On the Accuracy of Analyses for In-Plane Bending of Smooth Pipe Bends with End Constraints

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Abstract

The accuracy of theoretical analyses for in-plane bending of smooth pipebends with end constraints is discussed and investigated with a view to explaining and reducing the differences between the major works. An earlier theory of the authors is improved to give more accurate answers for bends with rigid flanges. Flanged bends are then examined in some detail, quantifying for the first time the important influence of the flange rigidity on the bend flexibility and stresses. A summary of some finite element analyses is presented from which it is clear that further work is desirable.

1. Introduction

Considerable research has been devoted to smooth curved pipes terminated by flanges or tangent pipes subjected to in-plane bending; however there is some divergence between the published results and this may confuse designers. For a review see ref.[1]. For tangent pipe constraints, most theories appear to be in broad agreement with about 10% variation in flexibility factors. Flanged bends show larger deviations, with differences up to an order in magnitude. The flanged geometry is given in fig 1 and theoretical and experimental results are summarised in figs 2 & 3. An attempt will be made here to briefly examine the differences with a view to clarifying the situation.

2. Flanged Bends

Fig 2 illustrates the main theoretical flexibility factors which have been obtained. The results of Whatham [3] and the theory published by the authors in [4] show similar trends but the latter values are about 10% lower.

When the theory [2] was first published, several assumptions were employed in order to reduce the level of computation, commensurate with reasonable accuracy but without invoking unnecessary complication, since it was intended to extend the work into the inelastic regime and to other loadings. One assumption set the meridional strain to zero for the rigid section displacements (see ref [2]) which causes the flexibility factors at high pipe factors to converge to $(1-\nu^2)$ and not unity, because it inhibits the natural Poisson effect. This may be removed by modifying certain of the displacement trigonometric series i.e. starting the meridional distortion functions $u_d(\varphi)$, $v_d(\varphi)$, $w_d(\varphi)$ from N = 1 instead of from N = 2 and omitting the even number terms in all of the rigid section displacement series. Another assumption in the original analysis set the shear strain to zero at the flange. This was actually removed in an earlier version but was re-incorporated for convenience to improve convergence. By selecting a circumferential function series which does not have zero slope at the flange, this assumption can be removed. Modified displacements are as follows,

Rigid Section displacements:

$$u_r = \sum_{j}^{JT} F_j \tfrac{1}{2}\left(\Theta - \frac{\alpha}{j\pi}\sin\left(\frac{j\pi\Theta}{\alpha}\right)\right)\left(1 + \frac{r}{R}\sin\varphi\right) + \tfrac{1}{2}\sum_{j}^{JT} D_j\left(\frac{j\pi}{\alpha}\right)\sin\left(\frac{j\pi\Theta}{\alpha}\right)\frac{r}{R}\sin\varphi$$

$$v_r = -\sum_{j}^{JT} D_j \sin^2\left(\frac{j\pi\Theta}{2\alpha}\right)\cos\varphi$$

$$w_r = -\sum_{j} D_j \sin^2\left(\frac{j\pi\Theta}{2\alpha}\right)\sin\varphi\,, \qquad j = 1, 3, 5 \ldots\ldots JT \tag{1}$$

Distortion Displacements:

$$u_D = \frac{r}{R}\sum_{m}^{MT}\sum_{n}^{NT} A_{mn}\left(\psi_{en}\frac{1}{n^2}\cos n\varphi + \psi_{on}\frac{1}{n^2}\sin n\varphi\right)\left(\frac{m\pi}{\alpha}\right)\sin\left(\frac{2m\pi\Theta}{\alpha}\right)$$

$$v_D = \sum_{m}^{MT}\sum_{n}^{NT} B_{mn}\left(-\psi_{en}\frac{1}{n}\sin n\varphi + \psi_{on}\frac{1}{n}\cos n\varphi\right)\left(\cos\left(\frac{m\pi\Theta}{\alpha}\right) - \cos\left(\frac{m\pi}{2}\right)\right)$$

