ABSTRACT

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In engineering design research the paradigm of market-based product design explores the integration of quantitative customer preference data and product design optimization problems. This method provides interdisciplinary approaches for connecting the marketing and engineering domains so that a diverse set of customer needs can be satisfied. This dissertation demonstrates that heterogeneous customer preferences models can be used to determine the most effective combination of product features in a product line optimization by exploring the formulation of effective product design search problems.

The first research topic addressed in this dissertation aims to understand the implication of consumer choice behavior and how the selection of a discrete choice model influences the results of a product design search problem. Motivation for this research topic comes from the challenges posed by Bayesian-based noncompensatory models. To overcome these challenges, the suitability of using a compensatory model in the presence of noncompensatory choices when conducting a product design search is explored.

The second research topic investigates how the choice of representing heterogeneous customer preferences effects the configuration of an optimal product line solution. Continuous and discrete representations of heterogeneity are available to design researchers, and the choice of model formulation can have a substantial impact on the final product design and the estimated values associated with different managerial objectives. The relative performance of each model form is compared in terms of model fitness and predictive ability. The structure of heterogeneous preferences is also explored to investigate why differences are observed in model estimates and product solutions obtained from an optimization.

The third research topic is focused on a quantitative method to use customer preference information to support mass customization design decisions. Despite advancements in consumer research, a quantitative assessment tool of customer preference has yet to be developed for mass customization environments. In particular, although conceptual definitions of sacrifice gap exist, a process for obtaining a quantitative measure of this metric is essentially unexplored. The main objectives of this topic are to propose a quantitative definition for

sacrifice gap and an optimization problem formulation that can be used to identify the customizable product features in a mass customization environment.

The fourth research topic quantitatively defines the reliability and robustness of a product design under uncertainty in discrete choice methods. Ignoring uncertainties associated with customer preferences and their estimation has caused concern about the reliability and robustness of an optimal product design solution. This study leverages Randomized First Choice (RFC) simulations to address stochastic preference coefficients. Based on the RFC, a quantitative way of measuring reliability and robustness of a product line design is proposed. A multi-objective problem formulation is developed to integrate the stochastic aspects into one framework. Then, a multi-attribute decision method for handling conflicts in decision making is used to demonstrate how a decision maker can choose a single design from the set of solutions under tradeoffs in reliability and robustness.

This dissertation not only provides foundational studies for integrating customer preference data into optimization problems, but it also introduces new quantitative approaches and optimization formulations in market-based product design. Through the first two topics, better understandings of how to integrate discrete choice models in product design optimization problems are established. The last two topics introduce new design search methods when preference heterogeneity and uncertainty are considered

Optimizing Product Line Offerings using Customer Preference Data

by

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BIOGRAPHY

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Chapter 1. Introduction

1.1. Motivation

The success of a product depends on making the correct decisions during the design process. Recent smartphone sales give a representative example of how product design decisions can play a central role in product success. The iPhone 6 and iPhone 6 plus launched in September 2014. These phones had larger screens than the iPhone 5 and 5s series. Apple announced it sold 46% more devices (74.5 million) during the holiday season in 2014 than the record 51 million iPhone 5s sold that quarter the year earlier (Apple Press Release 2015). This was the most successful quarter in Apple’s business history. To explain this success, many analysts said the key factor was the larger screens and multiple sizes. The design decision to adopt two variants with larger screens has made a significant contribution to the success of these new models. This leads to the following question. How can information about customer preference be used to make the most effective design decisions?

In a globally competitive market, a significant challenge for a company is understanding consumer preferences and leveraging that information to guide the introduction of desired products. The paradigm of market-based product design explores the integration of quantitative customer preference data and product design optimization problems. This method provides interdisciplinary approaches for connecting the marketing and engineering domains so that a diverse set of customer needs can be satisfied. The work completed in this dissertation demonstrations that heterogeneous customer preferences models can be used to determine the most effective combination of product features in a product line optimization by exploring the formulation of effective product design search problems.

1.2. Market-Based Product Design

Market-based product design uses quantitative methods to support design decisions at the interface of market systems and engineering design. This design methodology entails three stages: first, survey instruments like Choice-Based Conjoint (CBC) are fielded and data is collected that represents a respondent’s choice; second, preference coefficients are estimated using statistical techniques to create demand models; third, a product design search problem is formulated and optimization techniques are used to identify optimal product attributes associated with managerial objectives and subject to pre-defined constraints.

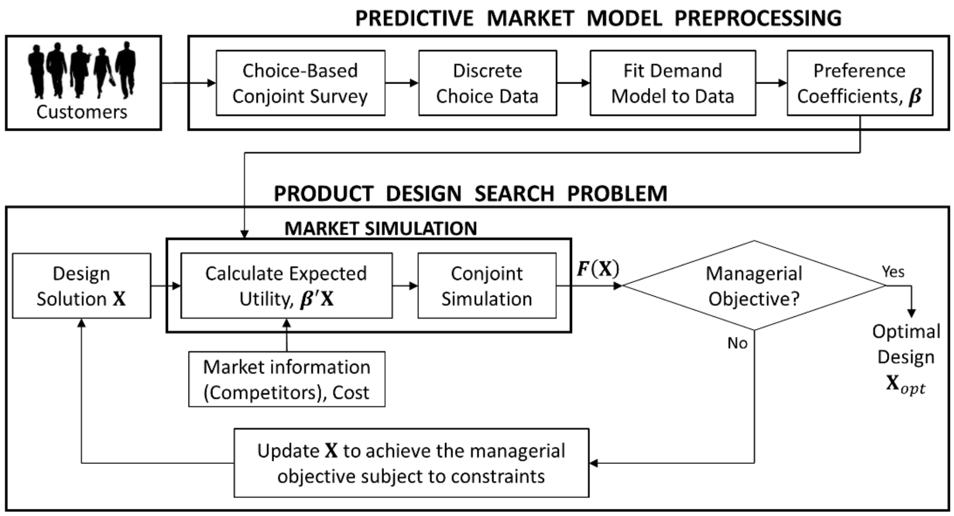

Figure 1.1. Framework of market-based product design

The first two stages of this process involve predictive market modeling, as are viewed as preprocessing steps for the product design search problem, as shown schematically in Figure 1.1. As one of the tools for estimating customer preferences, discrete choice analysis has been widely used by implementing variations of the generalized linear model - such as logit and probit models (Chen, Hoyle, and Wassenaar 2013; Train 2009). Recently, models that represent preference heterogeneity – providing the ability to represent variation in taste across individuals (Allenby and Rossi 1998) – have seen greater use.

The predictive market model is integrated into a market simulator. This simulator is then used to evaluate the performance of product offerings in the market. From an engineering perspective, this is core element of market-based product design frameworks. Market simulations use the part-worths obtained in preference modeling to create hypothetical market scenarios consisting of a subset of product alternatives to simulate purchase choices. Using results from a market simulator, optimization algorithms can search for a new design by exploring how changes to product configuration can positively affect managerial objectives.

Product design formulations can have many variants according to the managerial objectives and design constraints considered. In a product line optimization formulation, the design variable X can be defined as discrete numbers to encode product attributes. The objective function F(X) is typically defined as profit or preference share. Design constraints such as design infeasibility can be defined if needed. Overall, these problems often require discrete and mixed-integer problem formulations, leading to the use of heuristic optimization techniques. Genetic Algorithms (GA) have been shown to have advantages in that they require far fewer function evaluations to converge to a set of solutions that other methods (e.g. grid searches, iterative weighted sums). In addition, a GA is very robust to ill-conditioned problem formulations (discontinuous, discrete, etc.). Details about optimization techniques for the problem are discussed in Chapter 2.2.

1.3. Research Questions

This dissertation is focused on theoretical frameworks for supporting design decisions in product development by integrating customer preference data into design optimization problems. The objective of this dissertation is to explore how heterogeneous preference estimates can be used in product design optimization. The research topic can be classified into two broad categories: investigating how the selection of discrete choice model influences the optimal product line configuration, and developing design search methods under preference heterogeneity. The research questions explored in this work are summarized as:

- Research Question 1. When respondents make noncompensatory choices with conjunctive screening rules, how does the optimal product line solution differ when preferences are modeled using compensatory and noncompensatory models? (Chapter 3)
- Research Question 2. How does the optimal product line solution change when preferences are estimated using a latent class multinomial logit and a hierarchical Bayes mixed logit? (Chapter 4)

- Research Question 3. How can sacrifice gaps be quantified to formulate an optimization problem for mass customization environments? (Chapter 5)
- Research Question 4. How can the reliability and robustness of a product design be defined under uncertainty when using discrete choice methods, and how was these measures be used to support engineering design decision-making? (Chapter 6)

Research Question 1 is focused on investigating the limitations of the Bayesian-based noncompensatory models in design optimization problems, and exploring the feasibility of using a compensatory model when respondents make noncompensatory choices. In Research Question 2, continuous and discrete representations of heterogeneous preferences are empirically compared in the context of an optimal product line solution. Research Question 3 proposes a quantitative definition of customer sacrifice gap and develops a design problem formulation for mass customization environments. Research Question 4 explores how uncertainty can be managed when using discrete choice methods, and a multi-objective optimization problem considering the reliability and robustness of a product design is proposed.

1.4. Significance of Research Topics

Market-based product design is centered on the notion that customer needs can be mathematically modeled and integrated into the engineering design process. While the use of market modeling techniques is becoming more prevalent in product design research, the effect of demand models on design optimization is relatively unexplored. Answering Research Questions 1 and 2 will provide practical understanding about the ramifications of selecting discrete choice model form in product design problems. Based on the foundational study about the integration of market models into optimization problems, Research Questions 3 and 4 will focus on how optimization problems can be formulated and solved. The significance of each research topic is addressed in this chapter.

1.4.1. Significance of Research Question 1

Many forms of the discrete choice models used in market-based design assume that consumers make compensatory choices. Compensatory choices are based on an additive utility rule; that is, high levels on some features can compensate for low levels on other features. However, a number of papers have demonstrated that noncompensatory choice models often improve both model realism and accuracy in predicting consumers' choices (Desai and Hoyer 2000; Ding 2007; Erdem and Swait 2004; Gilbride and Allenby 2006). The noncompensatory choice rule supposes consumer's choice tasks are conducted using heuristic decision-making strategies.

Imagine a consumer, who does not want a manual transmission, shopping for a new car. The consumer first narrows his choices to a small set of cars equipped with an automatic transmission and then compares the cars in that set using an additive rule. This choice behavior is called a consider-then-choose process. The presence of a noncompensatory preference structure in the first stage forms a consideration set. The second stage compares product alternatives and selects one from this set, reflecting a compensatory choice. When designing products for noncompensatory consumers using a consider-then-choose process, one of the most significant challenges is launching products that are not being screened out of the consideration set.

Since the early 2000s, there has been increased development in modeling noncompensatory choices using computationally expensive methods like Bayesian inference and machine learning techniques. The performance of noncompensatory models has been proven in terms of model fitness and predictability by several other research projects (Arora et al., 2011; Gilbride and Allenby, 2004; Jedidi and Kohli, 2005; Swait, 2001; Yee et al., 2007). It has been shown that noncompensatory models can slightly outperform compensatory models in both hit rate for holdout tasks and likelihood function values.

Despite the increased development of noncompensatory choice models, however, compensatory models have been predominantly used in market-based product design. In addition, the effectiveness of noncompensatory models in a product design search has not yet been entirely explored. Morrow et al. proposed nonlinear programming relaxations for market system design problems to deal with the discontinuous likelihood functions of a consider-then-choose model (Morrow, Long, and MacDonald 2014). Long and Morrow investigated the impact of noncompensatory choice behavior when an optimal design was created using compensatory models (Long and Morrow 2014). Their studies, however, have been focused on evaluating predictive power and design error at the population-level without exploring individual-level part-worth estimates.

Research Question 1 is motivated by the potential challenges of the existing Bayesian-based noncompensatory model. These challenges include:

- Inadequate screening rule assumptions that may lead to an incorrect estimation of noncompensatory choices because the existing noncompensatory models are derived with specific screening rules.
- Aggregate part-worths have an inappropriate form that is difficult to use in optimization problems due to their probabilistic cutoff values.
- Discontinuous choice probability functions associated with noncompensatory models can cause numerical difficulty when precisely solving design constraints during optimization using a GA.

Considering these limitations, compensatory models may be a more suitable form for optimization problem formulations because they have valuable advantages that include:

- Compensatory models have generalized forms not affected by heuristic rules.
- Aggregate part-worths can be integrated into optimization problems.
- Likelihood functions are described as continuous functions using logistic or normal distributions.

The hypothesis behind the idea of modeling noncompensatory choices with a compensatory model is that if part-worths are extreme (take on large values when all part-worths are zero-centered) in compensatory models, the additive part-worths rule can act like a noncompensatory rule (J. Hauser 2009). Thus, noncompensatory choices could be inferred from the extreme part-worths estimated for compensatory models. To answer this research question, the suitability of the Bayesian-based compensatory modeling of noncompensatory choices is investigated.

1.4.2. Significance of Research Question 2

Effectively designing products for a market requires the modeling of variation in taste across individuals. Representing such variation in taste is significant because capturing different segments of the market leads to differentiated product design (Allenby and Rossi 1998). Two customer preference models capable of representing customer heterogeneity are the hierarchical Bayes mixed logit (HB-ML) model and the latent class multinomial logit (LC-MNL) model. The HB-ML model estimates part-worth values by assuming a continuous distribution of preference heterogeneity. The LC-MNL model describes heterogeneity with discrete distributions that are determined by assigning the probability that a respondent belongs to a particular class.

The two representations available to model preference heterogeneity can have a substantial impact on the final product design and the managerial objective. However, the impact of preference heterogeneity on product configuration has not received much attention in the market-based product design literature. Michalek et al. (2011) maintained that even though discrete choice models with continuous and discrete heterogeneity representations can predict choices almost equally well (Andrews et al., 2002a; Andrews et al., 2002b), the optimized designs from different models may be very different. In a recent study by Sullivan et al. (2011), part-worth estimates from both models were used to design an optimal product line. Significant differences between the design solutions were observed. Sullivan et al. (2011) explored why the solution differed by comparing correlations between the part-estimates and simulating first-choice at the respondent level.

To answer Research Question 2, the HB-ML and the LC-MNL are empirically compared in terms of model fitness and predictive ability. Impacts of model form selection on the output associated with product line optimization are also investigated. Notice that this study does not argue that one statistical model is superior to another, but rather investigates the applicability of the two models in product design problems.

1.4.3. Significance of Research Question 3

Mass customization is a product strategy that provides custom-tailored goods or services to meet a consumer's diverse needs at near mass production prices (Gilmore and Pine 2000; S. M. Ferguson, Olewnik, and Cormier 2014). When done correctly, mass customization is a win-win strategy for both customers and producers. Consumers can get a tailor-made product at a reasonable price that reflects their personal selection of product features. Producers can make their products more appealing to consumers by obtaining more accurate information about demand and saving costs by eliminating waste in their supply chains.

The term 'mass customization' was first used by Davis (1987) nearly 30 years ago, where mass customization was explained as having the capability of reaching the same number of customers as in a mass market while treating those customers "*individually as in the customized markets of pre-industrial economies*. (Davis 1987)" Later, Pine (1999) described mass customization as "*providing tremendous variety and individual customization, at prices comparable to standard goods and services*" to fill the market "*with enough variety and customization that nearly everyone finds exactly what they want*." From a manufacturing perspective, Du et al. (2001) describe mass customization as "*the technologies and systems to deliver goods and services that meet individual customers' needs with near mass production efficiency*." All definitions of mass customization are consistent in that they share two notions: 1) they discuss that customer needs can be met more effectively using customized product offerings, and 2) they aim to leverage innovation in product design, manufacturing, and logistics to maintain prices comparable to mass produced products. In this context, the benefit of mass customization is its ability to simultaneously increase value to both the customers and the company when compared to mass production.

For the 30 years that followed Davis's work mass customization has gained attention as an intriguing prospective strategy for manufacturing businesses. However, despite great efforts in all components necessary to make the new paradigm realizable, mass customization has largely not lived up to its promise (Zipkin 2001). Ferguson et al. (2014) presented a review of mass customization literature and suggested four primary challenges associated with the realization of mass customization.

i. Limitations of existing customer needs and preference assessment tools.
ii. The need to explore approaches for requirement specification and conceptual design.
iii. Insights from methodologies focused on the development of durable mass customization goods.
iv. Necessary enhancements in information mapping and handling.

Among the research opportunities, a tool for customer needs and preference assessment is important at the early stage of product development (S. M. Ferguson, Olewnik, and Cormier 2014). However, despite advancements in consumer research, a quantitative assessment tool of customer preference has yet to be developed for mass customization environments. In particular, although conceptual definitions of the sacrifice gap exist, a process for obtaining a quantitative measure of the term is almost unexplored. Research Question 3 aims to quantitatively define sacrifice gap and develop an optimization problem formulation capable of identifying the optimal mix of customizable product features for mass customization environments.

1.4.4. Significance of Research Question 4

Critical limitations of existing market-based product design methods arise from the common assumption that customer preferences are inherently deterministic. This occurs in spite of the fact that they are statistical estimates exhibiting estimation error. Existing approaches use point estimates of an individual's part-worths in market simulations, ignoring variability in these estimates. Therefore, a critical issue when solving product design search problem becomes the reliability and robustness of the optimal design under uncertainty.

Recent research recognizes this limitation and attempts to rectify it by conducting post-optimality robustness tests for the share-of-choice problem (Camm et al., 2006; X. Wang et al., 2009; Wang and Curry, 2012). Camm et al. (2006) repeatedly solved the deterministic optimization problem by using random draws and comparing the solutions. Results of this study were not reassuring, as the optimal design profile when using aggregate part-worths accounted for 23.5% of all solutions. Wang et al. (2009) implemented a sample average approximation (SAA) method for stochastic discrete optimization (Kleywegt, Shapiro, and Homem-de-Mello 2002) to the share-of-choice product-line problem. They suggested that a solution's uncertainty could be managed by taking a sufficient number of draws per person. Wang and Curry (2012) studied the concept of robustness in integer programming for the share-of-choice problem. They assumed that individual preferences are bounded, independent, and symmetric variables. In addition, the covariance matrix for individual level part-worths was assumed to be a diagonal matrix, preventing correlation among product features. However, these assumptions do not agree with their random-effects model estimated using the hierarchical Bayes technique. In a mixed logit model, no constraint exists on the covariance matrix and preference coefficients are not defined as interval variables.

To answer this question, a multi-objective optimization problem formulation is proposed for reliable and robust product line designs. Demand model variability is quantified using a Randomized First Choice (RFC) method. Robustness and reliability of market-based product design are defined and integrated into one framework. A multi-attribute decision method is then conducted to support design decision-making from the set of candidate solutions.

1.5. Dissertation Outline

To investigate the four research questions introduced in the previous section, this dissertation is divided into seven chapters. Chapter 1 discussed the motivation behind this research and introduced the research questions being addressed and their significance. Chapter 2 provides background knowledge to aid in the explanation of this research. Chapter 3 investigates core differences in compensatory models and noncompensatory models, with a specific focus on differences in optimal product solution. Chapter 4 explores solution differences to an optimization problem when using discrete (latent class) and continuous (hierarchical Bayes) representations of heterogeneous customer preferences. Chapter 5 explores the quantification of customer sacrifice gap in mass customization environments. The reliability and robustness of a product design under uncertainty is defined in Chapter 6, and a multi-objective problem formulation is proposed that integrates them into a single framework.

Finally, Chapter 7 concludes this dissertation by revisiting the research questions and describing avenues of future work.

Chapter 2. Background

2.1. Discrete Choice Models

Discrete choice analysis is used to create models of product demand by capturing customer's choice behavior in a series of choice questions (Louviere and Woodworth 1983). In most market-based product design research, consumer preferences are estimated using discrete choice models with a compensatory utility rule. The assumption behind this compensatory utility rule is that consumers weigh and make trades between attributes when making a purchasing decision. Product utility is calculated by adding the part-worth estimates associated with each product feature selected.

The necessary background knowledge about discrete choice models and their ability to represent customer heterogeneity is introduced in Chapter 2.2. Chapter 2.2.1 provides the overview of discrete choice methods to build predictive market models. Chapter 2.2.2 introduces the fundamentals of discrete choice models and logit models. Chapter 2.2.3 and 2.2.4 review Latent Class Multinomial Logit (LC-MNL) model and Hierarchical Bayes Mixed Logit (HB-ML) model, which are called as discrete and continuous representations of preference heterogeneity, respectively.

2.2.1. Discrete choice analysis in market-based product design

The survey technique used in Discrete Choice Analysis (DCA) is designed to closely mimic the purchase process of buyers in the real world – choosing among available offerings including a 'no-buy' option. The main characteristics distinguishing DCA from conventional conjoint value analysis (Orme 2006) is that respondents express preferences by choosing from sets of design concepts, rather than by rating or ranking each concept individually.

DCA consists of three core tasks:

- Collect discrete choice data: Ask discrete choice questions (Figure 2.1) that force respondents to make trade-offs among features.

Which product would you choose from these options?		
Alt 1	Alt 2	Alt 3
4.7” Silver ☐	5.5” Grey ☐	None ☐

Figure 2.1. Discrete choice survey question (choice task)

- Estimate part-worths: Determine the value respondents place on each feature by exploring the trade-offs they make (Figure 2.2). Chapters 2.2.2, 2.2.3, and 2.2.4 describe mathematical details about how to define and estimate part-worth values.

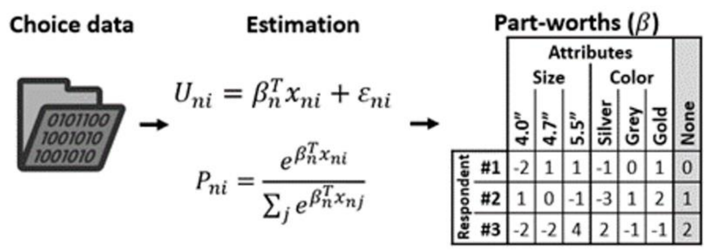

Figure 2.2. Estimating part-worths from a discrete choice survey

- Market simulation: Simulate how the market reacts to various feature trade-offs you are considering. ‘What-If’ scenarios can be tested in an optimization problem to predict the choice probability of a hypothetical design (Figure 2.3).

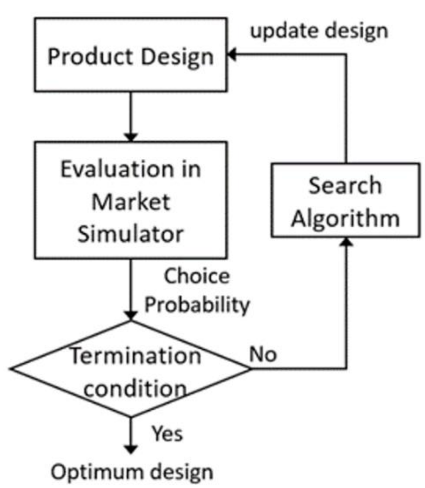

Figure 2.3. Product design optimization

2.2.2. Logit model

While conventional conjoint value analysis uses a linear regression to estimate part-worth utilities, DCA is mathematically based on the logistic regression of a utility function. A utility function is a mapping of a multi-dimensional attribute space into a single dimensioned choice probability space. A basic assumption is that each customer's choice decision can be modeled by a choice utility that person n obtains from the alternative j. This choice utility can be expressed as a sum of an observed utility V_{nj} and an unobserved random disturbance ε_{nj} as in (Train 2009; Ben-Akiva and Lerman 1985; Chen, Hoyle, and Wassenaar 2013; Rossi, Allenby, and McCulloch 2005).

$$U_{nj} = V_{nj} + \varepsilon_{nj} = \beta_n^T x_{nj} + \varepsilon_{nj} \tag{2.1}$$

In this equation, β_n is a vector of part-worths for the nth individual and x_{nj} is a vector of values that describes the configuration of product j. In generalized form, β_n and ε_{ni} are expressed as $\beta_n = \beta_n^* / \sigma_\beta$ and $\varepsilon_{ni} = \varepsilon_{ni}^* / \sigma_\beta$, where σ_β is a scale parameter that is typically set to 1. Usually, β_n is unknown to the researcher and estimated statistically.

The probability that decision maker n chooses alternative i in a binary choice scenario is defined by (Train 2009)

$$\begin{aligned} P_{ni} &= P\left(U_{ni} > U_{nj} \ \ \forall j \neq i\right) \\ &= P\left(V_{ni} - V_{nj} > \varepsilon_{nj} - \varepsilon_{ni} \ \ \forall j \neq i\right) \end{aligned} \tag{2.2}$$

The observable term, ε_{nj}, is represented by a Type-I extreme value distribution (Gumbel distribution) with an i.i.d. (independent and identically distributed) assumption. When the maximum utility rule is applied in simulations, the expectation is identical to the logit model because the difference between Gumbel distributions is logistically distributed (Train 2009). Using the binary logit probability in Eq. (2.2), any particular multinomial choice model can be derived given assumptions on the joint distribution of the unobserved utility. The choice probability (P_{ni}) that a person n chooses an alternative i from a set of products j where $j \in \{1 \ldots J\}$ can be obtained from the logit formula as (Train 2009)

$$P_{ni} = \frac{e^{\beta_n^T x_{ni}}}{\sum_j e^{\beta_n^T x_{nj}}}. \tag{2.3}$$

The choice probability is also known as preference share. For the multinomial logit model to estimate population-level preference, the part-worths are usually estimated using maximum likelihood estimation.

While compensatory models using an additive utility rule have been widely used due to their simplicity, the same models also impose several limitations. First, the additive utility rule may not accurately model real choice behavior because respondents often find it challenging to consider the entire set of product attributes when making a choice. Second, IIA (independence from irrelevant alternatives) becomes a challenge in compensatory models because the choice probability of an alternative is affected by the presence of other alternatives. This issue arises from the i.i.d. assumption of the error term ε_{ni}. Third, part-worth estimates and choice probabilities are sensitive to changes in model parameters, such as the scale parameter σ_β.

2.2.3. Latent class model

Stouffer et. al (1950) first introduced latent class analysis in 1950 as a tool to formulate latent variables from survey data. The basic idea underlying latent class analysis is that there are some parameters that can be used to reveal unobserved subgroups with similar preferences (Magidson and Vermunt 2004). Categorical indicators are assumed to define these groups. Extensions of latent class analysis toward discrete representations of heterogeneity have been achieved by including nominal variables and integrating the maximum likelihood algorithm

(DeSarbo, Ramaswamy, and Cohen 1995). In the design community, latent class analysis has been used as a tool for market segmentation by Besharati et al. (2006), Williams et al. (2008), and Turner et al. (2011).

Since a multinomial logit model is used to evaluate the likelihood of each subgroup, it is called an LC-MNL model. An LC-MNL model first classifies individuals into segments with similar preferences and then estimates the part-worths of each segment using a multinomial model. Simultaneously, individuals' probabilities of membership $\alpha(s)$ in each segment are estimated (DeSarbo, Ramaswamy, and Cohen 1995). The unconditional choice probability $P(j)$ that a respondent chooses an alternative j is obtained as

$$P(j) = \sum_{s=1}^{S} \alpha(s) \Lambda(j|s) \tag{2.4}$$

where $\alpha(s)$ is the probability of a respondent belonging to the class s, and $\Lambda(j|s)$ is the probability of a respondent choosing alternative j conditioned on belonging to the class s.

2.2.4. Hierarchical Bayes mixed logit model

The hierarchical Bayes mixed logit (HB-ML) model exemplifies a continuous representation of heterogeneity because the mixed logit model is used to define individual-level preferences as continuous distribution functions. Initially, the random coefficient model was not typically applied in the engineering design community because of the computational burden involved in both estimating these models and using them in market simulators. While the popular multinomial model has been widely applied (Wassenaar and Chen 2003; Wassenaar et al. 2004; Michalek, Feinberg, and Papalambros 2005; Resende, Grace Heckmann, and Michalek 2012a; Li and Azarm 2000; Tucker and Kim 2011; Kumar, Chen, and Kim 2006) as well as the nested logit model used by Kumar et al. (Kumar et al. 2009; Kumar, Chen, and Simpson 2009), applications of the HB-ML are more recent (Z. Wang, Kannan, and Azarm 2011; Shiau et al. 2007; Foster et al. 2014; Hoyle et al. 2011; Michalek et al. 2011; Grace Haaf et al. 2014).

The mixed logit probability can be obtained as the integrals of multinomial probabilities over a density of part-worths and is derived as Eq. (2.5). The individual-level part-worths, β_n, are assumed as a multivariate normal distribution with density $\phi\left(\beta \middle| b, W\right)$ and the parameters b and W are determined in estimation. However, analytic integration is impossible for the mixed logit probability because the multivariate distribution doesn't have a closed-form. Thus, numerical integration is employed using Bayesian inference with Markov-Chain Monte-Carlo (MCMC) methods.

$$P_{ni} = \int \left(\frac{e^{\beta^T x_{ni}}}{\sum_j e^{\beta^T x_{nj}}} \right) \phi\left(\beta \middle| b, W\right) d\beta \,. \tag{2.5}$$

The mixed logit model employing Bayesian inference is called a Hierarchical Bayes Mixed Logit (HB-ML) model because there are two levels. The assumption at the higher level is that an individual's preferences are normally distributed. At the lower level, a logit formula in assumed to quantify the choice probability. With this hierarchy, the posterior distribution of Bayes' rule for β_n, b, and W is expressed as (Train 2009):

$$K\left(b, W, \beta_n \,\forall n \middle| Y\right) \propto \prod_n L\left(y_n \middle| \beta_n\right) \phi\left(\beta_n \middle| b, W\right) k\left(b, W\right) \tag{2.6}$$

where the chosen alternatives for person n are denoted y_n and the choice of the entire sample are labeled $Y = \{y_1, \cdots, \mathrm{y}_N\}$. $k(b,W)$ is the prior of b and W which are the parameters of $\phi\left(\beta_n \middle| b, W\right)$ that represents preference heterogeneity. $L\left(y_n \middle| \beta_n\right)$ is the likelihood function of the mixed logit model obtained using the logit formula. By conducting numerical integration using Gibbs sampling, draws of the posterior can be generated (Train 2009). Detailed procedures of the numerical integration are not presented in this dissertation.

In practical use, aggregate part-worths values are often used by taking the mean value from a finite numbers of draws $r = 1, 2, \ldots, R$ obtained from the distribution of β_n. A corresponding aggregate choice probability is obtained as (Chen, Hoyle, and Wassenaar 2013)

$$\hat{P}_{ni} = \frac{1}{R}\sum_{r=1}^{R} P_{ni,r} = \frac{1}{R}\sum_{r=1}^{R} \frac{e^{\beta_{n,r}^T x_{ni}}}{\sum_j e^{\beta_{n,r}^T x_{nj}}} \tag{2.7}$$

where R is the number of random draws, $P_{ni,r}$ is the probability of respondent n choosing product i in the r th draw, and $\beta_{n,r}$ is the corresponding simulated random coefficients. However, using aggregate part-worths ignores sources of uncertainty associated with discrete choice methods, raising concerns about the reliability and robustness of optimal designs obtained when using market simulations.

2.2. Optimization Techniques for a Product Line Design Problem

Design optimization is a process of finding the best solution that satisfies design objectives under constraints. In market-based product design, the product line design problem using individual-level part-worths can be expressed as

$$\boxed{\begin{array}{l} \text{minimize} \quad \text{Preference Share} = \dfrac{1}{N}\dfrac{1}{I}\displaystyle\sum_{n=1}^{N}\sum_{i=1}^{I}\hat{P}_{ni} \\[2em] \text{with respect to} \;\; \text{X = Product configurations} \\ \qquad\qquad\qquad\qquad\quad \text{of own offerings in a product line} \\[2em] \text{subject to} \;\; \text{lower and upper bounds of each attribute} \end{array}} \tag{2.8}$$

I and N indicate the number of products offered by a manufacturer and the number of respondents, respectively. $\hat{P}_{ni}$ is the probability that respondent n chooses a product alternative i. Eq. (2.8) is a simplified representation of a product line search problem and many variants could be further developed. A product designer seeking an opportunity to adopt a market-based product design strategy would need to reflect limitations and decisions concerned with manufacturing, marketing, or engineering design. Design problems should be able to control any of these limitations and decisions in product feature configuration. In the design optimization problem, the limitations and decisions associated with these different aspects of the design problem can be specified as design variable constraints.

Many times the formulation of a product line design problem is combinatorial in nature. This leads to a mathematical optimization problem where an optimal solution is found from a finite set of objects. This specific type of combinatorial optimization problem is considered a computationally intractable (i.e., non-deterministic polynomial (NP) hard) problem, which means exact algorithm like integer programming and branch-and-bound (X. (Jocelyn) Wang, Camm, and Curry 2009) struggle to efficiently solve large-scale problems (Kohli and Sukumar 1990; Green and Krieger 1985).

To better handle the computationally intractable combinatorial problem, an increasing number of studies has adopted heuristic optimization techniques. For a single product problem, greedy approaches (Dobson and Kalish 1993) and rule-based techniques (Kohli and Krishnamurti 1987; Sudharshan, May, and Shocker 1987; Nair, Thakur, and Wen 1995) were studied. To identify product line solutions, other heuristic methods has been recently applied, such as simulated annealing (Fujita, Sakaguchi, and Akagi 1999), particle swarms (Tsafarakis, Marinakis, and Matsatsinis 2011), and Genetic Algorithms (GA) (Foster et al. 2014). GA is used to all product line optimization problems in this dissertation because GA works directly with the combinatorial problems and existing studies have shown the effectiveness of a GA in product line searches (Simpson 2004; Foster et al. 2014).

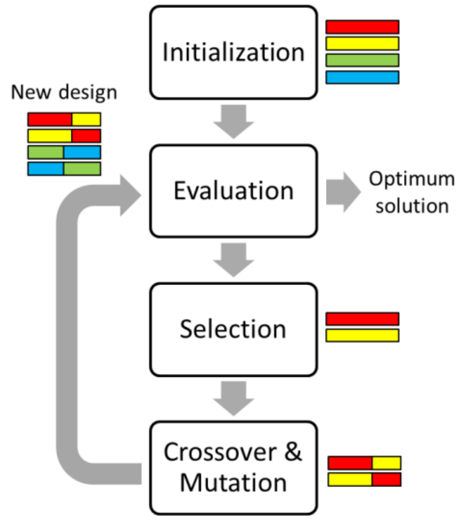

Figure 2.4. Flowchart of Genetic Algorithm

A genetic algorithm, first introduced by (Holland 1975), is a type of evolutionary algorithm that is based on the Darwin's natural selection theory. A typical genetic algorithm usually consists of four main steps: initialization, selection, crossover, and mutation as shown in Figure 2.4. Initialization is primarily focused on developing the initial population. Often, the initial population is randomly generated because priori information about the global optimum is usually not known. In product line optimization, generating a targeted population for a genetic algorithm using an individual's preference estimates can reduce computational cost and improve solution quality (Foster et al. 2014). Selection specifies which designs in the population are chosen for the genetic operators (crossover and mutation). Then, the genetic operators update designs to generate a new design set. Finally, the algorithm should stop if the pre-defined termination condition is satisfied.

