DYNAMIC STRUCTURAL ANALYSIS OF UNCOUPLED SUBSYSTEMS

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SUMMARY

Analytical techniques and results are presented for dynamic seismic analysis of uncoupled structures. It is concluded that an uncoupled subsystem dynamic analysis can produce results essentially identical to those from a coupled analysis. The techniques utilized and conclusions obtained are applicable in general for any system/subsystem configuration. A nuclear power plant primary structure and a pressurized water reactor coolant system are used as the system/subsystem example. To develop general and realistic results, soil-structure interaction, non-uniform damping and representative structural stiffnesses were considered in developing the results. Standard time-history model analysis techniques were used. However, the conclusions are not dependent on the solution techniques.

In performing dynamic seismic analyses of subsystems, a common approach is to include a simplified model which correctly represents the stiffness as well as the mass effects of the subsystem in the primary system dynamic model. The results are then used as forcing functions for separate and more detailed subsystem analyses. This approach enables the analyst to use more detailed subsystem mathematical models when performing either spectrum analysis or time history analysis of the subsystem in question.

Analyses of complex systems with hard-to-predict results have led to some concern regarding the accuracy of this method. This is especially true for the case in which subsystem stiffnesses are similar to the primary system stiffness. For a simple two-mass system/subsystem, it can be shown by close form solution that an uncoupled analysis of the subsystem, using forcing functions obtained from the coupled system, will produced results identical to those of the coupled system. However, for a complex multi-supported subsystem receiving different excitations at each support, it is impractical to perform a closed-form analysis to demonstrate the adequacy of this approach. This paper demonstrates, by example, that a properly uncoupled subsystem analysis will produce results consistent with those obtained from a coupled system/subsystem analysis, regardless of the complexity of the system.

Coupled model analyses performed during this study include both planar and 3-dimensional coupled building/reactor coolant system models, to which a "typical" ground acceleration forcing function was applied at the base of soil springs. Maximum forces and moments within the subsystem and at support interface locations, as well as response spectra, accelerations, and displacements at locations within the subsystem were calculated and used as "bench mark results". From the coupled planar model analysis, support interface time-history accelerations and displacements were extracted and used as the forcing functions for the uncoupled subsystem analyses.

Uncoupled subsystem models to which these acceleration and displacement forcing functions were applied consisted of both 3-dimensional and planar model representations of the reactor coolant system, identical to those coupled with the primary structure models in the coupled analyses. The results obtained from the uncoupled planar model analysis served as a basis for one-to-one comparison with the coupled planar model results.

Analysis of the uncoupled 3-dimensional model of the reactor coolant system was performed to demonstrate the validity of using planar coupled model results as input to a more detailed 3-dimensional subsystem by comparing the results with the coupled 3-dimensional model analysis. In both the planar and 3-dimensional uncoupled analyses, the subsystem exhibited natural frequencies significantly different than those shown in the coupled models. However, when primary system/subsystem stiffnesses are similar, these effects are expected and are shown to be proper by closed form solution of simple cases.

Excellent comparison of analytical results leads to the definite conclusion that a properly uncoupled subsystem dynamic analysis is correct and equivalent to a coupled analysis, regardless of the complexity of the systems treated.

1. Introduction

In performing dynamic seismic analyses of subsystems, a common approach is to include a simplified model of the subsystem, which correctly represents its stiffness and mass effects, in the primary system dynamic model. The resulting motions of the subsystem support locations are then used as forcing functions for separate and more detailed subsystem analyses. This approach enables the analyst to use more detailed subsystem mathematical models when performing either spectrum analysis or time history analysis of the subsystem in question, while accounting for dynamic interaction between the primary system and the subsystem.

Analyses of complex subsystems with hard-to-predict results may lead to some concern regarding the validity of this method, especially for the case in which subsystem stiffnesses are similar to the primary system stiffness. For a simple two-mass system/subsystem, it can be shown by closed form solution (Appendix A) that an uncoupled analysis of the subsystem, using forcing functions obtained from the coupled system, will produce results identical to those of the coupled system. For a complex multi-supported subsystem receiving different excitations at each support, however, it is impractical to perform a closed form analysis to demonstrate the validity of this approach.

