

## RATCHETING PHENOMENA

H. Hübel

Siemens AG, Friedrich-Ebert-Str., Bergisch Gladbach, Germany

### ABSTRACT

This paper is intended to provide an overview of different aspects of ratcheting under cyclic loading below the creep range. It distinguishes between material ratcheting and structural ratcheting, each being characterized by several different phenomena which appear in different configurations of materials, states of stress, structural geometries and loadings. The systematic compilation of these phenomena presented in the paper may help to improve understanding between material researchers, developers of inelastic methods of analysis, structural analysts and design code committees. Above all, a certain degree of knowledge about the different mechanisms of ratcheting is important for a structural analyst to be able to choose an appropriate analytical method for assessing the ratcheting phenomena involved in a specific design problem.

### 1 INTRODUCTION

Component strength analyses based on a nuclear design code (e.g. ASME Code Section III, KTA Safety Standards or RCC-M) require performance of a ratcheting analysis. In terms of calculational effort, ratcheting analyses are difficult.

As a phenomenon of inelastic material and structural behavior, ratcheting can, in principle, be investigated by means of detailed inelastic analyses, in which the evolution of stress and strain is calculated for a given load histogram on a step-by-step basis. The final objective of a detailed inelastic analysis of ratcheting is to demonstrate that the strains accumulated in one direction during the anticipated service life of a structure do not exceed the strain limits set by design codes. Clearly, the analyst must ensure that the constitutive law employed is capable of quantifying the specific ratcheting phenomena which he or she wishes to assess with the analysis. The methodology of detailed inelastic analysis involves relatively high engineering and computational costs.

Therefore simplified methods of analysis are desirable. Although a large number of simplified inelastic methods of analysis exist in the literature, their simplification compared with detailed inelastic analyses really only comprises a (usually quite considerable) reduction in computer costs - something which, however, can often only be achieved after a larger amount of engineering effort has first been invested (see Refs. 1 and 2 for a definition of "detailed", or "rigorous", vs. "simplified" inelastic methods).

It does not, therefore, come as a surprise that the methods given in the

codes and standards for ratcheting analysis are not detailed insofar as, for example, only a limited number of theoretically possible ratcheting mechanisms are examined, that certain methods of analysis apply only to specific geometries and loading configurations, and that certain material behavior characteristics which have a significant effect on ratcheting (such as kinematic hardening) are not considered. Such limitations naturally necessitate the incorporation of a considerable degree of conservatism in these methods which in turn can mean that a component might "fail" a ratcheting analysis performed with such a method while a closer examination (by a detailed elastoplastic analysis, for example) would reveal the existence of large margins to ratcheting.

The aim of the present paper is to describe various phenomena of plastic ratcheting (creep ratcheting is disregarded). This may help to improve understanding between material researchers, developers of material models (constitutive equations) or simplified inelastic methods of analysis, structural analysts and design code committees. Most of all the paper addresses the structural analyst who must select analytical methods (e.g. constitutive equations or simplified inelastic methods) appropriate for the ratcheting phenomena involved in a specific design problem.

## 2 DEFINITION OF TERM "RATCHETING"

There is no widely accepted definition of the term "ratcheting". Some structural analysts associate with ratcheting an increase in strain by a constant amount in each loading cycle - an idea supported by some nuclear design codes providing simplified methods to guard against ratcheting based on an elastic-perfectly plastic material model (e.g. Bree diagram). Others use the term "ratcheting" if the strain accumulated cycle-by-cycle tends towards infinity for an infinite number of cycles (which does not necessarily imply a constant strain increment in each cycle). Material researchers seem to prefer the interpretation that "ratcheting" means any accumulation of strain, even if the rate of accumulation decreases so rapidly with the number of cycles that stress-strain hysteresis reaches a steady state after a finite number of cycles - a situation often characterized by structural analysts as "plastic shakedown after some progressive deformation (or transient ratcheting) has occurred".

In the following, "ratcheting" is used synonymously with "progressive deformation". It occurs if, when a structure is subjected to cyclic loading, the mean strain (arithmetic mean of maximum and minimum strain during one loading cycle) changes during any one loading cycle at at least one point of this structure with respect to the mean strain induced during the preceding cycle due to inelastic material behavior. This does not rule out the possibility that the increase in mean strain during successive loading cycles decreases and even ultimately stops so that a state of saturation is approached or reached which is characterized by elastic or plastic shakedown (alternating plasticity). In order to characterize such a situation in which the accumulation of strain is bounded (even after an infinite number of cycles), the term "finite ratcheting" will be used.

Strain accumulation is, by definition, ruled out in strain-controlled loading conditions. In a configuration of material and geometry, etc., having potential for ratcheting (under displacement-controlled, stress-controlled or force-controlled loading), ratcheting will then manifest itself as a cycle-to-cycle change in the mean stress.