As future work, developing tailored GA algorithms for product line problems is recommended to enhance the quality and efficiency of a product design search. A GA is applied to all product search problems throughout this dissertation and it could effectively find solutions in most problems. However, when a design space becomes large, a GA can require significant run time and may not find the optimal solution. As demonstrated by the mass customization problem in Chapter 5 and the multi-objective problem presented in Chapter 6.4.4, enhancing computational efficiency is beneficial for large-scale design problems.

Chapter 3. Modeling Noncompensatory Choices with a Compensatory Model

The main goal of this chapter is to explore the quality of solution obtained to a product line design search when using a compensatory model in the presence of noncompensatory choices and a noncompensatory model with conjunctive screening rules. Motivation for this work comes from the challenges posed by Bayesian-based noncompensatory models: the need for screening rule assumptions, probabilistic representations of noncompensatory choices, and discontinuous choice probability functions. This chapter demonstrates how respondents making noncompensatory choices with conjunctive rules can lead to compensatory model estimations with distinct respondent segmentation and relative, large absolute part-worth values. Results from a product design problem suggest that using a compensatory model can provide benefits of smaller design errors and reduced computational costs. Product design optimization problems using real choice data confirm that the compensatory model and the noncompensatory model with conjunctive rules provide comparable estimates of a product not being screened out in a hypothetical noncompensatory choice simulation. While many different noncompensatory heuristic rules exist, the presented study is limited to conjunctive screening rules.

3.1. Introduction

Many discrete choice model forms assume that consumers make compensatory choices based on an additive utility rule; that is, high levels on some features can compensate for low levels on other features. However, market research papers have demonstrated that noncompensatory choice models often improve model realism and accuracy in predicting consumers' choices (Desai and Hoyer 2000; Ding 2007; Erdem and Swait 2004; Gilbride and Allenby 2006). The objectives of this chapter are to investigate the use of Bayesian-based noncompensatory models with conjunctive screening rules and to compare the results of a product optimization to the solution obtained when using a compensatory hierarchical Bayes mixed logit model. A conjunctive screening rule assumes that a customer has a required minimum level for a product feature, and the customer will screen out a product that doesn't meet the minimum level. This study is motivated by the inherent challenges associated with making screening assumptions and correctly inferring the screening rules used by a respondent population. Even when these screening rules can be correctly inferred, estimating a two-stage model can be challenging, leading to errors that can ultimately lead to sub-optimal design decisions. Further, optimization of a product line is made difficult because of the probabilistic representations of noncompensatory choice and the discontinuous choice probability functions that often accompany noncompensatory models.

One would need to know or be able to infer the screening rules in order to appropriately use a noncompensatory model. Even with adequate data with the screening rule assumption, errors associated with noncompensatory choice estimation could lead to sub-optimal design decisions. The suitability of using existing noncompensatory models is discussed in Chapter 3.2.1 by reviewing the existing models. In particular, Chapter 3.2.2 addresses the limitations of an existing noncompensatory model. Chapter 3.3 describes how to explore the performance of using a compensatory model in the presence of noncompensatory choices. This concept is examined in Chapter 3.4 by analyzing synthetic data. Chapter 3.5 deals with real choice data to explore differences in a product optimization using both the compensatory and noncompensatory models.

3.2. Background Knowledge

Compensatory models capable of estimating individual-level part-worths are described in Chapter 2.1. In Chapter 3.2, various heuristics of noncompensatory choices and their modeling methods are reviewed, mainly focusing on the HB multinomial probit model with conjunctive screening rule.

3.2.1. Noncompensatory choice models

Researchers in economics and psychology have demonstrated that consumers use various heuristics to simplify their choice decisions (J. Hauser 2009). By adopting heuristics, a two-stage decision process – referred to as a Consider-Then-Choose model – has received attention because of its added realism. By employing noncompensatory screening rules, consumers narrow their decisions to a small set of products called a consideration set. Then, they use a compensatory choice rule to evaluate the remaining products and make a selection.

Various heuristic decision rules for noncompensatory choices have been proposed, including conjunctive, disjunctive, lexicographic-by-aspect, elimination-by-aspects, and disjunctions of conjunctions (Hauser 2009). The existing studies about the Bayesian-based noncompensatory models (Gilbride and Allenby 2006; Gilbride and Allenby 2004) suggest the conjunctive rule model is effective in both model fitness and predictability at individual-level estimates. Hence, this article focuses on consider-then-choose models with the conjunctive screening rule, where consumer consider if the product has all "must have" and no "must not have" aspects. It is formed by multiplying an indicator function across the attribute of an alternative as in Eq. (3.1) (Gilbride and Allenby 2004):

$$\prod_{m} \mathrm{I}(l_{im} > \gamma_m) = 1 \tag{3.1}$$

Here, l_{im} is the level of attribute m for choice alternative i. The cutoff value γ_m, is the smallest level of the attribute that needs to be present for the consumer to consider the alternative (Swait 2001). The indicator function indicates whether a choice alternative is screened out or not in a noncompensatory choice. Thus, the indicator function, $\mathrm{I}(\cdot)$, is equal to 1 when a level l_{im} exceeds a threshold value γ_m, and this indicates the choice alternative i is not screened out by a conjunctive rule in a noncompensatory choice. If the alternative has a lower level of the attribute than the cutoff value, the product is screened out.

Advances in Bayesian inference, machine learning, and greedoid languages make it possible to quantify consider-then-choose scenarios for a variety of heuristics. Noncompensatory models of conjunctive screening rules have also been applied to Hierarchical Bayes Multinomial Probit (HB-MNP) models. The most significant difference from the HB-MNP is to additionally estimate the cutoff values in the upper level of the hierarchy, as in Eq. (3.2) (Gilbride and Allenby 2006; Gilbride and Allenby 2004).

$$P_{ni} = \text{Prob}\left(U_{ni} > U_{nj} \text{ for all } j \text{ such that } \prod_m \mathrm{I}(l_{njm} < \lambda_{nm}) = 1\right) \tag{3.2}$$

l_{njm} is the level of the attribute for respondent n for alternative j and attribute m. γ_{nm} is a respondent-level threshold of attribute m for respondent n. When an attribute is continuously distributed, it is assumed that the cutoff values are normally distributed. When an attribute consists of discrete levels, a multinomial distribution can be adopted such that $\gamma_{nm} \sim \text{Multinomial}(\theta_m)$, where θ_m is the vector of multinomial probabilities associated with the grid for attribute m. Each level is tested to determine the highest possible cutoff value (γ^*_{nm}) from allowable cutoff values (γ^a_{nml}) using the Metropolis-Hastings algorithm (Kruschke 2010) based on a probability given in Eq. (3.3) (Gilbride and Allenby 2004) where l indicates attribute levels:

$$\gamma_{nm} = \gamma^a_{nml} \text{ with probability } \frac{\mathrm{I}(\gamma^a_{nml})\theta_{ml}}{\sum_l \mathrm{I}(\gamma^a_{nml})\theta_{ml}} \tag{3.3}$$

This model returns an individual's part-worths, cutoff values, and cutoff probabilities at each draw. Disjunctive and elimination-by-aspects rules can also be modeled using Bayesian inference (Gilbride and Allenby 2006; Gilbride and Allenby 2004).

The choice probabilities can be expressed as $(J-1)$-dimensional integrals over the differences between the errors because probit models are not closed form (Train 2009). These differences are defined as $\tilde{V}_{nij} = V_{ni} - V_{nj}$ and $\tilde{\varepsilon}_{nij} = \varepsilon_{ni} - \varepsilon_{nj}$. Then, for the consider-then-choose process using a probit model the choice probability that individual n chooses any alternative i that is in the consideration set is given by Eq. (3.4) (Gilbride and Allenby 2004)

$$P_{ni} = \begin{cases} \int \mathrm{I}(\tilde{V}_{nij} + \tilde{\varepsilon}_{nij} > 0 \ \ \forall j \neq i)\phi(\varepsilon_n)d\varepsilon_n & j \in C_n \\ 0 & j \notin C_n \end{cases} \tag{3.4}$$

$\mathrm{I}(\cdot)$ is an indicator of whether the statement in parentheses holds, $\phi(\varepsilon_n)$ is the joint normal density with zero mean and covariance Ω, and C_n denotes a consideration set of a consumer n.

Notice that noncompensatory attributes used to form consideration sets are excluded from part-worth estimation. In other words, noncompensatory attributes are only used to determine whether a choice alternative is in a consideration set in the first stage. Only compensatory attributes are used in part-worth estimation. This is because calculating the choice probability relies on the utility difference obtained by an additive utility rule. Noncompensatory attributes are used to estimate the probability of cutoff based on Eq. (3.3).

3.2.2. Challenges of using noncompensatory models

The performance of noncompensatory models has been proven in terms of model fitness and predictability (Gilbride and Allenby 2004; Gilbride and Allenby 2006; Swait 2001; Arora, Henderson, and Liu 2011; Jedidi and Kohli 2005; Yee et al. 2007). However, from the standpoint of design optimization, noncompensatory models have some inherent limitations:

- Inadequate screening rule assumptions may lead to an incorrect estimation of noncompensatory choices. Further, there is no general form to describe all noncompensatory heuristics. For example, the HB-MNP model with conjunctive screening rules can only describe choices that screen out attribute levels lower than the minimum requirements. This form may be inappropriate for non-incremental levels such as color or brand.
- Aggregate part-worths are difficult to use in optimization due to their probabilistic cutoff values. Each draw for a noncompensatory model results in a set of part-worths and cutoff values. Since these outcomes are interconnected, aggregate part-worths and their corresponding cutoff probabilities have to be integrated into the optimization problem. Although MCMC can be used to consider all draws, this demands considerable computational power.

- Discontinuous choice probability functions (Eq. (3.4)) can cause numerical difficulty when precisely solving design constraints (Morrow, Long, and MacDonald 2014).

Considering these limitations, compensatory models have numerous advantages from a product optimization perspective: 1) generalized forms, 2) aggregate part-worths can be used, and 3) likelihood functions are continuous. Further, when estimating part-worths at the individual-level, large absolute part-worth values (relative to the other part-worth values estimated in a zero-centered formulation) can cause the additive part-worth rule to act like a noncompensatory rule (Hauser 2009). For these reasons, this article explores the challenges of using noncompensatory models and compares the results of a product optimization to those obtained when using compensatory models in the presence of noncompensatory choices.

3.3. Technical Approach

3.3.1. Synthetic choice data

The study explores how compensatory models can approximate a two-stage choice process, and examines the differences in an optimal design search when using compensatory and noncompensatory models. This approach is driven by the hypothesis that distinct population segments can be identified from the individual-level part-worths for specific attributes where noncompensatory decisions might be made. To verify this hypothesis, a two-stage process using the LC-MNL and HB-ML models is proposed, as shown in Figure 3.1.

Discrete choice data is generated to mimic the consider-then-choose process, and is mathematically modeled using both noncompensatory and compensatory models. The HB-MNP model with a conjunctive screening rule is used as the noncompensatory model. The results of the cutoff distributions and posterior estimates show how the noncompensatory model describes the consider-then-choose process. To construct and understand the compensatory model, both LC-MNL and HB-ML models are used. A latent class analysis is first conducted to segment the population, and then the individual-level preferences are further explored based on the segmentation information. The latent class results are investigated to determine how attributes where noncompensatory decisions are made yield segments in terms of membership probability and the attribute importance. Then, the obtained segments are evaluated using individual part-worths in terms of attribute importance and heterogeneity representation.

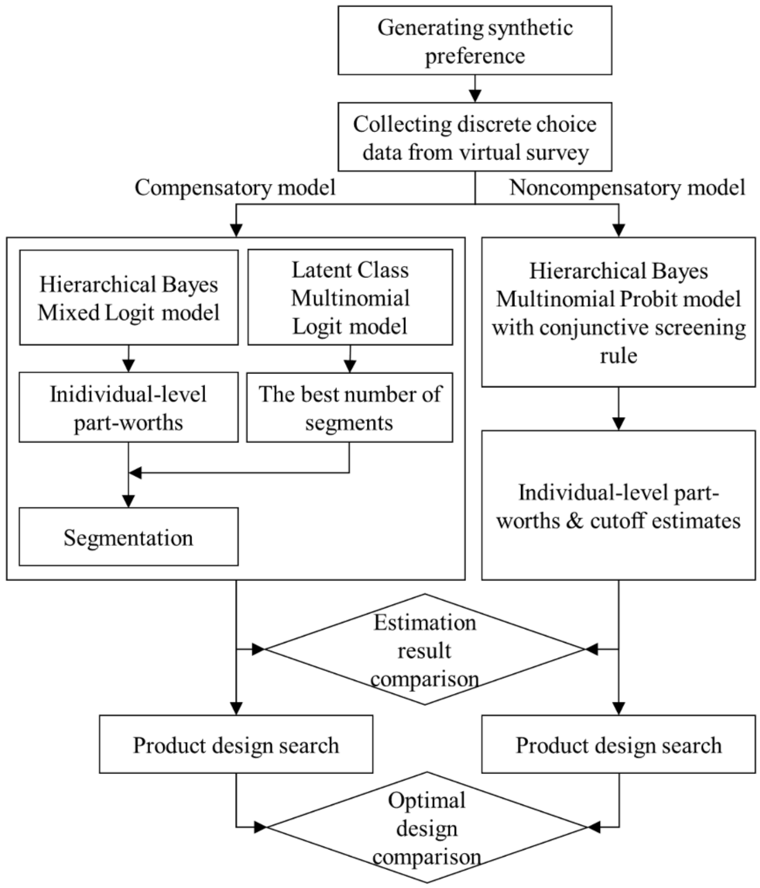

Figure 3.1. Flowchart for comparing compensatory models and a Bayesian-based noncompensatory model with conjunctive screening rules for synthetic choice data

A product design problem is then solved using a Genetic Algorithm (GA) for both the compensatory and noncompensatory models. So that individual-level preferences can be used in the optimization problem, the HB-ML and HB-MNP models are used as the compensatory and noncompensatory models, respectively. The design results are compared to show the suitability of compensatory modeling of the two-stage choice process for product search problems.

3.3.2. Real choice data

Real choice data does not provide information about which respondents actually made noncompensatory choices, or what heuristics they used to make those choices. Thus, it is impossible to apply the same technical approach used to analyze the outcomes from synthetic choice data because ‘true’ preferences are not available. For this reason, a different approach is proposed to assess and compare the optimum product designs of compensatory and noncompensatory models, as shown in Figure 3.2. To assess and compare the optimum product designs of compensatory and noncompensatory models, a hypothetical noncompensatory choice simulation is proposed.

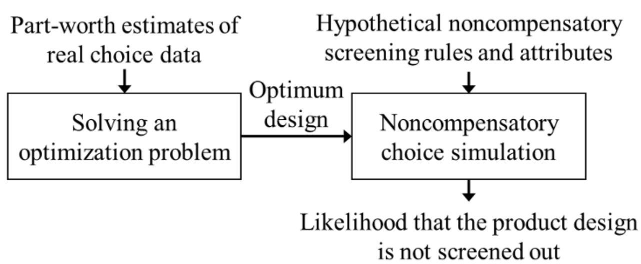

Figure 3.2. Conceptual procedure of noncompensatory choice simulation using hypothetical screening rules

The objective of this choice simulation procedure is to figure out whether a respondent screened out specific product features when making a choice decision. The simulation is based on discrete choice data from a real study. Given part-worth estimates for both a HB-ML model and a HB-MNP model with conjunctive rules, optimum product design solutions can be obtained by maximizing the choice probability of the solution. Then, a hypothetical noncompensatory screening rule is assumed, To consider many noncompensatory choice scenarios, the Disjunctions of Conjunctions (DOC) rule that generalizes conjunctive, disjunctive, and subset conjunctive rules (Hauser 2014) is used because the choice heuristics used by respondents are unknown. For an optimum product design, a noncompensatory choice simulation is conducted to calculate the likelihood (L_H) that the product design is not screened out at the noncompensatory choice stage. This likelihood is easily obtained by dividing the number of feasible screening rules by the total number of choice simulations.

An example of a hypothetical noncompensatory choice simulation when using real choice data is shown in Figure 3.3. Assume that respondents took a discrete choice survey consisting of 10 choice tasks involving products with 2 attributes and 5 total levels (three for attribute 1 and two for attribute 2). The data in Figure 3.3 shows the cumulative number of times each attribute level was chosen in the 10 choice tasks. For Product 1, created using level L3 of attribute A1 and level L1 of attribute A2, it is clear that this product was not screened out because the respondent chose these two attribute levels at least once when completing the choice tasks. However, for Product 2, created using level L1 of attribute A1 and level L1 of attribute A2, the analysis is more complicated.

Table: choice data of a respondent

	A1			A2	
Level	L1	L2	L3	L1	L2
Cumulative number of choices	0	2	8	4	6

A: attribute, L: level

<u>Noncompensatory choice simulation</u>

- Product 1 = (L3 of A1, L1 of A2)

  Cumulative number of choices = (8,4)

  → Product 1 was not be screened out in noncompensatory choice

- Product 2 = (L1 of Attribute 1, L1 of Attribute 2)

  Cumulative number of choices = (0,4)

  → It cannot be figured out what happened for Product 2 in noncompensatory choice

Figure 3.3. An example of a hypothetical noncompensatory choice simulation when using discrete choice data obtained from an actual survey

This analysis is made more challenging because level L1 of attribute A1 may pass the noncompensatory screening rule even though that attribute level was never chosen. From the data available, it cannot be determined if the respondent screened out this attribute level at the noncompensatory choice stage, or if this occurred because of the compensatory tradeoffs that occur after the consideration set has been created. Hence, when running a simulation for several screening rules, the minimum likelihood that a product is not screened out can be estimated in the hypothetical noncompensatory choice simulation.

3.4. Case Study using Synthetic Choice Data

As shown in Figure 3.3, choice data itself cannot be used to explicitly state respondents' choice processes. It is impossible to know who made noncompensatory choices and what heuristics were used. However, this information can be captured if synthetic data is created using pre-defined virtual agents. In this study, virtual respondents are generated using conjunctive screening rules because the existing studies of the Bayesian-based noncompensatory models suggest that the conjunctive rule model shows the best performance in both model fitness and predictability at individual-level estimates (Gilbride and Allenby 2004; Gilbride and Allenby 2006). Synthetic choice data is collected using a simulated discrete choice survey. Part-worth estimates from the compensatory model and the noncompensatory model with conjunctive screening rules are obtained and compared. Finally, a product design optimization is performed to compare differences in solution. Since only a conjunctive rule and a conjunctive model are used in this case study, the findings and discussions are limited to the conjunctive rule and its associated noncompensatory model. Exploring other forms of noncompensatory heuristic rules such as disjunctive, lexicographic-by-aspects, elimination-by-aspects, and disjunctions of conjunctions is recommended as a future research topic.

3.4.1. Generating synthetic choice data

To generate synthetic survey data, a choice-based conjoint survey is designed around a vehicle selection scenario. Attributes and levels used in this study are described in Table 3.1. The manual transmission, automatic transmission without shift, and automatic transmission with shift are called MT, AT1, and AT2, respectively. TM is used as an abbreviation of "transmission", and the capital letter A with a number stands is used to represent different product attributes. Survey questions are generated using Sawtooth SSI Web (Sawtooth

Software inc. 2011). Respondents are asked to evaluate 16 buying scenarios and 4 holdout questions. Each scenario contains four product alternatives and a fifth no-buy option.

Table 3.1. Car attributes and levels used in virtual survey

Level	Attribute						
	Transmission	Sunroof	A3	A4	A5	A6	Price
Level 1	MT	No	2 levels	4 levels	4 levels	4 levels	$21,000
Level 2	AT1	Yes					$20,000
Level 3	AT2						$19,000
Level 4							$18,000

Table 3.2 shows the pre-defined preferences of the 200 virtual respondents used to form consideration sets at the first choice stage. Respondents use only the transmission and sunroof attributes when making noncompensatory choices, and lower levels are screened out. For example, respondents cannot screen out AT1 because the conjunctive rule assumes there is a minimum requirement value. If a respondent screen out AT1 only, the noncompensatory model with the conjunctive rule cannot catch the behavior and the respondent is considered to make compensatory choices.

Table 3.2. Pre-defined preferences of virtual respondents

Group	Number of respondents	Screen out	Must-have feature in consideration set
1	40	MT & No Sunroof	AT1/AT2 & Sunroof
2	40	MT	AT1/AT2
3	40	MT & AT1	AT2
4	40	No Sunroof	Sunroof
5	40	Only perform compensatory choices	

Respondents in groups 1 through 4 exhibit noncompensatory behavior and narrow their choice alternatives into a consideration set. Then, they compare all remaining alternatives and choose one. To mimic a real choice situation, if no alternative in the consideration set satisfies the minimum utility requirement, the no-buy option is selected. Respondents in group 5 perform only compensatory choices. To mimic the additive part-worth rule in compensatory choices and introduce heterogeneity, respondents' preferences (excluding price) are generated based on uniform distributions with pre-defined intervals. Price preferences are manually generated and constrained so that respondents prefer lower prices. The virtual survey results in 3,200 observations for model estimation.

Attribute importance of the synthetic data for each group is shown in Table 3. Since attribute importance is calculated based on an additive rule assumption, the importance of noncompensatory attributes cannot be evaluated. However, maximum and minimum utility values of noncompensatory variables exist. These values are defined as:

Attribute importance of the synthetic data for each group in shown in Table 3.3. Since attribute importance is calculated based on an additive rule assumption, the importance of noncompensatory attributes cannot be evaluated. However, maximum and minimum utility values of noncompensatory variables exist. These values are defined as:

$$\max\left(V_{nc}\right)+\sum_{h}\min\left(V_{c,h}\right)>V_{threshold}>\min\left(V_{nc}\right)+\sum_{h}\max\left(V_{c,h}\right) \tag{3.5}$$

where V is a part-worth set for each attribute, h indicates the number of compensatory attributes, while nc and c indicate noncompensatory and compensatory attributes, respectively. $\max(V_{nc})$ indicates a part-worth of the noncompensatory attribute in the consideration set and $\min(V_{nc})$ indicates a part-worth set of the noncompensatory attribute excluded from the consideration set. From Eq. (3.5), the smallest range of $\max(V_{nc})-\min(V_{nc})$ is obtained as $\sum_{h}\max(V_{c,h})-\sum_{h}\min(V_{c,h})$. Hence, the minimum attribute importance of a noncompensatory attributes is described by Eq. (3.6):

$$\frac{\max(V_{nc})-\min(\mathrm{V}_{nc})}{\max(V_{nc})-\min(V_{nc})+\sum_{h}\{\max(V_{n,h})-\min(\mathrm{V}_{c,h})\}}\,. \tag{3.6}$$

From this calculation, the minimum attribute importance of a noncompensatory attribute is 50%. This value is used to compare the degree to which compensatory models describe noncompensatory choices in Chapter. 3.4.2 and 3.4.3.

Table 3.3. Attribute importance of the synthetic data

Group	TM	Sunroof	A3	A4	A5	A6	Price
1	50.0		4.2	9.6	10.1	9.6	16.6
2	50.0	4.7	5.0	8.2	8.8	8.5	14.9
3	50.0	5.7	4.2	8.6	9.6	7.5	14.4
4	7.1	50.0	4.8	8.0	8.3	7.8	14.0
5	11.7	8.1	8.7	14.9	15.1	16.1	25.3

3.4.2. HB-MNP model with conjunctive rules

Once the virtual survey results were collected, the HB-MNP with the conjunctive rule was fit using R (R foundation for statistical computing 2015). The inference was conducted using Bayesian MCMC methods. The chain was run for the first 5,000 iterations, with the final 5,000 iterations used to estimate the moments of the posterior distributions.

Table 3.4 shows the aggregate estimates of the cutoff probability obtained using the conjunctive model. For discrete attributes, cutoffs are reported in terms of multinomial point mass probabilities. Each level is recorded as an integer (e.g., 0, 1, 2) and the recorded values indicate l_{nim} in Eq. (3.2). Thus, a grid of possible cutoff values, γ_{nm}, are also specified (e.g., -0.5, 0.5, 1.5, 2.5). The lowest cutoff value indicates that all levels are acceptable and respondents made compensatory choices. The highest level indicates that none of the levels are acceptable (Gilbride and Allenby 2004). A cutoff value of 0.5 indicates that only the lowest level is unacceptable, and 1.5 indicates that level 1 and 2 (recorded values 0 and 1) are screened out.

Table 3.4. Threshold estimates for the posterior means of the conjunctive model

Attri-bute	Level (recorded value)	Possible cutoff	Probability of each cutoff		Attri-bute	Level (recorded value)	Possible cutoff	Probability of each cutoff	
			Pre-defined	Obtained				Pre-defined	Obtained
TM	MT (0)	-0.5	40 %	38.0 %	A5	1 (0)	-0.5	100 %	88.6 %
	AT1 (1)	0.5	40 %	38.8 %		2 (1)	0.5	0 %	3.6 %
	AT2 (2)	1.5	20 %	20.6 %		3 (2)	1.5	0 %	2.6 %
		2.5	0 %	2.7 %		4 (3)	2.5	0 %	2.6 %
							3.5	0 %	2.6 %
Sunroof	No (0)	-0.5	60 %	58.1 %					
	Yes (1)	0.5	40 %	39.2 %	A6	1 (0)	-0.5	100 %	88.5 %
		1.5	0 %	2.8 %		2 (1)	0.5	0 %	3.6 %
						3 (2)	1.5	0 %	2.6 %
A3	1 (0)	-0.5	100 %	94.5 %		4 (3)	2.5	0 %	2.6 %
	2 (1)	0.5	0 %	2.8 %			3.5	0 %	2.6 %
		1.5	0 %	2.7 %					
					Price	$21k (0)	-0.5	100 %	87.6 %
A4	1 (0)	-0.5	100 %	89.2 %		$20k (1)	0.5	0 %	4.3 %
	2 (1)	0.5	0 %	3.0 %		$19k (2)	1.5	0 %	2.9 %
	3 (2)	1.5	0 %	2.6 %		$18k (3)	2.5	0 %	2.6 %
	4 (3)	2.5	0 %	2.6 %			3.5	0 %	2.6 %
		3.5	0 %	2.6 %					

The probabilities of each cutoff obtained from the conjunctive model closely correspond with the pre-defined noncompensatory preferences in Table 3.2. For example, 40% of the total respondents were pre-defined to screen out MT in the virtual survey, and the conjunctive model results in 38% doing so. Estimates of cutoff probability equal to approximately 2% reflect the influence of the prior distribution and the inherent noise of MCMC. Therefore, respondents are shown to evaluate attributes 3, 4, 5, and price using a compensatory rule set.

For non-compensatory attributes, the part-worth estimates shown in Table 5 approach zero. This is because the choice probability is evaluated using only the alternatives in the consideration set. If an alternative is in the consideration set, then its choice probability is determined relative to the other alternatives in the set. If an alternative does not pass the screening rule, then its choice probability is zero. Thus, the screened alternative is not included in the posterior estimation processes. This leads to a competition of alternatives in the consideration set that only has acceptable product features. Eventually, distinctions in posterior estimates between features are diminished and the estimates approach zero. In other words, noncompensatory attributes are excluded from part-worth estimation as explained in Chapter 3.2.1. Because of this, the part-worths of noncompensatory attributes, such as transmission and sunroof, are estimated to be relatively flat in comparison to the other attributes, as shown in Table 3.5.

Table 3.5. Part-worth estimates for the noncompensatory model

Attribute	Level	Posterior mean	Attribute	Level	Posterior mean
TM	AT1	0.09	A5	2	-0.30
	AT2	0.05		3	-0.26
				4	0.16
Sunroof	Yes	-0.07			
			A6	2	0.07
A3	2	0.40		3	-0.41
				4	-0.23
A4	2	0.34			
	3	0.32	Price	$ 20,000	1.78
	4	0.15		$ 19,000	3.31
				$ 18,000	5.08

3.4.3. Latent class analysis

Segmentation of a population often occurs when respondents within a group have relatively similar preferences, but those preferences are quite different from group to group. It is hypothesized that if there are distinct attributes used to form consideration sets, these attributes will play the most significant role in defining different preferences from group to group. Latent class estimation was conducted using Sawtooth Software's CBC Latent Class module (Sawtooth Software inc. 2007). Statistical measures assess the goodness of fit (Nylund, Asparouhov, and Muthén 2007), but often provide conflicting information on the optimal number of classes in the model. Based on the low rate of change in these statistics for models with 5 classes or more, the latent class model fit with 5 classes was selected.

Table 3.6 shows a comparison between the predefined respondent groups originally defined in Table 3.2 and latent class estimation. As listed in Table 3.2, the five groups are expressed using Arabic numbers according to their noncompensatory choices. The groups obtained from the latent class analysis, expressed using Roman numbers, are nearly identical to the pre-defined groups. In particular, all respondents in groups 1, 2, and 3 are placed in segments I, II, and III. Also, the 40 respondents in group 4, who were defined to screen the no-sunroof feature, are divided into segments I and IV. Since the 'no sunroof' feature is screened out of segments I and IV, the 2 respondents moved from group 4 to segment I are still considered to maintain their preferences. The estimates of the respondents in group 5 depend on random preference generation and survey design. Even though these respondents did not make noncompensatory choices, if cumulative choices are rationally biased, it could be defined as a member of the segment that does make noncompensatory choices. This is because the latent class analysis does not estimate noncompensatory choices, but simply classifies respondents with similar preferences.

Table 3.6. Number of members in each group

		Latent class					
		I	II	III	IV	V	sum
Pre-defined group	1	40					40
	2		40				40
	3			40			40
	4	2			38		40
	5	1	4		10	25	40
	sum	43	44	40	48	25	200

Membership probability demonstrates how effectively respondents are categorized into groups. Latent class estimation assumes that each respondent has some non-zero probability of belonging to each group. If the segmentation strategy fits the data very well, membership probabilities approach one. As shown in Table 3.7, respondents effectively have membership probabilities in only one class. The average maximum membership probability is 99.36%.

Table 3.7. Membership probability of belonging to a group

Number of Respondent	Latent class				
	I	II	III	IV	V
43	99.88	0.00	0.00	0.12	0.00
44	0.99	98.68	0.00	0.00	0.33
40	0.00	0.01	99.99	0.00	0.00
48	0.00	0.00	0.00	99.96	0.04
25	0.00	1.67	0.00	0.01	98.31

The attributes used to form consideration sets may be inferred from the attribute importance associated with each group. As shown in Table 8, the noncompensatory attributes associated with each segment have greater than 50% importance, similar to the result found in Eq. (3.6). Additionally, the attributes that result in the formation of consideration sets have much greater importance than the other attributes. Despite the strong inference about the attributes used in noncompensatory choice, the latent class analysis does not explicitly identify if a noncompensatory screening is used. The result in Tables 3.7 and 3.8 show that if there are noncompensatory choices, and the latent class results in estimates with high membership probabilities, the importance for the attribute driving the noncompensatory rule will be higher than 50%.

Table 3.8. Attribute importance of latent class analysis

Segment	TM	Sunroof	A3	A4	A5	A6	Price
I	70.5		3.5	4.6	3.1	5.3	13.0
II	56.4	2.0	1.1	4.0	4.0	2.8	29.7
III	82.9	0.3	1.0	3.0	1.7	2.0	9.1
IV	3.4	50.7	2.6	5.1	4.9	2.5	30.8
V	5.7	11.6	5.1	9.9	10.4	9.1	48.2

3.4.4. HB-ML model

While the number of segments and the features forming consideration sets can be speculated using the LC-MNL model, individual-level preferences can be estimated using the HB-ML model. This section provides details on how the HB-ML model mimics the two-stage choice process by presenting part-worths, attribute importances, and rank orders of the feature levels.

Aggregate zero-centered part-worth estimates for the HB-ML model are shown in Table 3.9. The HB-ML model was fit using the Sawtooth Software CBC/HB module (Sawtooth Software inc. 2014). For each respondent 10,000 random draws were performed before averaging the next 10,000 random draws to create the posterior means. It is observable that the transmission and sunroof attributes have posterior means with larger deviations because they are used to mimic the behavior associated with creating the consideration sets. In contrast, the posterior means of the other attributes are relatively flat.

Table 3.9. Part-worth estimates for the HB-ML model

Attribute	Level	Posterior mean	Attribute	Level	Posterior mean
TM	AT1	9.57	A5	2	-0.69
	AT2	13.04		3	-0.72
				4	-0.09
Sunroof	Yes	8.37			
			A6	2	0.56
A3	2	0.57		3	0.19
				4	0.16
A4	2	0.64			
	3	0.53	Price	$ 20,000	3.65
	4	0.17		$ 19,000	5.69
				$ 18,000	8.22

The results in Table 3.9 are for all respondents. Borrowing segmentation information from the latent class analysis, the individual-level part-worth estimates obtained from the HB-ML model are grouped by segment. For brevity, only the attributes used when making noncompensatory choices are listed. From these results, the second hypothesis that large absolute part-worth values (with respect to the other attributes) would be captured in the individual-level estimates is verified using the segment-level part-worths and the distributions of the individual-level part-worths.

Table 3.10. Hit rate comparison between HB-MNP with conjunctive rules and HB-ML model

Model	Hit Rate
HB-MNP with conjunctive rule	69.19%
HB-ML	72.75%

A comparison of model performance using predictive accuracy is provided in Table 3.10. Predictive accuracy is defined by how well the model can predict a future set of observations. For synthetic choice data without added variability, a hit-rate measure also describes how well a model captures the pre-defined preferences of the virtual respondents. Using the four holdout questions, a hit-rate measure is obtained for each model. Table 3.10 shows that the compensatory model (HB-ML) has a slightly greater predictive accuracy than the noncompensatory model (HB-MNP with conjunctive rule). The original research paper that presented the noncompensatory HB-MNP model suggested that the HB-MNP model with conjunctive rule should have a greater predictive accuracy than a compensatory model (HB-MNP) (Gilbride and Allenby 2004). However, this study demonstrates that a compensatory model can be more in some scenarios, even though the comparison is between a logit model and a probit model. This result could be caused by the inherent noise of MCMC for the cutoff estimation of the noncompensatory model discussed in Chapter 3.2.2. Although the hit-rate difference is small, the larger hit rate for the HB-ML model is expected to reduce potential design errors due to incorrect preference estimation than the conjunctive rule model.