This paper presents the results of an investigation of dynamic seismic analyses of complex uncoupled substructures. The techniques used and conclusions obtained are applicable in general for any system/subsystem configuration. A nuclear power plant primary building structure and a pressurized water reactor coolant system are used as the system/subsystem examples. Methods of analysis are discussed and typical results are presented.

2. Discussion

2.1 General

Figure 1 is a simplified flow chart of the analyses performed and the comparisons of results which are made. Standard time-history modal analyses techniques described in references [1 and 2] are used.

Coupled model analyses performed during this study include both planar and three-dimensional coupled building/reactor coolant system models, to which a "typical" ground acceleration forcing function is applied at the base of soil springs. Maximum forces and moments within the subsystem and at support interface locations, as well as response spectra, accelerations, and displacements at locations within the subsystem are calculated and used as "bench mark results." From the coupled planar model analysis, support interface time-history accelerations and displacements are extracted and used as the forcing functions for the uncoupled subsystem analyses.

Uncoupled subsystem models to which the coupled model acceleration and displacement forcing functions are applied consist of both three-dimensional and planar model representations of the reactor coolant system, identical to those coupled with the primary structure models in the coupled analyses. The results obtained from the uncoupled planar model analysis serve as the basis for one-to-one comparison with the coupled planar model results. Analysis of the uncoupled three-dimensional model of the reactor coolant system is performed to demonstrate the validity of using planar coupled model results as input to a more detailed three-dimensional subsystem by comparing the results with the coupled three-dimensional model analysis.

2.2 Description of System/Subsystem Models

Figures 2 and 3 are schematic representations of the coupled system/subsystem models which are used in the analysis. Structural properties are defined by three-dimensional beam elements, using the STRUDL computer program [3]. Structural stiffnesses and weights representative of concrete supporting structures and containments are used for the building primary system. Subsystem properties are those of a 3410-Mwt pressurized water reactor coolant system. Soil-structure interaction effects, including significant rocking, are accounted for by the set of springs attached to the basemat of the primary system model.

In developing the coupled models care was taken to assure that proper member force and moment releases are given at the system/subsystem interface locations. It is important that the subsystem has the same geometric constraints in the coupled analysis as those which are imposed on it during the uncoupled analysis.

In the coupled planar model there are 33 lumped mass points with 46 dynamic degrees of freedom. The simplified planar model of the subsystem, representing the reactor coolant system, includes 6 lumped mass points with 9 dynamic degrees of freedom. The concrete supporting structure primary system model includes 27 lumped mass points with 37 dynamic degrees of freedom. The model provides two-dimensional response characteristics for excitation in the horizontal or vertical directions.

The three-dimensional model includes a total of 46 lumped mass points with 73 dynamic degrees of freedom. Of this total, 9 mass points and 15 dynamic degrees of freedom are associated with the subsystem representing the reactor coolant system. The model allows three-dimensional response for excitation in any direction.

The planar and three-dimensional subsystem model representations of the reactor coolant system used in the uncoupled subsystem analyses are identical to those coupled with the primary structure models in the coupled analyses.

2.3 Ground Excitation

Figure 4 is a plot of the horizontal ground acceleration time-history used in the analysis. Digitized data, defined at time intervals of 0.01 seconds, is used as the excitation forcing function applied at the base of the soil springs in the coupled planar and three-dimensional models. The acceleration response spectrum of this motion, having a maximum acceleration value of 0.33g, is shown in Figure 5.

2.4 Method of Analysis

The general time-history modal analysis techniques described in references [1 and 2] for structural systems excited at multiple support locations are used in this study. The equations developed in the above references are directly applicable for the uncoupled subsystem analyses. The equations are simplified for the coupled model analyses where the only excitation function is the ground acceleration applied at the base of the model soil springs. Thus there are no non-datum support accelerations or displacements to be considered in the coupled analyses.

Non-uniform modal damping is used in the coupled model analyses. Composite modal damping values are determined as follows:

$$\beta\omega = \frac{\Phi^T \beta \mu \Phi}{\Phi^T \mu \Phi} \qquad (1)$$

where: $\beta\omega$ = the resulting matrix of weighted modal damping factors

Φ = square matrix of mode shapes

μ = complete system diagonal mass matrix

β = the diagonal matrix of damping factors associated with the corresponding elements of the mass matrix

Subsystem damping values which are used to develop the equivalent modal damping are 1% for the reactor coolant system, 2% for the basemat and interior building, 2 1/2% for the containment shell and 8% for soil. Modes which reflect only rigid body motion such as rocking of the building on the soil springs are assigned modal damping values of 8%.