Structures exhibiting a homogeneous distribution of stress (i.e. the multiaxial state of stress and the level of stress are the same at each location of the structure), e.g. tension bars, are used as test specimens by material researchers to examine purely material behavior. In terms of

continuum mechanics, this means that the field equations (equilibrium and continuity conditions) defining the behavior of a structure in conjunction with the constitutive equations are a priori identically fulfilled so that these material tests can be used to set up constitutive equations (material models). Ratcheting appearing in material tests is termed "material ratcheting". It must be accounted for in constitutive equations. Furthermore, structural analysts need to be aware of the different phenomena of material ratcheting so that they are able to choose a suitable material model if ratcheting is to be assessed by a detailed inelastic analysis for a specific design problem.

Ratcheting phenomena which do not appear in material tests but require a nonhomogeneous distribution of stress are categorized as "structural ratcheting". Simplified inelastic methods of analysis aim at accounting for these effects which should be reflected in design codes. Structural analysts must have an understanding of these phenomena in order to be able to select a simplified method suitable for assessing ratcheting of a specific structural configuration.

### 3 MATERIAL RATCHETING

Material ratcheting is a form of ratcheting which can occur in the absence of structural effects, i.e. if stress is distributed homogeneously in a structure, and is thus a purely material-related effect which can be observed in material tests and modeled in constitutive equations. Some of the different mechanisms can be clearly illustrated in the stress space using a continuum mechanics approach.

The following assumptions are usually made for constitutive equations of metallic materials below the creep range:

- There exists a yield surface which encloses the elastic stress space.
- The yield surface is isotropic.
- The material satisfies Drucker's stability criteria (the yield surface must thus be convex).
- The hydrostatic state of stress does not affect the plastic behavior (so that we can concentrate on examining deviatoric states of stress  $\sigma'$ ).

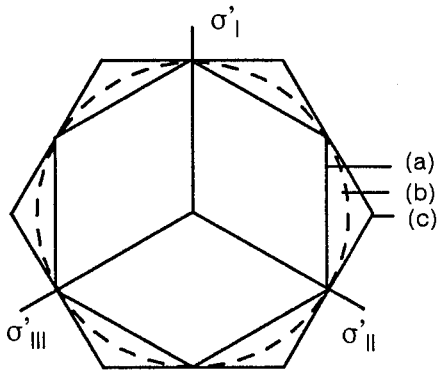
If, in addition to the above assumptions, the material displays identical behavior in tension and compression (i.e. no strength-differential effect), the yield surface has six axes of symmetry in the deviatoric stress space and must be enclosed by boundary lines (a) and (c) in Figure 1.

If we further assume a flow rule which is associated with the yield surface, the direction of a plastic strain increment will follow the normal to the yield surface at the associated stress point on the yield surface (normality rule). If cyclic loading is applied and if this loading causes elastically proportional changes in stress during a cycle, the loading path will pass through the origin of the stress space (Fig. 2). The directions of the plastic strain increments  $d\epsilon^{Pl}(1)$  and  $d\epsilon^{Pl}(2)$  at the maximum and minimum values of the loading cycle are exactly opposed. It is thus impossible for strain to accumulate in the direction normal to the loading path, i.e. ratcheting does not occur.

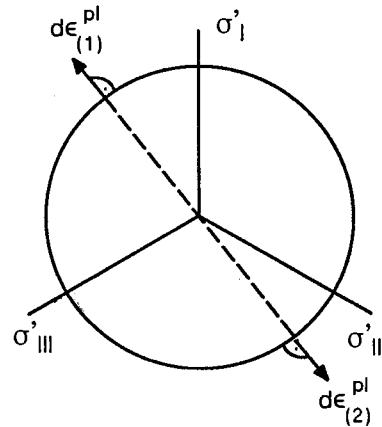
If the material is elastic-perfectly plastic, the yield surface does not change during inelastic action. If, however, the material hardens, the yield surface changes during inelastic processes. It can expand (isotropic hardening), move (kinematic hardening), and deform.

#### 3.1 Isotropic Hardening

If the hardening contains an isotropic portion, i.e. if the cyclic yield strength increases with the number of cycles, this can lead (Fig. 3) in the