Table 3.11. Zero-centered part-worth estimates of each segment obtained using HB-ML model

Latent class	Transmission			Sunroof	
	MT	AT1	AT2	No	Yes
I	-12.7	5.6	7.1	-9.9	9.9
II	-10.9	5.5	5.4	-0.2	0.2
III	-11.7	-1.4	13.1	-0.5	0.5
IV	-0.4	-0.1	0.5	-8.6	8.6
V	0.1	-0.5	0.5	1.1	-1.1

Table 3.11 shows that segments I through IV have large absolute part-worth coefficients compared to the relatively flat part-worths estimated for segment V. The large absolute part-worths at the individual-level are also observed in the histogram displayed in Figure 3.3.

The presence of large absolute part-worth values in a compensatory model is significant, because if a part-worth value for an attribute is large enough it can effectively mimic the upper stage of a noncompensatory screening rule. In the aggregate estimates of the HB-ML model it is also noticeable that the AT1 feature of segment III is relatively flat. However, MT has a large absolute value despite the fact that the two features were screened out at the same time in the virtual survey. This is not a special case in the commercial software used; rather is it likely an outcome of the prior distribution assumption.

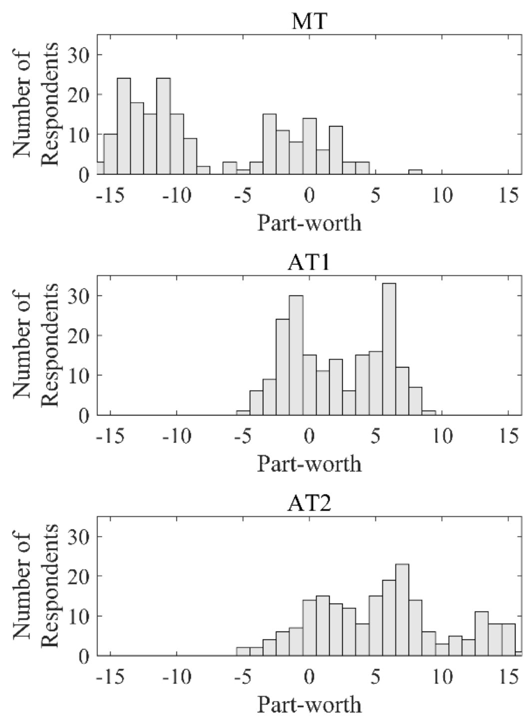

Figure 3.4. Histogram of aggregate posteriors for transmission attribute obtained using the HB-ML model

All three histograms in Figure 3.4 are closer to multimodal distributions than normal distributions. The distribution of heterogeneity has to be specified when estimating hierarchical

Bayesian models, and a multivariate normal distribution is most commonly used. The commercial software used in this study also adopts the multivariate normal distribution. However, when the true distribution of heterogeneity is as close to a finite mixture of normal distributions as the noncompensatory choices, it is inappropriate to use a multivariate normal. Thus, a hierarchical Bayes model may over-estimate the proportion of the part-worths near zero (Gilbride and Lenk 2010). Exploring appropriate prior distribution assumptions, or incorporating prior knowledge, is a source of future work.

Attribute importance values listed in Table 3.12 are obtained using the individual-level part-worths obtained from the HB-ML estimation. As described in Chapter 3.4.1, the noncompensatory variables have to have at least 50% importance; group 1 in the HB-ML model satisfies this condition. However, notice that groups 2, 3, and 4 contain importance values lower than 50%. This implies that the HB-ML model does not completely approximate a noncompensatory choice.

Table 3.12. Attribute importance of HB-ML model

Group	TM	Sunroof	A3	A4	A5	A6	Price
1	67.9		2.8	3.7	6.1	4.9	14.5
2	41.2	7.1	6.0	7.2	8.8	9.0	20.6
3	43.7	8.3	5.2	8.2	9.6	8.6	16.4
4	11.5	42.2	5.6	6.5	7.3	6.9	20.0
5	14.8	11.8	8.0	10.7	13.6	13.6	27.6

Importance values below 50% suggest a switching of products across the threshold of selection. For a respondent who screened out MT in each choice task, Figure 3.5 depicts the switching of a product having an MT feature and the largest part-worths of each compensatory attribute. Even though the respondent never selected the MT feature in the virtual survey, some products having MT can be selected in a market simulation. This is due to the absence of a strict heuristic consideration rule in compensatory models. However, since product search problems only focus on the several top products, the impact of this scenario may be minimal.

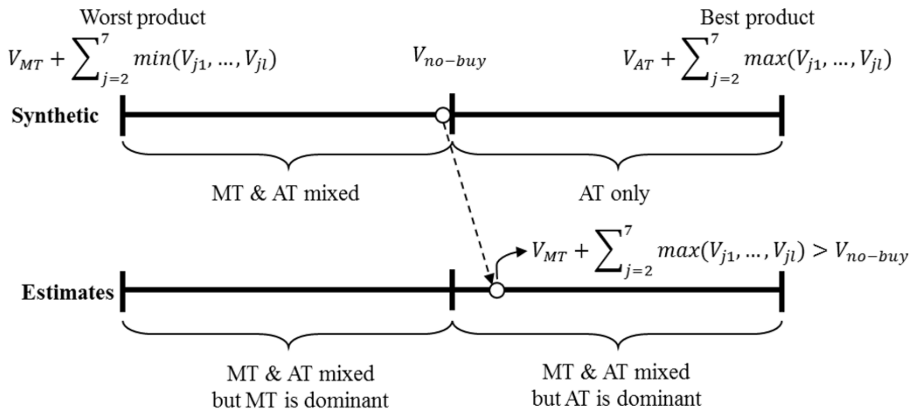

Figure 3.5. Conceptual diagram to show the absence of a strict threshold in compensatory modeling of noncompensatory choice

The results in Table 3.13 help explain why the absence of a strict threshold may have minimal impact in optimization problems. 160 respondents from groups 1 to 4 performed noncompensatory choices. Using the HB-ML part-worths, these screening rules can be reproduced for 150 of the 160 respondents. Further, the utility gaps between the best product and the switched product are significantly large. The effectively zero odds ratio values also suggest that the violated products likely have no significant effect on a product search.

Table 3.13. Part-worth comparison of the switched products

Pre-defined group	No. of respondents having switched products	Avg. utility of the best product*	Avg. utility of the best product among switched products**	Threshold (V_{no-buy})	Odds Ratio***
2	4	16.3	1.5	1.0	≅ 0
3	2	20.7	8.7	8.0	≅ 0
4	4	22.5	4.0	1.8	≅ 0

* $V_{nc,consideration} + \sum \max(V_{c1}, \dots, V_{cl})$, ** $V_{nc,screen\ out} + \sum \max(V_{c1}, \dots, V_{cl})$

*** $\frac{exp(V_{nc,screen\ out} + \sum \max(V_{c1}, \dots, V_{cl}))}{exp(V_{nc,consideration} + \sum \max(V_{c1}, \dots, V_{cl}))}$

3.4.5. Product design search

This section focuses on product configuration differences when using the compensatory and noncompensatory models in an optimization. Individual-level aggregate estimates of the

HB-ML model are used as the compensatory model. In contrast, as explained in Chapter 3.2.2, using the aggregate values of the conjunctive model is challenging because of the probabilistic representation of cutoffs and corresponding part-worth estimates. For this reason, 5,000 draws were generated using an MCMC process. To ensure independency between draws and to manage the computational cost of this procedure, every 10th draw was kept for use in the optimization, leading to 500 draws per individual. All draws were equally weighted when evaluating share of preference.

The pricing structure for each attribute is shown in Table 3.14. In addition to this pricing structure, a base price of $18,000 is added. A piecewise linear interpolation is used to calculate the price attribute part-worth a piecewise linear interpolation is assumed. The objective of the search is to maximize the choice probability of the optimized product configuration in a competitive market, using Eq. (2.7) and (3.4) for the compensatory and noncompensatory models, respectively. Competitor products are defined in Table 3.15.

Table 3.14. Pricing structure

	TM	Sunroof	A3	A4	A5	A6
Level 1	0	0	0	0	0	0
Level 2	800	500	500	100	200	100
Level 3	1000			200	300	200
Level 4				300	400	300

Table 3.15. Attribute levels of competitor products in the market

Competitor	TM	Sunroof	A3	A4	A5	A6	Price
Product 1	MT	No	1	1	1	1	$18,000
Product 2	AT1	Yes	1	2	3	3	$19,900
Product 3	AT2	Yes	2	4	4	4	$21,000

The first scenario considered was finding the best product to offer. This scenario does not necessarily require a search algorithm because only 768 product configurations had to be evaluated. The second scenario considered was finding the optimal configurations when offering two products, leading to a problem size of 589,056 product combinations. This required a search algorithm because one market simulation using the noncompensatory model took approximately 15 seconds on a laptop running an Intel i7 2.20 GHz with 16GB RAM. A genetic algorithm was used for both model formulations because a GA works directly with discontinuous choice probabilities and previous work has shown the advantages of this technique in a product line optimization (Z. Wang, Kannan, and Azarm 2011; Foster et al. 2014). The pool size was set at 200 and the stall generation limit was set to 50.

Table 3.16. Optimal product configuration for each model (Scenario 1)

Model	TM	Sun-roof	A3	A4	A5	A6	Price	SOP_o	SOP_t	Design error
True	AT2	Yes	1	3	1	2	$20,100	37.1%	37.1%	-
Compensatory	AT2	Yes	1	2	1	2	$20,300	52.4%	35.9%	3.2%
Noncompensatory	AT2	Yes	1	1	3	3	$20,350	41.3%	29.2%	21.3%

Table 3.17. Optimal product configuration for each model (Scenario 2)

Model	TM	Sun-roof	A3	A4	A5	A6	Price	SOP_o	SOP_t	Design error
True	AT1	Yes	1	1	3	3	$19,850	54.2%	54.2%	-
	AT2	Yes	1	2	1	2	$20,300			
Compensatory	AT1	Yes	2	2	1	3	$19,900	68.7%	47.8%	11.8%
	AT2	Yes	1	2	1	2	$20,300			
Noncompensatory	AT2	Yes	2	1	3	3	$20,550	61.8%	45.6%	15.9%
	AT2	Yes	1	4	2	4	$20,400			

Solutions to the two optimization scenarios are shown in Tables 3.16 and 3.17, respectively. Investigating solution differences is divided into two aspects – the inclusion of must-have features/attributes and design error. Design error is quantified by evaluating the objective function using both the synthetic (true) preferences generated in Chapter 3.3.1 and the estimated preferences from the model fits. SOP_o indicates the share of preference used as an objective function value. SOP_t indicates the share of preference evaluated in the synthetic (true) preferences. The design error metric is defined as (SOP_t of synthetic data $-$ SOP_t of estimated data)/ $\mathrm{SOP_t}$ of synthetic data. For instance, the design error of the compensatory model for the scenario 1 is obtained as $(37.1 - 35.9)/37.1 = 3.2\%$. Notice that the gap, $SOP_t - SOP_o$, does not directly provide an evaluation of a design solution because the value can change by adjusting the scale parameter σ_β and other settings associated with model estimation.

In the one product design scenario, the noncompensatory attributes (transmission and sunroof) are represented by the levels used to form the consideration sets. The compensatory model product configuration has three of the correct compensatory features, while the noncompensatory model product configuration only has one correct. These results imply that if there are strong noncompensatory choices, the noncompensatory attributes can be found regardless of model. In terms of design error, the compensatory model performs better than the noncompensatory model.

Similar outcomes are observed in the two product scenario. The true data and the compensatory model use the same transmission and sunroof features, though there is a discrepancy in some of the compensatory attributes. Although the noncompensatory model resulted in the only AT2 for the transmission feature, the crucial finding is that both models find solutions that would be included in the consideration sets.

The predictive accuracy listed in Table 3.10 provides an evidence that supports why the compensatory model (HB-ML) resulted in smaller design errors than the conjunctive model (HB-MNP with conjunctive rule) in both problems. The slighter greater predictive accuracy of the compensatory model implies that the compensatory model better reflects the true preferences of the respondents. However, this result is limited to the simulated data for this study and additional research is needed to understand design problem formulation influences this result.

For the compensatory attributes there exists both commonality and discrepancy in the optimal product configurations. To investigate how many differences exist between the three solutions, choice probabilities are tested at all attribute levels to assess the sensitivity of the share of preference calculation. The intervals between the maximum and minimum choice probabilities when only one attribute changes its level are shown in Table 3.18.

Table 3.18. Choice probability interval sensitivity study

Data	Interval between max and min choice probabilities (%)					
	TM	Sunroof	A3	A4	A5	A6
True	19.2	9.5	0.6	3.4	5.3	7.2
Compensatory	36.6	22.2	1.4	6.6	16.4	9.1
Noncompensatory	36.7	18.2	1.8	4.3	7.9	9.7

The results in Table 3.18 show that, for both models, the intervals between maximum and minimum choice probabilities are larger than the true data. Also, the noncompensatory attributes have significantly larger intervals than the compensatory attributes. Normalized intervals of choice probability for each model are displayed in Figure 3.6. This result is analogous to attribute importance in the product search. As a general result, the noncompensatory model tends to give more weight to the noncompensatory attributes while underweighting the compensatory attributes. The result for the compensatory model is less structured, with variability across all attribute types.

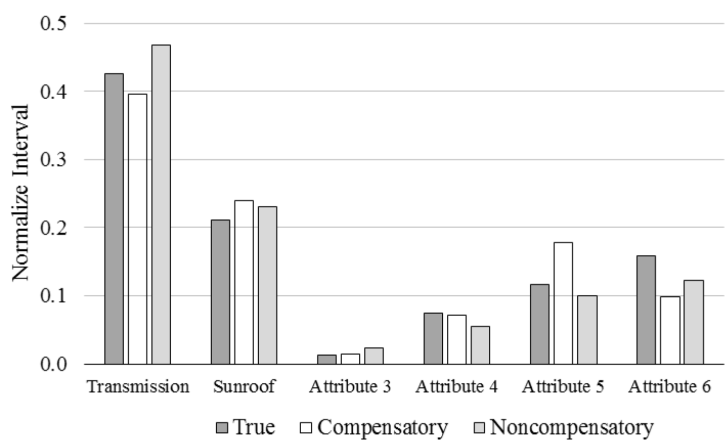

Figure 3.6. Interval comparison between the max. and min. choice probabilities of each attribute

In this study, the compensatory model is more accessible because of the reduced computational burden and no requirement for screening rules. Product configurations from both estimated models found solutions very similar to the optimal solution when the true preferences are used. When calculating design error, the compensatory model outperformed the noncompensatory solution in both the one and two product design scenario. To further support this finding in the product search problem, Chapter 3.5 explores a problem using choice data from real respondents.

3.5. Case Study using Real Choice Data

3.5.1. Survey design and modeling

The data presented in Chapter 3.4 applies to a survey with virtual respondents where the preferences are the respondents were known a priori. To explore solution differences when using choice data from an actual survey, a second case study was conducted. Motivation of this case study and technical approach are presented in Chapter 3.3.2. For this case study, a discrete choice survey with 12 choice tasks was completed by 205 respondents. For each question, the respondent was faced with four MP3 player configurations and a “No-Buy” option. Respondents were then asked to choose one product alternative they would be most likely to purchase. Each MP3 player was composed of 8 product attributes. Product attributes and their levels are listed in Table A1 of the Appendix.

Once the survey data was collected, the HB-MNP with a conjunctive rule was fit using R (R foundation for statistical computing 2015) and the HB-ML was fit using Sawtooth Software’s CBC/HB module. For both models 10,000 random draws were used for each respondent before averaging the next 10,000 random draws to estimate the moments of the posterior distributions. For the noncompensatory model, every 20th draw was kept for use in the optimization to ensure draw independency and to manage the computation expense. This resulted in 500 draws per individual.

Table A2 in the Appendix shows the aggregate estimates of the cutoff probability obtained using the conjunctive model. Conjunctive screening rules were estimated at 55.2% and 75.7% for the Storage size and Price attributes, respectively. They can be regarded as incremental attributes whose higher levels are preferred. This trend is shown as large absolute part-worth values of the compensatory model as shown in Table A3. In addition, this leads to the relatively large attribute importance values in the compensatory model, as listed in Table A4. This suggests that even though the compensatory model does not have an ability to capture noncompensatory choice behavior, biased preference in the noncompensatory attributes is maintained.

The cost structure in Table A5 is assumed with a base cost of $33. Price is assumed as one and a half times the cost. A piecewise linear interpolation is assumed to calculate the part-worths for the price attribute. The product features of the competing products are summarized in Table A6. The competitor products are based on the feature and price combinations for the iPod Nano, iPod Touch, and iPod Classic to match a possible real world market. In addition to the three competing products, the 'No-buy' option is also considered part of the market.

3.5.2. Product design search

This section focuses on the differences in optimal solution when using the compensatory and noncompensatory models. Individual-level aggregate estimates of the HB-ML model are used for the compensatory model. In contrast, simulations involving 500 draws were used for each respondent in the noncompensatory model. Three competitor products are defined. The objective of the search is to maximize the choice probability of the optimized product configuration in a competitive market, using Eq. (2.7) and (3.4) for the compensatory and noncompensatory models, respectively. In addition, the noncompensatory choice simulator explained in Chapter 3.2.2 was used to search for the best likelihood value that a design is not screened out. The hypothetical noncompensatory choice simulation used in this study assumes respondents focus on only a small subset of product attributes in the noncompensatory choice stage to simplify their choice decisions. Combinations of conjunctive and disjunctive rules are assumed in the simulation.

The maximum numbers of subset conjunctive and disjunctive rules are set as two and one. In set theory, the two subset conjunctive rules and one disjunctive rule are expressed as $(C_1 \cap C_2) \cup D_1$, where C and D indicate conjunctive and disjunctive rules, respectively. Thus, 332 different noncompensatory choice scenarios exist, which is obtained as

$\left({}_{8}C_{2}+{}_{8}C_{1}+{}_{8}C_{0}\right)\times\left({}_{8}C_{1}+{}_{8}C_{0}\right)-1$. By averaging the results of the 332 choice simulations, the minimum likelihood (L_H) that a product design is not screened out in the hypothetical noncompensatory choice can be estimated.

The first scenario was to find the best product to offer. This scenario leads to 393,216 possible product feature combinations. The second scenario considered was finding the optimal configurations when offering two products, leading to a problem size of about 1.54e+11 product combinations. A genetic algorithm was used for both problems. The pool size was set as 10 times ndv where ndv indicates the number of design variables and the stall generation limit was set to 100.

Table 3.19. Optimal product configuration for each model (Scenario 1)

Data	A1	A2	A3	A4	A5	A6	A7	Price	SOP	L_H
Simulator	3	5	3	3	2	6	1	$192	-	59.3%
Compensatory	5	8	3	3	2	1	4	$209	33.8%	49.4%
Noncompensatory	8	8	3	4	2	3	4	$246	29.5%	51.9%

Table 3.20. Optimal product configuration for each model (Scenario 2)

Data	A1	A2	A3	A4	A5	A6	A7	Price	SOP	L_H
Simulator	3	5	3	3	2	7	1	$192	-	72.7%
	6	6	4	6	6	6	4	$413		
Compensatory	5	8	3	3	2	1	4	$209	50.3%	65.7%
	8	5	3	4	5	8	3	$396		
Noncompensatory	5	8	2	4	2	3	4	$194	45.9%	66.7%
	8	5	3	5	5	6	3	$396		

Solutions to the two optimization scenarios are shown in Tables 3.18 and 3.19, respectively. The L_H value was obtained by evaluating the optimum design using the hypothetical noncompensatory choice simulator. In the one product design scenario, a noticeable result is that the optimum designs of compensatory and noncompensatory models have four common product features. In addition, the likelihood values are comparable to the largest likelihood value that is obtained in the optimization problem using the noncompensatory choice simulator. Similar outcomes are observed in the two product scenario; both solutions have some common product features and their likelihood values are similar.

These results demonstrate that a compensatory model can be advantageous and useful even if a product designer believes noncompensatory choices have been made. Without strict screening rules, it is possible for a compensatory model would be able to mimic the two-stage choice process using large absolute part-worth values to prevent product solutions with screened out product attributes.

3.6. Summary

The main purpose of this study was to explore the suitability of using compensatory models to mimic the consider-then-choose process when trying to design an optimal product offering. This work is motivated by the potential errors associated with assuming screening rules, probabilistic representations of noncompensatory choices, and discontinuous choice probability functions associated with existing Bayesian-based noncompensatory models. It was hypothesized that distinct segments would be captured where screening occurred, and that large absolute part-worth values would be found in the individual-level estimates of the HB-ML model.

To verify this hypothesis, this research first investigated segmentation techniques of the two-stage choice process using latent class analysis. Using latent class is based on the idea that noncompensatory choices would cause a distinct differentiation of population preference. The numerical results of latent class analysis confirm this hypothesis. The distribution of preference heterogeneity is explored to compare the true preference and the compensatory model at individual-level preference. The results of the individual-level preference analysis show that the HB-ML model can represent noncompensatory choices using large absolute values in part-worths despite the absence of strict thresholds. Lastly, implications of model choice between the two representations of the consider-then-choose process are discussed using the results of the product design search problem. The results of the product design search show interesting implications of model form choice. Although there are several insignificant differences between the two models in the market simulation, the compensatory model has some significant advantages such as the small design error and its relatively inexpensive computational burden. In the analysis of the real choice data, results also confirm the suitability of the compensatory model in product design search as its optimum design has an acceptable level in the likelihood gap associated with the proposed hypothetical noncompensatory choice simulation.

A limitation of this work is that the attributes used to make noncompensatory choices may not have the largest importance in a latent class model when only a small number of respondents make noncompensatory choices. However, this should not be a concern when searching for an optimal solution as the outcome reflects solutions capable of maximizing or minimizing an objective across all respondents. In this case, the small number of the respondents performing noncompensatory choices would not have a significant effect on the optimization problem. Further, a standard choice-based study was used. These findings should be explored using more complex survey tools like adaptive choice that refines the choice alternatives seen by a respondent as data from the choice tasks is gathered. Finally, it should be noted that the findings of this work cannot be generalized to all noncompensatory choices and all noncompensatory models that exist in the literature. Rather, this study was focused on conjunctive screening rules estimated using a Bayesian-based noncompensatory model.

Future work will focus on six different challenges: 1) developing optimization techniques capable of using the compensatory model of the two-stage choice process, 2) resolving the limitation of assuming normally distributed priors, 3) exploring differences between compensatory models when different noncompensatory heuristics are used by respondents, 4) investigating the implications of the discrepancy between the true preference distribution and the prior distribution assumption of Bayesian inference, 5) exploring the effect of modeling other noncompensatory heuristic rules with a compensatory model in a product design search, and 6) developing segmentation methods for noncompensatory models.

Chapter 4. Implications of Heterogeneous Preference Presentation

Market-based product design optimization is focused on designing products that satisfy a diverse set of customer needs. Accounting for the heterogeneous distribution of customer preferences allows an optimization algorithm to effectively explore market opportunities. Common approaches to modeling heterogeneity include the hierarchical Bayes mixed logit (HB-ML) model and the latent class multinomial logit (LC-MNL) model. The HB-ML model exemplifies a continuous representation of heterogeneity, while the LC-MNL model corresponds to a discrete representation. This study compares the relative performance of each model form in terms of model fitness and predictive ability. The structure of heterogeneous preferences is also explored to investigate why differences are observed in model estimates and product solutions obtained from an optimization. Results reveal a continuous representation leads to better model fitness, predictive ability, and reduced design error in the optimal solution due to its larger degree of freedom. If, however, market segmentation is of interest, a discrete representation would be more effective because it first derives segment-level preferences.

4.1. Introduction

The foundation of market-based product design is that customer needs can be mathematically modeled and integrated into the engineering design process. Representing variation in taste is important to this process, because capturing different segments of the market leads to differentiated product designs (Allenby and Rossi 1998). Two customer preference models capable of representing customer heterogeneity are the hierarchical Bayes mixed logit (HB-ML) model and the latent class multinomial logit (LC-MNL) model. The HB-ML model estimates part-worth values by assuming a continuous distribution of preference heterogeneity. The LC-MNL model describes heterogeneity with discrete distributions that are determined by assigning the probability that a respondent belongs to a particular class. In a recent study (Sullivan, Ferguson, and Donndelinger 2011), part-worth estimates from both models were used to design an optimal product line. Significant differences between the design solutions were observed, and they explored why the solution differed by comparing correlations between the part-worth estimates and simulating first-choice at the respondent level.

Building off this previous effort, this study empirically compares the relative performance of the HB-ML and LC-MNL models in terms of model fitness and predictive ability. Impacts of model form selection on the output associated with product line optimization are also investigated. The goal of this study is not to argue that one statistical model is superior to another, but to investigate the application of each model in product design problems.

Virtual respondents are created that allow for a comparison of estimated model part-worths to the "true" part-worths associated with the synthetic data. Chapter 4.2 introduces the technical approach used in this study to compare the two models, and their results are discussed in Chapter 4.3. Chapter 4.4 supports the argument in Chapter 4.3 by providing results of real data. Finally, Chapter 4.5 discusses conclusions and future work.

4.2. Technical Approach

The approach presented in this chapter generates virtual consumers who have synthetic preference structures. This permits the creation of a baseline that can be used as a "true preference." This is necessary because the study conducted by Sullivan et al. (Sullivan, Ferguson, and Donndelinger 2011) demonstrated that the discrete and continuous representations of heterogeneity led to different product designs, but they were not able to compare accuracies related to preference representation and design search because no true baseline has known. Using synthetic preference data as the true preference has been examined by several comparative studies (Foster and Ferguson 2014; Long and Morrow 2014; Andrews, Ainslie, and Currim 2002; Andrews, Ansari, and Currim 2002). In this study, synthetic preference data is used as illustrated in Figure 4.1 with the following steps:

(1) True preference: Generate respondents with synthetic part-worths for product features and attributes.
(2) Discrete choice data: Have the virtual respondents answer a suite of choice task

questions to generate discrete choice data.

(3) Modeling: Estimate individual-level part-worth estimates using LC-MNL and HB-ML models.

(4) Comparison: Compare the parameter estimates with the true part-worths and analyze the differences statistically.

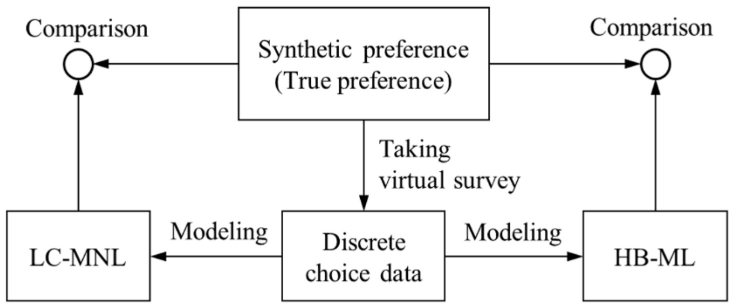

Figure 4.1. Strategy for model comparison

Model fitness and predictive ability are evaluated by performing first-choice and preference share analyses in Chapter 4.2.1. To quantify heterogeneity in product feature importance and attribute level preference, internal correlations are measured in Chapter 4.2.2. In addition, Chapter 4.2.3 provides a method for investigating implications of the model form choice for product line search. The technical approaches are applied to both synthetic and real choice data and results of the case studies are discussed in Chapter 4.3 and 4.4.

4.2.1. Approach to validate model fitness and predictive ability

This section provides a general approach for statistically comparing model fitness and predictive ability of part-worth estimates across model forms. Three analyses are performed: one for model fitness and two for predictive ability. Detailed approaches of performance measures are as follows.

(1) Model fitness: Fitness of a regression model is defined as a measure of how well a model fits a given set of data. Hit-rates for the choice tasks are used, and are

calculated as the percentage of choice tasks predicted correctly using the regression model.

(2) Predictive ability: Predictive ability relates to how well the regression model predicts a future set of observations. A virtual market is defined from a subset of all possible products, and a market simulation is performed. Results are analyzed using both first-choice share (maximum utility rule) and preference share predictions as described in (a) and (b) of Figure 4.2, respectively. The first-choice share is calculated as $n_{correct}/N$ where N is the total number of available options. This test is conceptually similar with a hit-rate measure for holdout questions. The preference share analysis is applied to investigate how well the models predict the true choice probability. The distribution of preference share errors and Root Mean Square Error (RMSE) are used as comparative measures.

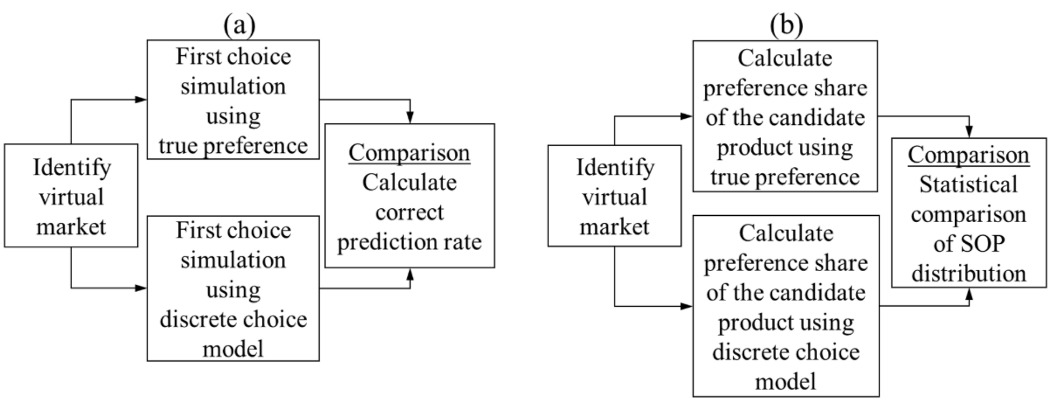

Figure 4.2. Strategy for validating predictive ability:
(a) First-choice analysis (b) Preference share analysis

4.2.2. Approach to investigate heterogeneity structure

It may be possible to explain differences in model performance by investigating model form influence on respondent preference representation. Distinctions in the heterogeneity of respondent preferences can be made at two tiers within a given set of part-worth utilities:

attribute importance and attribute level preference. Detailed approaches to quantifying heterogeneity are shown in Figure 4.3 and explained as follows.

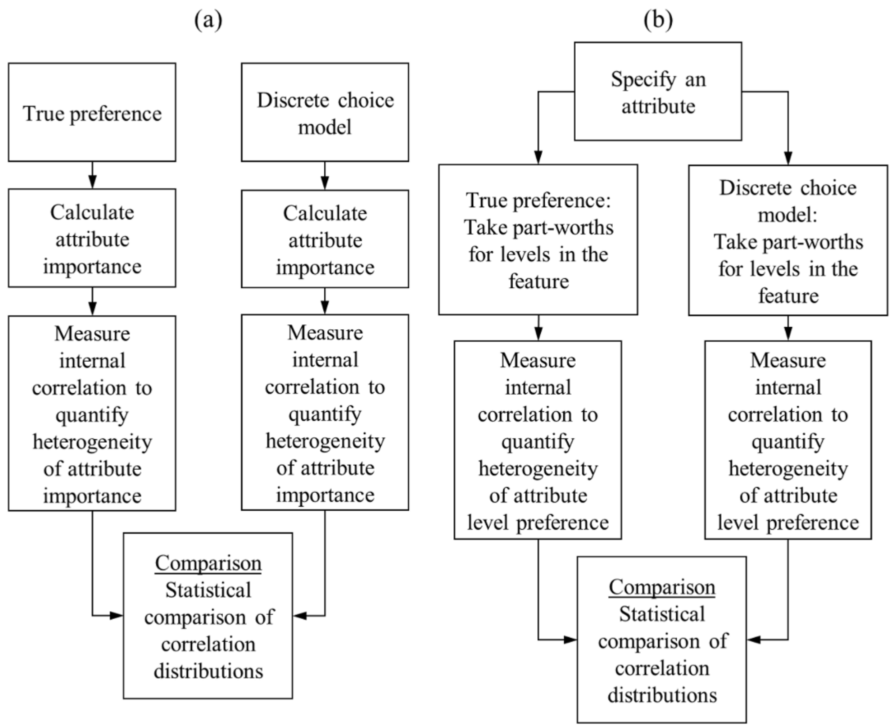

Figure 4.3. Heterogeneity quantification:
(a) Internal heterogeneity of attribute importance
(b) Internal heterogeneity of attribute level preference

(1) Internal heterogeneity of attribute importance: This measures the correlation of each respondent's importance to all other respondent's importance. As provided in Figure 4.3-(a), the first step is to calculate the feature importance for each set of part-worths. Next, Spearman's rank correlation coefficient is used to determine the correlation of each respondents' importance with respect to all other respondents' importance. This leads to $n(n-1)/2$ observations of the correlation coefficient for each model, where n is the number of respondents. The result is a distribution of correlation coefficients for each set of part-worths that represents the internal structure of preference heterogeneity. The two distributions obtained from the true preference and the estimates are compared using a statistical goodness-of-fit test.

(2) Internal heterogeneity of attribute level preference: Correlation analysis of feature level preference explores heterogeneity in respondent preference structure within each feature. As provided in Figure 4.3-(b), the first step is to specify an attribute to be analyzed. The part-worths of the feature levels within the attribute are ordered in rank. Then, Spearman's rank correlation coefficient is measured to determine the heterogeneity of attribute levels. This leads to $n(n-1)a/2$ observations of the correlation coefficient for each model, where n is the number of respondents and a is the number of attributes. The results consist of three correlation coefficient distributions, one for the true preference and two for the models. These distributions characterize the heterogeneity captured within the specified feature level preferences of each set of part-worths. Again, the statistical goodness-of-fit test is used to compare the test results between the true preference and estimates.

When comparing the hypothesized and empirical distributions, the most natural way is to check the adequacy of the empirical distribution by comparing the hypothesized and empirical distributions in terms of goodness-of-fit. There are several types of goodness-of-fit tests: Chi-square, Cramér-von Mises, and Kolmogorov-Smirnov (K-S), etc. In this study, the K-S test is used because it is applicable to all types of cumulative distribution functions (CDF) (Noh, Choi, and Lee 2010).

The K-S test measures the maximum distance between the hypothesized CDF (i.e. CDF of the internal correlation of the true preference) and the empirical CDF (i.e. CDF of the internal correlation of the estimated preference). Thus, the K-S statistics are obtained as given by: (Noh, Choi, and Lee 2010; Haldar and Mahadevan 2000)

$$D_n = \sup_x \left| F_X(x) - G_n(x) \right|, \tag{4.1}$$

where $F_X(x)$ and $G_n(x)$ are the hypothesized CDF and the empirical CDF, respectively. D_n follows the Kolmogorov distribution and its CDF is given by

$$P\left(D \le x\right) = 1 - 2\sum_{k=1}^{\infty}(-1)^{k-1}e^{-2k^2x^2} \quad . \tag{4.2}$$

In this study, Eq. (4.3) is approximated in the range of k from 1 to 1000 instead of the infinity. Since D_n is a mathematically random variable, the CDF of D_n is related to a significance level α as

$$P\left(D_n \le D_n^{\alpha}\right) = 1 - \alpha \tag{4.3}$$

for the confidence level, $1-\alpha$. D_n^{α} is a critical value when determining if the null hypothesis should be accepted between the K-S test statistic and its hypothesized distribution.

