicitly used for each component. Numerical difficulties such as those mentioned above must be excluded.

It is shown that it is impossible to define floor response spectra in such a way that they can serve as the basic spectra for a RSA of a component analogous to the ground spectra for a RSA of a building. The RSA procedure implemented does not mean any additional work for the user. It does, however, slightly increase the computation time.

A complete RSA must include two essential aspects:

- It must still be applicable in the case of resonances between building and component

- It must consider the contributions made by higher eigenforms in a suitable way.

The present report offers a satisfactory solution for both requirements.

2. General assumptions and symbols

It is agreed to define each supporting system suggestively as building and each supported system as component.

Σ_B is a linear elastic building structure with n_B degrees of freedom, m supports for a component Σ_K (see below) and the following dynamic values of the l-th eigenform ($l = 1, 2, \ldots, n_B$):

ω_l^B Eigenfrequency

ϕ_{jl}^B Eigenvector component at the building node j = component support j ($j = 1, 2, \ldots, m$)

$\Gamma_{\alpha,l}^B$ Participation factor for the direction α

D_l^B Modular damping

Further, Σ_K is a linear elastic component structure with n degrees of freedom and m supports (= fixed points in Σ_K), which is characterized by the following values of the p-th eigenform (p = 1,2,..., n):

ω_p Eigenfrequency

ϕ_{KP} Eigenvector component at the component node k (k = 1,2,..., n)

$\Gamma_{\alpha,p}$ Participation factor for the direction α

D_p Modular damping

We now go on to define a structure Σ_{B+K} , which consists of the two structures Σ_B and Σ_K , whereby the component supports are degrees of freedom for Σ_B , but, however, all the actual degrees of freedom of the component are not suppressed by Σ_B. We want to speak of the degrees of freedom of support j = 1,2,..., m, although they represent fixed points for the component.

The i-th eigenform (i = 1,2,..., $\bar{n} = n + n_B$) of the total structure Σ_{B+K} is characterized by:

$\bar{\omega}_i$ Eigenfrequency

$\bar{\Phi}_{ki}$ Eigenvector component at the "actual" component node k (k = 1,2,..., n)

$\bar{\Phi}_{ji}$ Eigenvector component at the component support j (j = 1,2, ..., m)

$\bar{\Gamma}_{\alpha,i}$ Participation factor for the direction α

$\bar{D}_i$ Modular damping

The equations of motion for the actual nodes $\bar{x}$ of the component and the component supports s in the system Σ_{B+K} are as follows:

$$M\ddot{\bar{x}} + K\bar{x} + K_m s = 0 \qquad (1)$$

$$M_s\ddot{s} + K_m^T\bar{x} + K_s s = 0 \qquad (2)$$

where

M : mass matrix } of Σ_K
K : stiffness matrix } of Σ_K
M_S : mass matrix } of the component supports in Σ_{B+K}
K_S : stiffness matrix } of the component supports in Σ_{B+K}
K_m : stiffness matrix which represents the connection between x and s
F_S : anchor force

M and M_s are assumed as being diagonal matrices. K_m^T represents the transposed matrix of K_m.

It is postulated that the dynamically relevant values (eigenfrequencies, eigenforms) of the building are only unimportantly influenced by the component (non-interacting component). The eigenforms of Σ_{B+K} then divide up into two types:

Type 1 (building modes)

The first type consists of eigenforms, in which the building contributions are equivalent with the eigenforms of Σ_B and in which the component contributions can be determined as a response to these building eigenforms, i. e. as forced vibrations. These eigenforms are termed building modes in the following.

Type 2 (component modes)

The second type consists of eigenforms, in which the component vibrates in the relevant eigenform of Σ_K, while the building remains still, and is only noticeable because of its infinitely large mass in comparison to the component mass. These eigenforms are termed component modes in the following.

The building modes are indexed with $i = 1,2,\ldots,n_B$, the component modes with $i = n_B+1, n_B+2,\ldots,\bar{n}=n_B+n$. Further, in each case, the modes are arranged according to increasing eigenfrequency.