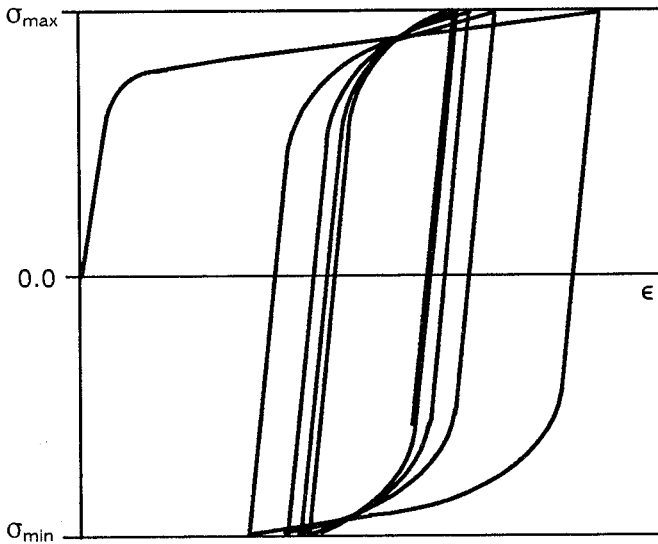


**Fig. 1:** Yield Surfaces in Deviatoric Stress Space:  
(a) Tresca, (b) Mises, (c) Exterior Limiting Yield Surface



**Fig. 2:** Direction of Plastic Strain Increments under Proportional Loading (Mises Yield Surface)

case of stress-controlled loading to an increase in the mean strain per cycle (while the maximum strain decreases) since the state of hardening at the maximum and minimum values of one loading cycle differs from the state of hardening at these values during the next cycle.



**Fig. 3:** Ratcheting (Referred to Mean Strain) under Cyclic Loading due to Isotropic Hardening;  $\sigma_{\min} = -\sigma_{\max}$

The strain accumulation is limited (finite ratcheting) because, in real materials, the proportion of isotropic hardening in the total hardening is limited and because isotropic hardening does not continue beyond a certain number of cycles.

This effect is accounted for (at least in qualitative terms) by all material models which describe isotropic hardening (even if only in the simplest of manners, such as in the case of the ORNL model of Ref. 3 in which all isotropic hardening is introduced at once during the first reversed yielding).

### 3.2 Irreversible Elastic Straining

It has been experimentally observed that unloading (from point A to point C in Fig. 4) may produce only elastic strains, but reloading produces plastic strains if carried beyond the level represented by point B in Figure 4 (Ref. 4). Thus strain is accumulated in each loading cycle. This effect may

be related to the Bauschinger effect (Ref. 5). Krempl and coworkers report (e.g. in Ref. 6) that time dependence of material behavior (viscoplasticity of austenitic steel at room temperature) is the driving force for that ratcheting phenomenon.

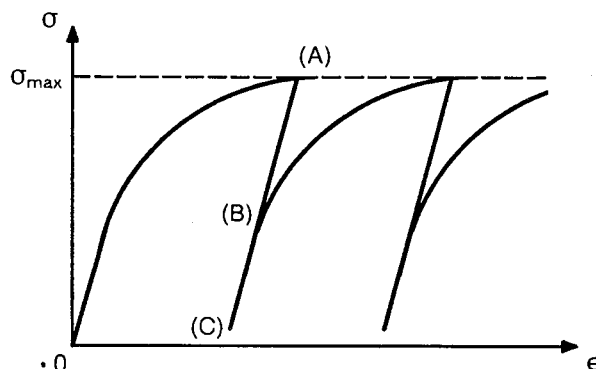


Fig. 4: Ratcheting due to Irreversible Elastic Straining

Material models can describe this phenomenon in a variety of ways: for example, by introducing a restraint on the movement of the yield surface by a so-called "α-reset" procedure (Ref. 7), or by means of a dynamic recovery term in the evolution law of the kinematic hardening variable (e.g. Chaboche model, Ref. 8), or by separating the yield and loading surfaces (multisurface theories, e.g. Ref. 4) or by using different flow velocities as a function of the rate of overstress (Interatom model, Ref. 9).

### 3.3 Temperature-History Effects

If the inelastic material properties are temperature-dependent, a temperature path history may cause ratcheting if the material does not respond instantaneously as under isothermal conditions at a given temperature following inelastic excursions at other temperatures (Refs. 10 and 11). Whether a material is temperature-history-dependent or temperature-history-independent may be examined in thermomechanical material tests. This phenomenon can be modeled in constitutive equations by adopting temperature-dependent terms in evolution laws of kinematic and isotropic hardening.

Let us assume, for example, a linear kinematic hardening model. For a uniaxial state of stress the kinematic hardening variable  $\alpha$  (position of center of yield surface in stress space) changes under monotonic isothermal loading with

$$d\alpha = C \, d\varepsilon^p \quad ; \quad C = \text{constant (plastic modulus)} \quad (1a)$$

which can be integrated as follows (with  $\alpha = 0$  if  $\varepsilon^p = 0$ ):

$$\alpha = C \, \varepsilon^p \quad (1b)$$

If one now considers non-isothermal loading, we may introduce in Eq. (1a) a temperature-dependent term (Ref. 12):

$$d\alpha = C(T) \, d\varepsilon^p + h(T) \, \alpha \, dT \quad (2)$$

For  $h(T) = 0$  we have

$$d\alpha = C(T) \, d\varepsilon^p \quad (3)$$

which cannot be integrated directly. Assuming

$$h(T) = \frac{\partial C(T)}{\partial T} \frac{1}{C(T)}$$

we have

$$da = C(T) d\varepsilon^{pl} + dC(T) \varepsilon^{pl} \quad (4a)$$

$$a = C(T) \varepsilon^{pl} \quad (4b)$$

Since Eq. (4b) is identical to Eq. (1b), it cannot describe a temperature-history dependence. Note that kinematic hardening changes even during elastic action ( $d\varepsilon^{pl} \neq 0$ ) after prior plastic straining ( $\varepsilon^{pl} = 0$ ).

