MECHANICAL FUEL-CAN INTERACTION EFFECTS ON FUEL ELEMENT THERMAL FREQUENCY RESPONSE

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ABSTRACT

A linear thermoelastic model has been used in a study of the effect of including the contact pressure dependence of the fuel can contact conductance, in the equations describing both the symmetric thermal frequency response to power and coolant or heat transfer fluctuations, and the asymmetric static problem of slightly tilted power and coolant temperature distributions. In the latter, analytical solution was feasible and formulae for the can surface heat flux are given. The former was reduced to coupled equations by Galerkin's method and solved by a general linear analysis computer code.

A specific example is discussed. In both cases the inclusion of the pressure dependent conductance was most significant for internal power rather than coolant temperature changes, and was influenced by the temperature dependent fuel conductivity. Errors of the order of 10% due to neglecting this effect were obtained for the example chosen.

The Galerkin method, using a third order polynomial for can temperature, and a sixth order even polynomial for the fuel was found to be adequate over a frequency range from 10^{-3} to 10^{2} radians per second.

1. INTRODUCTION

In thermal reactor fuel elements designed to have, under operating conditions, contact between fuel pellets and can in the individual fuel pins, symmetric temperature pin profiles are dependent on the magnitude of the fuel-can contact conductance, an uncertain function of contact pressure and gas gap properties. For situations other than static symmetry, variations in space and time of this conductance could be significant for detailed thermal analyses. The linear dynamics modelling of fuel element and coolant in water cooled reactors, and the static sub-channel performance analysis of a fuel pin cluster are such situations. In the latter, small temperature and surface heat flux differences are important in a detailed study of subcooled boiling, to locate points of bubble inception, bubble departure and bulk boiling.

The work reported here is an estimate of the effect of pressure dependent conductance for a particular pin geometry with specified thermal properties, under conditions of small symmetric dynamic, and small asymmetric static perturbations. A realistic fuel model must include temperature dependent conductivity, but no attempt was made to construct an elaborate rheological model. Linear thermoelasticity was assumed appropriate for a frequency response analysis, but a temperature dependence of the fuel element coefficient of linear expansion was included.

A further incentive for this study was the possibility of using frequency response or statistical noise analysis techniques as a diagnostic tool for identifying and quantitatively evaluating contact conductance in an experimental thermal fuel-can-coolant assembly.

2. PROBLEM FORMULATION

With zero axial heat conduction, no heat generation in a can of constant properties, and constant fuel heat capacity, a fairly general statement of the thermal problem of a canned solid cylindrical fuel pin is as follows:-

$$\nabla\cdot k(T)\nabla T(r,\theta,t) + Q(r,\theta,t) = \rho c\frac{\partial T(r,\theta,t)}{\partial t} \quad ; \quad 0 \leq r < a \tag{1}$$

$$k'\nabla^2 T'(r,\theta,t) = \rho' c'\frac{\partial T'(r,\theta,t)}{\partial t} \quad ; \quad a < r < b \tag{2}$$

$$-k(T)\frac{\partial T}{\partial r} = -k'\frac{\partial T'}{\partial r} = C(T,p)\,\{T^{-}(\theta,t) - T^{+}(\theta,t)\} \quad ; \quad r = a \tag{3}$$

$$k'\frac{\partial T'}{\partial r} + H(\theta,t)T' = H(\theta,t)\,T^{*}(\theta,t) \quad ; \quad r = b \tag{4}$$

$$p = F(T,T') \tag{5}$$

It is now assumed that in the steady state, small non-uniformities in Q, H and T^* exist, giving rise to temperatures varying with θ, and that thermal transients are excited by small symmetric perturbations. The distributions may therefore be represented as follows:-

$$Q(r,\theta,t) = Q_0(r) + \delta Q(r,\theta) + \delta Q(r,t)$$

$$T(r,\theta,t) = T_0(r) + \delta T(r,\theta) + \delta T(r,t)$$

$$T^*(\theta,t) = T_0^* + \delta T^*(\theta) + \delta T^*(t)$$

$$H(\theta,t) = H_0 + \delta H(\theta) + \delta H(t)$$

Allowing for the contact pressure dependence of the conductance at the fuel-can interface, and writing