Uniform modal damping of 1% is used for the uncoupled planar and three-dimensional subsystem analyses of the reactor coolant system models.

Time-histories of the acceleration and displacement responses at the system/subsystem interface locations, obtained during the coupled planar model analysis, are used as the forcing functions for the uncoupled analyses. Both translational and rotational support motions are required for the subsystem analyses, because rotations are significant due to the soil-structure interaction effects.

Results consist of maximum forces and moments, maximum accelerations, maximum displacements, and acceleration response spectra at selected locations for the analysis of the planar and three-dimensional coupled and uncoupled models.

3. Results

3.1 Planar Analyses

A summary of results of the eigenvalue analyses for the planar models is presented in Table I. A comparison between the coupled and uncoupled models is made for the frequency response of the reactor coolant system. Note that in the uncoupled analysis the subsystem exhibits natural frequencies significantly different than those shown in the coupled model. However, when primary system/subsystem stiffnesses are similar, these effects are to be expected and are shown to be proper by closed form solution of simple cases in Appendix A.

In addition to the modes for which a direct comparison of natural frequencies can be made, the coupled model analysis reveals that the motion of the steam generators becomes predominant in a low frequency mode associated with the primary structure.

Response spectra for all nine mass point dynamic degrees of freedom as well as selected non-mass point locations within the reactor coolant system were developed from the time-histories of the acceleration response produced during the analyses. Comparison between the coupled and uncoupled analyses is excellent for all locations. Response spectra for the steam generator upper mass point location, presented in Figures 6 and 7, are typical of the results. Note that not only the maximum accelerations but also the frequency content of the spectra from the two analyses are nearly identical.

Force and moment reactions at all system/subsystem interface locations and at locations within the subsystem were calculated for both coupled and uncoupled analyses. Maximum reactions are summarized and presented in Figure 8. Comparison of results is again considered to be excellent.

3.2 Three-Dimensional Analysis

Results from the analyses performed utilizing the three-dimensional models are summarized and compared in a manner similar to those from the planar analyses. Table II presents a comparison of the eigenvalue analyses' results. Some subsystem natural frequencies are again seen to be significantly different in the two models, as discussed in Appendix A.

Figures 9 and 10, response spectra for the upper mass point location on the reactor vessel, are typical of the spectra generated and the consistency of results. Figure 11 presents a comparison summary of the maximum forces and moments which were produced by the analyses. These comparisons are excellent, especially considering that responses from a more detailed three-dimensional subsystem analysis, utilizing forcing functions from a simplified planar system/subsystem analyses, are being compared with results from a coupled analysis utilizing a more detailed three-dimensional system/subsystem model.

4. Summary

Dynamic analyses have been performed, utilizing both coupled and uncoupled models of a complex system/subsystem configuration consisting of a nuclear power plant basemat, containment, and internal structure and a pressurized water reactor coolant system. To develop general and realistic results, soil-structure interaction, non-uniform damping and representative structural stiffnesses were considered in developing the results. Standard time-history modal analysis techniques were used. However, the conclusions are not dependent on the solution techniques nor on the specific details of the representative models used.

5. Conclusions

A. A properly uncoupled analysis of a multi-supported subsystem produces results which are essentially identical to those obtained from a coupled system/subsystem analysis.

B. Analysis results obtained by utilizing simplified models which correctly represent the stiffness and mass effects of the system and subsystem being analyzed, may be used as forcing functions for separate and more detailed subsystem analyses. A subsystem analysis, utilizing a detailed subsystem model, will then produce results consistent with those obtained from a coupled system/subsystem analysis in which detailed models are utilized.

C. Uncoupled subsystem analyses are correct, including cases in which structural stiffnesses are such that the coupled subsystem exhibits response characteristics significantly different than those shown in the coupled model, provided the excitations applied to the uncoupled model are developed from a coupled model to account for interaction effects.

D. Excellent comparison of analytical results leads to the definite conclusion that a properly uncoupled subsystem dynamic analysis is valid and equivalent to a coupled analysis, regardless of the complexity of the systems treated.