Both Eq. (3) and Eqs. (4) are frequently used in practical applications. Eq. (3) is implemented in the finite element program ADINA (Ref. 13), while Eqs. (4) are used in ABAQUS (Ref. 14). Their differences become obvious if we consider the stress-controlled loading of Figure 5 which, like the temperature, is applied cyclically (but phase-shifted for the sake of greater clarity) and is strictly symmetric in tension and compression ( $\sigma_{\min} = -\sigma_{\max}$ ). The elastic-plastic modulus is  $E_{th}$  at the "hot" temperature and  $E_{tc}$  ( $> E_{th}$ ) at the "cold" temperature.

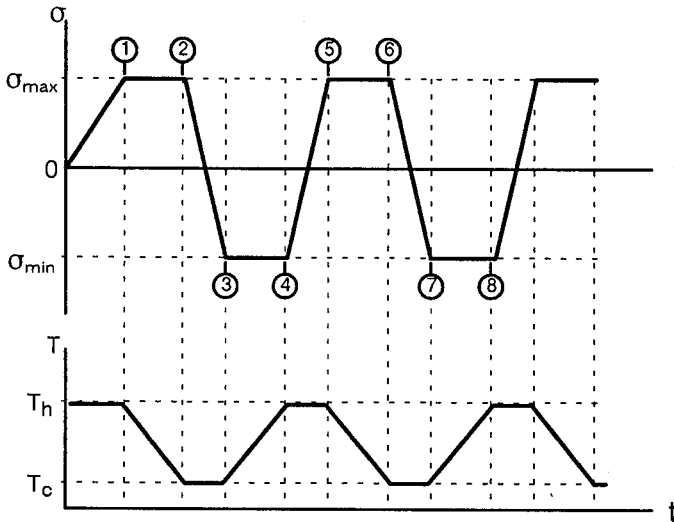


Fig. 5: Stress and Temperature Histogram;  $\sigma_{\min} = -\sigma_{\max}$

With Eq. (3) we obtain (infinite) ratcheting as can be seen from the stress-strain response in Figure 6a and the movement of the yield surface in the stress space in Figure 6b. The direction of strain accumulation is determined by the direction of the initial loading. Ratcheting is not obtained with Eqs. (4) (Figs. 7a and 7b) but instead a closed stress-strain loop in which the maximum and minimum strains are strictly symmetric in tension and compression ( $\varepsilon(t_4) = -\varepsilon(t_2)$ ) and the yield stress in compression appears to be reduced as follows:

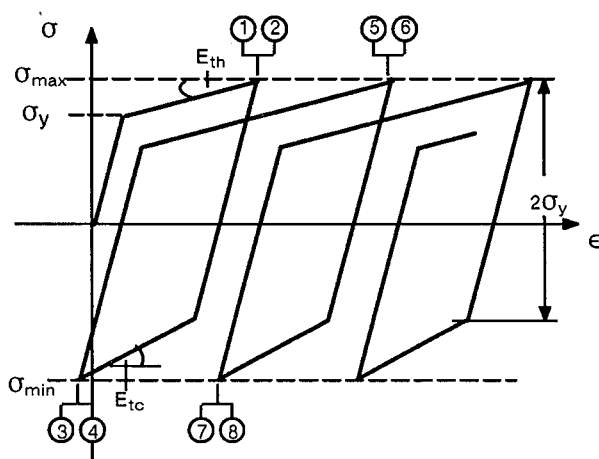
$$a = 2\sigma_y - \varepsilon^{pl}(t_1) \cdot (C_c - C_h)$$

as long as  $a \geq 0$  (for  $a < -2\sigma_y$ , i.e. for a large initial plastic strain, compressive yielding occurs at the maximum stress during the time period between  $t_1$  and  $t_2$ ). The strain amplitude in Figure 7a coincides with the larger of the two different strain amplitudes in two consecutive half cycles in Figure 6a.

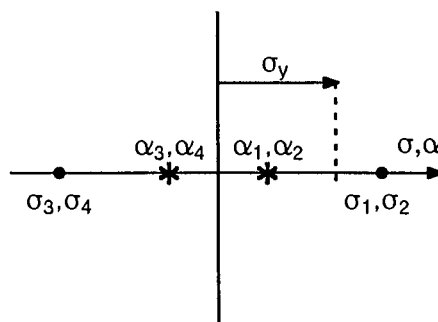
It is interesting to note that a temperature dependence of yield stress does not cause ratcheting.