$$C = C_0(T_0,p_0) + \left(\frac{\partial C}{\partial p}\right)_0 \{\delta p(\theta) + \delta p(t)\} \tag{6}$$

$$\text{with} \quad \delta p(\theta) = F\,(\delta T(r,\theta),\ \delta T'(r,\theta)\,)$$
$$\text{and} \quad \delta p(t) = F\,(\delta T(r,t),\ \delta T'(r,t)\,),$$

linearization of the governing equations for the prediction of $\delta T(r,t)$ and $\delta T(r,\theta)$, leads to three problems which may be stated as follows:-

(a) Symmetric steady state problem;

$$\nabla \cdot k(T_o)\nabla T_o + Q_o(r) = 0 ; \qquad 0 \leq r < a \qquad (7)$$

$$k'\nabla^2 T_o' = 0 ; \qquad a < r < b \qquad (8)$$

$$-k(T_o)\frac{dT_o}{dr} = -k'\frac{dT_o'}{dr} = C_o(T_o^- - T_o^+); \qquad r = a \qquad (9)$$

$$k'\frac{dT_o'}{dr} + H_oT_o' = H_oT_o^* ; \qquad r = b \qquad (10)$$

(b) Asymmetric static perturbation problem;

$$\nabla^2(k(T_o)T) + Q(r,\theta) = 0 \quad ; \quad 0 \leq r < a \qquad (11)$$

$$k'\nabla^2 T' = 0 \quad ; \quad a < r < b \qquad (12)$$

$$-\frac{d}{dr}(k(T_o)T) = -k'\frac{dT'}{dr} = C_o(T^- - T^+) + \left(\frac{\partial C}{\partial p}\right)_o \Delta T_o p(\theta) \quad ; \quad r = a \qquad (13)$$

$$k'\frac{dT'}{dr} + H_oT' = H_oT^*(\theta) \quad ; \quad r = b \qquad (14)$$

where $\Delta T_o = (T_o^- - T_o^+)$ at $r = a$, and $T^*(\theta)$ can be identified with

$$\delta T^*(\theta) + (T_o^* - T_o')\frac{\delta H(\theta)}{H_o}$$

(c) Symmetric transient perturbation problem;

$$\nabla^2(k(T_o)T) + Q(r,t) = \rho c\frac{\partial T}{\partial t} \quad ; \quad 0 \leq r < a \qquad (15)$$

$$k'\nabla^2 T' = \rho'c'\frac{\partial T'}{\partial t} \quad ; \quad a < r < b \qquad (16)$$

$$-\frac{d}{dr}(k(T_o)T) = -k'\frac{dT'}{dr} = C_o(T^- - T^+) + \left(\frac{\partial C}{\partial p}\right)_o \Delta T_o p(t) \; ; \quad r = a \qquad (17)$$

$$k'\frac{dT'}{dr} + H_oT' = H_oT^*(t) \quad ; \quad r = b \qquad (18)$$

where $$T^*(t) = \delta T^*(t) + (T_o'^* - T_o')\frac{\delta H(t)}{H_o} \qquad (19)$$

In problems (b) and (c), the perturbation $p(\theta)$ or $p(t)$ in the fuel-can contact pressure is assumed due to purely thermoelastic strains. Thus

$$p = F(E,E',\alpha,\alpha',T,T',\nu,\nu')$$

and its determination involves the thermal strain perturbations,

$$\varepsilon_T = \int_{T_o}^{T_o+T} \alpha(T)\, dT \simeq \alpha(T_o)T \tag{20}$$

For fuel and can, the models are

$$\varepsilon_T = (\alpha_o + \alpha_1 T_o)T \quad ; \qquad \varepsilon_T' = \alpha_o' T' \tag{21}$$