$$w_D = \sum_{m}^{MT}\sum_{n}^{NT} C_{mn}\left(\psi_{en}\cos n\varphi + \psi_{on}\sin n\varphi\right)\left(\psi_{om}\cos^2\left(\frac{m\pi\Theta}{\alpha}\right) - \psi_{em}\sin^2\left(\frac{m\pi\Theta}{\alpha}\right)\right)$$

$$+\sum_{m}^{MT} H_m\left(\psi_{om}\cos^2\left(\frac{m\pi\Theta}{\alpha}\right) - \psi_{em}\sin^2\left(\frac{m\pi\Theta}{\alpha}\right)\right)$$

$$m = 1, 2, 3, \ldots\ldots MT, \qquad n = 1, 2, 3, \ldots\ldots NT \tag{2}$$

This new series was employed as in ref. [2] but more terms were required in each series (JT = 6, MT = 6, NT = 7) (144 terms instead of the 105 terms in the original analysis). The new results are now too close to those of Whatham to be clearly illustrated on the graph; comparisons are given in table 1 where the average difference is only 2%.

2.1 Bends with tangent pipes

For connected tangent pipes the modifications are more complex and the original results are not as significantly improved because of an assumption in this case which allows some degree of discontinuity at the joint between bend and tangent. To remove this assumption would require a rather more sophisticated analysis. The difference between theoretical factors for tangent bends is not as dramatic and this has not been pursued.

2.2 Extensions using minimum energy approach

The major benefit of the energy approach is that its relative simplicity makes it possible to extend the work to examine other related problems, e.g the problem of combined bending and pressure [7] or the out of plane bending case [8]. Perhaps the most important extension is to inelastic behaviour and flexibility and stress factors have been determined for pure creep [9]. Creep rates for bends with flanges are considerably lower than for bends with tangents. It is worth considering that welded rings or flanges could deliberately be incorporated in high temperature pipework in order to reduce the damage caused by creep strain accumulation on a critical bend.

2.3 Flange rigidity

Basically, a flange will restrict the natural ovalisation of the curved pipe, but the degree to which it does so will be dependent on its radial and planer stiffness. Flanges have been assumed to be either totally rigid e.g. Whatham [3] and present authors [2], or semi-rigid, usually restricting radial distortion displacements only thus allowing some degree of warping of the plane of the flange or deformation of the adjacent shell wall. In ref. [10], an earlier theory was re-examined to show the importance of not assuming full rigidity. They presented relatively simple formulae for flexibility factors which can be reduced to

$$K = K_k + \frac{(1 - K_k)}{\ell} \quad \frac{\sinh \ell + \sin \ell}{\cosh \ell + \cos \ell}$$

$$\ell = \alpha\sqrt{\frac{R}{r}}\left(\frac{3\lambda^2}{(1 - \nu^2)} + \frac{1}{4}\right)^{\frac{1}{4}}$$

In the above K_k is the Karman flexibility factor for bends without end constraints and the pipe factor $\lambda = Rt/r$. Values are presented in fig. 4 and table 1 and considerably higher factors result. In an earlier paper [2] results from Axelrad's work were given but using only the first term Karman solution consistent with the first term solution used to derive the given equation but the fully converged Karman approximation was used here as suggested by the authors in [10].

2.4 Experimental comparisons

Clearly, rigidity has also influenced the range of experimental flexibility factors via the use of different thicknesses of flanges. Unfortunately some of the most extensively quoted work [11] appears to have employed relatively thin flanges; however the sparseness of the published details inhibits meaningful comparison. Tests conducted recently, including those by the present authors and J F Whatham, have demonstrated flexibilities significantly lower than [10] with typical industrial flanges; furthermore the stress distributions obtained from these tests have generally shown excellent conformance with the rigid flange theory [2,3].