6. Acknowledgement

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References

[1] KASSAWARA, R.P., PECK, D.A., "Dynamic Analysis of Structural Systems Excited at Multiple Support Locations," ASCE Specialty Conference on Structural Design of Nuclear Plant Facilities, Vol. 1, pp. 73-88 (1973) Combustion Engineering Publication TIS-3759.

[2] KASSAWARA, R.P., PECK, D.A., "Dynamic Analysis of Equipment Systems Excited by Multi-Directional Support Motions," ASCE Specialty Conference on Structural Design of Nuclear Plant Facilities, Vol. 1-A, pp. 169-187 (1975) Combustion Engineering Publication TIS-4538.

[3] ICES STRUDL-II, The Structural Design Language, "Engineering Users Manual, Vol. 1, Frame Analysis," R68-91, First Edition, MIT (November 1968).

TABLE 1

RESULTS OF EIGENVALUE ANALYSES

PLANAR COUPLED AND UNCOUPLED MODELS

COMPONENT NAME	DOMINANT DEGREES-OF-FREEDOM	DIRECTION	NATURAL FREQUENCY COUPLED	UNCOUPLED
STEAM GENERATORS	404, 3404	X	10.00	
STEAM GENERATORS	404, 3404	X	13.76	15.03
STEAM GENERATORS	404, 3404	X	12.41	15.64
STEAM GENERATORS & REACTOR VESSEL	409, 3409, 9996	X	17.77	18.80
REACTOR VESSEL	9996	Y	22.44	22.33
STEAM GENERATORS	404, 3404	Y	19.05, 21.74	25.13
STEAM GENERATORS	409, 3409	X	26.72	26.94
REACTOR VESSEL	9995	X	29.59	29.53
REACTOR VESSEL	9996	X	36.80	37.18

TABLE 2

RESULTS OF EIGENVALUE ANALYSES

3-DIMENSIONAL COUPLED AND UNCOUPLED MODELS

COMPONENT NAME	DOMINANT DEGREES-OF-FREEDOM*	DIRECTION	NATURAL FREQUENCY COUPLED	UNCOUPLED
STEAM GENERATORS	404, 3404	X	9.98	
STEAM GENERATORS	404, 3404	X	13.68	14.88
STEAM GENERATORS	404, 3404	X	12.34	15.56
STEAM GENERATORS & REACTOR VESSEL	409, 3409, 9996	X	16.05	18.62
REACTOR VESSEL	9996	Y	21.75	22.67
STEAM GENERATORS	404, 3404	Y	23.64, 24.60	25.32
STEAM GENERATORS	409, 3409	X	26.62	26.87
REACTOR VESSEL	9995	X	28.96	28.87
REACTOR VESSEL	9996	X	36.32	37.18