The temperature dependent fuel conductivity is taken to be

$$k(T_o) = (A + B\, T_o)^{-1}$$

For the steady state symmetric problem with a uniformly distributed heat source Q_o, the temperature and thermal conductivity distributions in the fuel are readily shown to be:-

$$T_o(r) = \frac{1}{B}\left(\frac{1}{K} e^{-BQ_o r^2/4} - A\right) \tag{22}$$

$$k(T_o(r)) = K\, e^{BQ_o r^2/4} \tag{23}$$

where $$K = k_o\, e^{-BQ_o a^2/4} \; ; \quad k_o = k(T_o(a)) \; ; \tag{24}$$

and $$T_o(a) = T_o^* + \frac{Q_o a}{2}\left\{ \frac{1}{C_o} + \frac{a}{K'} \ln\left(\frac{b}{a}\right) + \frac{a}{bH_o} \right\} \tag{25}$$

3. STATIC ASYMMETRIC PERTURBATIONS

For problem (b) the solution for the simple distributions

$$Q(r,\theta) = Q_1 r \cos\theta \; ; \quad T^*(\theta) = T_1^* \cos\theta$$

may serve to illustrate the effect of pressure dependent conductance on the can surface heat flux. The temperature distributions will have the form

$$T(r,\ ) = \frac{1}{k_o}\left(Ar - \frac{Q_1 r^3}{8}\right) e^{-X(1-(r^2/a^2))} \cos\theta \tag{26}$$

$$T'(r,\theta) = \left(Cr + \frac{D}{r}\right) \cos\theta \tag{27}$$

where $$X = BQ_o a^2/4 \tag{28}$$

In Appendix 1, an expression is derived for the thermoelastic contact pressure between fuel and can for this type of distribution, assuming equal Poisson's ratios to give a result independent of end conditions. Evaluation of the thermal strain moments with the assumed linear expansion coefficients presents no difficulty.

The solution for $\bar{q}_1$, the amplitude of the can surface heat flux, $q(\theta) = \bar{q}_1 \cos\theta$, involves a number of parameters, defined as follows:-

$\eta = k'/k_o$ = ratio of conductivities at interface

$\beta_o = H_o b/k'$ = can surface Biot number

$\beta_i = C_o a/k'$ = fuel-can interface Biot number

The remaining parameters are introduced by the coefficient $(\frac{\partial C}{\partial p})_o$, i.e.,

$$\gamma_c = (\frac{\zeta^2 - 1}{2})\, E_1\, \alpha_o'\, \Delta T_o\, (\frac{1}{C}\frac{\partial C}{\partial p})_o \tag{29}$$

$$\gamma_f = (\zeta^4 - 1)\, E_1\, \alpha_o\, \Delta T_o\, (\frac{1}{C}\frac{\partial C}{\partial p})_o\, G_f \tag{30}$$

$$\gamma_i = (\zeta^4 - 1)\, E_1\, \alpha_o\, \Delta T_o\, (\frac{1}{C}\frac{\partial C}{\partial p})_o\, G_i \tag{31}$$

where

$$G_f = (1 - \frac{A\alpha_1}{B\alpha_o})\, (\frac{2e^X-(2+2X+X^2)}{2X^3}) + \frac{\alpha_1}{\alpha_o Bk_o}\, (\frac{2e^{2X}-(2+4X+4X^2)}{16X^3}) \tag{32}$$

$$G_i = (1 - \frac{A\alpha_1}{B\alpha_o})\, (\frac{e^X-(1+X)}{2X^2}) + \frac{\alpha_1}{\alpha_o Bk_o}\, (\frac{e^{2X}-(1+2X)}{8X^2}) \tag{33}$$

As the problem is linear, the results may be given separately for the amplitude $\bar{q}_1$ due to Q_1 and T_1^* separately:-

$$\frac{4\bar{q}_1}{Q_1 a^2} = \eta\beta_o\beta_i\, (2 + 3\gamma_i - \gamma_f)\, \Omega^{-1} \tag{34}$$

$$\frac{-\bar{q}_1 b}{k'T_1^*} = \beta_o\left[\zeta^2-1+\zeta^2\beta_i\beta_o(4+\gamma_c(3+\zeta^2))\Omega^{-1}\right] / \left[\zeta^2-1+\beta_o(\zeta^2+1)\right] \tag{35}$$

$$\text{with } \Omega = \left[1+\eta\beta_i(1+\gamma_i)\right]\left[\zeta^2-1+\beta_o(1+\zeta^2)\right] + \beta_i\left[\zeta^2+1+\beta_o(\zeta^2-1)+\gamma_c(\frac{3\zeta^2+1}{2} + \beta_o(\frac{\zeta^2-1}{2}))\right] \tag{36}$$