2.5 Finite element analyses

Currently, the only attempt to quantify the influence of flange rigidity appears to be some work conducted by the authors using the finite element method which was mentioned in ref. [2]. Further work has now been undertaken in order to investigate the apparent discrepancies between rigid and semi-rigid flanges. The SUPERB finite element program was used with about 250 parabolic isoparametric shell elements in a manner similar to that illustrated in ref [2]. Initial convergence runs demonstrated acceptable results although it was noted that along the circumference of the bend the stress gradient was greatest at the flanges. As a consequence, element density was increased toward the flange. The following parameters were used; R=450mm, r=150mm, bend angle α =90 degrees and the bend thickness ,t, and the flange thickness ,T, were varied within the bandwidth of reasonable industrial usage. Figure 4 and table 2 summarise the flexibility and stress results for t = 3.0mm, 7.5mm and 15.0mm, and for T = 37.5mm, 75mm and 150mm. It was noted that the flexibility factors, obtained by dividing the shell rotation at the flange by $MR\alpha/EI$, varied around the flange circumference and were not constant as is usually assumed. In table 2 the flexibility factor at the flange intrados, centreline and extrados are quoted and it can be seen that the variation is minimal for the thickest flange. The flexibility factors at the centreline are shown in figure 4. The thickest flange produces results very similar to those of the rigid flange theories [2,3] and the thinnest flange produces results approaching those from [10]. Note that in applying these results to practical problems it would probably be necessary to consider the composite effect of the bend flange and the flange to which it is bolted by the use of a thickness somewhere between the single and double flange thicknesses.

The peak stresses at the bend centreline given in table 2 also demonstrate the important influence of flange rigidity. It should be noted that higher stresses (not reported here) were often encountered in the region local to the flange. Although these decayed rapidly away from the flange, they could be significant if fatigue was a consideration.

3. Discussion

In nuclear design work, it may be thought that the lowest flexibility factors should be assumed to estimate the maximum reaction loading back onto the components of the pipeline due to expansion and seismic type loadings. On this basis fully rigid flanges would give the lowest flexibilities. However, this ignores the fact that the stresses in a rigid flanged bend can be lower than that pertaining in actual flanged bends. Hence higher stresses may be present in the bend if the constraint is not as severe as assumed.

4. Conclusions

An attempt has been made to examine the effects of the major assumptions made in the analyses of bends with end effects. The theoretical results for flanged bends lie at two extremes which can be classified as rigid flanges or semi-rigid (thin) flanges. There can be considerable differences between them. It has been shown here that actual flanges give results between these extremes, varying with the flange thickness as shown in fig 4. One can only conclude that further work of this type is required to clarify and catalogue the important effect of flange stiffness.

5. References

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R/r	α (Deg)	λ	Flexibility Factors: Original Solution [1]	Modified Solution	Whatham [2]	Axelrad & Emmerling [9]
10	180	.1	11.84	12.02	12.3	14.4
		.2	6.36	6.45	6.56	7.33
		.5	2.64	2.72	2.75	3.08
		1	1.46	1.54	1.55	1.60
10	90	.1	7.10	7.47	7.69	12.14
		.2	4.52	4.68	4.78	6.40
		.5	2.25	2.34	2.37	2.86
		1	1.37	1.46	1.47	1.56
3	180	.06	10.59	13.15	13.7	20.70
		.15	6.07	6.67	6.83	8.60
		.3	3.63	3.84	3.89	4.56
3	90	.06	2.59	2.88	2.95	8.99
		.15	2.35	2.57	2.60	4.32
		.3	1.99	2.14	2.15	3.02