Building modes : $\bar{\omega}_1 < \bar{\omega}_2 < \bar{\omega}_3 < \ldots < \bar{\omega}_{n_B}$

Component modes: $\bar{\omega}_{n_B+1} < \bar{\omega}_{n_B+2} < \ldots < \bar{\omega}_{n_B+n}$

Of course, the relation between $\bar{\omega}_{n_B}$ and $\bar{\omega}_{n_B+1}$ is not limited, i. e. the three cases $\bar{\omega}_{n_B} < \bar{\omega}_{n_B+1}$ $\bar{\omega}_{n_B} = \bar{\omega}_{n_B+1}$ and $\bar{\omega}_{n_B} > \bar{\omega}_{n_B+1}$ are possible.

3. Eigenvectors, participation factors and modal displacements of the system Σ_{B+K}

The solutions can be found in [2].

3.1 Building modes ($i = 1,2\ldots, n_B$)

$$\bar{\Gamma}_{\alpha,i} = \Gamma_{\alpha,i}^B \tag{3a}$$

$$\bar{\omega}_i = \omega_i^B \tag{3b}$$

$$\bar{\mathcal{D}}_i = \mathcal{D}_i^B \tag{3c}$$

$$\bar{\Phi}_{ji} = \Phi_{ji}^B \tag{3d}$$

$$\bar{\Phi}_{Ki} = \sum_{p=1}^{n}\sum_{j=1}^{m} \phi_{Kp} P_{pj} \quad \frac{\omega_p^2}{\omega_p^2 - (\omega_i^B)^2} \phi_{ji}^B \tag{3e}$$

whereby the participation matrix

$$P = - \Phi^T M K^{-1} K_m \tag{4}$$

was introduced.

3.2 Component modes ($i = n_B + 1, n_B + 2,\ldots n_B + n$)

$$\bar{\omega}_i = \omega_{i-n_B} \tag{5a}$$

$$\bar{\mathcal{D}}_i = \mathcal{D}_{i-n_B} \tag{5b}$$

$$\bar{\Phi}_{ji} = 0 \tag{5c}$$

$$\bar{\Gamma}_{\alpha,i=\bar{1}+n_B}\,\bar{\Phi}_{Ki} = \sum_{l=1}^{n_B}\sum_{j=1}^{m} \phi_{K\bar{1}}\,P_{\bar{1}j}\,\phi^B_{jl}\,\Gamma^B_{\alpha,l}\,\frac{(\omega^B_l)^2}{(\omega^B_l)^2-\omega^2_{\bar{1}}} \tag{5d}$$

3.3 Calculation of the modal displacements

The maximum contribution $\bar{x}_{Ki}$ of the i-th eigenform of Σ_{B+K} to the total displacement at any component node k amounts to:

$$\bar{x}_{Ki} = \bar{\Gamma}_{\alpha,i}\,\bar{\Phi}_{Ki}\,\frac{S_\alpha(\bar{\omega}_i,\bar{D}_i)}{\bar{\omega}_i^{\,2}} \tag{6}$$

Accordingly, the maximum contribution $\bar{G}_i$ of the i-th eigenform to any value $\bar{G} = \bar{G}\,(\bar{x}_1,\bar{x}_2,\dots,\bar{x}_K,\dots,\bar{x}_{\bar{n}})$ (e. g. force, stress, etc.) is:

$$\bar{G}_i = \bar{G}\,(\bar{x}_{1i},\bar{x}_{2i},\dots,\bar{x}_{Ki},\dots,\bar{x}_{ni}) \tag{7}$$

In (6), $S_\alpha(\bar{\omega}_i,\bar{D}_i)$ represents the ground acceleration response spectrum for the direction α, the circular eigenfrequency $\bar{\omega}_i$ and the modal damping $\bar{D}_i$.

The RSA now assumes that

$$\bar{G}_{RSA} = \sqrt{\sum_{i=1}^{\bar{n}} \bar{G}_i^2} \tag{8}$$

represents the maximum value for $\bar{G}$.