In the absence of pressure dependent conductance,

$$\frac{4\bar{q}_1}{Q_1 a^2} = 2\eta\beta_o\beta_i\Omega_o^{-1} \tag{37}$$

$$\frac{-\bar{q}_1 b}{k'T_1^*} = \beta_o(\zeta^2-1+4\zeta^2\beta_i\beta_o\Omega_o-1) / \left[\zeta^2-1+\beta_o(\zeta^2+1)\right] \tag{38}$$

$$\Omega_o = (1+\eta\beta_i)(\zeta^2-1+\beta_o(1+\zeta^2)) + \beta_i(\zeta^2+1+\beta_o(\zeta^2-1)) \tag{39}$$

In the limiting case of an uncanned fuel element,

$$\frac{4\bar{q}_1}{q_1 a^2} \rightarrow \frac{-\bar{q}_1 b}{k_o T_1^*} \rightarrow \frac{\beta_o}{1+\beta_o} \tag{40}$$

4. SYMMETRIC TRANSIENT PERTURBATION

The plane strain thermoelastic analysis of a canned cylinder gives, for equal Poisson's ratios,

$$p = E_o \Delta\varepsilon_T \tag{41}$$

where $$1/E_o = (1 - 2\nu)\left(\frac{1}{E} - \frac{1}{E'}\right) + \frac{2\zeta^2(1-\nu)}{E'(\zeta^2-1)} \tag{42}$$

and $$\Delta\varepsilon_T = \langle\varepsilon_T\rangle_f - \langle\varepsilon_T\rangle_c \tag{43}$$

i.e. $\Delta\varepsilon_T$ is the difference between average fuel and can thermal strains. The contribution to the interfacial heat flux from the variable conductance can then be written

$$T_o\left(\frac{\partial C}{\partial p}\right)_o p = \Theta P \; ; \qquad \Theta = E_o\left(\frac{\partial C}{\partial p}\right)_o \; , \qquad P = \Delta T_o \Delta\varepsilon_T \tag{44}$$

The reduction of problem (c) to a set of coupled linear equations, from which the effect of Θ on the frequency response characteristics (with Q and T* as inputs) can be studied, is readily achieved using Galerkin's weighted residual method.

The fuel and can equations,

$$L(T) + Q = 0 \; ; \qquad L'(T') = 0 \tag{45}$$

and the approximations

$$T = \sum_1^n T_j(t) f_j(r) \; ; \qquad T' = \sum_J^N T_J(t) g_J(r) \tag{46}$$

lead to the (N + n) equations:-

$$\left.\begin{aligned} \int_0^a r\, f_i \,\{L\,(\sum_j T_j f_j) + Q\}\, dr &= 0 \,, \qquad i = 1,\ldots n \\ \int_a^b r\, g_J \,\{L'(\sum_K T_K g_K)\,\}\, dr &= 0 \,, \qquad J = 1,\ldots N \end{aligned}\right\} \tag{47}$$

Using Green's theorem, the coupled linear equations for the time dependent T_j and T_J coefficients have the following form, assuming Q spatially uniform:-

$$\sum_{j=1}^{n} \left(A_{ij}\frac{dT_j}{dt} - K_{ij}T_j + F_{ij}T_j\right) - \sum_{J=1}^{N} X_{iJ}T_J = Qq_i - p_i\Theta P \tag{48}$$

$$\sum_{K=1}^{N} \left(A_{JK}\frac{dT_K}{dt} - K_{JK}T_K + F_{JK}T_K + P_{JK}T_K\right) - \sum_{j=1}^{n} X_{Jj}T_j = T^*h_J + p_J\Theta P \tag{49}$$

$$P = T_0 \left(\sum_i T_i < \alpha_0 f_i + \alpha_1 T_0(r) f_i > f - \alpha_0' \sum_J T_J < g_J > c \right. \qquad (50)$$

The constant coefficients may be written explicitly in terms of the f(r) and g(r) trial functions.