### 3.4 Shape of Yield Surface

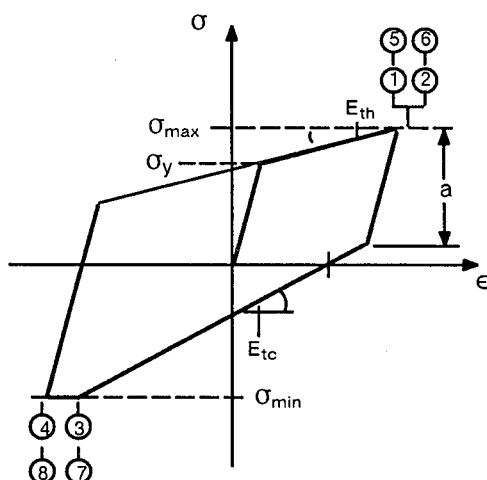
So far, only uniaxial states of stress have been examined. In the following, ratcheting phenomena will be presented which only arise with



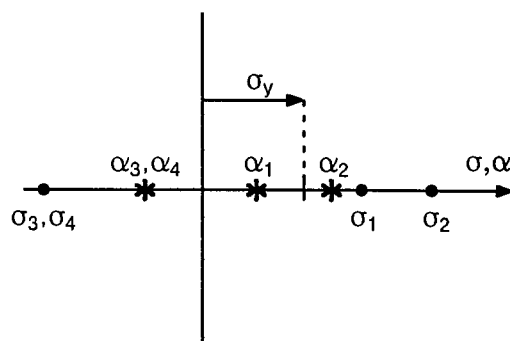
**Fig. 6a:** Stress-Strain Response with Eq. (3)



**Fig. 6b:** Position of Yield Surface with Eq. (3)



**Fig. 7a:** Stress-Strain Response with Eq. (4)



**Fig. 7b:** Position of Yield Surface with Eq. (4)

multiaxial states of stress. In this section it will continue to be assumed that the changes in stress are proportional, i.e. that they lie on a straight line which passes through the origin of the stress space (cf. Fig. 2). This is the case, for example, with thin-walled tubes subjected to cyclic stress-controlled torsional loading. The role played by the shape of the yield surface in respect of a potential ratcheting mechanism will be investigated.

Material models for metals do not, as a rule, account for such effects because they are usually based on a Mises yield surface.

### 3.4.1 Strength-Differential Effects (SDE)

If the yield strength in tension is different from that in compression (SDE), three of the axes of symmetry of the yield surfaces shown in Figure 1 disappear. As long as it retains its isotropy and convexity, the

yield surface can, under exceptional circumstances, assume the shape shown in Figure 8.

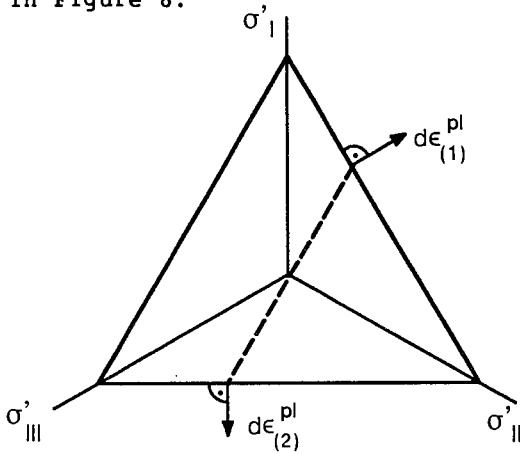


Fig. 8: Yield Surface with SDE

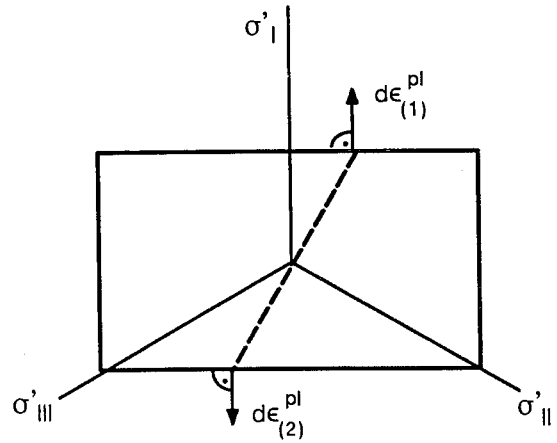


Fig. 9: Yield Surface with Anisotropy

If plastic flow can still be assumed to be associated, the direction of a plastic strain increment will be normal to the yield surface. It can be seen in Figure 8 that the directions of the plastic strain increments at the maximum and minimum values, (1) and (2), of the loading cycle are no longer exactly opposed and that with each cycle there is a strain increase in the direction normal to the loading path. This therefore leads to ratcheting (which is infinite even if the material hardens kinematically), causing a tube to increase in length under cyclic torsional loading.