$\bar{G}_{RSA}$ may be calculated with formulae (7) and (8) using the modal displacements (6). Further, $\bar{x}_{Ki}$ may be easily determined using (6), as

$$\bar{\Gamma}_{\alpha,i}\,,\ \bar{\Phi}_{Ki}\,,\ \bar{\omega}_i\,,\ \bar{D}_i$$

are already known according to sections 3.1 and 3.2.

The relative coordinates

$$x_{rel} = x + K^{-1}K_m S \tag{9}$$

are more suitable for many problems than the absolute coordinates.

The following is valid

$$S_{ji} = \begin{cases} \Gamma^B_{\alpha,i}\,\phi^B_{ji}\,\dfrac{S_\alpha(\omega^B_i,D^B_i)}{(\omega^B_i)^2} & \text{for the building modes } (i = 1,2,\dots,n_B) \\ 0 & \text{for the component modes } (i=n_B+1,n_B+2,\dots,n_B+n) \end{cases} \tag{10}$$

If one inserts $\bar{x}_{Ki}$ and S_{ji} in (9) and applies formula (6) in conjunction with the formulae (3) and (5) for $\bar{x}_{Ki}$, one obtains:

Building modes (i = 1,2..., n_B)

$$\bar{x}_{Ki} = \sum_{j=1}^{m} \phi^B_{ji}\,\Gamma^B_{\alpha,i}\,\frac{S_\alpha(\omega^B_i,D^B_i)}{(\omega^B_i)^2}\sum_{p=1}^{n}\phi_{Kp}\,P_{pj}\,\frac{\omega_p^2}{\omega_p^2-(\omega^B_i)^2} \tag{11a}$$

Component modes (i = n_B+1, n_B+ 2,...n_B+ n):

$$\bar{x}_{K,i=\bar{1}+n_B} = \sum_{j=1}^{m}\phi_{K\bar{1}}\,P_{\bar{1}j}\,\frac{S_\alpha(\omega_{\bar{1}},D_{\bar{1}})}{\omega_{\bar{1}}^2}\sum_{l=1}^{n_B}\phi^B_{jl}\,\Gamma^B_{\alpha,l}\,\frac{(\omega^B_l)^2}{(\omega^B_l)^2-\omega^2_{\bar{1}}} \tag{11b}$$

4. Consideration of higher eigenforms

Higher eigenforms can be taken into account in formula (11) analogous to the procedure in the classical RSA.

In the case of building modes (11a), the sum

$$\sum_{p=1}^{n} \frac{\Phi_{Kp} P_{pj}}{\omega_p^2 - (\omega_i^B)^2}$$

is replaced by

$$A = \sum_{p=1}^{n_o} \frac{\Phi_{Kp} P_{pj}}{\omega_p^2 - (\omega_i^B)^2} + \sum_{p=n_o+1}^{n} \frac{\Phi_{Kp} P_{pj}}{\omega_p^2} \tag{12}$$

In the case of the component modes (11b), only the component modes $i = n_B + 1, n_B + 2, \dots, n_B + n_o$ are considered, while the remaining component modes $i = n_B + n_o + 1, n + n_o + 2, \dots, n_B + n_o$ are neglected.

We then write (12) in the form

$$A = \sum_{p=1}^{n_o} \Phi_{Kp} P_{pj} \left(\frac{1}{\omega_p^2 - (\omega_i^B)^2} - \frac{1}{\omega_p^2} \right) + (x_{Stat})_{Kj} \tag{13}$$

whereby

$$(x_{Stat})_{Kj} = \sum_{p=1}^{n} \frac{\Phi_{Kp} P_{pj}}{\omega_p^2} \tag{14}$$

represents the static solution of

$$K x_{Stat,j} = - M K^{-1} K_m e_j \tag{15}$$

Thereby the vector e_j with m components is defined by:

$$(e_j)_q = \begin{cases} 1 & \text{for } q = j \\ 0 & \text{for } q \neq j \end{cases} \tag{16}$$

To summarize, the following results:

Building modes ($i = 1, 2 \dots n_B$)

$$(x_{Ki})_{rel} = \sum_{j=1}^{m} \Phi_{ji}^B \Gamma_{\alpha,i}^B S_\alpha(\omega_i^B, D_i^B) \left[\sum_{p=1}^{n_o} \frac{\Phi_{Kp} P_{pj}}{\omega_p^2 - (\omega_i^B)^2} \left(\frac{\omega_i^B}{\omega_p^2} \right)^2 + (x_{Stat})_{Kj} \right] \tag{17a}$$

Component modes ($i = n_B + 1, n_B + 2, \dots n_B + n$)

$$\underset{i=\bar{i}+n_B}{(x_{Ki})_{rel}} = \begin{cases} \sum_{j=1}^{m} \Phi_{K\bar{i}} P_{\bar{i}j} \frac{S_\alpha(\omega_{\bar{i}}, D_{\bar{i}})}{\omega_{\bar{i}}^2} \sum_{l=1}^{n_B} \frac{\Phi_{jl}^B \Gamma_{\alpha,l}^B (\omega_l^B)^2}{(\omega_l^B)^2 - \omega_i^2} & \text{for } i = n_B+1, n_B+2, \dots n_B+n_O \\ 0 & \text{for } i > n_B + n_O \end{cases} \tag{17b}$$

5. Resonance

We now turn to the resonance case

$$\omega_{i_o} \approx \omega_{l_o}^{B}$$

Using (17), we obtain the following equations in this case:

$$(\bar{x}_{K l_o})_{rel} = \sum_{j=1}^{m} \Phi_{K i_o} P_{i_o j} S_\alpha(\omega_{i_o}, D_{l_o}^{B}) \Phi_{j l_o}^{B} \Gamma_{\alpha, l_o}^{B} \frac{1}{\omega_{i_o}^2 - (\omega_{l_o}^{B})^2} \tag{18}$$

$$(\bar{x}_{K, i_o + n_B})_{rel} = \sum_{j=1}^{m} \Phi_{K i_o} P_{i_o j} S_\alpha(\omega_{i_o}, D_{i_o}) \Phi_{j l_o}^{B} \Gamma_{\alpha, l_o}^{B} \frac{1}{(\omega_{l_o}^{B})^2 - \omega_{i_o}^2} \tag{19}$$

One sees that both $(\bar{x}_{K l_o})_{rel}$ and $(\bar{x}_{K, i+n_B})_{rel}$ increase infinitely in the limit

$$\lim_{\omega_{i_o} \to \omega_{l_o}^{B}}$$

In [1], it was shown that it is meaningful to use

$$\lim_{\omega_{i_o} \to \omega_{l_o}^{B}} \left[(\bar{x}_{K l_o})_{rel}^2 + (\bar{x}_{K, i_o + n_B})_{rel}^2 \right]^{1/2} = \frac{\Phi_{K i_o} \Gamma_{\alpha i_o} \Phi_{j l_o}^{B} \Gamma_{\alpha, l_o}^{B} S_\alpha(\omega_{i_o}, D_{l_o}^{B})}{2 \omega_{i_o}^2 \sqrt{D_{i_o} (D_{i_o} + D_{l_o}^{B})}} \tag{20}$$

in the Onefloor Response Spectrum Analysis.

We now wish to replace the factor in (18) and (19)

$$\frac{1}{\omega_{i_o}^2 - (\omega_{l_o}^{B})^2} \tag{21a}$$

by

$$\frac{1}{\omega_{i_o}^2 - (\omega_{l_o}^{B})^2 + g} \tag{21b}$$

whereby the function g is determined in such a way that (20) is analogously fulfilled ($\Gamma_{\alpha, i}$ can only be replaced by $\sum P_{i_o j}$ on the right side).