Arising from the fuel:-

$$A_{ij} = \rho c \int_0^a r f_i f_j dr \; ; \qquad K_{ij} = \int_0^a k(T_0) f_j \nabla^2 f_i r \, dr$$

$$F_{ij} = a\, f_j \left(C_0 f_i + k(T_0) \frac{df_i}{dr}\right) \quad \text{at} \quad r = a$$

$$X_{ij} = a\, C_0\, f_i(a)\, g_J(a)$$

$$q_i = \int_0^a r\, f_i\, dr \; ; \qquad p_i = a\, f_i(a) \qquad (51)$$

Arising from the can:-

$$A_{JK} = \rho' c' \int_a^b r g_J g_K dr \; ; \qquad K_{JK} = k' \int_a^b r g_K \nabla^2 g_J dr$$

$$F_{JK} = a\, g_K \left(C_0\, g_J - k' \frac{dg_J}{dr}\right) \quad \text{at} \quad r = a$$

$$R_{JK} = b\, g_K \left(H_0\, g_J + k/\frac{dg_J}{dr}\right) \quad \text{at} \quad r = b$$

$$X_{Jj} = a\, C_0\, g_J(a)\, f_j(a)$$

$$h_J = b\, H_0\, g_J(b) \; ; \qquad p_J = a\, g_J(a) \qquad (52)$$

With the functions

$$f_i(r) = \left(1 - \frac{r^2}{a^2}\right)^{i-1} \; ; \qquad i = 1, \ldots n$$

$$g_J(r) = \left(\frac{r - a}{b - a}\right)^{J-1} \; ; \qquad J = 1, \ldots N \qquad (53)$$

all coefficients can be evaluated analytically.

The computer programme CLASSIC, Thompson & Godfrey(1), was used to evaluate frequency response functions relating average fuel and can temperatures, can surface temperature, and can surface heat flux to Θ and T^*, with Θ as the variable system parameter.

5. NUMERICAL RESULTS

The following gives the relevant data for the UO_2 - Zircalloy 2 fuel pin studied:-

a	=	0.763 cm
b	=	0.801 cm.
ρc	=	2.6059 joules/cm^3 ^oC
Q_o	=	300 watts/cm^3
$k(T_o)$	=	$(10.575 + 0.02115\ T_o)^{-1}$ watts/cm.^oC
α	=	$(8.4217 + 4.2962\ T_o \times 10^{-3}) \times 10^{-6}$ /^oC
C_o	=	1.3 watts/cm^2 ^oC
$\rho' c'$	=	1.7924 joules/cm^3 ^oC
k'	=	0.1555 watts/cm.^oC
α'	=	6×10^{-6} /^oC
H_o	=	4.0 watts/cm^2 ^oC
T^*	=	270^oC

Frequency response functions were calculated, using CLASSIC, with numerical values of the parameter θ equal to 0, 75 and 150. This range was chosen from the result that a θ of 76 corresponded to the values

$$E = 30 \times 10^6 \text{ p.s.i.a.}, \qquad E' = 11.6 \times 10^6 \text{ p.s.i.a.}$$

$$\nu = 0.3, \qquad \frac{\partial C_o}{\partial p} = 9.4 \times 10^{-5} \text{ watts/cm. } ^oC\text{. p.s.i.a.}$$

Table I gives the amplitude of the symmetric frequency response functions relating the average fuel temperature, average can temperature, can surface temperature and can surface heat flux to internal power and coolant temperature, computed for four trial functions in both fuel and can regions. Other results showed that two trial functions in both fuel and can, while not sufficient to reproduce the correct zero frequency and high frequency response, gave the same trends, and fractional changes due to the θ parameter of the same order.