*MODES WITH DOMINANT RESPONSE IN X AND Y DIRECTIONS ARE GIVEN IN TABLE

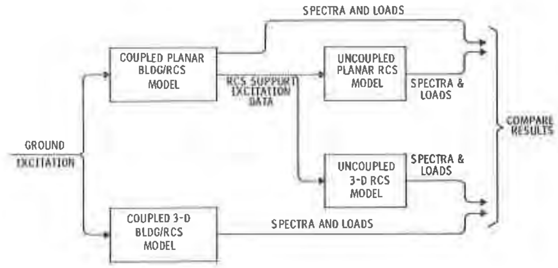

Fig. 1: Flow chart of coupled and uncoupled seismic analyses

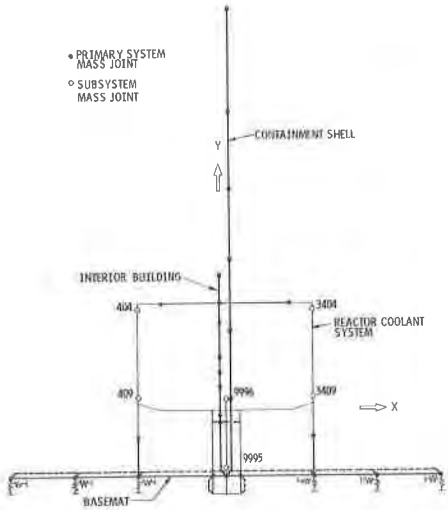

Fig. 2: Coupled system/subsystem planar seismic analysis model

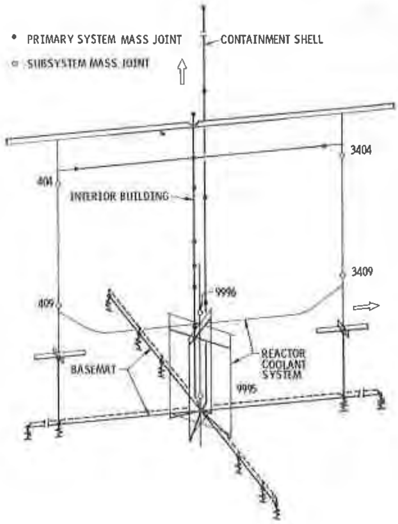

Fig. 3: Coupled system/subsystem three-dimensional seismic analysis model

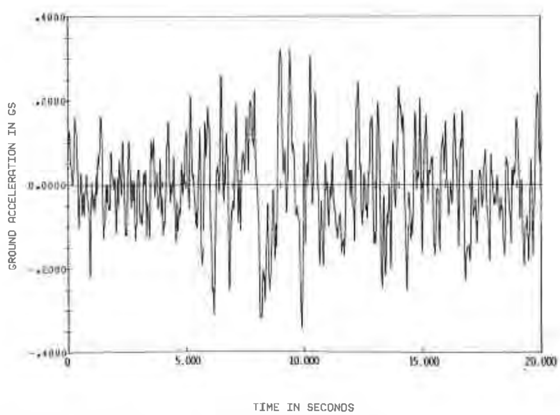

Fig. 4: Horizontal ground acceleration

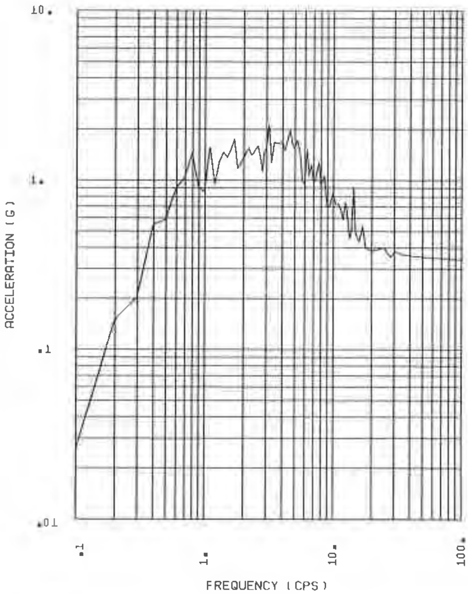

Fig. 5: Horizontal ground acceleration spectrum - 1% damping

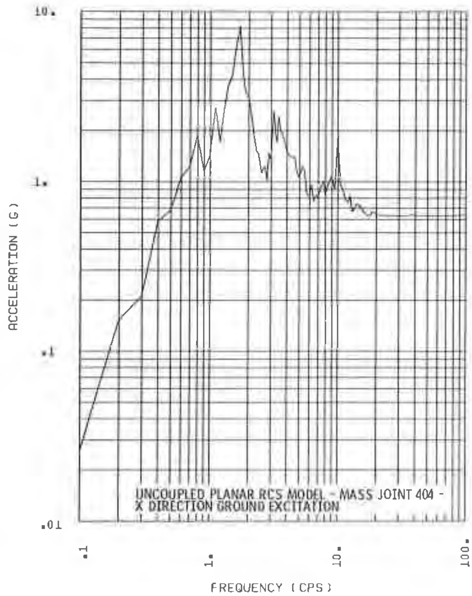

Fig. 6: Steam generator response spectrum - X direction, 1% damping

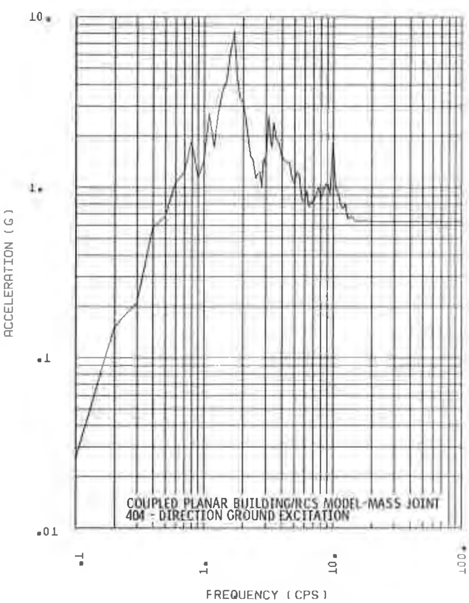