In this study, the K-S test for two samples is used. Suppose the first sample $X_1, \ldots, X_m$ of size m has a distribution with CDF $F(x)$, and the second sample $Y_1, \ldots, Y_n$ of size n has a distribution with CDF $G(x)$. The null hypothesis between the K-S test statistic and its hypothesized distribution is rejected at level α if

$$D_{mn} > \sqrt{\frac{m+n}{nb}}D_{mn}^{\alpha} \tag{4.4}$$

where D_{mn}^{α} is found from Eq. (4.2) and (4.3). In this study, m and n are equal since the number of product levels are constant in both the true and estimated preferences. If the null hypothesis is rejected, this implies that the estimated data is inconsistent with the hypothesized data obtained from the true preference. On the other hand, accepting that the null hypothesis is accepted suggests that there is no meaningful difference between the two data sets at the defined significance level.

In addition to the goodness-of-fit test, a visual diagnostic can be used to determine if two data sets come from populations with a common distribution. A Quantile-Quantile plot (Q-Q plot) is provided to compare two probability distribution by plotting their quantiles against each other (Rice 2006). In this study, the empirical distribution obtained from the estimates is compared against the hypothesized distribution on the Q-Q plot.

4.2.3. Approach to investigate implications of model form choice

The next step is to explore the implications of different heterogeneity representations when designing products. The objective function used in this work is maximizing the market share of preference of a product line. As shown in Figure 4.4, each set of part-worth estimates may result in different product line designs.

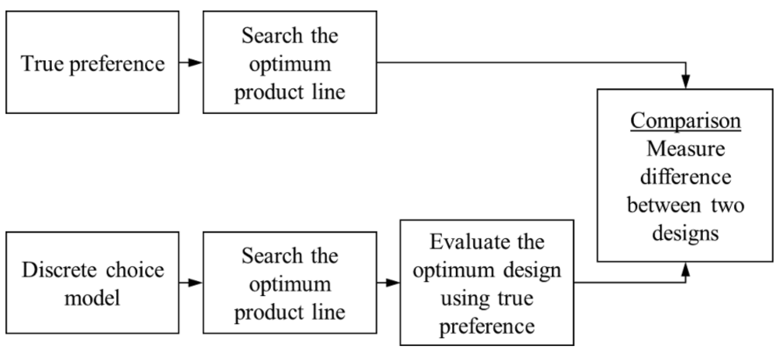

Figure 4.4. Strategy for investigating implications of model form choice for product line design

Design error can be quantified by evaluating a design error metric given by:

$$\frac{SOP_{true} - SOP_{model}}{SOP_{true}}, \tag{4.5}$$

where SOP_{true} and SOP_{model} indicate the share of preference (SOP) of the designs obtained using the true preference and the estimated preference, respectively. When evaluating the optimum design of the estimated preference using the true preference, SOP_{model} is always less than or equal to SOP_{true}. It is obvious that the model with a smaller design error among the two models would be preferred in product design search problems.

In addition to the comparison of the design errors in terms of the objective function values, the optimum product line solutions are also analyzed at the product feature level. From a product development standpoint, the type and number of product features developed has a significant impact on both business and engineering decisions. Comparison metrics are created around quantifying the number of unique features included in the product line solution, and the number of common and uncommon features when comparing a solution to that obtained from the true preference-based solution.

4.3. Case Study using Synthetic Choice Data

To compare the effects of using discrete and continuous representations of heterogeneity, a vehicle configuration scenario is created. Synthetic preferences are generated that serve as the true preferences of the respondents, and discrete choice data is collected based on the choice task questions. Then, part-worth estimates are obtained for each representation. Additionally, a product line design problem is solved to investigate implications of model form selection for product design.

4.3.1. Synthetic preference and virtual survey

Attributes and levels used in the vehicle selection scenario are described in Table 4.1. Manual and automatic transmission are called MT and AT, respectively. A capital letter A with a number stands for the other product attributes considered. The choice-based conjoint study for this problem was created using Sawtooth SSI Web 7.0 (Sawtooth Software inc. 2011). Two-hundred virtual respondents evaluated 12 buying scenarios per survey, where each scenario contained four product alternatives and a fifth no-buy option. The survey design was imported into Matlab to conduct choice tasks by virtual respondents.

Table 4.1. Attributes and levels used in virtual survey

	Engine	TM	A3	A4	A5	A6	Price ($)
Level 1	Gasoline	MT	2 levels	4 levels	4 levels	4 levels	18,000
Level 2	Gasoline Turbo	AT					19,000
Level 3	Diesel						20,000
Level 4							21,000

Table 4.2. Synthetic preference generation

Preference distribution	**Synthetic data**
Uniform	All preferences except for price are generated from uniform distributions
Normal	All preferences except for price are generated from normal distributions
Weibull	All preferences except for price are generated from Weibull distributions
Finite mixture of normal distributions (Segmented)	Preferences in engine and transmission are generated as finite mixtures of normal distributions to mimic the strongly segmented market. Other preferences except for price are generated from uniform distributions.

Four discrete choice data sets based on the different heterogeneity assumptions shown in Table 4.2 were generated to serve as the true preference baselines. An HB-ML model typically is estimated using a multivariate normal assumption because it is relatively simple to estimate and works well in many applications (Gilbride and Lenk 2010). The commercial software used in this study, Sawtooth Software's CBC/HB module (Sawtooth Software inc. 2014), also assumes a multivariate normal distribution. However, since preference heterogeneity is not restricted to a multivariate normal distribution, the data-generation process should not consider only a normal distribution. When generating the true preferences for price levels, it is assumed that a lower price is more preferred to a higher price.

Figure 4.5 shows histograms of the part-worth coefficients for a Gasoline engine drawn for a population of virtual respondents from the four experimental conditions. The normal distribution matches the assumption of the HB-ML model (Figure 4.5-(b)). The uniform distribution represents a population where part-worths are essentially evenly distributed across the range without a distinct peak point (Figure 4.5-(a)) and the Weibull distribution represents a population who has a distinct population-level preference but is non-normally distributed (Figure 4.5-(b)). The finite mixture of the normal distributions was specified to describe a population that has strongly segmented preferences, and it is observable that this distribution has a larger interval than the other distributions. For the finite mixture case, the engine and TM attributes were generated using a segmented distribution while the other attributes were generated from a uniform distribution. All mean and variance values of the distributions were randomly selected at predefined intervals. A total of 200 virtual respondents was generated for each case.

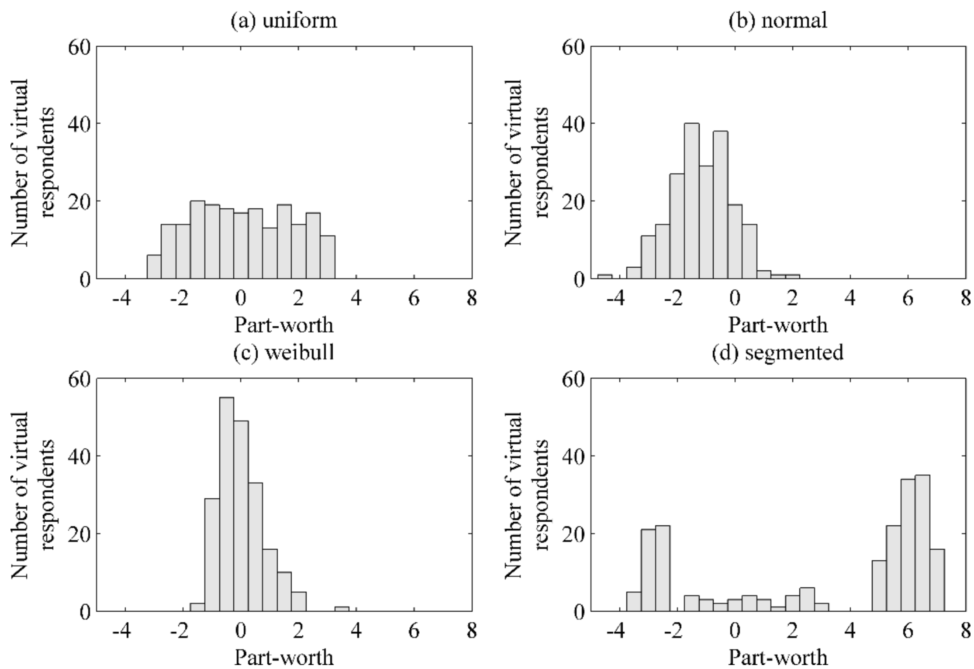

Figure 4.5. Histogram of true preference distribution for Gasoline engine feature: (a) Uniform (b) Normal (c) Weibull (d) Segmented market

The HB-ML models were fit using Sawtooth Software's CBC/HB module (Sawtooth Software inc. 2014). 10,000 random draws for each respondent were completed before assuming convergence, and the next 10,000 random draws were averaged to minimize error. The latent class estimation was conducted using Sawtooth Software's Latent Class module (Sawtooth Software inc. 2007) that performs a maximum likelihood solution. Then, the individual-level part-worths of the LC-MNL model were obtained using Eq. (2.4).

4.3.2. Validating model fitness and predictive ability using first-choice analysis

The performance of the two models is measured in terms of model fitness and predictive ability as discussed in Chapter 4.2.1. As shown in Table 4.3, the hit rate results for the 12 choice task questions strongly support that an HB-ML model provides a superior model estimation compared to the LC-MNL model. For all distributions, the HB-ML models provide a near-flawless estimations of original choice. Additionally, the performance of the HB model only varies slightly as a function of preference distribution. This implies that even though a normal distribution of heterogeneity is assumed for the HB-ML model, model performance is not decreased when a population is represented by another distribution. Both models are estimated to provide the best hit rate when a segmented preference distribution defines the respondent population. This outcome is supported by the large degree of freedom associated with an HB-ML model and that the estimation process of a LC-MNL model first classifies individuals into segments with similar preferences.

Table 4.3. Hit rate for the choice tasks used in the estimation

Synthetic data	Hit rate	
	HB-ML	LC-MNL
Uniform	99.3%	50.3%
Normal	99.3%	56.8%
Weibull	98.3%	58.3%
Segmented	99.6%	65.3%

Nevertheless, the nearly perfect hit rate values of the HB-ML model may not be reflected when using real choice data. A significant cause of estimation error is that human choice behaviors are often not consistent across choice tasks (J. J. Louviere 2001). In this study, choice task inconsistency is ignored so that the focus can be placed on differences in model form.

Having explored hit rate, a virtual market experiment is designed to measure the predictive ability of each model. In each market simulation, three products are introduced: two products as competitors (Table 4.4) and one from the manufacturer. A compensatory choice is made, and the number of correct predictions is used to calculate hit-rate. For this analysis, products are created by using the levels of the attributes fielded in the survey, only. No interpolation between attribute levels is allowed. Since the possible number of products created from this set is 768, and 200 virtual respondents were created, the experiment resulted in 153,600 observations for each model. When the experimental products are the same as one of the competitor products, preference share is equally divided in market simulation.

Table 4.4. Competitor products in market simulation

Competitor	Engine	TM	A3	A4	A5	A6	Price ($)
Product 1	Gasoline Turbo	AT	2	2	2	2	20,000
Product 2	Diesel	MT	1	1	1	1	19,000

The results of the virtual market simulation shown in Table 4.5 indicate the HB-ML model is more powerful in predictive ability than the LC-MNL model regardless of heterogeneity distribution. Another noticeable result is that both models have the highest hit rates when respondent preferences are segmented. When respondent preferences are uniformly distributed, the lowest hit rates are achieved. This suggests that uniformly distributed true preferences would be approximated in both models because the HB-ML model assumes a multi-variate normal distribution of heterogeneity and the LC-MNL model estimates subgroups with similar preferences. Higher hit rates of the HB-ML model than the LC-MNL model for a uniform distribution implies that the normal distribution assumption of mixed logit models is more flexible and able to capture uniformly distributed preferences.

Table 4.5. Hit rate obtained using entire products

Synthetic data	Hit rate	
	HB-ML	LC-MNL
Uniform	74.4%	43.8%
Normal	86.2%	75.5%
Weibull	84.1%	71.9%
Segmented	89.5%	82.6%

4.3.3. Validating model fitness and predictive ability using preference share analysis

Preference share analysis of the virtual market simulation described in Figure 4.2 examines choice probability recovery. Figure 4.6 displays the cumulative distribution functions of the magnitude of preference share errors calculated as $\left|SOP_{true} - SOP_{model}\right|$.

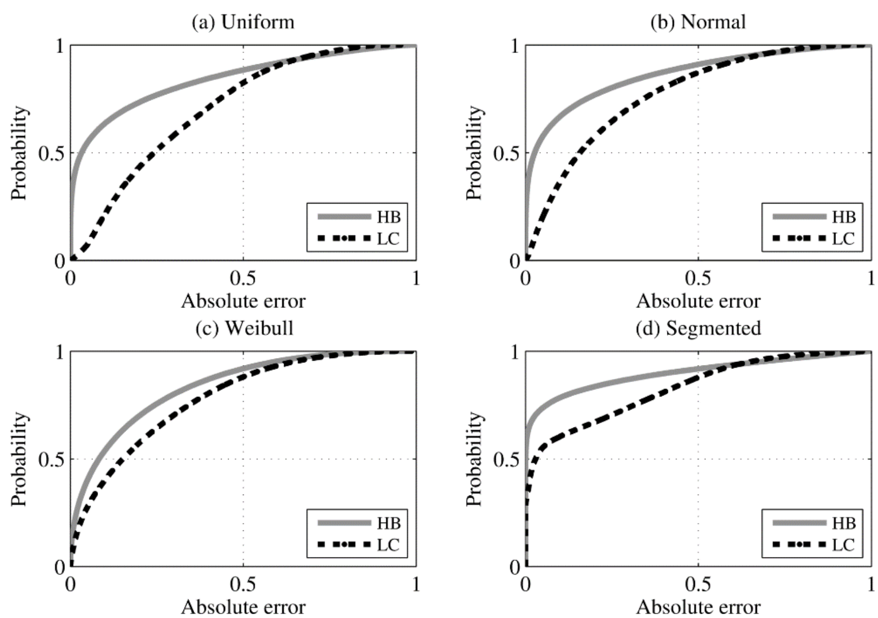

Figure 4.6. CDF of preference share error:
(a) Uniform (b) Normal (c) Weibull
(d) Finite mixture of normal distribution (Segmented)

For all data, it is observable that the HB-ML models have steep upslopes near zero error, while the LC-MNL models have relatively gentle slopes. This indicates the HB-ML models usually have smaller preference share simulation errors than the LC-MNL models regardless of the true preference distribution. The superiority of the HB-ML models in predictive ability is confirmed by the RMSE comparison shown in Table 6. The HB-ML models result in 20% to 30% lower RMSEs than the LC-MNL models. Also, both models lead to the smallest errors for the segmented population, while a uniform distribution has the largest errors.

Table 4.6. RMSE of preference share

Synthetic preference	HB-ML	LC-MNL
Uniform	0.282	0.357
Normal	0.252	0.309
Weibull	0.252	0.301
Segmented	0.239	0.286

4.3.4. Investigating heterogeneity structure using attribute importance

The internal heterogeneity of attribute importance is analyzed using 19,900 ($\frac{200!}{2!(200-2)!}$, choose 2 respondent from 200 respondents) Spearman correlation coefficient observations. Figure 4.7 gives an example of the coefficient distributions for the uniform distribution of true preference. The true preference and the HB-ML model demonstrate very similar structure, while the LC-MNL model has a totally different shape. Notice that the LC-MNL model results in distinct differences in correlation coefficient values, taking on the appearance of a mixture of groups. This behavior is likely caused by the segmentation that occurs within a latent class estimation, where discrete segments are first created. The differences in correlation distributions can be evaluated using the K-S test and Q-Q plot as explained in Chapter 4.2.2.

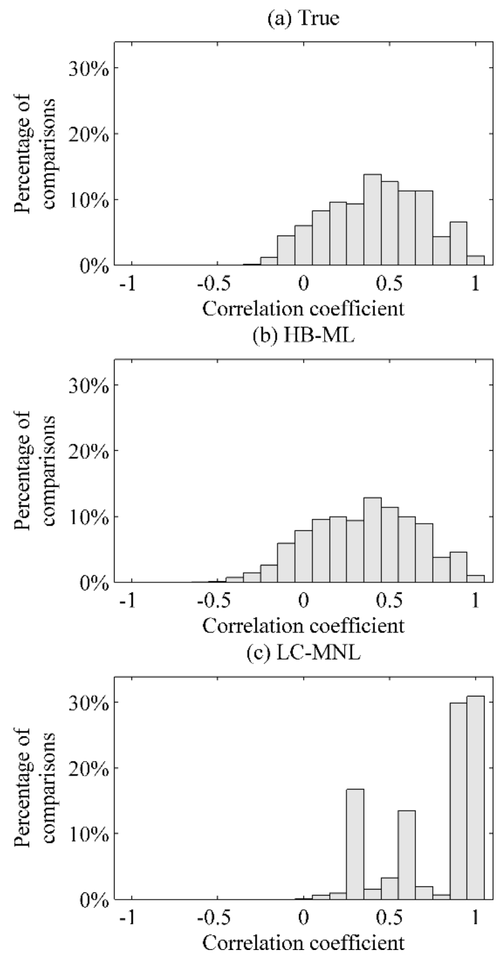

Figure 4.7. Histogram of correlation coefficient quantifying heterogeneity in feature importance (Synthetic data: Uniform distribution)

In a K-S test of Spearman correlation for feature importance, the HB-ML models result in significantly smaller statistics than the LC-MNL models. These results are shown in Table 4.7. A noticeable result is that both models lead to the smallest K-S statistics for the segmented population, while the uniform distribution results in the largest K-S statistics. Figure 4.8 displays the empirical CDF for each distribution. The HB-ML model correlates closely to the reference probability curve, while the LC-MNL model does not.

Table 4.7. KS test of Spearman correlation in feature importance

Synthetic preference	K-S statistics	
	True vs. HB-ML	True vs. LC-MNL
Uniform	0.088	0.530
Normal	0.067	0.434
Weibull	0.084	0.356
Segmented	0.071	0.326

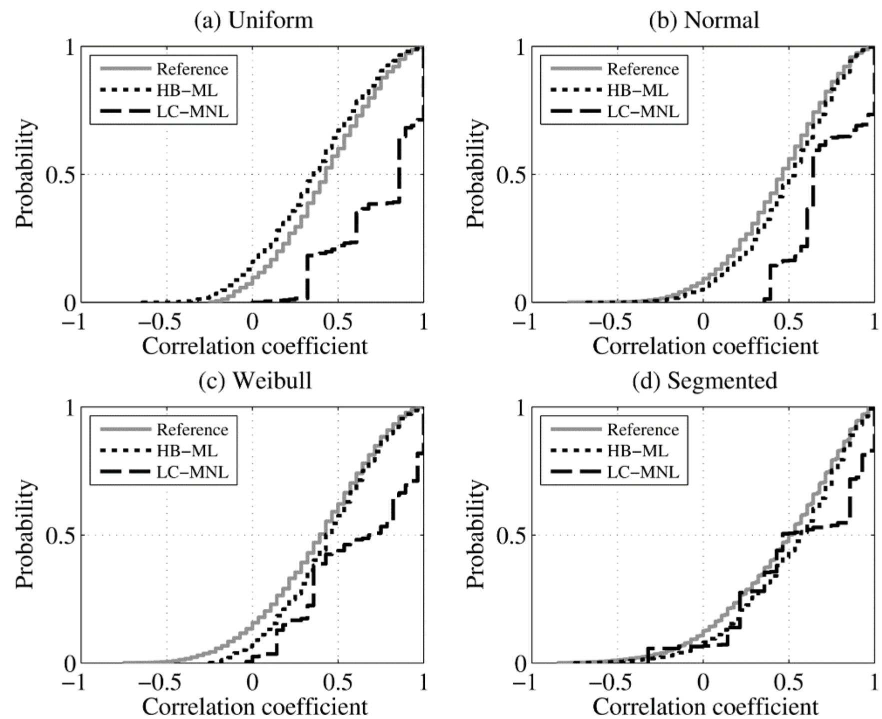

Figure 4.8. Visual comparison of CDFs for importance heterogeneity

Even though the HB-ML model provides a similar CDF curve to those of the reference distributions, all tests reject the null hypothesis at the 5% significance level. From Eq. (4.4), the critical value $\sqrt{2n^{-1}}D_{mn}^{\alpha}$ is obtained as 0.0136 where $n = 19{,}900$ and $\alpha = 0.05$. Since, all K-S statistics in Table 4.7 are larger than 0.0136, no test accepts the null hypothesis. This implies that the estimated data is inconsistent with the hypothesized data obtained from the true preference for both models. Nevertheless, it is apparent that the HB-ML model is more accurate than the LC-MNL model at representing population heterogeneity for feature importance.

A Q-Q plot also provides a visual comparison by which to check the validity of a distributional assumption for a data set. Figure 4.9 displays Q-Q plots for each pre-defined population. If the reference distribution obtained from the true preference and the sample distribution obtained from the estimates agree, then the Q-Q plot follows the reference line. For the correlation coefficient distribution, it is apparent that the reference distribution resembles the HB-ML model more than the LC-MNL model in all data sets. Even if longer and shorter tails on negative reference quantiles are observed in the quantiles of the HB-ML models for the uniform and Weibull population, respectively, they are relatively insignificant compared with the larger discrepancies associated with the LC-MNL models.

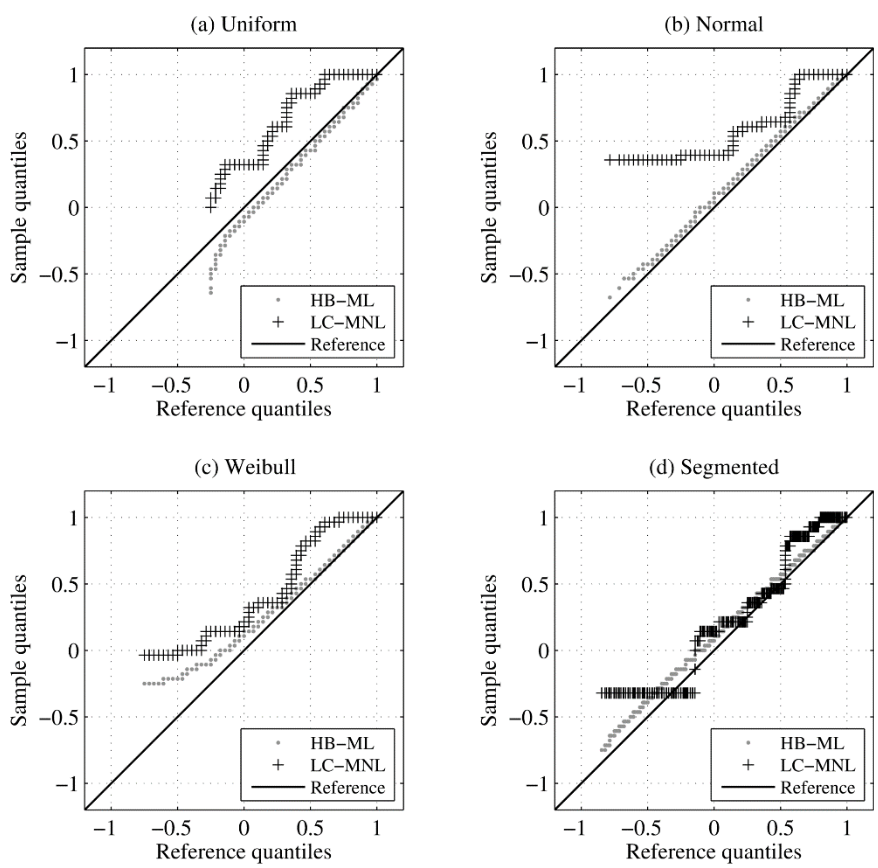

Figure 4.9. Q-Q plot of attribute importance heterogeneity

4.3.5. Investigating heterogeneity structure using attribute level preference

Correlation analysis of feature level preference captures heterogeneity in respondent preference structure within each feature. The internal heterogeneity of feature level preference is analyzed using 19,900 Spearman correlation coefficient observations for each attribute. K-S tests and Q-Q plots are provided to make an argument that the HB-ML model can capture a more accurate representation of heterogeneous preference than the LC-MNL model.

Table 4.8. KS test of Spearman correlation in feature level preferences

Synthetic preference	Attribute	K-S statistics		Synthetic preference	Attribute	K-S statistics	
		True vs. HB-ML	True vs. LC-MNL			True vs. HB-ML	True vs. LC-MNL
Uniform	Engine	0.008†	0.137	Normal	Engine	0.135	0.066
	TM*	0†	0.008†		TM*	0.007†	0.118
	A3*	0.007†	0.129		A3*	0.023	0.405
	A4	0.015	0.298		A4	0.139	0.705
	A5	0.017	0.279		A5	0.155	0.473
	A6	0.003†	0.440		A6	0.132	0.473
Weibull	Engine	0.008†	0.381	Segment	Engine	0.229	0.248
	TM*	0.005†	0†		TM*	0.004†	0.092
	A3*	0.002†	0.119		A3*	0.003†	0.018
	A4	0.032	0.464		A4	0.085	0.218
	A5	0.014	0.249		A5	0.039	0.243
	A6	0.051	0.234		A6	0.011†	0.300

* indicates binary feature

† indicates the null hypothesis is accepted.

The HB-ML model results in significantly smaller K-S statistics than the LC-MNL models for 22 of 24 features, as shown in Table 4.8. This implies that the HB-ML model more closely resembles the true preference in terms of feature level preference. In particular, the null hypothesis tests for 11 of 24 features are accepted at 5% of significance level for the HB-ML model where the critical value $\sqrt{2n^{-1}}D_{mn}^{\alpha}$ is obtained as 0.0136. Conversely, only 2 features are accepted for the LC-MNL model. As explained in Chapter 4.2.3, accepting the null hypothesis implies that there is no meaningful difference between the two test distributions at the specified significance level. Therefore, the results of the null hypothesis suggest that the

HB-ML model can accurately capture the true preference for 11 product features, while the LC-MNL model can only capture 2 product features at the defined significance level.

Figures 4.10 and 4.11 provide a visual diagnostic for the quality of the correlation distribution. These figures display a comparison of feature level heterogeneity for the uniformly distributed synthetic data. The other distributions the graphs are excluded as general similarities were observed. In the comparison of CDFs displayed in Figure 4.10, the correlation distributions of the HB-ML model are in very good agreement with the reference. Conversely, the LC-MNL model results in significant discrepancies, except for the transmission attribute. In particular for the CDFs of attributes 4, 5, and 6 it is noticeable that the correlation coefficients of the LC-MNL model are less distributed and gathered around a few coefficient values. A hypothesis for this behavior is that it is an artifact of the segmentation associated with a latent class model.

Similar to the CDFs, Q-Q plots for attributes 4, 5, and 6 indicate that the correlation distributions of the HB-ML model are comparable to the reference. Because the engine attribute has only three levels, and binary options are available for transmission and attribute 3, a reduced number of correlation coefficient values are returned: five ($\rho \in \{-1, -0.5, 0, 0.5, 1\}$) for the engine attribute and two ($\rho \in \{-1, 1\}$) for the binary attributes, as seen in Fig. 4.10. Thus, comparing quantiles for these attributes is an ineffective strategy.

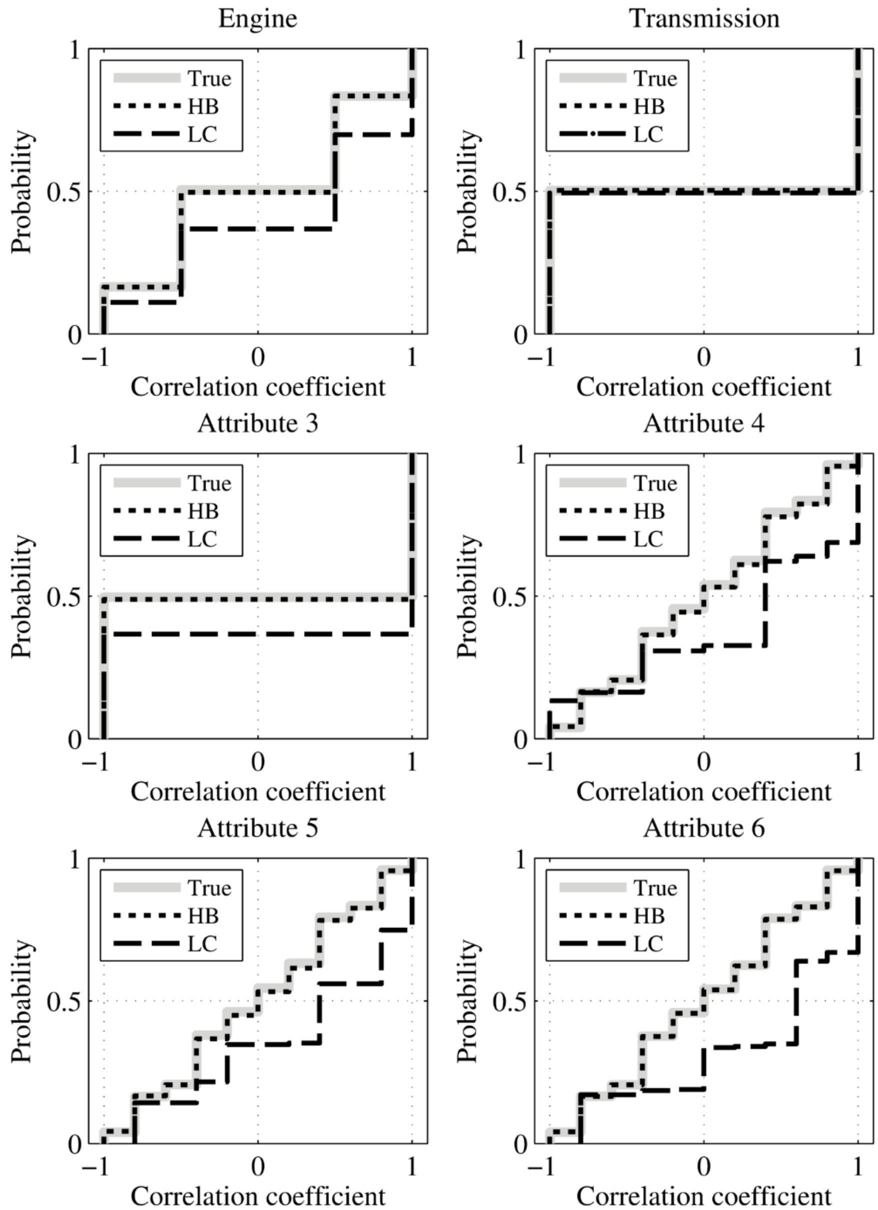

Figure 4.10. Visual comparison of CDFs for level preference heterogeneity
(Synthetic data: Uniform distribution)

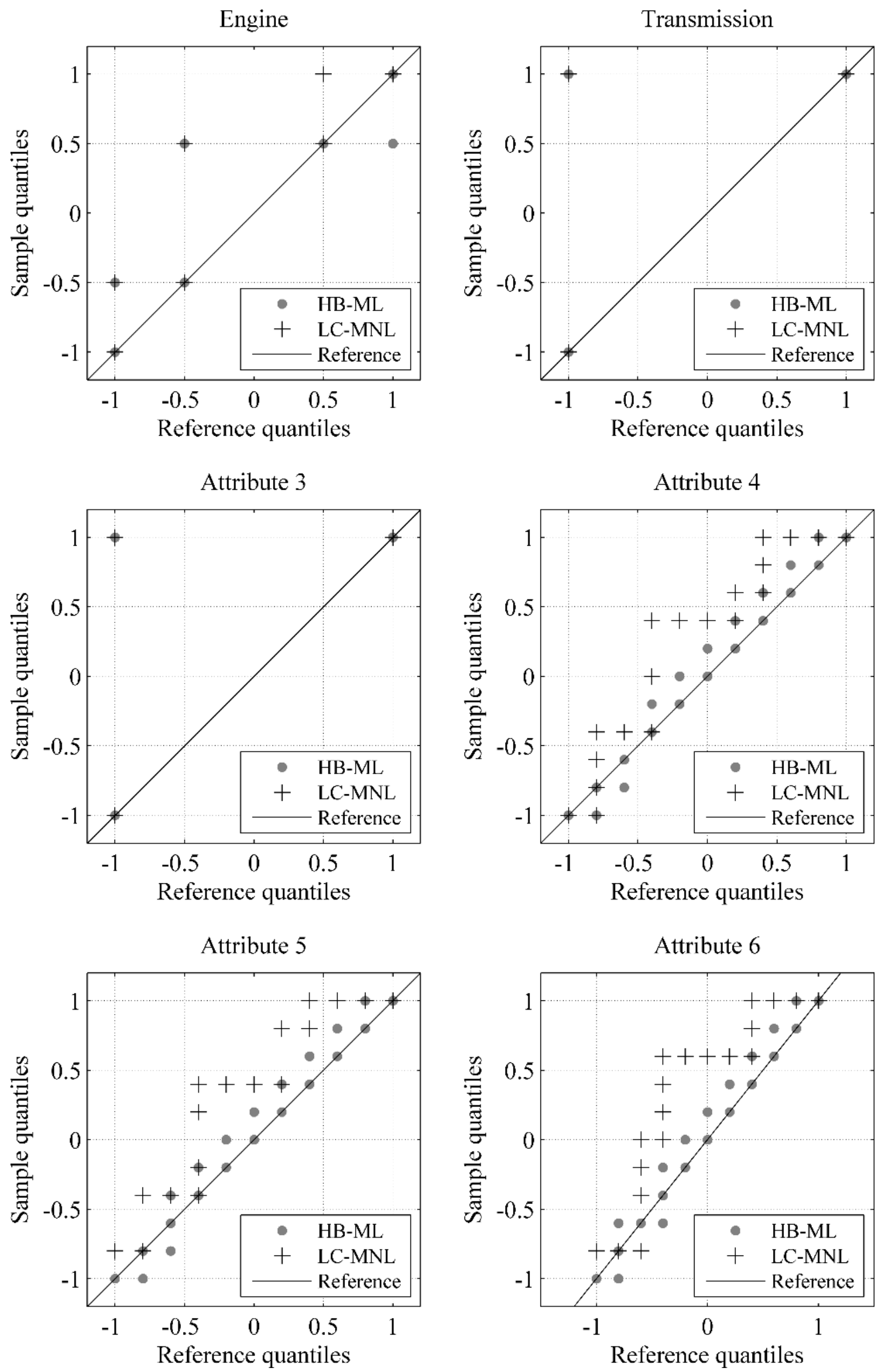

Figure 4.11. Q-Q plot of level preference heterogeneity
(Synthetic data: Uniform distribution)

4.3.6. Investigating implications of model form choice

The dramatic differences observed in the heterogeneity of model parameters in the HB-ML and LC-MNL models have significant ramifications for product line design problems. This section investigates the implications of model form on the outcome of a product line optimization problem. As discussed previously, the objective of the optimization problem is to maximize the market share of preference share of the line. A genetic algorithm was written in Matlab to conduct the optimization. The size of the population at each generation was set as $50np$ where np is the numbers of products considered.