The effect of the pressure dependence of the fuel-can conductance, as shown in Table I, is mainly on the response to internal power changes. The maximum changes, compared with values for $\theta = 0$ may be summarized as follows:

θ	T_{ave}	T'_{ave}, $T'(b)$, $q(b)$
75	10.6% ($\omega \to 0$)	+ 7.1% (ω = 1.0)
150	19.2% ($\omega \to 0$)	+ 14% (ω = 1.0)

As expected, the effect of finite θ on the can temperatures and heat flux amplitudes is less significant, the major effect being on the very small low frequency heat flux response. For the fuel however, the temperature dependence of the thermal conductivity, and the conductance, combine to give significant changes in the low frequency, average fuel temperature response. For constant conductivity the static fuel temperature would be

TABLE I

θ	ω rad/sec	$\left\lvert \frac{T_{ave}}{Q} \right\rvert$	$\left\lvert \frac{T'_{ave}}{Q} \right\rvert$	$\left\lvert \frac{T'(b)}{Q} \right\rvert$	$\left\lvert \frac{q(b)}{Q} \right\rvert$	$\left\lvert \frac{T_{ave}}{T^*} \right\rvert$	$\left\lvert \frac{T'_{ave}}{T^*} \right\rvert$	$\left\lvert \frac{T'(b)}{T^*} \right\rvert$	$\left\lvert \frac{q(b)}{T^*} \right\rvert$
0	10 - 3	3.617	1.356-1	9.084-2	3.634-1	1.644	1	1	1.623-3
	10 - 2	3.588	1.346-1	9.020-2	3.608-1	1.631	9.994-1	9.996-1	1.611-2
	10 - 1	2.240	9.133-2	6.119-2	2.448-1	1.063	9.761-1	9.839-1	1.058-1
	1	3.365-1	2.924-2	1.959-2	7.836-2	2.643-1	9.212-1	9.466-1	2.902-1
	10 + 1	3.756-2	5.943-3	3.981-3	1.592-2	4.718-2	8.166-1	8.697-1	8.384-1
	10 + 2	3.824-3	5.691-4	3.779-4	1.460-3	4.167-3	3.875-1	4.972-1	2.581
75	10 - 3	3.235	1.356-1	9.084-2	3.634-1	1.521	1	1	1.506-3
	10 - 2	3.213	1.348-1	9.032-2	3.613-1	1.511	9.995-1	9.997-1	1.497-2
	10 - 1	2.117	9.598-2	6.431-2	2.572-1	1.038	9.780-1	9.851-1	1.037-1
	1	3.336-1	3.131-2	2.098-2	8.392-1	2.697-1	9.203-1	9.459-1	2.959-1
	10 + 1	3.751-2	6.331-3	4.241-3	1.696-2	4.833-2	8.128-1	8.671-1	8.460-1
	10 + 2	3.823-3	6.052-4	4.-19-4	1.553-3	4.278-3	3.863-1	4.970-1	2.578
150	10 - 3	2.923	1.356-1	9.084-2	3.634-1	1.421	1	1	1.412-3
	10 - 2	2.907	1.349-1	9.040-2	3.616-1	1.413	9.996-1	9.997-1	1.405-2
	10 - 1	2.002	1.000-1	6.700-2	2.680-1	1.015	9.797-1	9.863-1	1.016-1
	1	3.306-1	3.335-2	2.234-2	8.936-2	2.750-1	9.194-1	9.453-1	3.013-1
	10 + 1	3.747-2	6.715-3	4.498-3	1.799-2	4.946-2	8.091-1	8.646-1	8.536-1
	10 + 2	3.822-3	6.411-4	4.257-4	1.645-3	4.388-3	3.851-1	4.967-1	2.574

independent of the conductance. In this example, a $1^{o}C$ rise in coolant temperature increases the fuel temperature by $1.644^{o}C$ at a power density of 300 watts/cm^3 when θ = (and this is reduced by 7.5% and 13.5% respectively, as θ increases to 75 and 150. Fr (19), these figures also refer to constant coolant temperature, but a 4% reduction in c surface heat transfer coefficient.