#### 3.4.2 Anisotropy

If the material is anisotropic, certain symmetry conditions of the yield surface shown in Figure 1 likewise disappear. Under exceptional circumstances, the yield surface could assume something like the shape shown in Figure 9 (which still satisfies the convexity condition in the absence of SDE).

Still assuming associated flow (so that the normality rule holds), the directions of the strain increments at the maximum and minimum values of the loading path are exactly opposed. Anisotropy cannot therefore be cited as a cause of ratcheting. It can, however, enhance the effect of ratcheting resulting from other causes.

#### 3.4.3 Second-Order Effects

Elongation of a tube under torsion observed experimentally is attributed to so-called second-order effects. It has long been known that, under cyclic loading, second-order strain accumulation can occur in isotropic materials not exhibiting SDE (Ref. 15). A quadratic relation between axial strain accumulated and the shear strain amplitude was found experimentally to be characteristic of this effect (Ref. 16). The axial strain accumulated can, after a certain number of cycles, become larger than the shear strain applied - even with small ranges of shear strain.

A continuum-mechanics interpretation is that the description of the yield surface by the classical theory of plasticity (e.g. by Mises and Tresca) is incomplete (Refs. 16 and 17). In fact, the yield surface can display certain, if only relatively minor, irregularities even if the six conditions of symmetry (cf. Fig. 1) in the absence of SDE and anisotropy are retained. They can be accounted for by introducing in the yield condition the third invariant of the deviatoric stresses (omitted in the



Mises criterion) in a particular manner (different from the Tresca criterion), maintaining the assumption of associated flow (and the normality rule).

### 3.5 Non-Proportional Loading

Again, a multi-axial state of stress will be examined, except that in this case it does not change proportionally but is characterized by two mutually independent stresses, as shown in the example of Figure 10.

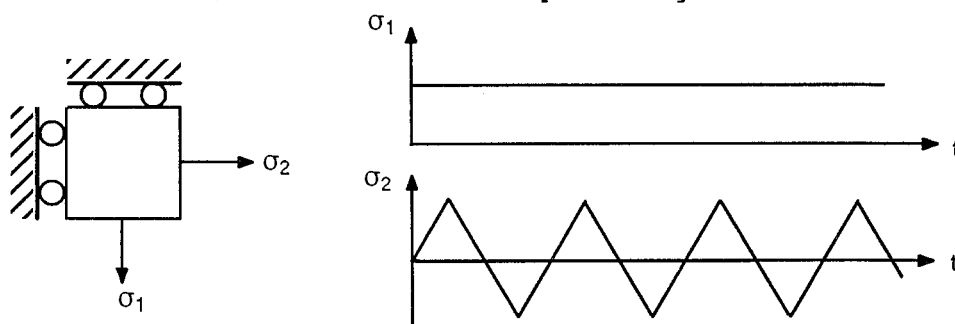


Fig. 10: Example of Non-Proportional Loading

This produces the following situation in the stress space (Fig. 11):

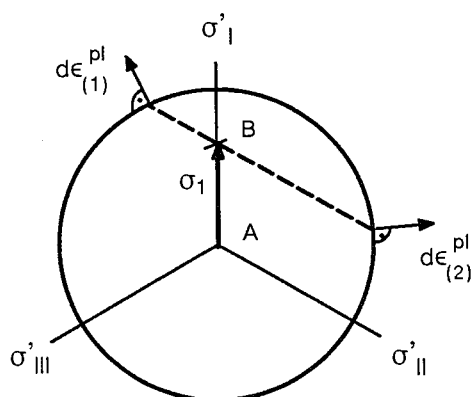


Fig. 11: Non-Proportional Loading in Stress Space (Mises Yield Surface)

Due to the non-proportionality of the cyclic loading path, the strain increments at the maximum and minimum values of the loading cycle have a common component normal to the loading path, thereby producing an increase in strain in that direction with each cycle. This ratcheting mechanism is covered by all material models.

As long as there is no hardening, ratcheting will be infinite. If the material hardens only isotropically, ratcheting will also be infinite, since isotropic hardening is limited in real materials. If kinematic hardening takes place, the center of the yield surface gradually shifts from point A to point B. During this process which, depending on the load level and the material parameters, can last for a very large number of cycles, the strain increase becomes smaller with each cycle. Strain stops accumulating when the center of the yield surface reaches point B (finite ratcheting).

This ratcheting mechanism can be more pronounced if the material is anisotropic or if the yield strength is temperature-dependent (although Secs. 3.3 and 3.4.2 showed that anisotropy and a temperature-dependent yield strength do not produce any ratcheting mechanism).