Using the latent class models the optimum number of products in a product line for each distribution type are 3, 3, 3, and 5, for the uniform, normal, Weibull, and segmented population, respectively. These numbers are determined based on the CAIC (Consistent Akaike Information Criterion) value that is typically used to determine the best number of classes in the latent class analysis (Nylund, Asparouhov, and Muthén 2007).

For the genetic search, the selection operator was random and the crossover operator was uniform with elites selected as 5% of the population. The mutation operator was Gaussian with the default value of 1 for both scale and shrink parameter. Finally, the convergence criterion was 10e-12 of the function tolerance or a 200 stall generation limit in 1000 generations.

Table 4.9. Results of product line design optimization

Synthetic preference	*np**	True	HB-ML		LC-MNL	
		SOP	SOP	error**	SOP	error**
Uniform	3	54.97%	53.00%	3.58%	41.08%	25.27%
Normal	3	68.64%	68.55%	0.13%	62.35%	9.16%
Weibull	3	80.00%	79.69%	0.39%	64.74%	19.08%
Segmented	5	95.14%	93.52%	1.70%	86.30%	9.29%

* *np* stands for the number of products in a line
** Eq. (4.5)

Table 4.9 displays the market share of preference results of the optimum product lines. Details about the optimum product lines are listed in Appendix B. The optimum designs obtained from evaluating the HB-ML and LC-MNL models are evaluated using the true preferences to evaluate the effectiveness of the solutions. A striking outcome is that the product line solutions from the HB-ML model are much better than the LC-MNL model in terms of design errors. In particular, the HB-ML model results in negligible design errors below 1% for normal, Weibull, and segmented true preference distributions. These results are reasonable in indicating that the model fitness, predictive ability, and heterogeneity structure play a significant role when searching for an optimal solution.

Table 4.10. Analysis of product line design

Synthetic preference	Data	Number of distinct features in product line	Number of common features to the design of the true data	Number of uncommon features to the design of the true data
Uniform	True	14	-	-
	HB-ML	14	11	3
	LC-MNL	8	8	0
Normal	True	11	-	-
	HB-ML	10	10	0
	LC-MNL	10	9	1
Weibull	True	11	-	-
	HB-ML	11	10	1
	LC-MNL	8	8	0
Segmented	True	19	-	-
	HB-ML	17	15	2
	LC-MNL	15	14	1

In addition to the analysis of design errors, feature commonality between each design solution and the solution associated with the true data is compared. Table 4.10 shows the commonality analysis results. The numbers of distinct features in LC-MNL's product lines are fewer than those of both the true and HB-ML's product lines. When using the uniformly distributed synthetic preference data, for example, both the true and HB-ML solution uses 14 distinct product features. However, the optimal product line when using the LC-MNL model uses only 8 distinct features. This suggests that the HB-ML model introduces more product line variety in response to the more complete representation of heterogeneity. The inability of the LC-MNL model to capture the same extent of heterogeneity leads to sacrificed feature variety, leading to possible missed market opportunities.

These results, however, should not be interpreted to suggest that the LC-MNL model has no value in product search problems. The objective function driving the optimization problem in this study aims to maximize share of preference, but engineering companies may be interesting in exploring market segmentation opportunities or creating targeted products for segments of a market. In these cases, the LC-MNL model may work better than the HB-MNL model because of the segmentation strategy associated with model formulation. Further studies should explore the role of LC-MNL models when specifically designing for specific market segments.

4.4. Case Study using Real Choice Data

The second case study deals with a real discrete choice data to validate the differences observed in the synthetic data. Since the true preference is not available in the real data, errors between the true and estimated preferences cannot be obtained. Therefore, this case study focuses on observing similar trends with the results of the synthetic data.

4.4.1. Discrete choice design

The data used in this study was obtained from an automotive company where 1618 respondents each answered 24 choice task questions. This suite of choice task questions included 4 hold-out questions. Each scenario contains four product alternatives and a fifth no-buy option. This problem has 25 unique product attributes spanning a total of 115 levels, and a price attribute with 8 levels.

The LC-MNL and HB-ML models were fit using the Sawtooth Software Latent Class and CBC/HB modules, respectively. The best number of classes was determined to be 5. Then, the individual-level part-worths of the LC-MNL model were obtained using Eq. (2.4). The CBC/HB module performed 20,000 random draws for each respondent before assuming convergence. The next 20,000 random draws were averaged to minimize the error.

4.4.2. Validating model fitness and predictive ability

To assess model fitness and predictive ability of each model, hit rate tests are conducted. Hit rates for the 20 choice tasks yielded 32,360 choice observations that could be used to evaluate model fitness. Predictive ability is compared using the hit rate associated with the 4 hold-out questions, yielding 6,472 choice observations for each model.

Table 4.11 compares hit rates for the choice tasks used in estimation and the hit rates for the hold-out questions. The hit rate for the 20 choice tasks approaches 100% for the HB-ML model and is dramatically higher than the hit rate of the LC-MNL model. Yet, the hit rate for the 20 tasks for the LC-MNL model is comparable the hold-out task hit rate. Similar to the results seen with the synthetic data, the performance gap between the two models decreases when the focus turns to predictive power.

Table 4.11. Hit rate comparison for real data

Hit rate test	HB-ML	LC-MNL
Hit rate for the choice tasks used in estimation	99.85%	55.20%
Hit rate for hold-out questions	59.65%	55.39%

4.4.3. Investigating heterogeneity structure using attribute importance

The internal heterogeneity of attribute importance is analyzed using 1,308,153 Spearman correlation coefficient observations for each model. Since the true preference is not available for real choice data, the coefficient distributions cannot be evaluated in terms of the resemblance to the true distribution. Analyses using real data more focus on observing similar trends with the results of the synthetic data.

Figure 4.12 displays histograms and CDFs of the correlation coefficients to compare the two models at the upper tier heterogeneity. The distinct characteristics watched in the synthetic data are observed in real data again. Notice that the LC-MNL model results in distinct differences in correlation coefficient values as it showed in the synthetic data result. It looks like mixtures of two correlation groups, while the distribution of the HB-ML model is close to the normal distribution. In addition, the coefficient values of the LC-MNL model are more biased toward a high positive correlation than the values of the HB-ML model. This difference is clearly viewed in the CDFs. In the LC-MNL result, 6.68% of the observations have the correlation coefficient of 1 and 22.29% have the correlation coefficient of above 0.9. This suggests that the LC-MNL model captures less heterogeneity than the HB-ML model at the attribute importance level as seen in the synthetic data result.

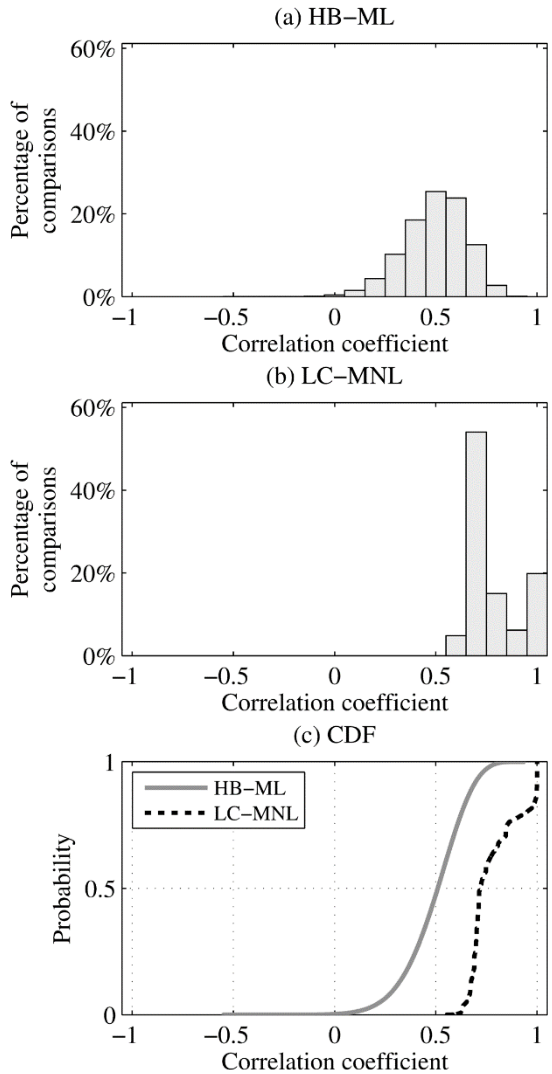

Figure 4.12. Comparison of HB-ML and LC-MNL models
using attribute importance correlation
(a) Histogram of correlation coefficient of HB-ML model
(b) Histogram of correlation coefficient of LC-MNL model
(c) Empirical CDFs

4.4.4. Investigating heterogeneity structure using attribute level preference

The internal heterogeneity of feature level preference is analyzed using 1,308,153 Spearman correlation coefficient observations for each attribute. Correlation coefficients for attribute 1 and 4 are compared in Figs. 4.13 and 4.14, respectively using histograms and CDFs. Attribute 1 is selected because it has the largest importance values for both models, when price is excluded. Attribute 4 was selected since it has the most attribute levels (9 levels) among all attributes. It is expected that distinct differences between the two models will exist when capturing the internal heterogeneity of attribute level preferences.

The most remarkable difference between the two models in Figures 4.13 and 4.14 is that the coefficients of the LC-MNL model are less distributed and gathered around a few coefficient values. In contrast, the correlation coefficients of the HB-ML models are smoothly distributed. This trend is also observed in the synthetic data result. This result should be expected because of the nature of the latent class estimation and the uniform distribution assumption associated with a HB-ML model.

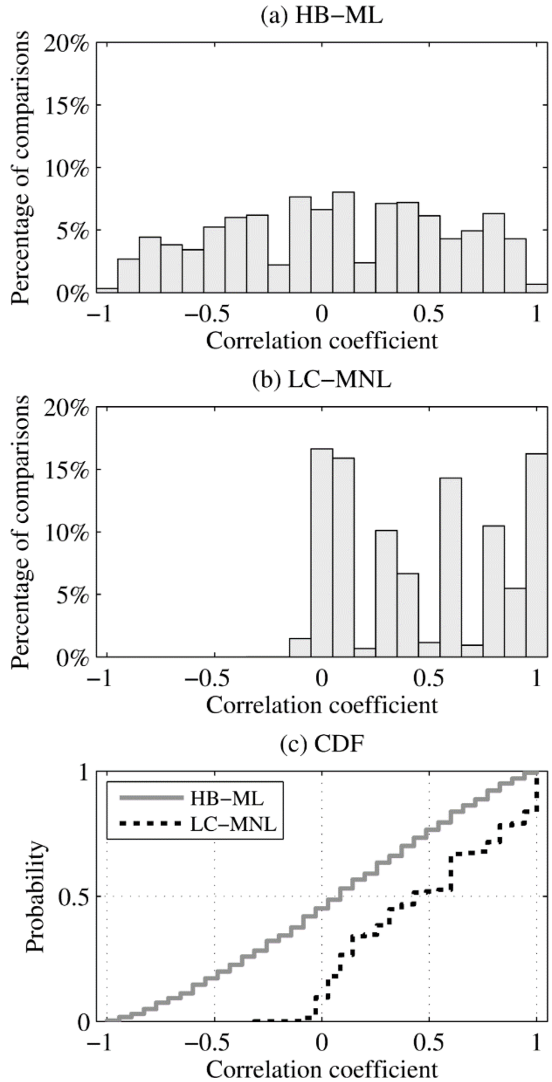

Figure 4.13. Attribute 1. Comparison of HB-ML and LC-MNL models
using feature level preference structures:
(a) Histogram of correlation coefficient of HB-ML model
(b) Histogram of correlation coefficient of LC-MNL model
(c) Empirical CDFs

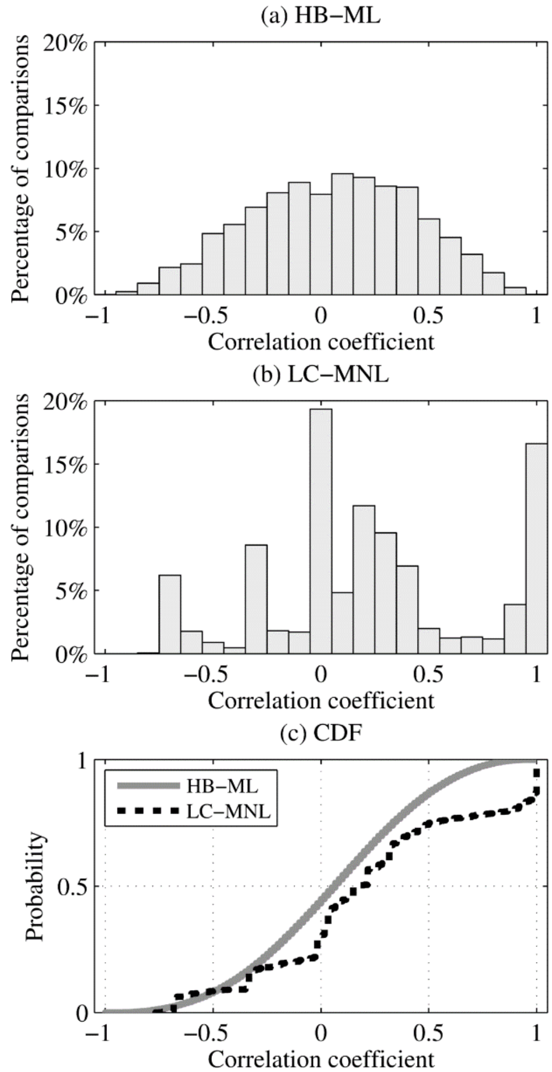

Figure 4.14. Attribute 4. Comparison of HB-ML and LC-MNL models using feature level preference structures:
(a) Histogram of correlation coefficient of HB-ML model
(b) Histogram of correlation coefficient of LC-MNL model
(c) Empirical CDFs

Another interesting observation is that the HB-ML model captures more heterogeneity than the LC-MNL model. The histograms for the two attributes show the coefficients of the LC-MNL models are more biased toward a high positive correlation than the values of the HB-ML model. Table 4.15 provides additional evidence to support this argument. The results include not only the descriptive statistics but also the percentages of the observation having high internal correlations ($\rho = 1$ and $\rho > 0.9$). For attribute 1, 20.94% of the observations have correlation coefficients greater than 0.9 for the LC-MNL model. For the HB-ML model, only 3.04% of the observations have such a large correlation. In particular, 16.23% of the LC-MNL model results have $\rho = 1$, which means their rank orders in attribute level preference are the same. This implies that the LC-MNL model captures less heterogeneity than the HB-ML model. For all attributes, this trend is consistently observed.

Table 4.12. Analysis of Spearman correlation coefficient of attribute level preference

Attribute	Population mean of Spearman's rho (standard deviation in parentheses)		Percentage of comparisons resulting in $\rho = 1$		Percentage of observations resulting in $\rho > 0.9$	
	HB-ML	LC-MNL	HB-ML	LC-MNL	HB-ML	LC-MNL
1	0.06 (0.52)	0.47 (0.37)	0.67	16.23	3.04	20.94
2	0.13 (0.56)	0.57 (0.35)	2.92	16.70	2.92	16.81
3*	0.05 (1.00)	1 (0)	52.36	100	52.36	100
4	0.06 (0.38)	0.23 (0.49)	0.00	13.02	0.20	19.34
5	0.04 (0.39)	0.33 (0.43)	0.00	16.72	0.35	20.38
6	0.07 (0.50)	0.56 (0.39)	0.39	18.44	2.10	21.01
7	0.14 (0.45)	0.47 (0.40)	0.33	16.19	1.86	20.56
8	0.04 (0.58)	0.19 (0.61)	4.92	21.29	4.92	21.29
9*	0.06 (1.00)	0.65 (0.76)	52.82	82.55	52.82	82.55
10	0.07 (0.46)	0.60 (0.35)	0.23	19.68	1.35	21.68
11	0.08 (0.58)	0.38 (0.55)	5.39	22.36	5.39	23.17
12	0.06 (0.59)	0.18 (0.71)	5.53	24.91	5.53	24.95
13	0.27 (0.67)	1 (0)	27.18	100	27.18	100
14	0.10 (0.51)	0.56 (0.38)	1.41	17.14	1.41	17.39
15	0.04 (0.59)	0.25 (0.55)	4.97	23.95	4.97	24.05
16*	0.18 (0.98)	1 (0)	58.86	100	58.86	100
17	0.02 (0.59)	0.37 (0.49)	4.81	15.00	4.81	15.61
18	0.17 (0.46)	0.72 (0.22)	0.39	18.69	2.21	30.34
19	0.17 (0.58)	0.73 (0.32)	7.86	56.43	7.86	56.43
20	0.12 (0.58)	0.77 (0.23)	6.56	35.57	6.56	35.63
21*	0.15 (0.99)	1 (0)	57.65	100	57.65	100
22	0.03 (0.58)	0.30 (0.57)	4.69	21.08	4.69	21.08
23	0.02 (0.40)	0.18 (0.54)	0.01	15.80	0.35	19.87
24	0.02 (0.59)	0.30 (0.57)	4.77	16.98	4.77	17.01
25	0.14 (0.69)	0.84 (0.23)	21.36	67.14	21.36	67.14

* indicates binary feature

4.5. Summary

This study compares models that provide continuous and discrete representations of heterogeneity, and provides empirical data describing the effectiveness of model choice. Prior work has documented that a continuous representation can more richly represent heterogeneity than a discrete representation (Sullivan, Ferguson, and Donndelinger 2011). In this study, synthetic preferences are created that allows for the quantification of differences between the true and estimated part-worths. Model fitness and predictive ability are used as performance measures. They are quantified using both first-choice analysis and preference share analysis using virtual market scenarios. Then, the experimental results are supported by analyzing heterogeneity structures. In addition, implications of model form choice for product line design are also investigated by comparing optimum designs obtained using the part-worth estimated from each model. The major findings from the experiment are summarized as follows:

(1) The HB-ML model outperforms the LC-MNL model in model fitness and predictive ability.
(2) The internal heterogeneity structure of the HB-ML model is very similar to that of the true preference, while the LC-MNL model results in wide discrepancies in heterogeneity structure.
(3) For LC-MNL model, the internal preference structure is less heterogeneous than that of the HB-ML model and the correlation coefficients are gathered around a few values.
(4) The HB-ML model – constructed around a normal distribution assumption - performs well even if the true preferences are distributed according to uniform, Weibull, and mixtures of normal distributions.
(5) The degree of differentiation in the true preference can impact model fitness, predictive ability, and heterogeneity of estimates. When preferences are created around a segmented population structure, the best model performance values are obtained. The uniform distribution assumption results in the worst values.
(6) The HB-ML model leads to smaller design errors than the LC-MNL model in product

line design. These findings suggest that the LC-MNL model should not be used to meet the business objective of maximizing preference share in a market with diverse customer preferences.

According to the findings, the HB-ML model is preferential to the LC-MNL model when representing heterogeneous preferences and making engineering design decisions. However, these findings do not imply that the LC-MNL model has no value when trying to design an optimal product line. The research presented in this work allows for interesting expansions in market-based product design research. To continue investigating implications of the HB-ML model on product design search problems, the inherent uncertainty internal to estimating the HB model must be explored. This topic is explored in Chapter 6.

Chapter 5. Quantifying Customer Sacrifice for Mass Customization Environments

By adopting mass customization, a manufacturer allows a consumer to reduce their sacrifice gap, defined as the gap between "what a customer wants" and "what the customer settles for." The driving force behind product customization is that it yields products with higher customer utility than mass-produced products because a closer preference fit to an individual's need is delivered. Although the narrative idea of mass customization exists, processes for quantifying sacrifice gap are almost unexplored. The absence of a quantitative definition for sacrifice gap has limited the advancement of optimization-focused design strategies that can be used to help determine which product attributes should be customized. This chapter proposes a quantitative definition for sacrifice gap and formulates an optimization problem for mass customization environments. Discrete choice theory is used to derive the quantitative definition of customer preference. The use of sacrifice gap is compared with a share of preference problem formulation to investigate changes in product line solutions for mass customization environments. Respondent-level product search problems and product line search problems are introduced and solved to demonstrate how approaching the optimization using a definition of sacrifice gap provides flexibility in configuring product features for the mass customization. Results of the optimization problem for mass customization environments confirm that the quantitative definition of sacrifice gap can effectively minimize the average sacrifice gap across respondents by providing optimal product feature offerings to deliver a closer preference fit to an individual's need.

5.1. Introduction

Preference heterogeneity describes the variation in taste across individuals. Understanding the impact of variation in customer preference is a fundamental challenge associated with optimal product design, and has been a driving focus of market-based product design research. In 1970, the concept of market-based product design was described in Alvin Toffler's famous book *Future Shock* as "*Companies are discovering wide variations in consumer wants and are adapting their production lines to accommodate them*. (Toffler 1970) In the engineering design community, the product development strategy described by this statement has been realized in the form of product line design problems (Green and Krieger 1985; Green and Krieger 1987; Kohli and Sukumar 1990). In these works, mass production strategies are explored that provide both manufacturers and consumers the greatest opportunity to take advantages of efficiency and cost savings.

Mass production strategies accept that some customers will be presented with product offerings that are not particularly tailored toward their individual preferences. Conversely, mass customization is a product development approach that provides custom-tailored goods or services that meet consumers' diverse needs at prices near that of mass production (Gilmore and Pine 2000; Pine 1999). Mass customization enables a consumer to minimize their Sacrifice Gap (SG), or the difference between a mass production offering and what the consumer truly wants. The ability to minimize sacrifice gap is based on the assumption that customized products create higher consumer utilities than mass produced products because customized products more closely match an individual's needs. In 1980, the concept of mass customization was described in another famous book by Toffler called *The Third Wave* (Toffler 1980), where the futurist predicted that information technology advancements would allow manufacturers to provide custom-cut products for individual users.

The term 'mass customization' was first used by Davis (Davis 1987) nearly 30 years ago, where mass customization was explained as having the capability of reaching the same number of customers as in a mass market while treating those customers "*individually as in the customized markets of pre-industrial economies*." (Davis 1987) Later, Pine (1999) described mass customization as "*providing tremendous variety and individual customization, at prices comparable to standard goods and services*" to fill the market "*with enough variety and customization that nearly everyone finds exactly what they want*." From a manufacturing perspective, Du et al. (2001) describe mass customization as "*the technologies and systems to deliver goods and services that meet individual customers' needs with near mass production efficiency*." All definitions of mass customization are consistent in that they share two notions: 1) they discuss that customer needs can be met more effectively using customized product offerings, and 2) they aim to leverage innovation in product design, manufacturing, and logistics to maintain prices comparable to mass produced products. In this context, the benefit of mass customization is its ability to simultaneously increase value to both the customers and the company when compared to mass production.

For the 30 years that followed Davis's work mass customization has gained attention as an intriguing prospective strategy for manufacturing businesses. However, despite great efforts in all components necessary to make the new paradigm realizable, mass customization has largely not lived up to its promise (Zipkin 2001). Ferguson et al. (2014) presented a review of mass customization literature and suggested four primary challenges associated with the realization of mass customization.

i. Limitations of existing customer needs and preference assessment tools.
ii. The need to explore approaches for requirement specification and conceptual design.
iii. Insights from methodologies focused on the development of durable mass customization goods.
iv. Necessary enhancements in information mapping and handling.

Among the research opportunities, a tool for customer needs and preference assessment is important at the early stage of product development (Ferguson, Olewnik, and Cormier 2014). However, despite advancements in consumer research, a quantitative assessment tool of customer preference has yet to be developed for mass customization environments. In particular, although conceptual definitions of the sacrifice gap exist, a process for obtaining a quantitative measure of the term is almost unexplored.

This study is focused on a quantitative method to assess customer needs and preference for mass customization. The main objectives of this study are to quantitatively define sacrifice gap and propose an optimization problem formulation that can be used to identify the customizable product features in a mass customization environment. Discrete choice theory is used to develop the quantitative definition of customer preference. In Chapter 5.2, sacrifice gap is defined using customer preference estimates. Implications of a sacrifice gap metric in product search problems are investigated in Chapter 5.3. In particular, an optimization problem formulation to search for the product feature offerings in customization environment is proposed in Chapter 5.4. Finally, Chapter 5.5 discusses conclusions and future work.

5.2. Quantitative Definition of Sacrifice Gap

Quantifying sacrifice gap, defined (Pine and Gilmore 2000; Gilmore and Pine 2000) as "what a customer wants exactly" less "what the customer settles for", is critical to mass customization success. The definition of sacrifice gap suggests that it has to be measured at the level of the individual rather than at a population-level. A discrete choice model is attractive for quantifying sacrifice gap because of its ability to represent preference heterogeneity across customers. In recent work, Ferguson et al. (2011) investigated the use of discrete choice methods as a mapping tool from the marketing to engineering domains so that the ideal product architectures for mass customization could be identified. Liechty et al. (2001) developed a discrete choice model to investigate the use of experimental choice menus for assessing customer preferences in web-based mass customization environments. Fogliatto and da Silveira (2008) proposed a method for market segmentation and choice menu design for mass customization using a discrete choice method. In this study, individual's preferences are quantified using discrete choice theory and a quantitative definition of sacrifice gap is proposed based on preference estimates. This section proposes a new quantitative definition of sacrifice gap that has an appropriate form for use in optimization problems.

5.2.1. Datum

Referring to the sacrifice gap definition, the concept of "what a customer wants exactly" (or the ideal product), must be explored in terms of its configuration and observed utility value. Ferguson et al. (2011) examined preference tendencies to determine each respondent's ideal product. They defined the ideal product as the combination of product attribute values that yielded the highest cumulative observed utility. Although price is inherently associated with the features combined to create a product, the customer preference estimates for price were excluded from the quantification of observed utility. This decision was made under the argument that the focus of customization is driven by product configuration.

Assuming a designer is capable of identifying the right customers and quantifying their preferences, the next task is defining a respondent's ideal product so that sacrifice gap can be measured. In this work, an ideal product is defined with a consideration of product price using the preference estimates obtained from a discrete choice survey. The challenge is determining the price of the ideal product; from a consumer perspective the cheaper product is most preferred, while from the perspective of the firm the lowest acceptable product price should be greater than product cost. Using these perspectives on product price, the ideal product is defined by the most preferred product features, provided at manufacturer cost. The observed utility of the ideal product for a respondent n is given by Equation (5.1).

$$V_{I,n} = \sum_i \max \mathbf{V}_{i,n} + V_{\text{cost},n}, \tag{5.1}$$

where $\mathbf{V}_{i,n}$ is a vector of part-worths for product attribute i estimated for respondent n. $V_{\text{cost},n}$ is the observed utility corresponding to the lowest available price (the cost) of the configured ideal product. Defining the "what the customer wants exactly" product in this manner provides a stationary datum from which to measure other goods in the market.

5.2.2. Sacrifice gap

Sacrifice gap is defined as the quantitative value between the ideal product and any product provided in the market. In earlier work by the authors (Porterfield and Ferguson 2012), an individual's sacrifice gap was quantified using the difference in utility values to show that a customized product reduced sacrifice. However, since utility differences are not directly proportional to differences in choice probabilities, an individual's utility differences cannot be integrated into a linear space. In other words, utility differences cannot be used as a measure of quantifying sacrifice across designs (and respondents). For example, when the utility of a respondent's ideal product is $V_I = 3$ and two products $(V_{y_1} = 1$ and $V_{y_2} = 2)$ are provided in the market, the utility difference between the ideal and offered products are $V_I - V_{y_1} = 2$ and $V_I - V_{y_2} = 1$, respectively. Yet, if using utility differences to quantify sacrifice gap, the sacrifice gap of product y_1 is twice that of product y_2. However, the choice probability of product y_1 is not twice that of product y_2.

It is desirable to have sacrifice gap measured as a dimensionless quantity that lies on a linear space. This allows the difference in choice probability to be consistent across all respondents and designs. In this study, ratios of logit probabilities between the ideal product and any product are used because there is a direct relationship between the observed utility ratios and the choice probabilities as $V_y - V_I = \ln(\mathrm{p}_y / \mathrm{p}_I)$, where V_I is the utility of the ideal product, V_y is the utility of any product y, and p_y and p_I indicate the logit probabilities of product y and the ideal product, respectively. Thus, the exponential value of the utility difference, $e^{V_y - V_I}$, can be used to quantify the ratio of choice probabilities between the ideal product and any design as $\mathrm{p}_y / \mathrm{p}_I$. A sacrifice gap measure is proposed as

$$SG = \begin{cases} 1 - e^{(V_y - V_I)} & \text{if } V_I \geq V_y \\ 0 & \text{otherwise} \end{cases}. \tag{5.2}$$

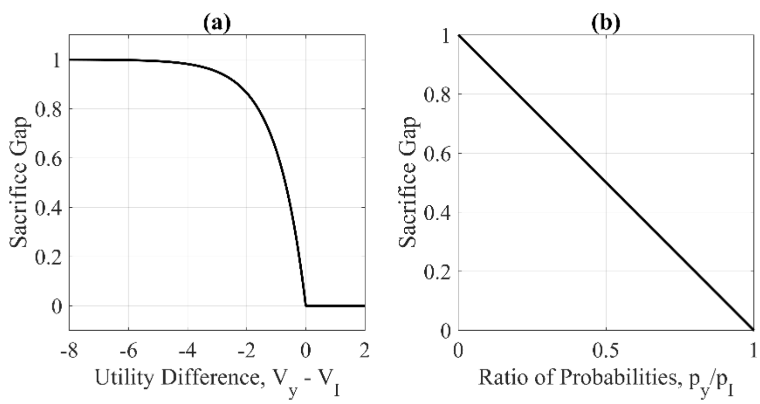

Figure 5.1. Representations of sacrifice gap:
(a) using utility differences (b) using odd ratios

Under this formulation, sacrifice gap is zero at the ideal product. It decreases as a design has an observed utility smaller than the ideal product. Fig. 5.1-(a) displays a graphical representation of sacrifice gap when considered using utility difference, and Fig. 5.1-(b) where sacrifice gap is defined using the logit probability ratio.

Returning to the definition of sacrifice gap, it is also necessary to define the largest utility product in the market, or “what the customer settles for.” The largest utility product has the largest share of preference (SOP) of all products modeled in a market simulation. Therefore, sacrifice gap is obtained using the utility difference between the ideal product and the largest utility product with the assumption that a customer would settle for the largest utility product in a market simulation. Using this first choice assumption is reasonable because the discrete choice theory basically assumes respondents make the first choice with an additive utility rule.

Using the largest utility product in measuring SG value brings up an interesting scenario. It is possible for the largest utility product in a market simulation to have a larger utility than the ideal product. For example, imagine that a customer's ideal feature configuration for a MP3 player consists of 256GB storage and blue color. In one scenario, blue color is available but the largest available storage size is 128 GB in market. In this situation, the customer would experience a sacrifice gap between the ideal product and the best available MP3 player (Blue and 128GB) in market due to storage size. In this market, the customer would settle for the best available product. What if a company offers 30% discount on the blue MP3 player with 128GB? In this case, the utility value gained from the discount may compensate the sacrifice gap. However, a company will not sell the product unless the discounted price is greater than or equal to cost. Thus, the ideal product of the customer is a MP3 player with 256GB and blue color offered at manufacturer cost.

Now, suppose that a MP3 player with 256GB and blue color is offered at \$100 for the same customer, but an identical red one is \$80. Then, even though the red color is not the most preferred attribute level, the total utility value of the red MP3 player may be greater than the blue MP3 player when price is considered. Depending on the customer's preference estimates for price, the red MP3 player may have a larger utility than the ideal product. This occurs when the utility difference, $V_y - V_I$, is a positive number. For this case, rather than using a negative sacrifice gap, a sacrifice gap of zero is given in the proposed sacrifice gap definition. The utility area above the ideal product is called the zero-SG range in this study. The area can be interpreted using marketing language that if a product has more utility than “what a customer exactly wants”, the customer would consider that the product has no sacrifice to meet one’s need.

If a respondent’s largest utility product has a smaller observed utility than his/her ideal product, all products in the market require some degree of sacrifice. This is shown in Fig. 5.2 for Group A, where none of the products in the market have an observed utility greater than the ideal product. For Group B, conversely, some products on the market have a utility greater than the ideal product, and they would have no sacrifice for the products in the zero-SG range compared with the ideal product because they have larger utilities than the datum. Hence, all respondents can fall into two groups according to the existence of products above the ideal product in optimization problems.

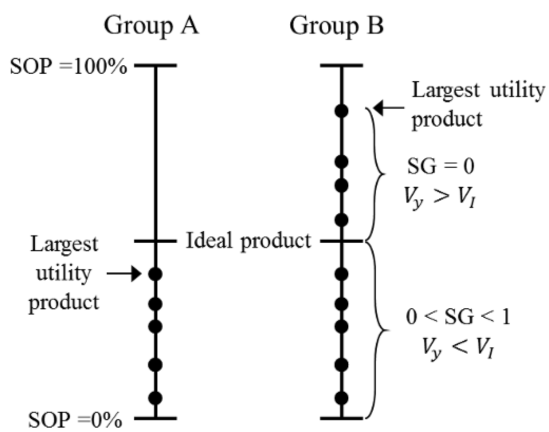

Figure 5.2. Conceptual diagram of product utilities in sacrifice gap metric

5.3. Implications of Sacrifice Gap on Product Search Problem

The premise of this case study is that an MP3 player company who is interested in mass customization aims to better meet the needs of the consumers. The firm wants to identify the product features that should be customizable from a larger set of considered product features. A choice-based conjoint survey if fielded and estimates of individual part-worth utilities are obtained from the HB-ML model as described in Chapter 2.2.4. Sacrifice gap will then be calculated as a metric by which decisions can be made regarding the product features they should and should not customize in the market. In advance of defining the mass customization problem, differences between minimizing SG and maximizing SOP are explored in both individual- and population-level product search problems to investigate the potential of SG for use in product design search problems in this chapter.

5.3.1. Survey design

For this numerical study a discrete choice survey with 12 choice tasks was fielded. The survey was completed by 205 respondents, and 10 choice tasks used to estimate the preference model while 2 were fixed for all respondents and used as hold-out questions. In each choice task question, a respondent was shown four full-profile MP3 player configurations and a "None" option. They were then asked to choose the product alternative they would be most likely to purchase. Each MP3 player was configured using the 8 product attributes, and associated levels, listed in Table A1 of Appendix A. The product attributes considered can generate a total of 393,216 unique product configurations. The choice-based conjoint study was created using Sawtooth Software's SSI Web (Sawtooth Software Inc., 2011).