Fig. 7: Steam generator response spectrum - X direction, 1% damping

MAXIMUM REACTIONS IN KIPS AND INCH-KIPS
X-DIRECTION GROUND EXCITATION

SUPPORT REACTIONS - GLOBAL COORDINATES

SUPPORT		COUPLED ANALYSIS	UNCOUPLED ANALYSIS
R.V. COL BASE	-Fx	217.9	219.6
	-Fy	307.7	314.6
	-Mz	449.3	512.2
R.V. UPR COL. SUPT	-Fx	915.5	952.4
S.G. BASE	-Fy	502.3	502.3
	-Mz	19,230.	20,990.
S.G. SNUBBER	-Fx	594.8	592.5

MEMBER FORCES AND MOMENTS - LOCAL COORDINATES

SUPPORT		COUPLED ANALYSIS	UNCOUPLED ANALYSIS
R.V. OUTLET NOZZLE	-Fx	493.3	511.3
	-Fy	152.2	152.4
	-Mz	14,420	14,430
R.V. LWR KEY	-Fx	211.9	213.1
S.G. INLET NOZZLE	-Fx	514.8	531.6
	-Fy	17.3	21.5
	-Mz	6,224	6,176
H.L. PIPING AT Jt. 800	-Fx	493.3	511.3
	-Fy	152.2	152.4
	-Mz	4,045	4,050
S.G. SHELL AT ELEV. 237.	-Fx	434.8	432.0
	-Fy	50.4	53.7
	-Mz	9,022	9,142