If a cyclic shear strain  $\epsilon_{12}$  is applied instead of a cyclic stress  $\sigma_2$ , this case corresponds to that of a thin-walled tube subjected to a constant axial force and cyclic displacement-controlled torsion. This forms the basis of the ratcheting efficiency diagrams (Ref. 18). However, other material ratcheting phenomena are also involved in the tests upon which these diagrams are based, e.g. isotropic hardening (Sec. 3.1), irreversible elastic straining (Sec. 3.2) and second-order effects (Sec. 3.4.3).

#### 4 STRUCTURAL RATCHETING

Structural ratcheting (cf. Sec. 2) can occur due to inelastic material behavior under cyclic loading even if there is no material ratcheting. It is possible, for example, with a purely kinematic hardening material in uniaxial stress states at a constant temperature. This type of ratcheting is actually produced by the nonhomogeneity of the state of stress in a structure. It can be examined by detailed inelastic analyses or by simplified inelastic analyses (Ref. 19).

In accordance with Reference 20, a distinction is made between two mechanisms of structural ratcheting: Type A and Type B. These two types can be distinguished by imagining that a volume  $V_F$  of material is removed from a structure.  $V_F$  is the volume in which, in the absence of a primary stress, the conditions of elastic shakedown under cyclic secondary stress cannot be satisfied. If the reduced structure is now incapable of carrying any applied load whatsoever, we have a Type B problem. If it can carry some applied load, regardless of the size of the secondary stress range, we have a Type A problem. If it can carry some applied load, but only within a specific secondary stress range, we have a transitional Type A/B problem.

Whether or not a ratcheting mechanism develops in a particular configuration of structure geometry, type of loading and load level, and which type of ratcheting is involved, can be illustrated in a ratcheting interaction diagram (Fig. 12) based on a maximum secondary stress  $\sigma_t$  and a (constant) primary stress  $\sigma_p$ , both normalized to the yield stress  $\bar{\sigma}_y$ . Such diagrams can be established by means of detailed or simplified inelastic analyses. They cannot, however, provide any information on strains accumulated in a certain number of cycles and can thus not be used to satisfy inelastic strain limits set by design codes. Nevertheless, they do turn out to be a useful design tool because they indicate a potential for excessive strain accumulation.

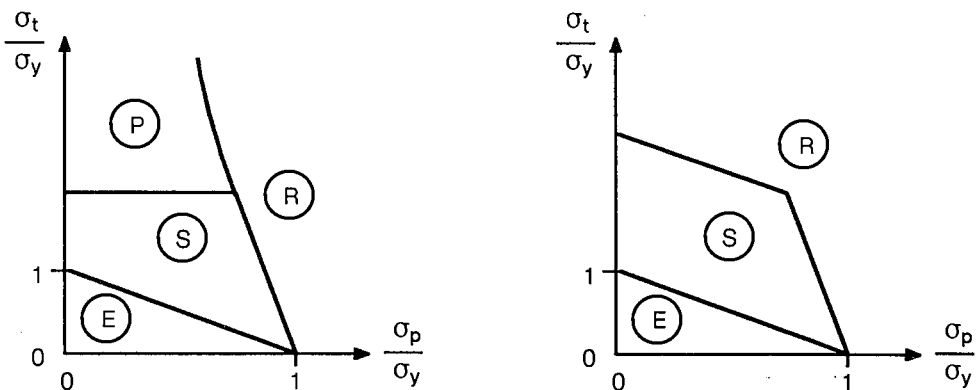


Fig. 12: Ratcheting Interaction Diagrams Typical of Type A (Left Side) and Type B (Right Side)

If the loading parameters are in region E, the structure responds entirely elastically. In region S the structure shakes down elastically

after some plastic straining during the first cycle. In region P plastic shakedown occurs (alternating plasticity), while ratcheting occurs in region R. It should be noted that a missing region P is typical of Type B problems and that the border between regions P and R intersects the  $\sigma_t/\sigma_y$ -axis somewhere above the S region in transitional Type A/B problems so that ratcheting is possible without a primary stress - a feature which has found little attention in design codes to date.

Figure 12 is based on an elastic-perfectly plastic material model. It is interesting to note that, contrary to widespread opinion, this may not be a conservative approximation of a hardening material for ratcheting analyses (Ref. 23).

Ratcheting interaction diagrams become more complicated than indicated in Figure 12 if constructed employing a hardening material model. With an elastic-perfectly plastic material model, strain accumulates infinitely in region R (constant increment per cycle). If hardening is introduced, elastic or plastic shakedown may eventually occur so that strain accumulation is bounded (finite ratcheting).

#### 4.1 Examples of Type A

A cylinder subjected to internal pressure and cyclic radial temperature gradients, which forms the basis of the Bree diagram (Ref. 22) adopted in several design codes (e.g. ASME III-NB, RCC-M and KTA 3201.2) is a practical example of a Type A problem.