This results in

$$\lim_{\omega_{i_o} \to \omega_{l_o}^{B}} g^2 = 4 D_{i_o} (D_{i_o} + D_{l_o}^{B}) \omega_{i_o}^4 \left[1 + \frac{S_\alpha^2(\omega_{i_o}, D_{i_o})}{S_\alpha^2(\omega_{i_o}, D_{l_o}^{B})} \right] . \tag{22}$$

As one can easily show, the factor (21a) arises in the case of an undamped forced vibration, while, in the case of a damped vibration, it is replaced by (21b) with the purely imaginary function

$$g(\omega_{i_o}, \omega_{l_o}^{B}) = \underbrace{i}_{\text{imaginary unit (not to be confused with the index i!)}} \cdot 2 \omega_{i_o} \omega_{l_o}^{B} D_f(\omega_{i_o}, \omega_{l_o}^{B})$$

In this case, the fictitious damping $D_f(\omega_{i_o}, \omega_{l_o}^{B})$ must be defined as

$$D_f(\omega_{i_o}, \omega_{l_o}^{B}) = D_{i_o} (D_{i_o} + D_{l_o}^{B}) \sqrt{1 + \frac{S_\alpha^2(\omega_{i_o}, D_{i_o})}{S_\alpha^2(\omega_{i_o}, D_{l_o}^{B})}}$$

so that no inconsistency is caused to (22).

By now including the resonance case too, we have the final version of the modal displacements:

Building modes ($i = 1,2 \dots n_B$)

$$(\bar{x}_{ki})_{rel} = \sum_{j=1}^{m} \phi_{ji}^{B} \Gamma_{\alpha,i}^{B} S_{\alpha}(\omega_i^B, D_i^B) \left[\sum_{p=1}^{n_o} \frac{\phi_{kp} P_{pj} (\omega_i^B/\omega_p)^2}{\omega_p^2 - (\omega_i^B)^2 + i2D_f(\omega_p, \omega_i^B)\,\omega_p\,\omega_i^B} + (x_{stat})_{kj} \right] \tag{23a}$$

$$s_{ji} = \phi_{ji}^{B} \Gamma_{\alpha,i}^{B} S_{\alpha}(\omega_i^B, D_i^B)/(\omega_i^B)^2 \tag{23b}$$

Component modes

for $i = n_B+1,\ n_B+2,\ \dots\ n_B+n_O$:

$$(\bar{x}_{k,i=\bar{1}+n_B})_{rel} = \sum_{j=1}^{m} \phi_{k\bar{1}} P_{\bar{1}j} \frac{S_{\alpha}(\omega_{\bar{1}}, D_{\bar{1}})}{\omega_{\bar{1}}^2} \left[\sum_{1=1}^{n_B} \frac{\phi_{j1}^{B} \Gamma_{\alpha,1}^{B} (\omega_1^B)^2}{(\omega_1^B)^2 - \omega_{\bar{1}}^2 + i2D_f(\omega_{\bar{1}}, \omega_1^B)\,\omega_{\bar{1}}\,\omega_1^B} \right] \tag{23c}$$

for $i > n_B + n_O$:

$$(\bar{x}_{ki})_{rel} = 0 \tag{23d}$$

for $i > n_B$:

$$s_{ji} = 0 \tag{23e}$$

The modal displacements (23) are complex. Formula (8) must therefore be replaced by

$$\sigma_{RSA} = \sqrt{\sum_{i=1}^{\bar{n}} |\bar{\sigma}_i|^2}$$

6. Conclusions

The present report contains a consistent Response Spectrum Analysis which, as far as we know, has been formulated in this way for the first time. A consistent RSA is so important because today this method is applied as an important tool for the earthquake proof of components in nuclear power plants.

The present report, however, still requires one completing point, namely the statistic consideration of the RSA superimposition procedure. The RSS (Root Sum Square) superimposition (8) of the individual modes was simply taken as being correct outside the resonance ranges, while the resonance contributions were arranged in such a way that they are in agreement with the preceeding statistic considerations [1] if the superimposition applied is of the same form as that ouside the resonance ranges.

A further task is the programming of the present theory and the calculation of some examples to be able to indicate the quantitative differences to previous practice.

References

[1] D. Schmitz, K. Peters, "Direct Evaluation of Floor Response Spectra from a Given Ground Response Spectrum", 4th SMIRT, K4/10

[2] D. Schmitz, K. Peters, "A Consistent Response Spectrum Method", 6th SMIRT, K10/5

[3] Bathe, Wilson, Peterson, EERC Report No. 72-10, Nov. 72, Berkeley, Calif.