Fig. 8: Maximum reactions for planar model analyses

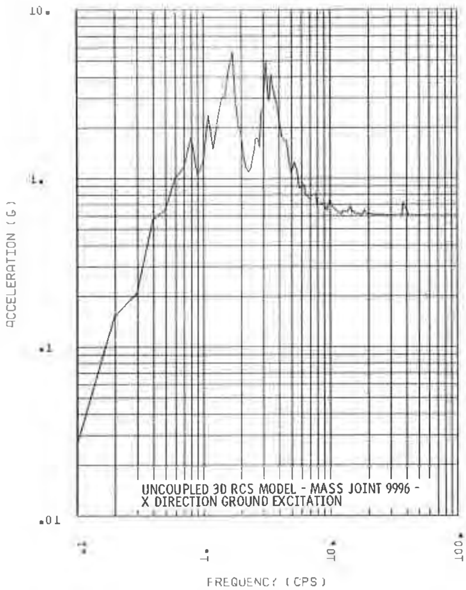

Fig. 9: Reactor vessel response spectrum - X direction, 1% damping

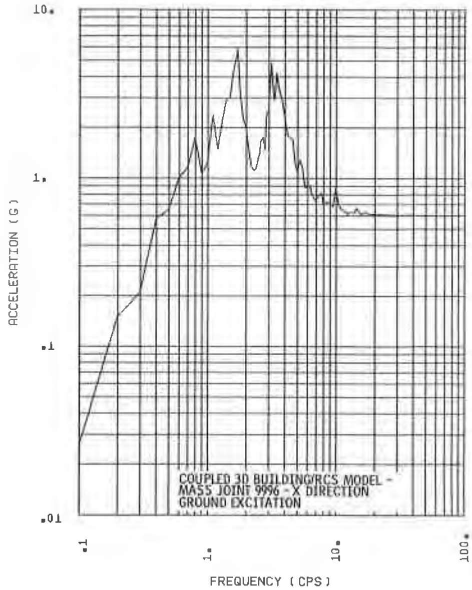

Fig. 10: Reactor vessel response spectrum - X direction, 1% damping

MAXIMUM REACTIONS IN KIPS AND INCH-KIPS
X-DIRECTION GROUND EXCITATION

SUPPORT REACTIONS - GLOBAL COORDINATES			
SUPPORT		COUPLED ANALYSIS	UNCOUPLED ANALYSIS*
R.V. COL. BASE	-Fx	97.8	100.8
	-Fy	178.6	182.8
	-Mz	885.	908
R.V. UPR COL.SUPT.	-Fh	525.8	542.5
S.G. BASE	-Fy	453.6	481.9
	-Mz	15,230	17,420.
S.G. SNUBBER	-Fx	614.3	615.2
MEMBER FORCES AND MOMENTS - LOCAL COORDINATES			
SUPPORT		COUPLED ANALYSIS	UNCOUPLED ANALYSIS
R.V. OUTLET NOZZLE	-Fx	474.8	492.4
	-Fy	99.1	107.6
	-Mz	8,270.	9,104.
R.V. LWR. KEY	-Fz	116.1	119.9
S.G. INLET NOZZLE	-Fx	482.8	500.7
	-Fy	50.8	51.3
	-Mz	3,455	4,059.
H.L. PIPING AT Jt. 800	-Fx	474.8	492.4
	-Fy	99.1	107.6
	-Mz	1,553	1,778
S.G. SHELL AT ELEV. 237.	-Fx	415.5	432.6
	-Fy	33.6	43.0
	-Mz	13,500	13,460.

*PLANAR COUPLED MODEL SOURCE OF INPUT EXCITATIONS

Fig. 11: Maximum reactions for three-dimensional model analysis

Appendix A

Normal Mode Solution of the Equations of Motion for Structures Subjected to Multiple Boundary Excitations

by
R. P. Kassawara

A1. Introduction

This paper presents the proper method of calculation of natural frequencies and mode shapes for an equipment system which is supported at several points in a structure, each support exhibiting a different motion during an earthquake excitation.

A2. Derivation of Equations

The undamped equations of motion for a multiply excited system are given in matrix form in equation (1) below [reference A1]:

$$\begin{bmatrix} M_M & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} X_M \\ X_S \end{Bmatrix} + \begin{bmatrix} K_{MM} & K_{MS} \\ K_{SM} & K_{SS} \end{bmatrix} \begin{Bmatrix} U_M \\ U_S \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_S \end{Bmatrix} \quad \text{(A1)}$$

where:

M_M = a diagonal matrix of the lumped masses of the system model.

K = the stiffness matrix of the system model condensed in such a way that only mass point elements (subscript m) and active, non-released, non-datum support elements (subscript s) remain in the matrix. The datum support eliminates rigid body motions.

U_M = displacement of mass point dynamic degrees of freedom relative to the datum support.

U_S = displacement of support points relative to the datum support.

X_M = absolute acceleration of mass degrees of freedom.

X_S = absolute acceleration of system support points.

The first of equations (A1) can be written as follows:

$$M_M U_M + K_{MM} U_M = M_M \gamma X_D - K_{MS} U_S \quad \text{(A2)}$$

where:

X_D = absolute acceleration of the datum support in a given direction.

γ = a vector defining the direction of excitation.

The quantities on the left hand side of equation (A2) are the unknown displacements and accelerations of the mass points of the equipment system, and the right hand side is a function of known constants M, γ, and K_{MS} and known time dependent variables X_D and U_S.

Combining the known quantities as:

$$F(T) = - M_M \gamma X_D + K_{MS} U_S \quad \text{(A3)}$$

Equation (2) becomes:

$$M_M U_M + K_{MM} U_M = F_T \qquad (A4)$$

Equation (A4) can be solved in its coupled form using standard numerical integration schemes, or the equations can be uncoupled through a modal synthesis. The modal synthesis is accomplished in the usual manner, as follows:

The homogeneous solution to equation (A4) leads to the relationship

$$K\Phi = M\Phi\omega^2 \qquad (A5)$$

where

ω = diagonal matrix of natural circular frequencies of the constrained system.

Φ = square, full matrix of characteristic mode shapes of the constrained system.

The transformation:

$$U_M = \Phi Q \qquad (A6)$$

leads to an uncoupling of equations (A2) into the form:

$$Q_I + \omega_I^2 = \alpha_I X_D - \beta_{IJ} U_{SJ} \qquad (A7)$$

where α and β are constant functions of M, K_{MS}, γ, ω, Φ and ω and Φ satisfy equations (A5).

Each of equations (A6) can be solved using standard techniques. The mass point displacements and accelerations are then obtained using equation (A6).

Thus, although the boundary points of the system are independently excited, the resulting equations of motion are correctly and uniquely solved on a time history basis using a modal analysis with mode shapes and frequencies derived from the constrained boundary configuration [reference A2].