Once survey data was collected, the Sawtooth Software's CBC HB module (Sawtooth Software inc. 2014) was used to estimate the part-worths for each respondent. For the price attribute, constraining utility estimates involves forcing the utility of high prices to be lower than the utility for lower prices. The cost structure in Table A5 is assumed with a base cost of $33. Price is assumed as one and a half times the cost. A piecewise linear interpolation is assumed to calculate the part-worths for the price attribute.

5.3.2. Respondent-level product search

Maximizing Share of Preference (SOP) is a popular objective function metric for product line search problems because SOP is a surrogate representation of customer behavior and market penetration. As introduced in Chapter 5.2.2, the SG measure defined is also based on choice probability. This section investigates how minimizing SG is different from the SOP optimization problem at the respondent-level in terms of product search results. The sacrifice gap value is obtained for the largest utility product in the market, which results in the smallest SG among all available products.

In SOP optimization problems, maximizing SOP corresponds to identifying product configurations that can be offered that have larger observed utilities than those products that are already on the market. For Group A, whose best product currently has a smaller utility than the ideal product, minimizing SG can lead to similar solutions to a maximize SOP optimization problem. However, for Group B, where the best product currently has a larger utility value than the ideal product, the two problem formulations may yield different optimum solutions. This is because all SG values are zero for products above the ideal product, resulting in multiple optimum solutions in that case.

This section runs respondent-level SG and SOP optimization problems to demonstrate how the SG definition yields different solutions from a SOP optimization problem. The product features of the competing products that represent the current market are summarized in Table A6. The competitor products are based on the feature and price combinations for the iPod Nano, iPod Touch, and iPod Classic. The 'None' option is also included. A Genetic Algorithm (GA) was used to find the optimal solutions because a GA has been shown to be effective in previous product line searches (Foster et al. 2014).

Table 5.1 shows the average objective function values across all respondents at the optimum for each problem formulation. It is noticeable that the average SG values are the same, but the SG problem formulation yields a lower average SOP. This difference can be explained by dividing the population into groups, as shown in Table 5.2. Group A has 152 members, and respondents belonging to this group are those where the best currently available product has a smaller observed utility than their ideal product. For respondents in this group, the two optimization problem formulations yield the same product configuration. This is because minimizing SG is not different than maximizing SOP if all available products have lower utility values than the ideal product. Group B has 53 members, and here the best product has an observed utility that is greater than or equal to their ideal product. However, for this group the two optimization problem formulations yield different optimum designs. The utility value of the SG optimum design is less than or equal to that of the SOP design because there are multiple optimum solutions in the zero-SG range for the SG problem. In other words, while

Group A has a one-to-one mapping between SG and SOP problems, while minimizing SG does not map directly to maximizing SOP for Group B.

Table 5.1. Function value of SG and SOP problems

objective function	Avg. SG (SD)	Avg. SOP (SD)
minimize SG	0.5580 (0.3943)	93.80% (19.21%)
maximize SOP	0.5580 (0.3943)	97.70% (7.85%)

* SD stands for Standard Deviation

Table 5.2. Function value of SG and SOP problems for each group

objective function	Group A (152 members)		Group B (53 members)	
	Avg. SG (SD)	Avg. SOP (SD)	Avg. SG (SD)	Avg. SOP (SD)
minimize SG	0.7526 (0.2503)	97.69% (8.55%)	0 (0)	82.65% (32.62%)
maximize SOP	0.7526 (0.2503)	97.69% (8.55%)	0 (0)	97.70% (5.46%)

* SD stands for Standard Deviation

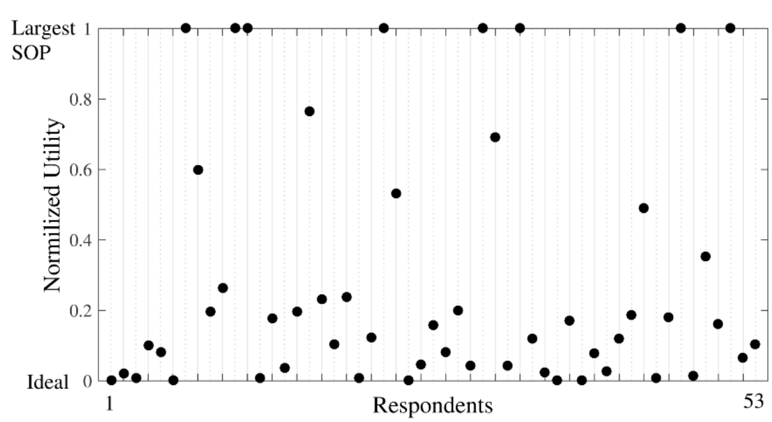

Figure 5.3. Utility comparison of SG optimum designs for Group B

Normalized observed utility values of the SG optimum designs for the 53 respondents in Group B are shown in Figure 5.3. The black dot indicates the normalized observed utility value of the SG optimum design, which is calculated as $(V_{SG} - V_I)/(V_B - V_I)$, where V_{SG}, V_I, and V_B are the observed utility values of the SG optimum solution, the ideal product, and the best product, respectively. Even though the 53 designs have different observed utility values and SOPs, their SG values are all zero. Also, the SG optimum design obtained in this study is one of the many available products between the ideal and best products.

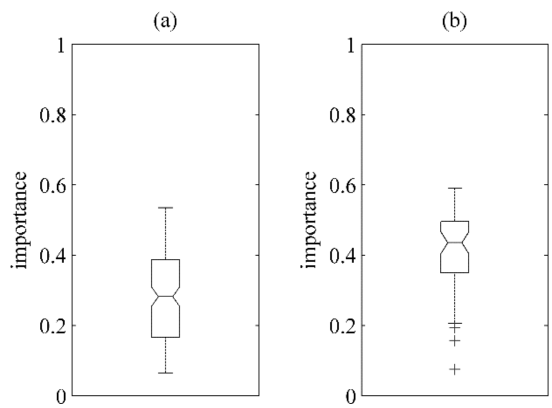

Figure 5.4. Price importance: (a) Group A (b) Group B

A noticeable difference between the two groups is the importance of price for each group, as shown in Figure 5.4. Respondents in Group A put less weight on price, compared to other the attributes in the survey, than those in Group B. This implies that respondents in Group A are more interested in product feature configuration than their counterparts in Group B who are more influenced by product price.

Literature in the market research community has demonstrated that mass customization is a more effective strategy when customers can be identified who are willing to pay a premium for increased product satisfaction (Du, Jiao, and Tseng 2006; Kuo and Cranage 2012; Berman 2002). Additionally, if a customer is not strongly driven by product price, that consumer may be willing to pay extra money for personalized goods (Kratochvíl and Carson 2005). In this study, targeting respondents in Group A may be a more effective design strategy when determining customization options that should be offered. This does not exclude Group B from the optimization problem formulation, however.

5.3.3. Product line search

This section explores the impact of formulating a product line search problem using objective functions associated with minimizing population-level SG and maximizing average SOP. An optimization algorithm is used to identify the configuration of five products, and the remainder of the marketplace includes the competitor products listed in Table A6 and the no-buy option.

Table 5.3 displays the optimum product line solutions obtained from each optimization problem formulation. The SG and SOP solutions have 14 common features among the 20 or 17 unique features that comprise each solution, respectively. This commonality between solutions is expected, as both formulations increase the observed utility of the products in the line, particularly creating more preferred products for Group A. In support of the decision presented in Chapter 5.3.2, if the optimization problems are run only when considering Group A, the two solutions are identical.

Table 5.3. Solution of product line search

Objective	Attribute								SG	SOP (%)
	1	2	3	4	5	6	7	Price ($)		
min. SG	8*	3*	1*	1	2*	3*	4	148.50	0.8654	72.46
	8*	5*	1*	3*	2*	3*	2*	189.75		
	4	7*	3*	4*	2*	1*	3*	199.50		
	7	5*	3*	5*	1	3*	3*	201.00		
	7	5*	3*	4*	2*	8	2*	231.00		
max. SOP	8*	3*	1*	3*	2*	3*	3*	178.50	0.9028	82.49
	8*	5*	3*	4*	2*	3*	3*	234.75		
	8*	8	3*	4*	2*	1*	3*	234.75		
	8*	8	3*	5*	2*	4	2*	246.00		
	8*	7*	3*	4*	3	1*	3*	276.00		

* stands for common features

Table 5.4. Group analysis of product line solution

objective function	Group A (152 members)		Group B (53 members)	
	Avg. SG (SD)	Avg. SOP (SD)	Avg. SG (SD)	Avg. SOP (SD)
minimize SG	0.9879 (0.0346)	70.49% (34.34%)	0.5203 (0.4627)	78.12% (27.14%)
maximize SOP	0.9880 (0.0406)	84.73% (22.12%)	0.6584 (0.4300)	76.07% (27.53%)

SG and SOP values of the two solutions are displayed in Table 5.4 for the two groups of respondents. This analysis demonstrates that the SG and SOP measures work differently in population-level optimization problems. For Group A, there is no difference between the two objective functions in SG values of their optimum solutions. For Group B, the SG design resulted in smaller SG and larger SOP values. These results suggest that if a five-product solution using the SG problem formulation was introduced into the market to reduce respondent sacrifice, the solution would have limited effectiveness with Group A due to their relative insensitivity to price because a smaller price sensitivity implies a less chance to have a larger utility product above the ideal product according to the respondent-level product search results. On the contrary, the SG optimization problem formulation works well for Group B who is more sensitive to price because Group B would have more chances to minimize their SG values. In the zero-SG range, Group B would have more flexibility in configuring product features while keeping zero sacrifice gap and this characteristic is caused by the advantage of the one-to-multiple mapping of Group B explained in Chapter 5.3.2. For this reason, the SG problem formulation resulted in cheaper products than the SOP problem formulation.

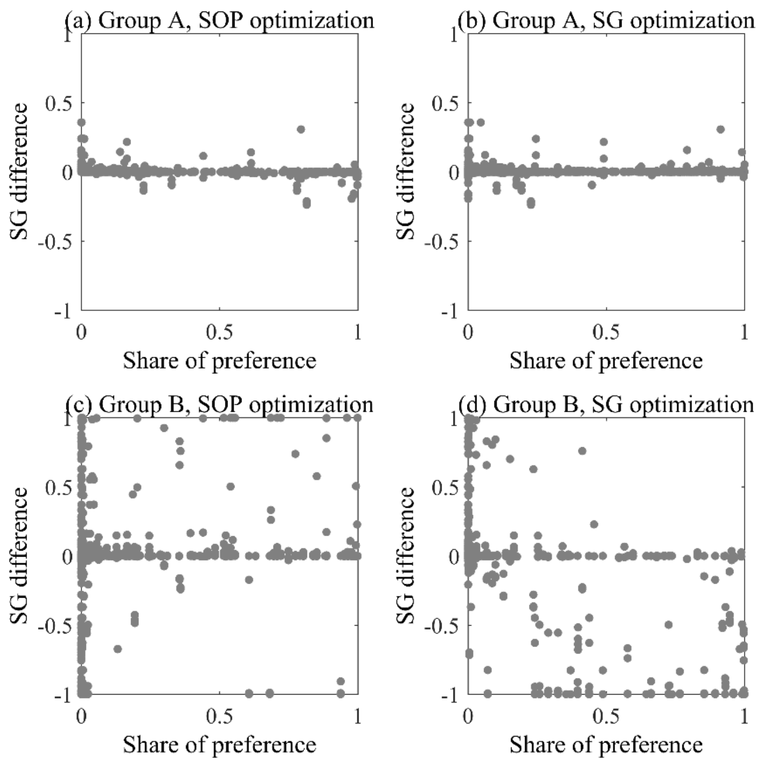

Figure 5.5. Sacrifice gap difference versus preference share:
(a) SG optimization for Group A (b) SG optimization for Group B
(c) SOP optimization for Group A (d) SOP optimization for Group B

A visual comparison of sacrifice at the respondent level is provided in Figure 5.5 to demonstrate that the proposed definition for SG provides an optimization problem formulation that works effectively for respondents represented by Group B. However, there is minimal advantage for Group A. The y-axis indicates the difference in sacrifice between the two optimal product line configurations. These differences are calculated by subtracting the SG values associated with the SOP solution from the SG values associated with the SG solution as SG of SG solution – SG of SOP solution. Thus, positive y-axis values mean that a SOP design is better than a SG design in terms of observed utility because smaller SG values are preferred. In contrast, negative y-axis values mean the SG design has a larger utility than a SOP design. This data consists of 5,125 (5 SG optimum designs × 5 SOP optimum designs (to compare each SG design against) × 205 respondents) observations, of which 4,225 belong to Group A and 900 belong to Group B. The x-axis indicates the share of preference values for each design.

Figure 5.5-(c) and (d) for Group B show distinct differences in SG difference values. In Figure 5.5-(d) there are more negative SG difference values, implying that the product line solution minimizing SG obtained reduced respondent-level sacrifice. When optimizing for share of preference, as in Figure 5.5-(c), there are more positive SG difference values which implies that maximizing SOP effectively works for Group B at the population-level optimization problem. From these figures it is verified that the optimization problem formulations have significant respondent-level effects for Group B, even though they are formulated at the population-level.

Conversely, Figure 5.5-(a) and (b) show that for Group A the SG definition did not make a significant improvement in minimizing SG or maximizing SOP. This could be due to the fact that minimizing SG is not different than maximizing SOP at the individual-level for Group A. To use the SG definition in mass customization problems, however, the SG optimization problem has to be effective at minimizing sacrifice for both groups. Utility values were then analyzed at the respondent-level to explore how the SG definition could be more effectively used for mass customization.

A respondent-level analysis provides interesting insight into the flexibility created by the proposed SG definition. Figure 5.6 displays the respondent-level utility analysis of Respondent #2, who is a member of Group B. The histogram shows the utility distribution for all available 393,216 ($=8\times8\times4\times6\times8\times8\times4$ obtained using the number of each levels) products. The observed utility of the ideal product is 16.7, and that of the best product is 19.8. Utility values of each optimum design are also plotted. Both optimization problem formulations resulted in one product residing in the zero sacrifice range.

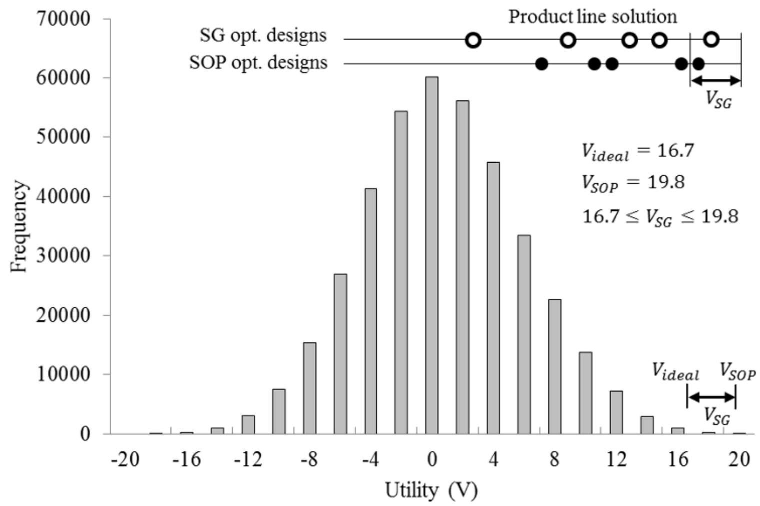

Figure 5.6. Respondent-level analysis of two product line solutions

There are 96 products in the zero-SG range (V_{SG}) for this respondent. This provides solution flexibility in that a company can choose to offer any of the 96 products in this range without impacting that respondent's sacrifice. Table 5.5 displays the active product features that fully cover the zero-SG range for this respondent. When a company offers the full combination of these active features, 3,240 unique combinations can be created from which the respondent can choose 96 with no sacrifice. By offering a subset of the total feature set, a manufacturer can also allow other respondents to reduce (or minimize) their sacrifice gap by selecting from one of the 3,240 product combinations. From this result, Chapter 5.4 proposes a problem formulation where the SG metric can be used to identify the set of product features that should be offered. This approach will be especially beneficial for those respondents in Group A who may be more willing to pay a premium for a customized product.

Table 5.5. Product feature for the products in the zero-SG range

Level	Attribute						
	1	2	3	4	5	6	7
1	OFF	OFF	ON	ON	OFF	ON	ON
2	ON	OFF	ON	ON	ON	OFF	ON
3	ON	OFF	ON	ON	OFF	ON	ON
4	ON	OFF	OFF	ON	OFF	OFF	ON
5	ON	ON		OFF	OFF	OFF	
6	ON	OFF		ON	OFF	OFF	
7	OFF	ON			OFF	OFF	
8	ON	ON			OFF	ON	

5.4. Mass Customization

This section formulates an optimization problem using the proposed definition of sacrifice gap to search for the product feature levels that should be offered in a mass customization environment.

5.4.1. Customization charge

To account for the additional financial burden to the firm associated with supply chain management, development of new product features, purchase of new machinery, etc., an extra charge is added to the feature price. It is assumed that the financial burden of customization is passed directly to the consumers as a customization charge. The structure of the customization charge presented in this work is hypothetical and not based on industry data. Table 5.6 presents two different customization charge structures. All product attributes are categorized into simple and complex customization attributes. Simple customization attributes include storage space, background color, and background overlay. Complex customization attributes include PVC (photo, video, and camera), WAP (web, app, and pedometer), input type, and screen size. The customization charge for complex attributes is due to a larger increase in design and manufacturing costs passed on to the consumer. The charge is twice as high for complex attributes as compared to simple attributes because complex attributes impact both design and manufacturing while simple attributes are likely to impact manufacturing considerations. The customization charge structures are applied to each attribute in turn, and then summed to yield a single customization charge value.

Table 5.6. Customization charge structure

No. of additional features	Simple attributes	Complex attributes
1	$1	$2
2	$2	$4
3	$3	$6
4	$4	$8
5	$5	$10
6	$6	$12
7	$7	$14

5.4.2. Constraints of design variables

A company seeking an opportunity to adopt a mass customization strategy would need to reflect limitations and decisions concerned with manufacturing, marketing, or engineering design. For example, if a company has already launched mass-produced products, developing new product features for mass customization would be burdensome due to additional costs. Under this circumstance, the company would want to provide customers with customizable features within already developed feature categories. Additionally, the company may want to control the number of customizable features or consider design prohibitions arising from technical infeasibilities (Foster and Ferguson 2014).

5.4.3. Optimization procedure

The optimization problem formulation proposed in this work minimizes aggregate SG by determining which options should be included in the customization set. This process also involves pricing the customized products accordingly and calculating a market average sacrifice gap based on all available alternatives in the market. The additional customization charge creates a trade-off between reducing sacrifice gap and the number of allowed build combinations. A genetic algorithm is used to determine to determine the appropriate feature mix as described in Figure 5.7.

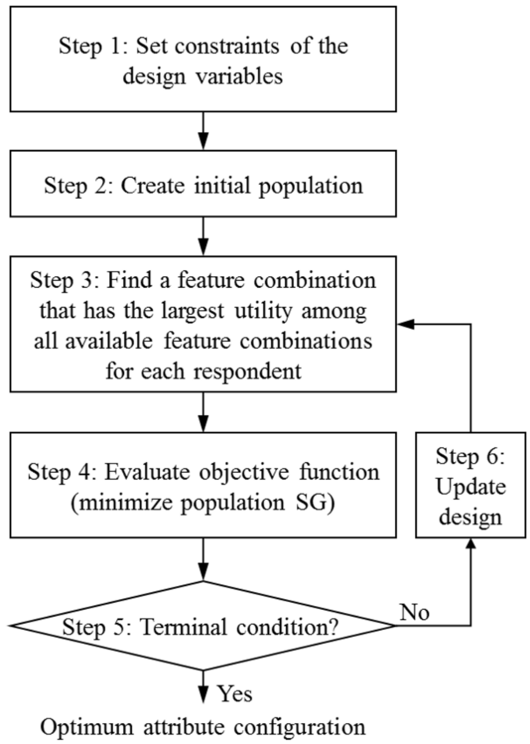

Figure 5.7. Iterative procedure of solving mass customization problem

- Step 1: Set design variable constraints. Such constraints should be added as needed, and include limitations and decisions concerned with manufacturing, marketing, or engineering design. No constraints are used in this problem.
- Step 2: Create the initial population for the GA. Three hundred initial designs are randomly generated in this problem.
- Step 3: Find the product design with the largest utility (the smallest sacrifice gap) among all available products configured by the current design variables for each respondent. This design will be used to calculate SG calculation in Step 4.
- Step 4: Calculate the sacrifice gap of the product design found in Step 3 for each respondent. Then, evaluate the market average sacrifice gap (the objective function for this optimization problem).

- Step 5: Compare terminal conditions such as the maximum number of total generations, the maximum number of generations without improvements in a function value, and the function tolerance. If one of the terminal conditions is met, the GA is terminated and returns the optimum solution.
- Step 6: Update designs by genetic search (selection, crossover, and mutation).

Step 3 is a computationally expensive task because all possible designs have to be evaluated to find the best feature combination for each respondent. If there are no design variable constraints, an enumeration of all possible products yields a maximum of 80,609,280 SG calculations (the maximum number of available products $\times$ 205 respondents) for one customization design. This will demand an extremely expensive computational cost. Compared to the population-level product line search optimization, the optimization problem to search for product features that should be offered in a mass customization environment has a significantly larger size. In recent work, to reduce computational cost and improve solution quality of genetic product line search, a targeted initial population method was developed and this could be also adopted in the optimization problem for a mass customization environment (Foster et al. 2014). As a future research topic, computationally efficient techniques for genetic product search are needed to be further explored.

The population size at each generation was 300, and the design space is represented by 59,935,452,609,375 possible designs obtained as $\left(\sum_{i=1}^{4}\frac{4!}{i!(4-i)!}\right)^{2}\left(\sum_{j=1}^{6}\frac{6!}{j!(6-j)!}\right)^{1}\left(\sum_{k=1}^{8}\frac{8!}{k!(8-k)!}\right)^{4}$ where the maximum level number of each attribute is {8,8,4,6,8,8,4}. The selection operator was random, and the crossover operator was uniform with 15 elites. The mutation operator was Gaussian with a default value of 1 for both scale and shrink parameters in Matlab. Finally, the convergence criterion was met when 50 generations were performed with no improvement in the objective value, or when the function tolerance is less than 10e-12. The maximum number of iterations before the algorithm halts was 500. The genetic search problem roughly took four days on a desktop computer running at Intel i7 3.40 GHz with 8 GB RAM.

5.4.4. Results

Table 5.7 displays the optimal product feature set for the developed mass customization problem. As expected, the solution does not prescribe all attributes to be "on." This indicates that the optimization problem reflects the trade-off between product price (including the customization charge) and feature utility. Minimizing market average sacrifice gap yielded a configuration with 30 of the 46 attribute levels turned on and a $36 customization charge. Among the 30 product levels turned on, 19 levels are also included in the product lines solutions reported in Chapter 5.3.3. In detail, 19 and 16 product features are common to the product line design of the SG and SOP problems, respectively. This means a firm must accommodate the inclusion of 11 or 14 more product features than the mass-produced product line of the SG and SOP problems, respectively.

Table 5.7. Optimum product feature solution for mass customization

Level	Attribute						
	1	2	3	4	5	6	7
1	ON†	OFF	ON*	ON*	ON*	ON*	OFF
2	OFF	ON†	ON†	OFF	ON*	OFF	ON*
3	ON†	ON*	ON*	ON*	ON*	ON*	ON*
4	ON*	OFF	OFF	ON*	OFF	OFF*	ON*
5	OFF	ON*		ON*	ON†	ON†	
6	OFF	OFF		OFF	ON†	OFF	
7	ON*	ON*			OFF	ON†	
8	ON*	ON*			OFF	ON*	

* stands for features included in mass-produced products
† stands for features that are new for mass customization

Table 5.7 provides a comparison of the mass customization solution with the other optimal designs. Implementing the optimum product features for mass customization decreases market average sacrifice gap from 0.8654 or 0.9028 in the mass production market to 0.7882, despite the added customization charge. The optimum feature offerings also increase the average preference share from 72.46% or 82.49% to 89.76%. The results of the individual-level product search can be regarded as reference values of the ideal customization market. Notice that the complete removal of sacrifice cannot be achieved in a mass customization environment due to the customization charge.

Table 5.8. Comparison of mass customization solution

Optimization problem	Avg. SG	Avg. SOP
Individual-level SG	0.5580	93.80%
Individual-level SOP	0.5580	97.70%
Aggregate SG for product line	0.8654	72.46%
Aggregate SOP for product line	0.9028	82.49%
Aggregate SG for mass customization	0.7882	89.76%

Table 5.8 shows group-level comparisons of the mass customization solution with the other optimal solutions. For Group A mass customization decreases market average sacrifice gap from 0.9879 or 0.9880 in the mass production market to 0.9119. The optimum feature offerings increase the average preference share from 70.49% or 84.73% to 92.22%. Similar trends are observable in the results for Group B.

Table 5.9. Group-level comparison of mass customization solution

Optimization problem	Group A		Group B	
	Avg. SG	Avg. SOP	Avg. SG	Avg. SOP
Individual-level SG	0.7526	97.69%	0	82.65%
Individual-level SOP	0.7526	97.69%	0	97.70%
Aggregate SG for product line	0.9879	70.49%	0.5203	78.12%
Aggregate SOP for product line	0.9880	84.73%	0.6584	76.07%
Aggregate SG for mass customization	0.9119	92.22%	0.4332	82.69%

5.5. Summary

This study explores more direct incorporation of customer preference data into engineering design problems to support product development decision-making for mass customization. This chapter specifically aimed to develop a quantitative definition of sacrifice gap and proposed an optimization problem formulation. A sacrifice gap measure is defined using the preference estimates obtained using discrete choice. Numerical studies confirm that the sacrifice gap definition can be effectively applied to design search problems for mass customization environments.

The numerical study for the respondent-level product search reveals that consumers are divided into two groups according to price importance: Group B places a large importance on price, while Group A is less sensitive to price and is more willing to pay a premium for a customized product. In addition, Group A has the largest utility product below the ideal product, but Group B has some products in the zero-SG range above the ideal product. Therefore, Group B has flexibility in configuring product feature levels in the zero-SG range while the SOP and SG definitions work in exactly the same way for Group A at the respondent-level. The product line designs include many common product features between the SG and SOP designs. However, the group-level analysis reveals that the SG definition has no distinct advantage over the SOP definition for Group A in minimizing sacrifice gap in population-level optimization problems. To make the SG definition effective for Group A in mass customization environments, an optimization problem that identifies the optimal set of product features to offer is proposed.

The advantage of the SG definition in a group-level optimization problem is that the flexibility of choosing products in the zero-SG range for Group B can provide Group A with a chance to configure product features in the mass customization optimization. Recall that minimizing a SG value does not give any chance to configure product feature levels for Group A at respondent-level optimization because Group A does not have zero-SG range. However, the sacrifice gap definition can provide much wide product alternatives for Group A in the mass customization problem. The increased variety of product alternatives is made by the flexibility in feature configuration of Group B. Despite adding a customization charge, the optimization result suggests that the mass customization environment can much more reduce customer’s sacrifice gap than the mass-produced product line.

Future works from this research will focus on three aspects. First, the proposed mathematical definition of sacrifice gap could be further applied to decision-making and its evaluation in the mass customization environment. In particular, if the willingness to pay a premium for customization is quantified using the discrete choice methods, it could bring more reality into sacrifice gap evaluation. Second, computationally efficient methods would be required for practical use in industry. Lastly, a new design problem integrating mass-produced product and mass customization environment could give a firm to explore more realistic product development opportunity.

According to the findings, mass customization is realizable by managing an individual's sacrifice gap using the HB model and optimization techniques. To continue developing product search problems focusing on individual-level preferences, one of the most significant challenges is to manage the inherent uncertainty in preferences. This topic is explored in the next chapter.

Chapter 6. Reliability and Robustness of Product Line Offerings under Uncertainty when Using Discrete Choice Methods

Point estimates of part-worth values in customer preference models have been used in market-based product design under the simplifying assumption that customer preferences can be treated as deterministic. However, customer preferences are not only inherently stochastic, but are also statistical estimates that exhibit random errors in model formulation and estimation. Ignoring uncertainty in customer preferences and estimation variability has caused concern about the reliability and robustness of an optimal product design solution. This study quantitatively defines reliability and robustness of a product design under uncertainty when using discrete choice methods. These metrics are then integrated into a multi-objective product search problem formulation. A multi-attribute decision method of handling conflicts in decision making is conducted to help a decision maker choose a single design from the set of solutions.

6.1. Introduction

According to (de Palma et al. 2008), *"human decision-making involves trading off costs or benefits, which are known now with certainty, with risky outcomes in the future." From a social science perspective, such individual decisions are associated with varying levels of probability (risk) and uncertainty because of missing information (ambiguity)* (Hsu 2005)*."* In discrete choice models, certainty and uncertainty can be discussed as estimation issues with observed and unobserved heterogeneity in preferences.

Uncertainty in customer preference estimates is a matter of individual-level decision behavior. Mixed logit models have typically been used to estimate preference heterogeneity, allowing variation in taste across individuals. Parameters of random utility models consist of a vector of preference coefficients (observable) and a random error term (unobservable). The observable vector of the preference coefficients is specified to be a multivariate normal distribution. The error term reflects specification errors, omitted factors, non-observable factors, and unobserved heterogeneity of preferences (de Palma et al. 2008). If the error term is independent and identically distributed (i.i.d.) with a Type I extreme value distribution, and the maximum utility rule is applied in simulations, the expectation is identical to the logit model (Train 2009). The hierarchical Bayes mixed logit (HB-ML) model assumes preference heterogeneity as a continuous distribution and Bayesian inference is employed to estimate posterior distributions of the preference coefficients. While taking numerical integration using in the estimation procedure, many draws of posterior distributions are generated. Instead of using the whole posterior distributions, point-estimates have been usually adopted in market simulation, because of their reduced computational cost, by taking mean values of the posterior distributions.

While many who use market simulators to make decisions commonly use point-estimates of individual's part-worths, this ignores uncertainties that are inherent to discrete choice methods. Customer preferences are not only inherently stochastic, but also are statistical estimates that exhibit errors in model form and estimation procedure. For this reason, there are concerns about the reliability and robustness of an optimal design solution under the presence of uncertainty in discrete choice methods.

Figure 6.1 illustrates how variations in part-worth estimates can influence the objective function in market-based design problems. Consider a single-objective problem where the goal is to maximize market share of the product line. Now, assume that point estimates of preference are used, as shown in Figure 6.1-(a). The best design is solution A. However, when variability within the preference estimates is considered, solution A is no longer the clear winner. Solutions B or C may capture more market share than the solution A, as visualized by the box plots. In multi-objective formulations, variation in the part-worth estimates leads to objective function values better described as surfaces, as shown in Figure 6.1-(b). Hence, choosing the optimal design is more difficult when variations in preference estimate are considered.

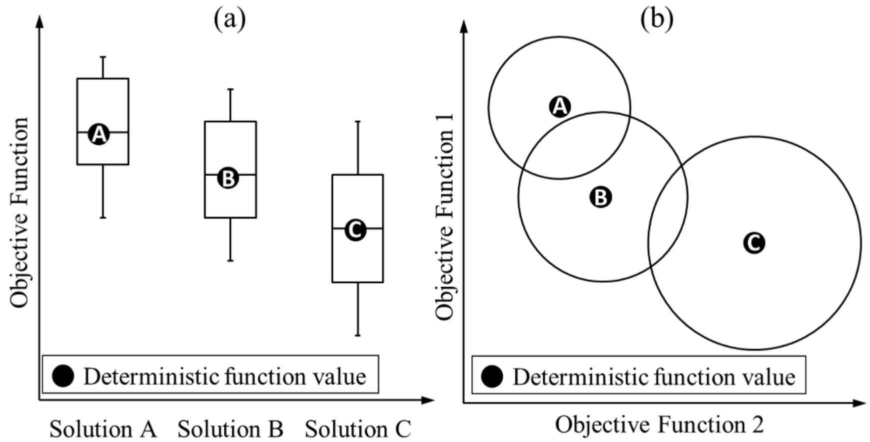

Figure 6.1. Variability in hypothetical solutions:
(a) Single-objective (b) Multi-objective

The objective of this chapter is to propose an optimization problem formulation to search for product line solutions by considering reliability and robustness under uncertainty when using discrete choice methods. When uncertainty in discrete choice methods is considered, selecting a final solution is made more difficult by the variability that occurs in objective function values. Therefore, additional criteria are required to evaluate the tradeoffs between design solutions when uncertainty sources are considered. With this purpose, the reliability and robustness of a product line design are characterized. Draws from a Bayesian-based mixed logit model are used with a Randomized First Choice (RFC) simulation to investigate how part-worth variation impacts the optimal design solution. A multi-objective optimization problem is then developed that incorporates reliability and robustness in product line design optimization. Chapter 6.2 provides background knowledge about discrete choice models and the quantification of uncertainty in market-based product design. Chapter 6.3 introduces definitions of reliability and robustness. Chapter 6.4 presents a numerical study to investigate how demand uncertainty impacts the results of a product line search, and a multi-objective problem formulation is proposed. Conclusions, limitations, future work are discussed in Chapter 6.5.

6.2. Background

Chapter 6.2.1 provides the literature on product design under uncertainty when using discrete choice methods. Chapters 6.2.2 and 6.2.3 introduces HB draws and RFC as method to quantify variation in demand.

6.2.1. Uncertainty in discrete choice methods

In engineering, ambiguity and vagueness of system variables or parameters are considered primary sources of uncertainties (Ayyub and McCuen 2011). Ambiguity is generally due to noncognitive (aleatory) sources that include: inherent physical randomness, statistical uncertainty, and model uncertainty (Haldar and Mahadevan 2000). Thus, it is irreducible uncertainty. Vagueness is due to cognitive (epistemic) sources such as limited knowledge and human factors (Haldar and Mahadevan 2000). Thus, these are reducible.

When using discrete choice methods, aleatory uncertainty can be caused by dynamics associated with demand and cost (Raman and Chatterjee 1995), inherent preference inconsistency (MacDonald, Gonzalez, and Papalambros 2009), and response variability (J. J. Louviere 2001). Epistemic uncertainty is caused by choice context (J. Louviere et al. 2002; Swait et al. 2002), sampling errors in Bayesian inference (X. (Jocelyn) Wang and Curry 2012; Von Hagel and Ferguson 2015; Besharati et al. 2006), and demand model misspecification (Gilbride and Lenk 2010; Abramson et al. 2000; Montgomery and Bradlow 1999).

Recent research, as listed in Table 6.1, recognizes the limitation of using deterministic preference estimates and attempts to rectify these limitations. In literature, there are three methods to manage uncertainty when using discrete choice methods: using HB draws, interval variables, and moment estimation.