Table 1 : Comparison of Theoretical Flexibility Factors

α = 90° R/r = 3		Flexibility Factors			Peak Stress Factors	
λ	T (mm)	Intrados φ = 90°	Centre φ = 0°	Extrados φ = + 90°	σ_φ	σ_θ
.06	150	3.11	3.16	3.20	1.66	2.85
	75	3.42	3.70	4.00	1.81	2.96
	37.5	4.61	5.79	7.80	3.82	3.27
0.15	150	2.54	2.65	2.71	1.79	2.25
	75	2.78	3.20	3.69	2.54	2.33
	37.5	2.70	4.17	6.95	4.65	3.18
0.3	150	2.08	2.22	2.35	2.05	2.14
	75	1.98	2.51	3.10	2.66	2.22
	37.5	0.81	2.64	5.33	3.39	2.19

Table 2 : Variation in Flexibility Factors with Flange Thickness (T)
Based on Finite Element Analysis. R=450mm, r=150mm

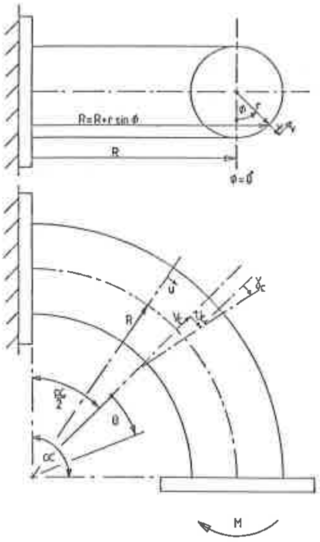

Fig.1 Flanged Bend Geometry

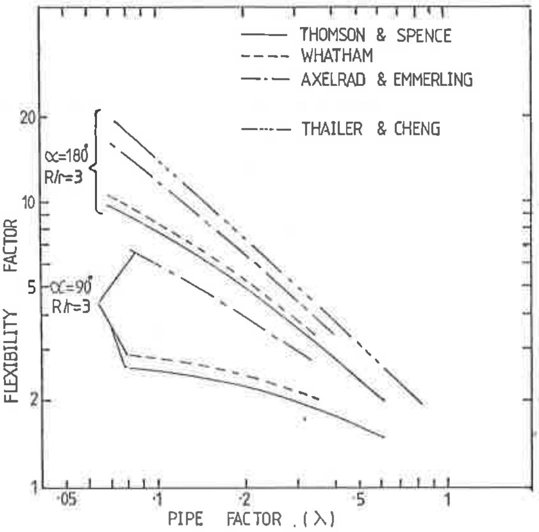

Fig.2 Theoretical Flexibility Factors

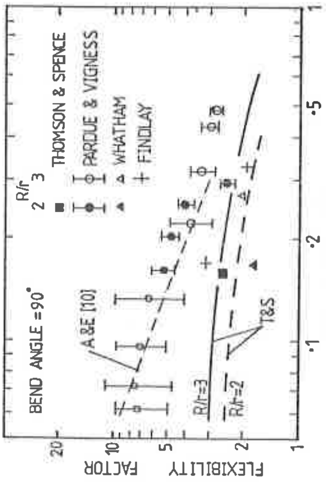

FIG.3

Fig.3 Experimental Flexibility Factors

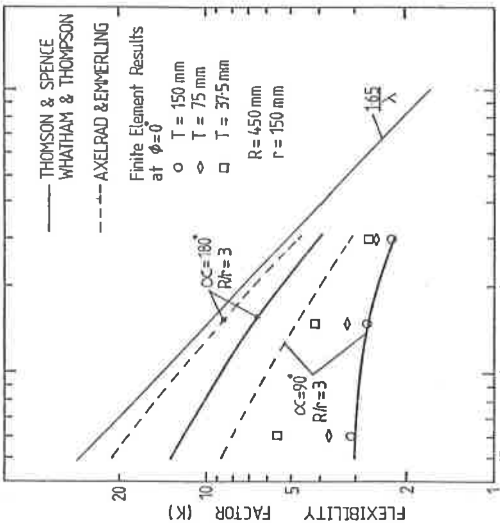

FIG.4

Fig.4 Effect of Flange Thickness on Bend Flexibility