Using the same data, the effect of the pressure dependent conductance on the surf heat flux due to simple tilted power and coolant temperature distributions has been cal ated, with the following results:-

	θ = 0	θ = 75
$\frac{4\bar{q}_1}{Q_1 a^2}$	0.880	0.941 (+6.9%)
$\frac{-\bar{q}_1 b}{k_o T_1^*}$	0.323	0.289 (-10.5%)

6. CONCLUSION

A first order linear perturbation theory has shown that the pressure dependence o fuel-can conductance can have a significant effect on the symmetric thermal frequency r ponse of a zircalloy clad UO_2 fuel pin, and on the temperature and heat flux distributi due to non uniformities in power density or surface heat transfer boundary conditions. suggests that the problem is worth further investigation, particularly as regards the structural mechanics of the assembly and the inclusion of temperature effects at the in face.

REFERENCE

(1) THOMPSON, J. J. & GODFREY, M., "CLASSIC - a Code for the stability and response analysis of linear systems." Australian Atomic Energy Commission Report, AAEC/TM551, July 1970.

NOTATION

a, b	outer radii of fuel and can
c, c'	specific heat of fuel and can
k, k'	thermal conductivity of fuel and can
p	contact pressure at interface
q	heat flux to coolant
r	radial co-ordinate
t	time
A, B	coefficients for temperature dependence of fuel conductivity
C	fuel can interface conductance
E, E'	Young's modulus of fuel and can
E_0, E_1	parameters relating thermal moments and contact pressure
H	heat transfer coefficient at can surface
M, M'	thermal moments in fuel and can regions
Q	heat source density
T, T'	temperature in fuel and can
T^*	bulk coolant temperature
α, α'	coefficient of linear thermal expansion in fuel and can
β_0, β_i	Biot numbers at outer and inner can surfaces
δX	small perturbation in X
ΔT	temperature drop at interface
ϵ_T, ϵ_T'	thermal strains in fuel and can
η	ratio of can to fuel surface conductivity
θ	angular co-ordinate
Θ	parameter based on pressure derivative of conductance
ν	Poisson's ratio
ρ, ρ'	density of fuel and can
ω	angular frequency
Suffix o	denotes steady state
1	denotes cos θ dependence
f	fuel
c	can
< >	spatial averages

APPENDIX I

ASYMMETRIC ELASTIC FUEL-CAN PRESSURE

With the temperature distribution $T(r,\theta,z) = T(r)\cos\theta$ the assumption $\varepsilon_{zz} = \Omega r \cos\theta$ leads to the two-dimensional displacement formulation

$$\mu \nabla^2 u_i + (\lambda + \mu) \frac{\partial}{\partial x_i} \left(\frac{\partial u_j}{\partial x_j}\right) = \frac{\partial}{\partial x_i} \left((3\lambda + 2\mu)\varepsilon_T - \lambda \varepsilon_{zz}\right)$$

where ε_T is the thermal strain $\int \alpha(T)dT$.

A solution may be sought as a linear combination of stress fields associated with a displacement potential ψ, i.e., $u_i = \frac{\partial\psi}{\partial x_i}$, and a stress function ω. Then

$$\nabla^2\psi = \left(\frac{1+\nu}{1-\nu}\right)\varepsilon_T - \left(\frac{\nu}{1-\nu}\right)\varepsilon_{zz}$$

and $$\nabla^4\omega = 0$$

For both fuel and can, the harmonic function $\nabla^2\omega$ is $r\cos\theta$, avoiding self straining effects. Writing

$$\psi = \psi(r)\cos\theta, \qquad \omega = W(r)\cos\theta$$

$$\phi = (W - 2\mu\psi)\cos\theta = \phi(r)\cos\phi$$

results in the basic differential equation for the radial function $\phi(r)$:-

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r\phi)\right) = Ar - \left(\frac{E}{1-\nu}\right)\varepsilon_T + \left(\frac{E\nu}{1-\nu^2}\right)\Omega r$$