An elementary example studied by many authors to illustrate the performance of simplified inelastic analysis methods is the configuration shown in Figure 13 in which two parallel bars of different cross sections are attached to a rigid plate restrained against rotation, the plate being subjected to a constant force causing a primary stress  $\sigma_p$  in the bars. Bar 1 is loaded with a cyclic thermal strain ( $\sigma_t$  being the maximum thermoelastic stress).

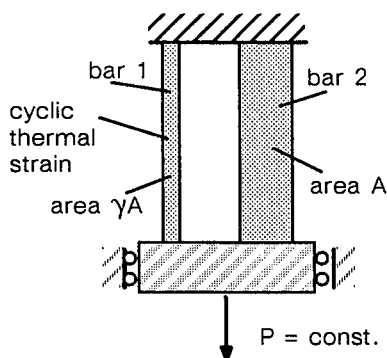


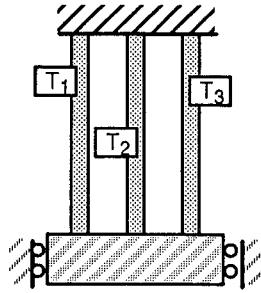
Fig. 13: Elementary Example of Structure-Load Configuration Exhibiting Type A Ratcheting

Ratcheting is characterized here by tensile yielding in bar 2 and elastic behavior in bar 1 during one half cycle, and elastic behavior in bar 2 and tensile yielding in bar 1 during the subsequent half cycle. Strain accumulates in the direction of the primary stress - regardless of the sign of the first thermal loading.

#### 4.2 Examples of Type B

A practical example of a Type B problem is a cylinder subjected to an axial load and an axial temperature history which fluctuates between a uniform temperature and a maximum temperature (Ref. 20). Another practical example is a cylinder subjected to a moving axial temperature gradient of constant level (Ref. 20).

The three-bar model of Figure 14, in which the bars (of equal cross sections) are subjected successively and periodically (period  $t_0$ ) to a thermal strain (temperatures  $T_1$ ,  $T_2$ ,  $T_3$ ) is an elementary example of a Type B problem.



Loading:  $T_1$  ( $0 \leq t \leq \frac{1}{3}t_0$ )

$T_2$  ( $\frac{1}{3}t_0 \leq t \leq \frac{2}{3}t_0$ )

$T_3$  ( $\frac{2}{3}t_0 \leq t \leq t_0$ )

**Fig. 14: Elementary Example of Structure-Load Configuration Exhibiting Type B Ratcheting**

## 5 CONCLUSIONS

A summary has been given in this paper of different ratcheting phenomena (as defined in Sec. 2). It is useful to characterize some phenomena as material ratcheting and others as structural ratcheting. The former can be examined in appropriate material tests and theoretically described by constitutive equations (used for detailed inelastic analyses), while the latter can be identified by detailed or simplified inelastic analyses of a structure.

Some of the material ratcheting phenomena can be deduced from standard material tests (e.g. isotropic hardening, Sec. 3.1), while others require nonstandard experimental techniques and have therefore so far attracted little attention for many nuclear materials (e.g. temperature-history effects, Sec. 3.3, and second-order effects, Sec. 3.4.3). A difficulty encountered in identifying the different phenomena of material ratcheting is that not all of them can be isolated in material tests since some of them appear simultaneously in real materials, meaning that careful interpretation of the test results is necessary.

Even simple constitutive equations such as a linear kinematic hardening model are capable of reflecting some of the material ratcheting phenomena (e.g. due to non-proportional loading, Sec. 3.5), while a relatively complicated theoretical framework is required to describe other phenomena, and is still the subject of current research (e.g. ratcheting due to irreversible elastic straining, Sec. 3.2, cf. Refs. 23 and 24).

Simplified inelastic analysis methods often merely aim at identifying whether or not a structural ratcheting mechanism can develop in a given structural geometry and loading configuration, employing an elastic-perfectly plastic material model. However, they cannot provide estimates of the strain accumulated in a certain number of cycles as is needed to satisfy limits on elastic strain set by design codes. Only a few simplified inelastic methods attempt to quantify accumulated strains and account for material hardening having a large effect on strain accumulation (Ref. 19).

The methods for analysis of structural ratcheting set forth in the design codes appear limited. The simplified inelastic methods offered therein often consider only Type A ratcheting (Sec. 4) and are only valid for a specific geometry and loading configuration without accounting for the (mostly beneficial) effects of material hardening (e.g. Bree diagram, Sec. 4.1) and without quantifying inelastic strains accumulated. As far as detailed inelastic analyses are concerned, design codes rarely give guidance on the choice of constitutive equations for use in a potential ratcheting situation. Moreover, there is a large potential for improving the rather crude 1%, 2% and 5% strain limits in these codes (Ref. 25).

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