Table 6.1. Literature that consider uncertainty in demand model

References	Methods to treat uncertainty in discrete choice methods	Design problem is to search	Design variables	Design objective
Camm et al. (Camm et al. 2006)	Using HB draws	A single product configuration	Discrete product attributes	Maximize FCS
Wang et al. (X. (Jocelyn) Wang, Camm, and Curry 2009)	Using HB draws	Product configurations of a product line	Discrete product attributes	Maximize FCS
Wang and Curry (X. (Jocelyn) Wang and Curry 2012)	Manually defining intervals of part-worths	A single product configuration	Discrete product attributes	Maximize FCS
Luo et al. (Luo et al. 2005)	Interval estimates of part-worths using 95% confidence levels	A single product configuration	Discrete product attributes	Maximize nominal SOP, Minimize SOP variance, Minimize worst-case performance
Besharati et al. (Besharati et al. 2006)	Interval estimates of part-worths using 95% confidence levels	A single product configuration	Discrete product attributes	Maximize nominal SOP, Minimize SOP variance, Maximize engineering design performance
Resende et al. (Resende, Grace Heckmann, and Michalek 2012b)	Moment estimation of market share based on continuous probability function of part-worths	Engineering and marketing variables of a single product	Continuous product attribute	Maximize profit at specified downside risk tolerance

FCS: First Choice Share
SOP: Share Of Preference

Camm et al. (Camm et al. 2006) and Wang et al. (X. (Jocelyn) Wang, Camm, and Curry 2009) used draws of HB model. They proposed a post-optimality robustness test that assesses the degree to which a design is negatively impacted by uncertainty in part-worths. Camm et al. (Camm et al. 2006) repeatedly solved deterministic optimization problems using random draws of HB-ML and compared their solutions. Results were not reassuring because the best design profile that corresponds to the solution of the aggregate part-worths accounted for 23.5% of all solutions. Wang et al. (X. (Jocelyn) Wang, Camm, and Curry 2009) implemented a sample average approximation method for stochastic discrete optimization (Kleywegt, Shapiro, and

Homem-de-Mello 2002) to the share-of-choice product line problem. They suggested that uncertainty could be integrated out of a solution by taking many draws per person.

Wang and Curry (X. (Jocelyn) Wang and Curry 2012), Luo et al. (Luo et al. 2005), and Besharati et al. (Besharati et al. 2006) defined variation in part-worths using interval variables and investigated the best and worst cases of product utility. Wang and Curry (X. (Jocelyn) Wang and Curry 2012) studied the concept of robustness in integer programming by explicitly capturing variation in part-worths for the share-of-choice problem. This study assumes that individual preferences are bounded, independent, and symmetric variables. Also, the covariance matrix for individual level part-worths is assumed to be a diagonal matrix, meaning no correlation among product features is allowed. However, these assumptions do not agree with the random effects model estimated using the hierarchical Bayes technique used in these works. In the mixed logit model, not only do no constraints exist on the covariance matrix, but the preference coefficients also are not defined as interval variables. Luo et al. (Luo et al. 2005) and Besharati et al. (Besharati et al. 2006) used confidence intervals of segment-level part-worths to determine whether or not one alternative dominates another alternative by calculating the lower and upper bounds of product utility. The idea of determining design robustness using intervals is conceptually reasonable, but is still limited by the inability to quantify the degree of robustness because it can only compare the best and worst cases. To quantify the robustness of a solution under uncertainty in demand modeling, it is necessarily required to address stochastic preference coefficients and directly use their probability distributions in a design problem.

Resende et al. (Resende, Grace Heckmann, and Michalek 2012b) proposed an analytical approach to estimating the effect of demand uncertainty on the final performance function. They applied the well-known delta method (Ver Hoef 2012) to the multinomial logit model to estimate the first and second moments of the performance function using a first-order Taylor series approximation. Although the closed-form function to compute the market share is only available to the multinomial logit model at a pre-specified risk level, analytical methods have the potential to be further developed in the future.

6.2.2. HB draws

Estimating an HB model requires a number of iterations before convergence is assumed. The hierarchical Bayesian framework, which is implemented using MCMC techniques, yields complete posterior distributions of the preference coefficients at the individual-level. Thus, HB yields multiple estimates, called draws, that form the posterior distributions of each respondent's preference coefficients. These draws are then averaged for each respondent to create a single vector of part-worths that represent preferences for each attribute level included in the study. Then, the single value is used as a best guess of random parameters ignoring variability in parameters. In market-based product design, the point-estimates of customer preference coefficients have been typically used as a simplifying assumption to reduce the computational burden of market simulations. To represent variability in Bayesian procedure, the draws themselves can be used in market simulations, instead of using point-estimates (Orme and Baker 2000; Von Hagel and Ferguson 2015; Shin and Ferguson 2015).

6.2.3. Randomized first choice simulation

A first choice model assumes respondents choose the product that has the highest utility value from the competitive set (maximum utility rule) (Orme 2006). RFC modifies this process by introducing error terms to the utility value during the simulation phase. Multiple part-worth values can be created by adding random errors to the aggregate part-worths obtained from the HB model. Then, a choice simulation is conducted using these modified part-worths following the maximum utility rule. RFC was first introduced as a simulation technique in response to product similarity challenges by Orme and Huber (Orme and Huber 2000; Huber, Orme, and Miller 1999) in 1999. They demonstrated that this formulation of the market simulator outperformed four commonly used models in predicting holdout choice shares: an aggregate multinomial logit model, a latent class model, an individual choice analysis of the latent class, and a HB-ML model. However, since RFC is computationally demanding, it may be too expensive to use in large-scale market-based design problems.

RFC adds two kinds of variability to the individual-level part-worths. The utility of alternative i for an individual n is derived as (Huber, Orme, and Miller 1999)

$$U_{ni} = (\beta_n + E_{a,n})^T x_{ni} + E_{p,i} \tag{6.1}$$

Here, E_a is a vector of attribute variability added to the part-worths and E_p is a product variability added to the product i. Intuitively, attribute variability represents inconsistency in a respondent's relative weights or part-worths applied to product attributes (Huber, Orme, and Miller 1999). The attribute variability term reflects variation in taste (Hausman and Wise 1978; Revelt and Train 1998). Product variability occurs when a customer evaluates choice alternatives with inconsistency in several different choice tasks (Huber, Orme, and Miller 1999). In the logit model, product variability is mathematically equivalent to the unobserved random disturbance in Eq. (2.1). Either a Gumbel or normal distribution can be selected for E_a, but Gumbel distribution has to be used to define E_P for logit models. Hence, a tuning process is required to determine the degree of variability. After introducing the variabilities to the point-estimates of part-worths, the first choice rule is simulated to predict choice shares.

While preference share methods are tunable for scale and usually more precise than a first choice simulation, they suffer from IIA (independence from irrelevant alternatives) issues (Orme 2006). First Choice Share (FCS) can resolve IIA issues but usually results in biased predictions and not be tunable for scale (Orme 2006). The RFC model combines the desirable aspects of the first choice and share of preference choice rules by introducing variations in point-estimates and simulating choices using the maximum utility rule many times. RFC simulation can resolve the extreme choice share issue by tuning the extent of attribute and product variation in Eq. (6.1).

6.3. Technical Approach

This chapter introduces a quantitative measurement of reliability and robustness of a product line design in market-based design using RFC simulation. A multi-objective problem formulation is proposed to integrate the stochastic aspects into one framework. Reliability and robustness are quantitatively defined in Chapter 6.3.1 and 6.3.2, respectively. Chapter 6.3.3

describes a multi-objective problem formulation used to search for a non-dominated set of product line solutions under uncertainty in discrete choice models.

6.3.1. Reliability of a product design in market

In engineering terminology, reliability is defined as "*the probability of successful performance*"; thus it is the converse of the term probability of failure (Haldar and Mahadevan 2000). In this study, reliability indicates satisfactory performance of a product line design. Reliability-Based Design Optimization (RBDO) (Tu, Choi, and Park 1999) is "*a method to achieve the confidence in product reliability at a given probabilistic level* (Lee et al. 2008)." Various methods have been developed to advance reliability analysis and design methods: sampling-based design using MCS (Monte Carlo Simulation) (Lee, Choi, and Zhao 2011; Lee, Shin, and Choi 2013), MPP (Most Probable Point) (Xiaoping Du and Chen 2000) based double-loop RBDO using FORM (First-Order Reliability Method) (Shin and Lee 2014; Shin and Lee 2015; Chiralaksanakul and Mahadevan 2005) and SORM (Second-Order Reliability Method) (Lee, Noh, and Yoo 2012), single-loop method called SORA (Sequential Optimization and Reliability Assessment) (Xiaoping Du and Chen 2004), and time-dependent reliability analysis (Xiaoping Du 2014). This study adopts sampling-based reliability analysis using MCS because no analytical method has been developed to define uncertainty in discrete choice models and market simulations.

Using the notion of failure to characterize unwanted behavior, two different types of "simulation failures" are proposed when designing a product line using discrete choice methods and individual-level choice behavior:

Failure-I*: A respondent fails to choose one of the products associated with the product line design solution in a single RFC replication.*

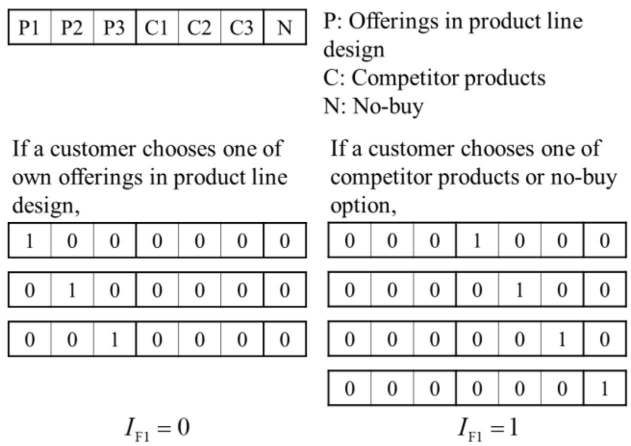

Figure 6.2. Visualization of Failure I for a Single RFC Replicate

Figure 6.2 illustrates Failure-I. Offerings in a product line, which are determined in an optimization problem, are called *Own Offerings* or *Own Products* in the rest of the chapter. There are three *Own Offerings*, three competitor products, and a no-buy option in a simulated market. The numbers 1 and 0 represent a respondent's product selection in a first choice simulation. The bit string $\mathbf{c}$ expresses the first choice result using RFC replicates. For example, if a consumer chooses *Own Product* 1 (P1) in the first choice simulation using RFC replicates, the choice result is saved as $\mathbf{c} = [1000000]$.

$I_{\mathrm{F1}}(\mathbf{c})$ is an indicator function to count Failure I and defined as

$$I_{\mathrm{F1}}(\mathbf{c}) \equiv \begin{cases} 0, & \mathbf{c} \in \Omega_{PL} \\ 1, & \text{otherwise} \end{cases} . \tag{6.2}$$

Ω_{PL} is a product line set that consists of *Own Offerings*. If one of *Own Offerings* is not selected in the first choice simulation using RFC replicates, $I_{\mathrm{F1}}(\mathbf{c}) = 1$. This means the product line design fails under uncertainty in discrete choice methods.

The probability of Failure-I is defined as

$$P_{\mathrm{F1}} \equiv P\left[\mathbf{c} \notin \Omega_{PL}\right] = \frac{1}{N}\frac{1}{R}\sum_{n=1}^{N}\sum_{r=1}^{R} I_{\mathrm{F1}}^{n,r}(\mathbf{c}_{n,r}) \tag{6.3}$$

N and R indicate the number of respondents and RFC replicates, respectively. Reliability-I, defined as $1-P_{\mathrm{F1}}$, represents how much market share is expected under demand uncertainty when introducing the product line solution into the market. Thus, Reliability-I represents the first choice share of a product line under uncertainty.

Failure-II: *A respondent changes their product choice decision (made using deterministic preference coefficients) to a different product or 'none' within the choice alternatives when the respondent is given an identical choice again.*

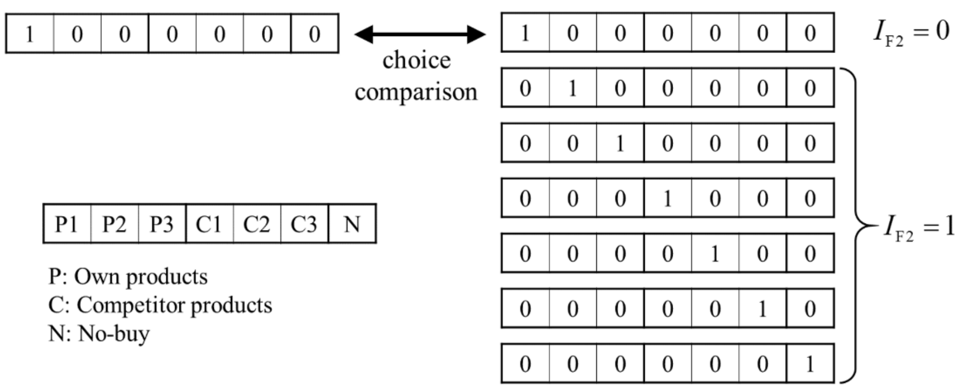

Figure 6.3. Visualization of Failure II for a Single RFC Replicate

Figure 6.3 illustrates Failure-II. $I_{\mathrm{F2}}(\mathbf{c})$ is an indicator function to count Failure-II and defined as

$$I_{\mathrm{F2}}(\mathbf{c}) \equiv \begin{cases} 0, & \mathbf{c} \in \Omega_D \\ 1, & \text{otherwise} \end{cases} \tag{6.4}$$

Ω_D is a deterministic choice set that is the optimum product line solution obtained using point estimates. If a respondent changes their choice decision made using aggregate part-worths to a different product in RFC simulation, $I_{\mathrm{F2}}(\mathbf{c})=1$. Thus, Failure-II is associated with choice inconsistency. The probability of Failure-II is defined as

$$P_{\mathrm{F2}} \equiv P\left[\mathbf{c} \notin \Omega_D\right] = \frac{1}{N}\frac{1}{R}\sum_{n=1}^{N}\sum_{r=1}^{R} I_{\mathrm{F2}}^{n,r}(\mathbf{c}_{n,r}) \tag{6.5}$$

Reliability II, defined as $1-P_{\mathrm{F2}}$, suggests how effectively the deterministic product line design works in the simulated market when uncertainty is considered in discrete choice methods.

6.3.2. Robustness of a product design in market

In engineering terminology, robustness is defined as "*the ability to tolerate the effect of uncertainty or variation in design parameters without eliminating the source of the uncertainty or variation* (Phadke 1995; Kalsi, Hacker, and Lewis 2001)." Robust design is a method to improve the design quality by minimizing the effect of uncertainty of the output performance function (Lee et al. 2008). It can be categorized into two types based on the source of variation (Chen et al. 1996):

Type I: "*minimizing variations in performance caused by variations in uncontrollable parameters* (Chen et al. 1996)" as described in Figure 6.4.

Type II: *"minimizing variations in performance caused by variations in design variables* (Chen et al. 1996)*."*

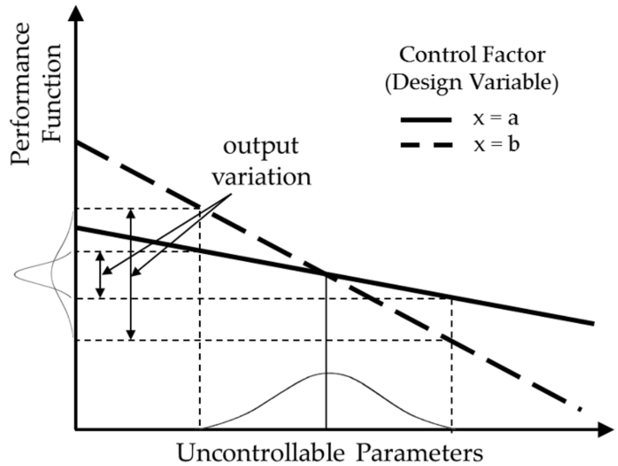

Figure 6.4. Robust design Type I **(Chen et al. 1996)**

Robustness in market-based product design can be defined as the ability to tolerate the effect of uncertainty in discrete choice methods. In product line optimization problems, design variables are commonly defined as the factors that embody product features. Preference coefficients are used as parameter values to define a market model and are not controlled in the optimization problem. Thus, robust design for market systems belongs to Type-I (if the design variables are not subject to uncertainty) because demand uncertainty is uncontrollable. A robust design problem for market-based product design can therefore be defined by minimizing objective function variability under demand uncertainty. In this study, First Choice Share predictions of a product line design using RFC replicates is chosen as the objective function, and variability is measured using the standard deviation (SD) of FCS values.

6.3.3. Product line design under uncertainty in discrete choice methods

A product line search can be developed as a triple-objective optimization problem that contains the reliability and robustness definitions discussed in Chapter 6.3.1 and 6.3.2. Thus, the problem aims at maximizing an average choice share (Reliability I) and its robustness while minimizing the probability of Failure II under uncertainty in demand model. The formulation of the product line search problem is expressed as

$$\boxed{\begin{array}{l}\text{minimize } P_{\text{F1}}, \sigma_{\text{FCS}}, P_{\text{F2}} \\ \text{with respect to Design variable = Product cofigurations} \\ \qquad\qquad\qquad\qquad\qquad\qquad\text{of own offerings in a product line} \\ \text{subject to lower and upper bounds of each attribute}\end{array}} \tag{6.6}$$

Eq. (6.6) is a simplified representation of product line search problems, and many variants could be further developed. A company seeking an opportunity to adopt a market-based product design strategy would need to reflect limitations and decisions concerned with manufacturing, marketing, or engineering design. Design problems should be able to control any of these limitations and the decisions associated with product feature configuration. The limitations and decisions associated with these other domains could be formulated as design variable constraints.

The triple-objective product line search problem returns Pareto optimal set and dominated solutions consisting of many different product configurations. After these results are returned, a process is needed to help the decision maker choose the best design from this set.

A decision procedure proposed in this study consists of two steps. First, product design selections are narrowed down to a small set of design alternatives. Infeasible designs are classified as those solutions with a lower predicted market share than an established baseline target market share (e.g., $\text{FCS}^{tar} = 55\%$). Also, a target probability of failure (e.g., $P_{\text{F2}}^{tar} = 30\%$) can be set. Then, designs that have lower reliability levels than the target reliability would be excluded from consideration. Next, a formal multi-attribute decision method is adopted to support a decision maker to make the best design decision under multi-attribute tradeoff. The hypothetical equivalents and inequivalents method (HEIM) (See, Gurnani, and Lewis 2004) is applied to consider a decision maker's preferences in a design process. Notice the proposed decision support procedure is an example to show how to utilize the optimization result in product design process. While many other decision procedures would be further developed, the discussion of decision support procedure is beyond the scope of this work.

6.4. Case Study

Task procedures of the numerical study are described in Figure 6.5. Generating synthetic choice data is presented in Chapter 6.4.1. The HB-ML model is fit using the synthetic discrete choice data and RFC replicates are drawn in Chapter 6.4.2. Deterministic design and the analysis of reliability and robustness are conducted in Chapter 6.4.3. Finally, Chapter 6.4.4 discusses a multi-objective product line search problem considering reliability and robustness under uncertainty in discrete choice methods and corresponding multi-attribute decision making.

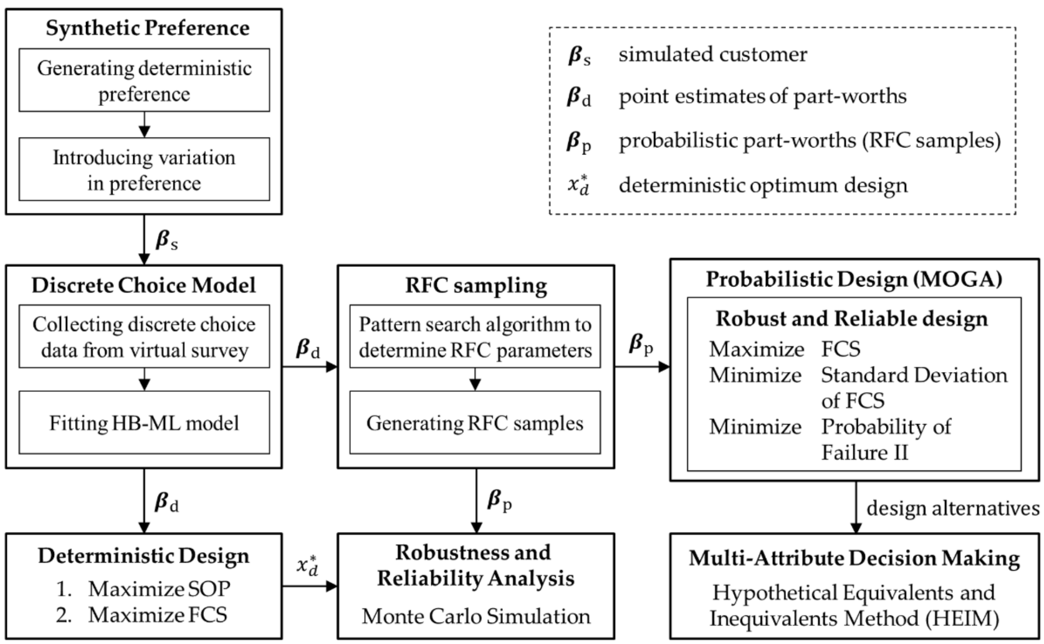

Figure 6.5. Flowchart of the presented study

6.4.1. Generating synthetic choice data

A simulated discrete choice data is used in this study because real choice data is usually too big to deal with using RFC replicates as an academic research purpose. To generate synthetic survey data, a tablet PC selection scenario is introduced. Attributes and levels used in this study are described in Table 6.2. The capital letter A with a number stands for an attribute. Survey questions are generated using Sawtooth SSI Web (Sawtooth Software inc.

2011). Respondents are asked to evaluate 15 buying scenarios including five hold-out questions. Each scenario contains three product alternatives and a fourth no-buy option.

Table 6.2. Tablet PC attributes and levels

Level	A1	A2	A3	A4	Price
	Connectivity	Processor	Screen Size	Storage	
1	Wi-Fi	Entry	7 inch	16 GB	\$ 200
2	Cellular	Mid	8 inch	32 GB	\$ 400
3		High-End	10 inch	64 GB	\$ 600
4			12 inch	128 GB	\$ 800
5				256 GB	

Synthetic preference data is generated as follow steps.

Step 1. Generate deterministic synthetic preference

- Generate μ_β (mean) for each attribute: $U(-1,1)$
- Generate σ_β (standard deviation) for each attribute: $U(0.5,1)$
- Generate β_n (individual's preference) for each attribute: $N(\mu_\beta, \sigma_\beta)$
- Generate β_{no-buy} (no-buy threshold) for each respondent: $U(0.6, 0.8)$

Step 2. Variation in the deterministic preference

For each choice task and each respondent

- Generate taste variation for each attribute: $U(-0.5, 0.5)$
- Generate variation in the no-buy threshold: $U(-0.1, 0.1)$

6.4.2. Quantifying variation in demand model

The individual-level part-worths of the HB-ML model are obtained using the Sawtooth Software CBC/HB module (Sawtooth Software inc. 2014) and 10,000 burn-in iterations are performed. 10,000 draws after burn-in are saved (skipping every hundredth draw) per respondent, for a total of 2,000,000 (200 respondents×10,000 draws) sets of 19 part-worths.

Determining how many draws of HB, and how many sampling replications for RFC, to use is significant as it affects the accuracy and efficiency of simulation. An existing study (Orme and Baker 2000), whose survey size is similar to the synthetic data generated in this study, used at least 100 draws per respondent based on a reliability test between replicates. The study compared MAE (Mean Absolute Error) values to its 95th percentile value. 100 replicates per respondent may be acceptable in terms of the internal reliability test, however, 100 replicates would not be enough to guarantee accuracy of the probability of failure in Monte Carlo Simulation (MCS). In MCS using 100 samples the smallest probability interval of failure is 1%. For a reliable estimate, at least ten times the minimum is usually recommended (Haldar and Mahadevan 2000). In this work, 10,000 replicates per respondent are used to predict the probability of failure at a level of 0.1%.

Another challenge is determining the RFC parameters that represent variations in the aggregate part-worths. Since there are 14 product attribute levels, there are 14 normal distributions for E_a and a Gumbel distribution for E_p. Their mean values should be exactly same as the point estimates. Therefore, 14 standard deviation values must be determined for the normal distribution. In Table 6.3, the standard deviation of each preference coefficient is represented as $\sigma_{\text{attribute,level}}$. For the Gumbel distribution, the mean is given by $\mathrm{E}(\mathrm{X}) = \upsilon + \gamma\alpha^{-1}$, where υ is a location parameter, α is a scale parameter, and γ is Euler-Mascheroni constant that is approximately equal to 0.5772 (Feller 2008). By setting $\mathrm{E}(\mathrm{X}) = 0$ to maintain the point estimates, the two parameters can be divided into independent and dependent variables. The scale parameter is searched as $\sigma_{\mathrm{P}} = \alpha^{-1}$ to determine E_p in Eq. (6.1).

Table 6.3. Data comparison

Data	N	RFC parameters															MAE	Enhancement
		$\sigma_{1,1}$	$\sigma_{1,2}$	$\sigma_{2,1}$	$\sigma_{2,2}$	$\sigma_{2,3}$	$\sigma_{3,1}$	$\sigma_{3,2}$	$\sigma_{3,3}$	$\sigma_{3,4}$	$\sigma_{4,1}$	$\sigma_{4,2}$	$\sigma_{4,3}$	$\sigma_{4,4}$	$\sigma_{4,5}$	σ_{P}		
Point estimates	1																9.60	datum
HB draws	10k																6.58	31.5%
RFC 1k	1k	0.12	0	3.04	0	0	0	0.54	0.36	0	0	0.26	0	3.06	3.32	0.16	3.63	62.2%
A-RFC 10k	10k																3.68	61.7%
RFC 10k	10k	0.13	0	3.12	0	0	0	0.55	0.36	0	0	0.26	0	3.15	3.38	0.15	3.66	61.9%

A pattern search algorithm is used to search for the RFC parameters that minimize the mean absolute error (MAE) in predicting choice shares using holdout questions (Huber, Orme, and Miller 1999). For example, suppose there are three products and they have 20, 30, and 50 choice shares, respectively. If we obtains the predicted first choice shares as 10, 20, and 70, respectively, the MAE value is calculated as $\left(|20-10|+|30-20|+|50-70|\right)/3=13.3$. A smaller MAE for the holdout questions implies better predictive ability. Table 3 shows MAE values of each data. As suggested in Orme and Baker's study (Orme and Baker 2000), using HB draws does not result in a smaller MAE value than RFC data in simulation, despite the simplified assumptions about the attribute and product variation distributions. According to (Orme and Baker 2000), *a reverse number of levels effect, and an excluded level effect*, can explain why RFC is more effective than using HB draws when considering the predictive power of market simulation.

The mechanism of the search process is simple but computationally expensive due to the size of the replicates. When taking 10,000 replicates per respondent, one pattern search iteration takes approximately 2.4 hours using a desktop running an Intel i7-2600 Quad-Core Processor 3.40 GHz with 8GB RAM. The total run time is approximately 19.4 days with 198 iterations to obtain the optimum parameter values listed in Table 6.3. In market-based design problems dealing with much larger data sets, determining RFC parameters using the presented method may become computationally intractable.

To reduce computational cost, four techniques are explored:

(1) Decrease the number of RFC samples: 1,000 replicates per respondent are generated instead of 10,000 to search for RFC parameters. Then, 10,000 random samples are generated using the parameters obtained using 1,000 replicates. This approach is called Augmented-RFC (A-RFC) in this study.

(2) Reduce the design space by dummy encoding the dependent variables: Among the 14 random variables, four are dependent variables because levels in an attribute are correlated. So, the four parameters can be determined by the other parameters searched in the optimization problem because $\sigma_{1,2}$, $\sigma_{2,3}$, $\sigma_{3,4}$, and $\sigma_{4,1}$ are set using dummy encoding to decrease the size of the search problem.

(3) Reduce design space by manipulating the design variables and stopping criteria: The lower and upper bounds are set as 0 and 5, respectively. The tolerance of design variables in the form of stopping criteria is set as 0.01, which results in discretized design space at 0.01 interval between 0 and 5.

(4) Use parallel computing.

By applying the techniques described above, computational run time was reduced from 19.4 days to 7 hours (119 iterations and 3.5 minutes per iteration). Also, the feasibility of the augmented RFC samples in product search problems is investigated. As shown in Table 6.4, the RFC 1k data results in exactly the same product line design as the RFC 10k data. This result suggests that reducing sample size in a parameter search problem would be acceptable in terms of product search objectives. The augmented RFC data also results in exactly the same product line solution with the RFC 1k data. This implies the A-RFC 10k data can maintain the original variation information of the RFC 1k in the design problem. However, to have confidence in the data augmentation technique for RFC sampling, increased validation is necessary for future work.

Table 6.4. Product line of each RFC data

Data	Objective	FCS (%)	Product 1				Product 2				Product 3			
			A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4
RFC 1k	maximize FCS	65.23	2	2	1	4	2	2	4	3	2	1	3	4
A-RFC 10k		65.38												
RFC 10k		65.58												

6.4.3. Single-objective product line search

This section investigates the effect of uncertainty by quantifying the reliability and robustness of a product line solution obtained using part-worth point estimates. Then, A-RFC 10k data is used to evaluate the product line design to introduce uncertainty. Reliability and robustness of the deterministic design are quantified using the definitions in Chapter 6.3.1 and 6.3.2.

Table 6.5. Pricing structure

Level	A1	A2	A3	A4
1	0	0	0	0
2	40	80	40	40
3		160	80	80
4			160	120
5				160

Table 6.6. Attribute levels of competitor products in the market

Competitor	A1	A2	A3	A4	Price
Product 1	2	1	1	5	$ 400
Product 2	1	3	2	3	$ 480
Product 3	2	2	4	4	$ 600

The pricing structure for each product attribute is shown in Table 6.5, and a base price of $200 is added. To calculate the part-worths for the price attribute, a piecewise linear interpolation is assumed. The competitor products in a simulated market are defined as shown in Table 6.6.

Latent class analysis (Sullivan, Ferguson, and Donndelinger 2011; Sawtooth Software inc. 2007), implies that containing three *Own Offerings* in a product line is the best scenario. The solution to the deterministic product line problem is shown in Table 6.7. Two different objectives are set: maximizing FCS and maximizing SOP. One *Own Product* is identical in the two solutions, but two *Own Products* are different. Notice that *Own Product* 3 of the SOP solution is identical to the third competitor product, because the share of preference simulation cannot resolve the IIA issue.

Table 6.7. Deterministic optimal product line design

Data	Objective	Product 1				Product 2				Product 3				Function value
		A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4	
Point-estimate	maximize FCS	2	2	4	3	2	2	1	5	2	2	4	2	FCS = 78.50 %
	maximize SOP	2	2	4	3	2	2	1	2	2	2	4	4	SOP = 77.79 %

Table 6.8 shows the reliability and robustness of the design solutions obtained in Table 6.4 and Table 6.7. Each design solution is evaluated using A-RFC 10k replicates. Solution A indicates the optimum design of FCS problem using A-RFC 10k data and is shown in Table 6.3. Solutions B and C represent the optimum designs of FCS and SOP problems, respectively, which are shown in Table 6.7.

Table 6.8. Reliability and Robustness analysis

Solution	Data	Objective	FCS (%)				P_{F2} (%)
			Avg	Min	Max	SD	
A	A-RFC 10k	maximize FCS	65.38	54.5	76.0	2.69	35.42
B	Point estimates	maximize FCS	63.46	51.5	75.0	2.98	38.35
C		maximize SOP	57.25	47.0	70.0	3.11	32.42

Solution C has a smaller FCS than Solution B. This is because the *Own Product* 1 of solution C and competitor product 3 divide FCS equally due to the presence of a duplicate offering in the choice set. The FCS value of Solution B decreases from 78.50% for the point-estimates to 63.46% for the RFC replicates. While first choice share predictions are usually too extreme and not tunable, RFC simulations can resolve the extreme choice share issue (Orme and Baker 2000). For this reason, first choice share decreases when evaluated using RFC replicates.

If a decision-maker is only concerned about average FCS values, the best solution is A because it has the largest choice share. However, when variability in the demand model is considered, Solution A is no longer the clear winner. Figure 6.6 describes distributed FCS values of the three design alternatives under variation in the demand model. It is obvious that there is a chance that Solutions B or C can capture more market share than Solution A as described in box plots. For the error bars, the central mark indicates the median value, the box indicates the 25th and 75th percentiles, the whiskers extend to approximately ±2.7σ with normal distribution assumption. Outliers outside ±2.7σ are individually plotted.

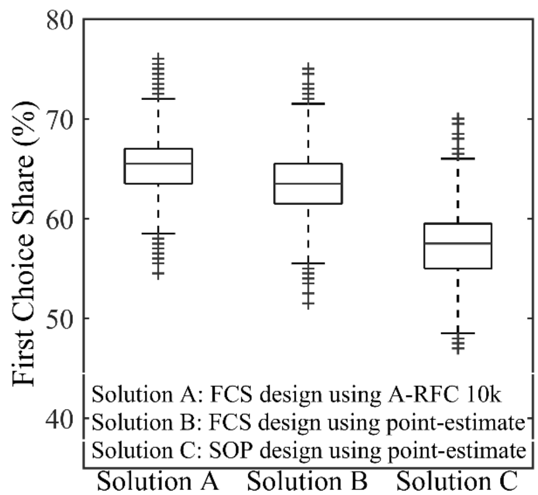

Figure 6.6. Variability in performance function

To support design decisions under variation in the demand model, this study adopts the reliability and robustness definitions explained in Chapter 6.3.1 and 6.3.2. Results of reliability and robustness analysis for the three alternatives are listed in Table 6.8. All numbers in Table 6.8 are evaluated using A-RFC 10k data to quantify reliability and robustness under the uncertainty of A-RFC 10k data.