The stresses and displacements, required for the application of boundary conditions, are

$$\sigma_{rr} = \bar{\sigma}\cos\theta\ ; \qquad \sigma_{r\theta} = \bar{\sigma}\sin\theta$$

$$u_r = \bar{u}\cos\theta\ ; \qquad u_\theta = \bar{v}\sin\theta$$

where

$$\bar{\sigma} = \frac{d}{dr}\left(\frac{\phi}{r}\right)$$

$$\bar{u} = -\frac{1}{2\mu}\frac{d\phi}{dr} + \left(\frac{1-\nu^2}{E}\right)\frac{Ar^2}{2}$$

$$\bar{v} = \frac{1}{2\mu}\frac{\phi}{r} + \left(\frac{1-\nu^2}{E}\right)\frac{Ar^2}{2}$$

In the fuel, $(0 \le r < a)$,

$$\phi(r) = \left(A + \left(\frac{E\nu}{1-\nu^2}\right)\Omega\right)\frac{r^3}{8} - \left(\frac{E}{1-\nu}\right)\frac{1}{r}\int_0^r r_1 \int_0^{r_1} \varepsilon_T(r_2)\, dr_2 dr_1$$

and in the can, $(a < r \le b)$

$$\phi'(r) = (A' + (\frac{E\nu}{1-\nu^2})'\Omega)\,\frac{r^3}{8} - (\frac{E}{1-\nu})'\,\frac{1}{r}\int_a^r r_1 \int_a^{r'} \varepsilon_T(r_2)\, dr_2 dr_1 + C'r + \frac{D'}{r}$$

The term $C'r$ in $\phi'(r)$ produces zero stress and rigid body displacement parallel to the x axis. Allowing for such a can displacement, the requirement that the displacements u_r and u_θ be continuous at the interface $(r = a)$ leads to the single condition, independent of C',

$$\bar{u}(a) + \bar{v}(a) = \bar{u}'(a) + \bar{v}'(a)$$

Continuity of radial and shear stress at $r = a$ and zero stresses on the can surface $r = b$. require

$$\bar{\sigma}(a) = \bar{\sigma}'(a) \qquad \text{and} \qquad \bar{\sigma}'(b) = 0$$

Development of these equations introduces the moments of the thermal strain, defined by

$$M_1 = \frac{1}{a^3}\int_0^r \varepsilon_T\, r^2 dr$$

$$M_1' = \frac{1}{b^3}\int_a^b \bar{\varepsilon}_T'\, r^2 dr$$

Evaluation of the constants enables the amplitude of the fuel-can contact pressure, $\bar{p} = -\bar{\sigma}(a)$ to be determined in terms of the elastic constants, the thermal strain moments, and the geometrical factor $\zeta = \frac{b}{a}$. The result is

$$\bar{p} = E_1 \left[(1+\nu)M_1(\zeta^4 - 1) - (1+\nu)'\, M_1'\, \zeta^3 + (\nu' - \nu)\,\frac{\Omega a}{4}(\zeta^4 - 1) \right]$$

where

$$E_1^{-1} = (\zeta^4 - 1)\left[(\frac{3-\nu-4\nu^2}{4E}) - (\frac{3-\nu-4\nu^2}{4E})' \right] + (\frac{1-\nu^2}{E})'\, \zeta^4$$

The constant Ω may now be determined from the assumed end conditions on the element. For the reasonable assumption that $\nu = \nu'$, the contact pressure $\bar{p}$ becomes independent of Ω and hence independent of the end conditions.

Thus for $\nu = \nu'$,

$$\bar{p} = E_1 \left[(\zeta^4 - 1)M_1 - \zeta^3 M_1' \right]$$

where

$$E_1^{-1} = (\frac{1-\nu}{E'})\,\zeta^4 + (\frac{3-4\nu}{4})(\frac{1}{E} - \frac{1}{E'})\,(\zeta^4 - 1)$$

DISCUSSION

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The problem could be solved by application of finite element methods, but the temperature dependence of fuel conductivity would require a great number of elements for accurate presentation. However, as synthesis techniques of particular interest to our work, this approach was adopted. Although the model presented in the paper has not considered the effect of the gas pressure in the gap, this could be easily included. Only "single tilt" in power and coolant temperature distribution ranging as $\cos\Theta$ has been considered in the paper, general formulation for an arbitrary variation is available, expressed in terms of $\cos\Theta$ and $\sin\Theta$.