The standard deviation of FCS values are used as a measure of design robustness, and the probability of Failure-II represents the reliability of a design under demand variation. For these results, there is no clear preferred solution alternative and the solutions can be ordered in each metric as:

- Reliability-I (FCS): A > B > C
- Reliability-II: C > A > B
- Robustness (SD of FCS): A > B > C

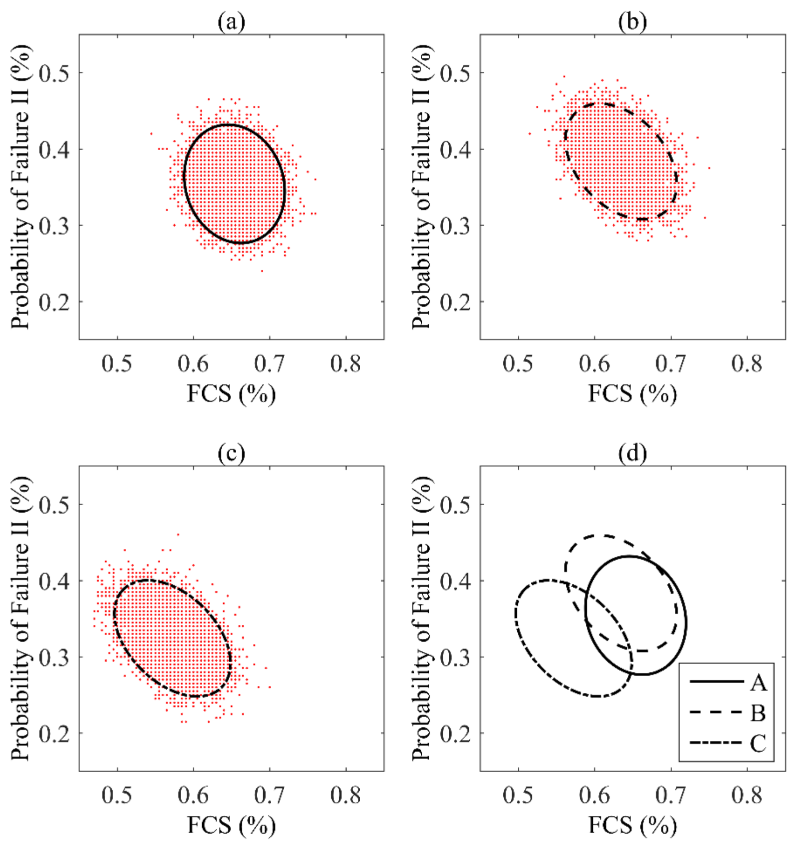

Figure 6.7. Distributed FCS and probability of Failure II:
(a) Solution A (b) Solution B (c) Solution C (d) 95% confidence ellipse

Figure 6.7 shows conflicts of solution options in terms of reliability and robustness by describing FCS values of 10,000 sets of RFC replicates and the corresponding reliability of

Failure-II. Each circle indicates a 95% confidence ellipse of all samples that can be drawn from the underlying normal distribution. In Figure 6.7-(d), it is difficult to determine a clear preferred solution alternative as there is considerable overlap between Solutions A and B. Also, there is a chance that Solution C may capture more share and have less simulation failure than the other designs. For this reason, considering multiple objectives was proposed in Chapter 6.3.3 and the results of this analysis are presented in the next section.

6.4.4. Multi-objective product line search considering reliability and robustness

To formulate a multi-objective product line search problem under uncertainty, the optimization algorithm is changed to the elitist non-dominated sorting GA (NSGA-II) (Deb 2001; Deb et al. 2002) to take an advantage of its sorting algorithm. The size of the population at each generation is 120 because it is ten times the number of design variables. To enhance solution quality, a targeted initial population (Foster et al. 2014) is generated. Two elite individuals are guaranteed to survive each generation. Binary tournament based on crowding distance is used as a selection operator, while crossover occurs using arithmetic means. The mutation operator is based on the Gaussian distribution. For integer encoding, each component is rounded to the nearest integer. Finally, convergence is met when 50 generations are performed with no improvement in the best fitness function value. When using A-RFC 10k data, one generation took approximately 50 minutes using a desktop running an Intel i7-2600 Quad-Core Processor 3.40 GHz with 8GB RAM. The total run time is approximately 10.3 days with 300 generations.

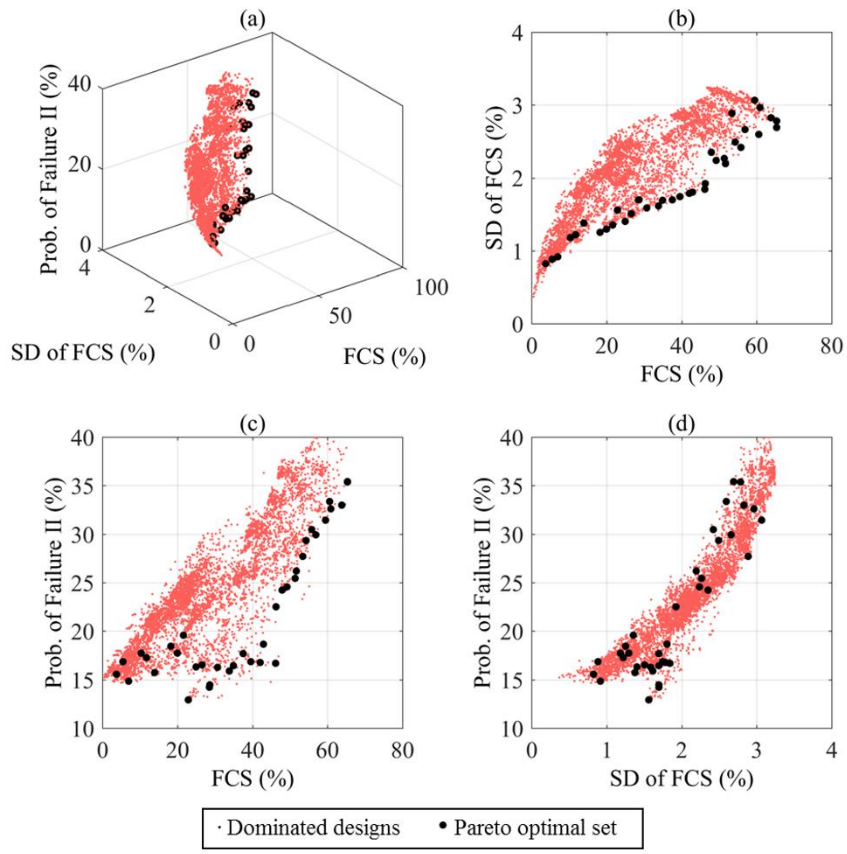

Figure 6.8. Pareto design alternatives of product line search problem:
(a) 3-D plot (b) FCS vs SD of FCS (c) FCS vs Prob. of Failure II
(d) SD of FCS vs Prob. of Failure II

The triple-objective product line search problem returns a Pareto optimal set and dominated designs consisting of many different product configurations. Figure 6.8 displays the Pareto optimal set and the dominated designs evaluated in the evolutionary algorithm. There are 37 unique solutions (black dots) in the Pareto optimal set and 6,020 unique designs (red dots) in the dominated set. The solutions obtained using the multi-objective optimization problem represent candidate product line designs. Thus, selecting one design solution from all candidates in Fig. 6.8 is important to consider trade-offs between reliability and robustness in decision-making process.

To narrow down the number of possible design alternatives, solutions capturing a predicted choice share below ($FCS^{tar} = 55\%$) are eliminated. Also, solutions that do not meet a target reliability level ($P_{\mathrm{F2}}^{tar} = 34\%$) are excluded from consideration sets. After applying these filters, 71 candidates remain, including six non-dominated solutions as shown in Figure 6.9.

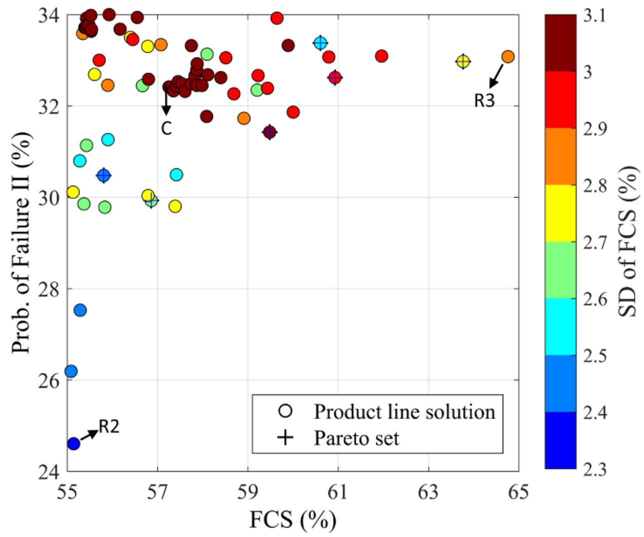

Figure 6.9. Refined design solutions

Selecting one solution among the set of alternatives in the feasible range is required but difficult because there are tradeoffs between FCS, SD of FCS, and the probability of Failure-II. For example, selecting one solution among solutions C, R2 (reference solution 2 in HEIM presented in Appendix), and R3 (reference solution 3 in HEIM presented in Appendix C) in Figure 6.9 is difficult because of tradeoffs between function values as shown in Table 6.9. The solution C is the optimum design maximizing SOP obtained using point-estimates. The solution R2 has the largest FCS value among the candidate solutions in Figure 6.9. The solution R3 is the best option when considering robustness and Failure-II. To finalize design decision considering reliability and robustness in a multi-objective framework, a multi-attribute decision method is necessary to ensure that a rational decision is made.

Table 6.9. Multi-objective solution comparison

Solution	FCS (%)	σ_{FCS} (%)	P_{F2} (%)
C	57.25	3.11	32.42
R2	64.76	2.83	33.08
R3	55.14	2.32	24.60

An optimization problem is constructed to solve for the attribute weights. The complete optimization problem is defined as

$$\begin{array}{lll} \text{minimize} & f = \left[1-\left(w_1+w_2+w_3\right)\right]^2 & \\ \text{subject to} & & \\ & \text{H3} > \text{H1:} & g_1 = -w_1 + w_2 + 0.5w_3 + \delta \le 0 \\ & \text{H1} > \text{H2:} & g_2 = 0.5w_1 - 0.5w_2 - w_3 + \delta \le 0 \\ & \text{H6} > \text{H5:} & g_3 = -0.5w_1 - w_2 + w_3 + \delta \le 0 \\ & \text{H5} > \text{H4:} & g_4 = -0.5w_1 + 0.5w_2 - 0.5w_3 + \delta \le 0 \\ & \text{H9} > \text{H8:} & g_5 = -0.5w_1 + 0.5w_2 - 0.5w_3 + \delta \le 0 \\ & \text{H8} > \text{H7:} & g_6 = -0.5w_1 - w_2 - 0.5w_3 + \delta \le 0 \\ & & 0 \le w_1, w_2, w_3 \le 1 \end{array} \tag{6.7}$$

where δ is 10e-4, to ensure the inequality of the two values. w_1, w_2, and w_3 are preference weights of FCS, SD of FCS, and the probability of Failure-II, respectively. The six inequality constraints are formulated from a decision maker's stated preference that H3>H1>H2, H6>H5>H4, and H9>H8>H7 for the hypothetical alternatives listed in Table C2 in the appendix.

Table 6.10. A single robust product line solution

A1	A2	A3	A4	Price	FCS (%)	σ_{FCS} (%)	P_{F2} (%)
2	2	1	2	$ 360			
2	2	3	4	$ 520	64.76	2.83	33.08
2	2	4	3	$ 560			

The solution for the attribute weights is obtained using the generalized reduced gradient (GRG) method. Using GRG and given a single starting point, (0, 0, 0), one feasible solution set of weights is obtained as (0.46, 0.39, 0.14). With the attribute weights, all 71 candidate solutions can be assessed and a solution with the highest score can be selected. The preferred product line design is listed in Table 6.10. Notice that the winning product line is not a solution in the Pareto optimal set because a decision maker's preference has been reflected in HEIM.

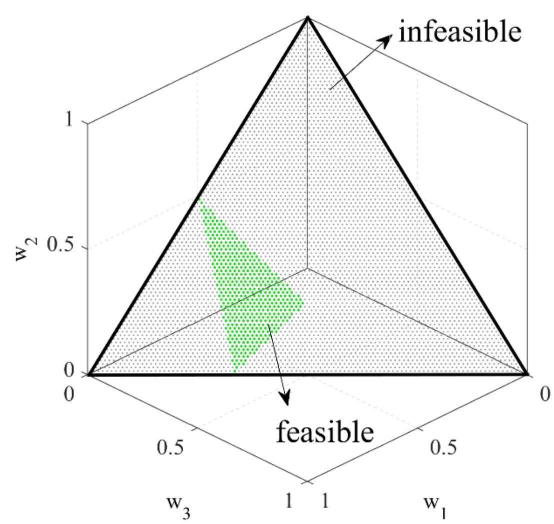

Figure 6.10. 3-D space visualization with $w_1 + w_2 + w_3 = 1$ plane

However, since the sum of the weights is set as an objective function, there are many possible sets of weights that minimize the objective function to zero and satisfy all constraints. Using another starting point, (1, 1, 1), a different set of weights is found (0.67, 0, 0.33). Hence, it is important to investigate if different weight values yield different winning alternatives. The robustness issue with respect to changes in the weight values is explored using visualization. A large number of attribute weights whose sum equals one are generated and plotted in Figure 6.10. The weight sets on the $w_1 + w_2 + w_3 = 1$ plane can be divided into feasible and infeasible designs according to the preference constraints. The green area indicates the feasible set of weights that satisfies all constraints. When evaluating the 71 candidates for all weight sets in the feasible area, only one winning alternative is observed, which is listed in Table 6.10. Thus, the product line solution is robust with respect to changes in the weight values. If several winning alternatives exist, more constraints are needed by creating new hypothetical alternatives (See, Gurnani, and Lewis 2004).

6.5. Summary

The main contribution of the study lies in specifying the reliability and robustness of a product design under uncertainty in discrete choice methods, and integrating these measures into a multi-objective optimization framework. In the proposed approach, an RFC model is used to introduce variation in the point-estimates of an HB-ML model. By simulating many choices, the RFC simulation can resolve issues associated with IIA and extreme choice share. An efficient search procedure is then proposed to quantify the degree of variation. A multi-objective optimization problem formulation is introduced using reliability and robustness to search for a set of design alternatives. Then, a multi-attribute decision method is conducted to support making design decisions considering tradeoff in the alternatives.

This work demonstrates that a product line decision can be enhanced to exhibit greater realism by considering uncertainty when using discrete choice models. The definition of Reliability-I can improve the choice probability of a firm's own product line design under uncertainty. Also, the definition of Reliability-II can enhance the effectiveness of the product line design in the market under uncertainty. A product line exhibiting robustness would have the ability to tolerate perturbations. In conclusion, the solution quality associated with the non-dominated set is enhanced by considering uncertainty in discrete choice methods using RFC simulation and multi-objective optimization techniques.

The proposed work has some limitations and future work is recommended. The degree of variation in RFC replicates depends on holdout question design because the parameter search problem in Chapter 6.4.2 aims to minimize the mean absolute error in prediction of holdout choice shares. Thus, methodologies to design holdout questions are necessary to enhance the validity of the uncertainty quantification method.

Another potential shortcoming is that the proposed optimization problem would be computationally intensive because of using many RFC replicates, though an efficient parameter search procedure and a targeted initial population for the multi-objective problem are applied. Thus, enhancing computational efficiency would be further developed for practical use. If demand uncertainty and market simulation can be expressed using correlated continuous distributions at the individual-level preference, analytical methods could be further explored to reduce computational burden. As computationally less expensive methods, analytical reliability analysis methods can be explored such as FORM and SORM when using a mixed logit model if: 1) demand uncertainty could be expressed using continuous distributions with correlations in preference structure, 2) closed-form choice probabilities are available, and 3) market simulators could handle continuous distributions.

Furthermore, developing tailored GA algorithms to draw designs close to Pareto set as many as possible is recommended to enhance the quality of product design search. Drawing quality designs is significant because a winning design may not be in the Pareto optimal set when considering a decision maker's preference in a multi-attribute decision-making method. Supervised genetic algorithms need to be explored to produce elaborated offspring.

Chapter 7. Conclusions

7.1. Research Summary

Market-based product design is a new engineering design paradigm that provides quantitative solutions to support decision-making in product design. It is an interdisciplinary approach that incorporates a predictive market model into a design optimization problem. This dissertation is focused on two topics associated with this integration. The first topic is to investigate how the form of the selected discrete choice model influences the optimal product line configuration. This topic includes modeling noncompensatory choices using compensatory models and the implications of heterogeneous preference presentation in product design optimization. The second topic is exploring the ramifications of considering preference heterogeneity and uncertainty in the problem formulation. Toward this goal, a customer's sacrifice gap is quantitatively defined and a product configuration search problem formulation for mass customization environments is proposed. Also, reliability and robustness measures of a product design under uncertainty when using discrete choice models are introduced and a multi-objective problem formulation is developed that considers both reliability and robustness.

The main contributions of the dissertation are 1) establishing foundational knowledge about the integration and use of discrete choice models in market-based product design and 2) defining optimization problem formulations that consider product development and manufacturing strategies while accounting for model uncertainties. Each research question is revisited and discussed in the next chapter.

7.2. Discussion of Research Questions

7.2.1. Research Question 1

Market-based product design research has explored the use of discrete choice analysis as one of the tools for estimating customer preferences. Many discrete choice model forms assume that consumers make compensatory choices based on an additive utility rule. However, recent market research papers have demonstrated that noncompensatory choice models often improve model realism and accuracy in predicting consumers' choices. Advances in Bayesian inference, machine-learning, and greedoid languages make it possible to quantify consider-then-choose scenarios for a variety of heuristics. In this circumstance, the first research question asked:

When respondents make noncompensatory choices with conjunctive screening rules, how does the optimal product line solution differ when preferences are modeled using compensatory and noncompensatory models?

Research Question 1 is motivated by the challenges of existing Bayesian-based noncompensatory models in design optimization: potential errors in screening rule assumptions, probabilistic representations of noncompensatory choices, and discontinuous choice probability function. To answer the research question, this research explores the suitability of using compensatory models to mimic the consider-then-choose process for a product design search. Two hypotheses are made and verified for compensatory modeling of noncompensatory choices: 1) distinct segments would be captured in latent class analysis and 2) large absolute part-worth values would be the result in individual-level estimates of the HB-ML models. The results of the product design search show that although there are several insignificant differences between the two models in the market simulation, the compensatory model has some significant advantages such as the small design error and its relatively inexpensive computational burden. The results of the real choice data analysis confirm the suitability of the compensatory model in product design search as its optimum design has an acceptable level in the likelihood gap in the hypothetical noncompensatory choice simulation.

7.2.2. Research Question 2

Customer preferences are naturally heterogeneous across individuals. Understanding the heterogeneity plays a central role to effectively explore market opportunities. Representing such variation in taste is important because capturing different segments of the market leads to differentiated product design. The second research question asked:

How does the optimal product line solution change when preferences are estimated using a latent class multinomial logit and a hierarchical Bayes mixed logit?

This study compares continuous and discrete models of heterogeneity and provides empirical data describing the effectiveness of model choice. Model fitness and predictive ability are measured by both first-choice analysis and preference share analysis. The experimental results are supported by analyzing heterogeneity structures. Implications of model form choice for product line design are also investigated by comparing optimum designs obtained using the part-worths of each model. According to the analysis results, the continuous model seems to be preferred to the discrete model when representing heterogeneous preference because the continuous model describes heterogeneity using a continuous distribution but the discrete model derives individual's preference from discrete subgroups.

7.2.3. Research Question 3

Mass customization is a product development approach that provides custom-tailored goods for minimizing customers' sacrifice gap. In market research, although conceptual definitions of the sacrifice gap exist, a process for obtaining a quantitative measure of the term is almost unexplored. The third research question asked:

How can sacrifice gaps be quantified to formulate an optimization problem for mass customization environments?

This study is focused on a quantitative method to assess customer needs and preference for mass customization. Customer sacrifice is defined using the discrete choice methods that are popularly used in consumer preference research. The individual-level product search problem reveals that consumers are divided into two groups according to the importance of price. This result implies that there exists a group who are willing to pay a premium for a customized product. In a population-level problem, it is observed that flexibility in configuring product feature levels can be provided in mass customization environments. Therefore, an optimization problem that is able to turn on or off product features is proposed and its effectiveness in mass customization environments is verified in a numerical study.

7.2.4. Research Question 4

Market simulations commonly use point estimates of individual's part-worths, ignoring sources of uncertainty associated with discrete choice methods. However, consumer preferences are not only inherently stochastic, but also are statistical estimates that exhibit errors in model form and estimation procedures. For this reason, there has been concern about the reliability and robustness of an optimal design solution under the presence of uncertainty in discrete choice methods. The last research question asked:

How can the reliability and robustness of a product design be specified under uncertainty in discrete choice methods and utilized for supporting design decision-making?

In this study, RFC method is used to deal with uncertainty in discrete choice methods. To reduce computational burden when determining RFC parameters, an efficient search procedure is proposed. Reliability and robustness are specified for an RFC replication and Monte Carlo simulation is conducted to measure them. Based on the reliability and robustness measures, a multi-objective optimization problem is introduced to search for a set of design alternatives. Then, a multi-attribute decision method is processed to support making design decisions considering tradeoff in candidate solutions.

The main contribution of this study is that a product line decision can be enhanced to exhibit greater realism by considering uncertainty when using discrete choice models. The solution quality associated with the non-dominated set is enhanced by considering uncertainty in discrete choice methods using RFC simulation and multi-objective optimization techniques.

7.3. Future Research Topics

Several future research topics were introduced in the previous chapters by considering limitations of the presented studies in this dissertation. Two major areas of future research are proposed in this section.

First, computational efficiency of GA-based product search problems has to be enhanced. When solving the mass customization problem in Chapter 5 and the multi-objective optimization problem in Chapter 6, the GA usually takes several days to return solutions. This is because there were a large number of design variables in the mass customization problem and millions of RFC samples were used to run Monte-Carlo simulation in the multi-objective problem. However, problems in industry may be significantly larger. To reduce computational burden and realize effective solutions to large-scale product search problems in industry, developing tailored GA algorithms or analytical methods could be explored.

Second, the framework of market-based product design can be further developed by integrating engineering attributes such as product sustainability into the constraints. To make this possible, quantitative definitions of those attributes have to be first explored. In addition, strategies in marketing can be also formulated and integrated into the framework. Then, new design problems could provide more effective solutions to help product development decision-making.

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APPENDICES

APPENDIX A. MP3 player survey data and modeling result

Table A1. MP3 attributes and levels

Level	Attributes							
	A1	A2	A3	A4	A5	A6	A7	Price
	Photo/Video/ Camera	Web/App/ Ped	Input	Screen size	Storage size	Backgro-und color	Backgro-und overlay	
1	None	None	Dial	1.5 in diag	2 GB	Black	No pattern / graphic overlay	$699
2	Photo only	Web only	Touch-pad	2.5 in diag	16 GB	White	Custom pattern overlay	$599
3	Video only	App only	Touch-screen	3.5 in diag	32 GB	Silver	Custom graphic overlay	$499
4	Photo and Video only	Ped only	Buttons	4.5 in diag	64 GB	Red	Custom pattern and graphic overlay	$399
5	Photo and lo-res camera	Web and App only		5.5 in diag	160 GB	Orange		$299
6	Photo and hi-res camera	App and Ped only		6.5 in diag	240 GB	Green		$199
7	Photo, video and lo-res camera	Web and Ped only			500 GB	Blue		$99
8	Photo, video and hi-res camera	Web, App, and Ped			750 GB	Custom		$49

Table A2. Threshold estimates for the posterior means of the conjunctive model (MP3 data)

Attribute	Level (recorded value)	Possible cutoff	Probability of each cutoff	Attribute	Level (recorded value)	Possible cutoff	Probability of each cutoff
A1	1 (0)	-0.5	65.2%	A5	1 (0)	-0.5	44.8%
	2 (1)	0.5	12.7%		2 (1)	0.5	27.5%
	3 (2)	1.5	4.4%		3 (2)	1.5	8.1%
	4 (3)	2.5	3.7%		4 (3)	2.5	6.0%
	5 (4)	3.5	2.7%		5 (4)	3.5	3.9%
	6 (5)	4.5	3.6%		6 (5)	4.5	2.6%
	7 (6)	5.5	2.8%		7 (6)	5.5	2.4%
	8 (7)	6.5	2.4%		8 (7)	6.5	2.4%
		7.5	2.4%			7.5	2.4%
A2	1 (0)	-0.5	56.2%	A6	1 (0)	-0.5	73.3%
	2 (1)	0.5	20.7%		2 (1)	0.5	7.0%
	3 (2)	1.5	5.6%		3 (2)	1.5	3.5%
	4 (3)	2.5	4.1%		4 (3)	2.5	3.6%
	5 (4)	3.5	3.3%		5 (4)	3.5	3.0%
	6 (5)	4.5	2.6%		6 (5)	4.5	2.4%
	7 (6)	5.5	2.4%		7 (6)	5.5	2.4%
	8 (7)	6.5	2.7%		8 (7)	6.5	2.4%
		7.5	2.4%			7.5	2.5%
A3	1 (0)	-0.5	83.8%	A7	1 (0)	-0.5	87.6%
	2 (1)	0.5	5.5%		2 (1)	0.5	4.0%
	3 (2)	1.5	5.4%		3 (2)	1.5	3.1%
	4 (3)	2.5	2.6%		4 (3)	2.5	2.7%
		3.5	2.7%			3.5	2.7%
A4	1 (0)	-0.5	64.0%	Price	$699 (0)	-0.5	24.3%
	2 (1)	0.5	15.6%		$599 (1)	0.5	13.7%
	3 (2)	1.5	9.5%		$499 (2)	1.5	12.4%
	4 (3)	2.5	3.4%		$399 (3)	2.5	14.6%
	5 (4)	3.5	2.4%		$299 (4)	3.5	10.8%
	6 (5)	4.5	2.6%		$199 (5)	4.5	12.4%
		5.5	2.5%		$99 (6)	5.5	6.2%
					$49 (7)	6.5	3.1%
						7.5	2.4%

Table A3. Part-worth estimates of compensatory and noncompensatory models (MP3 data)

Attribute	Level	Posterior mean		Attribute	Level	Posterior mean	
		Compensatory	Noncompensatory			Compensatory	Noncompensatory
A1	2	1.20	-1.13	A5	2	5.21	1.00
	3	1.50	-0.53		3	5.10	0.31
	4	2.31	-0.29		4	6.85	1.57
	5	3.04	0.83		5	7.76	1.82
	6	3.19	0.58		6	6.45	0.99
	7	2.42	0.29		7	7.27	2.37
	8	4.88	2.49		8	6.94	1.46
A2	2	4.14	1.40	A6	2	-0.72	-0.75
	3	2.97	-0.35		3	-0.05	0.39
	4	0.63	-2.20		4	-0.52	-0.54
	5	6.24	3.11		5	-1.09	-1.17
	6	3.70	0.31		6	-1.07	-0.60
	7	5.25	1.88		7	-1.05	-1.47
	8	6.73	3.53		8	-1.03	-0.95
A3	2	-1.01	-0.37	A7	2	0.32	0.13
	3	2.57	2.43		3	1.00	0.52
	4	-0.34	-0.47		4	1.07	0.71
A4	2	1.25	-0.08	Price	$599	-0.51	-2.87
	3	3.69	0.64		$499	4.52	0.13
	4	3.94	1.36		$399	5.88	-0.87
	5	3.35	1.07		$299	8.92	1.20
	6	4.12	1.43		$199	14.07	4.26
					$99	15.92	5.16
					$49	18.13	6.46

Table A4. Attribute importance of compensatory model (MP3 data)

	A1	A2	A3	A4	A5	A6	A7	Price
Average	10.8	14.7	8.6	9.9	14.4	7.4	4.4	29.8
Maximum	23.0	27.3	26.7	22.6	23.7	18.3	10.9	53.1
Minimum	3.3	3.7	0.9	1.8	5.9	2.2	0.3	8.7

Table A5. Cost structure

Level	Attribute						
	A1	A2	A3	A4	A5	A6	A7
	Photo/ Video/ Camera	Web/ App/ Ped	Input	Screen size	Storage size	Background color	Background overlay
1	0	0	0	0	0	0	0
2	2.5	10	2.5	12.5	22.5	5	2.5
3	5	10	20	22.5	60	5	5
4	7.5	5	10	30	100	5	7.5
5	8.5	20		35	125	5	
6	15	15		40	150	5	
7	16	15			175	5	
8	21	25			200	10	

Table A6. Configuration of competing market products

Attribute	Competitor product 1	Competitor product 2	Competitor product 3
Photo/ Video/ Camera	Photo, video and hi-res camera	Photo, video and hi-res camera	Photo, video and hi-res camera
Web/App/Ped	App only	Web and App only	Web and App only
Input	Dial	Touchscreen	Touchscreen
Screen size	1.5 in diag	4.5 in diag	4.5 in diag
Storage	16 GB	32 GB	160 GB
Background color	Black	Black	Silver
Background overlay	Custom graphic overlay	Custom pattern and graphic overlay	Custom graphic overlay
Price	$ 169	$ 369	$ 504

APPENDIX B. Optimum product line solutions used in Chapter 4.3.6

Table B1. Optimum product line solution (Uniform distribution)

Synthetic preference	Attribute						Price
	Engine	TM	A3	A4	A5	A6	
True	1	1	1	3	2	1	18,250
	1	1	2	1	1	4	18,500
	2	1	1	2	3	2	18,900
HB-ML	1	1	1	1	1	2	18,100
	1	1	2	4	2	1	18,500
	1	2	1	2	3	3	19,500
LC-MNL	1	1	1	1	1	1	18,000
	1	1	1	1	1	2	18,100
	1	1	1	1	1	4	18,300

* Underlines indicate the uncommon product features to the design of the true data

Table B2. Optimum product line solution (Normal distribution)

Synthetic preference	Attribute						Price
	Engine	TM	A3	A4	A5	A6	
True	1	1	2	2	1	4	18,600
	2	1	1	4	1	2	18,800
	3	1	2	4	1	2	19,500
HB-ML	1	1	2	4	1	2	18,500
	1	1	2	2	1	4	18,600
	3	1	1	4	1	2	19,300
LC-MNL	2	1	2	2	1	4	19,100
	2	1	2	4	3	4	19,400
	3	1	2	4	1	2	19,500

* Underlines indicate the uncommon product features to the design of the true data

Table B3. Optimum product line solution (Weibull distribution)

Synthetic preference	Attribute						Price
	Engine	TM	A3	A4	A5	A6	
True	1	1	2	2	1	1	18,300
	1	1	1	1	1	4	18,300
	1	1	1	4	4	1	18,500
HB-ML	1	1	1	1	4	1	18,300
	1	1	2	2	1	1	18,300
	1	1	1	1	2	4	18,400
LC-MNL	1	1	1	1	1	1	18,000
	1	1	1	1	4	1	18,300
	1	1	1	2	4	1	18,400

* Underlines indicate the uncommon product features to the design of the true data

Table B4. Optimum product line solution (Finite mixtures of normal distributions)

Synthetic preference	Attribute						Price
	Engine	TM	A3	A4	A5	A6	
True	1	1	2	4	1	3	18,600
	1	2	1	1	2	1	19,100
	3	1	1	2	2	2	19,300
	2	2	2	2	2	4	20,200
	2	2	2	4	4	1	20,200
HB-ML	3	1	1	2	1	2	19,200
	1	2	2	3	3	1	19,550
	1	2	2	4	1	4	19,700
	2	2	1	2	2	3	19,900
	2	2	1	1	4	2	19,900
LC-MNL	1	1	1	4	2	3	18,500
	3	1	1	1	4	3	19,500
	2	2	2	3	2	1	19,950
	2	2	1	3	4	1	19,950
	2	2	1	2	4	3	20,100

* Underlines indicate the uncommon product features to the design of the true data

APPENDIX C. Multi-attribute decision making using HEIM

The multi-attribute decision-making procedure using HEIM follows the original HEIM paper (See, Gurnani, and Lewis 2004) that handles an individual decision maker. If one wants to deal with issues like consensus and compromise in group members, the group HEIM (See and Lewis 2006) could be adopted in the design process. Details about HEIM used in Chapter 6 follow.

Step 1. Identify the attributes. The first step is to identify the attributes that are important in the decision problems. In the presented example, FCS, SD of FCS, and the probability of failure II are selected. If required, more attributes such as commonality can be included.

Step 2. Determine the strength of preference within each attribute. Assessing a decision maker's strength of preferences for each attribute is necessary. In this study, a quadratic equation, $v = a_2u^2 + a_1u + a_0$, is supposed to describe the strength of preference, where v indicates the strength of preference, u is a normalized attribute value, and a_2, a_1, and a_0 are coefficients to be determined. The maximum and minimum attribute values used for normalization are listed in Table C1. Figure C1 shows the strength of preference for each attribute and their coefficient values.

Table C1. Maximum and minimum of attribute value for reference alternatives

Reference alternatives	Attribute value (%)			Remarks
	FCS	σ_{FCS}	P_{F2}	
R1	55.01*	2.81	34.83	Smallest FCS
R2	64.76**	2.83	33.08	Largest FCS
R3	55.14	2.32*	24.60*	Smallest σ_{FCS} and P_{F2}
R4	57.35	3.11**	32.33	Largest σ_{FCS}
R5	60.65	2.89	34.95**	Largest P_{F2}

* Minimum value of each attribute
** Maximum value of each attribute

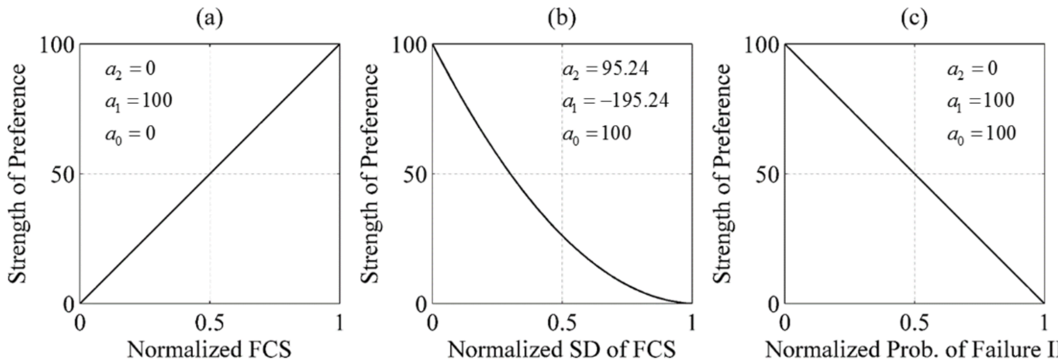

Figure B1. Strength of preference: (a) FCS (b) SD of FCS (c) Prob. of Failure-II

Step 3. Set up hypothetical alternatives and formulate the preference structure as an optimization problem. This step aims to elicit decision maker's preferences to construct constraints in an optimization problem described in Eq. (6.6). A 3^{3-1} fractional factorial design is used with three levels (0, 50, and 100 scores from the strength of preference curves in Figure B1) as listed in Table C2. Assume a decision maker feels rating nine alternatives at once is difficult. So, three alternatives are rated at a time as H3>H1>H2, H6>H5>H4, and H9>H8>H7. The inequivalent can be formulated to the inequality constraints in Eq. (6.6).

Table C2. Strength of preference score for hypothetical alternatives

Hypothetical alternative	Strength of preference, v		
	FCS	σ_{FCS}	P_{F2}
H1	0	0	0
H2	50	50	100
H3	100	100	50
H4	0	50	50
H5	50	100	0
H6	100	0	100
H7	0	100	100
H8	50	0	50
H9	100	50	0