Two points should be emphasized here. First, the boundary motions are completely independent quantities and are not altered by the motion of the system masses. Either the equipment system can be uncoupled from the supporting structure or, if some energy feedback is suspected to occur, the system can be represented in a simplified form in the supporting structure, in order to calculate the boundary point motions of the equipment system. In either case, the final detailed analysis of the equipment system involves the use of independent boundary motions.

Secondly, the solution of equations (A2) yields the time history motions of the mass point degrees of freedom compatible with the boundary motions at each time step to give the true deformed configuration of the structure. These mass point motions are not applied to the structure with fixed boundary points as shown in Figure A1a, but in the manner shown in Figure A1b.

A3. Solutions

The following is a (closed form) example for a simple structure-equipment system analyzed first as a coupled system (Fig. A2) and then as an uncoupled system (Fig. A3). The example illustrates that, using modal analysis, the solutions for the equipment mass are identical for the coupled model and for

A3.1 Coupled Solution

$$K = \begin{matrix} K_1 + K_2 & - K_2 \\ - K_2 & K_2 + K_3 \end{matrix} \qquad (A8)$$

$$\begin{matrix} - Mx_1 \\ 0 \end{matrix} = \begin{matrix} K_1 + K_2 & - K_2 \\ - K_2 & K_2 + K_3 \end{matrix} \begin{matrix} U_1 \\ U_2 \end{matrix} \qquad (A9)$$

Solving the second of Equations (A9) for U_2 gives:

$$U_2 = \frac{K_2}{K_2 + K_3} U_1 \qquad (A10)$$

The first of Equations (A9) then reduces to:

$$MU_1 + K_M U_1 = MX_D \qquad (A11)$$

where:

$$K_M = \frac{K_1 K_2 + K_1 K_3 + K_2 K_3}{K_2 + K_3} \qquad (A12)$$

$$\omega \text{ coupled} = \omega_c = \begin{matrix} K_M \\ M \end{matrix} \qquad (A13)$$

$$U_1 + \omega_c^2 U_1 = - X_D \qquad (A14)$$

For simplicity, let:

$$X_D = C = \text{CONSTANT} \qquad (A15)$$

$$U_1 = a \cos \omega_c t + b \sin \omega_c t + U_{1_{PARTICULAR}} \qquad (A16)$$

The solution of the coupled system is:

$$U_1 = - \frac{C}{\omega_c^2} \quad 1 - \cos \omega_c t \qquad (A17)$$

$$U_2 = \frac{- K_2}{K_2 + K_3} \quad \frac{C}{\omega_c^2} \quad 1 - \cos \omega_c t \qquad (A18)$$

A3.2 Uncoupled Solution

$$MX_1 + K_1 + K_2 \quad U_1 + -K_2 U_2 = 0 \qquad (A19)$$

$$U_1 + \omega_u^2 U_1 = -Xd + \frac{K_2 U_2}{M} \quad \text{(A20)}$$

where:

$$\omega_u = \frac{K_1 + K_2}{M}$$

Substituting X_d and U_2 from the coupled solution gives:

$$U_1 + \omega_u^2 U_1 = \Lambda - \Psi \cos \omega_c t \quad \text{(A21)}$$

where:

$$\Lambda = -C \quad 1 + \frac{K_2^2}{K_1 K_2 + K_1 K_3 + K_2 K_3}$$

and

$$\Psi = -C \frac{K_2^2}{K_1 K_2 + K_1 K_3 + K_2 K_3}$$

The solution of Equation (A21) is of the form:

$$U_1 = A \cos \omega_u t + B \sin \omega_u t + U_{1_{PART.}} \quad \text{(A22)}$$

where $U_{1_{particular}}$ is of the form:

$$U_{1_{PART.}} = D + E \cos \omega_c t \quad \text{(A23)}$$

The solution is of the form:

$$U_1 = \frac{-\Lambda}{\omega_u^2} + \frac{\Psi}{(\omega_u^2 - \omega_c^2)} \cos \omega_u t + \frac{-\Psi}{(\omega_u^2 - \omega_c^2)} \cos \omega_c t + \frac{\Lambda}{\omega_u^2} \quad \text{(A24)}$$

Expansion of the Λ and Ψ shows that:

$$\frac{-\Lambda}{\omega_u^2} = \frac{-\Psi}{(\omega_u^2 - \omega_c^2)} = \frac{C}{\omega_c^2} \quad \text{(A25)}$$

Therefore Equation (A24) reduces to:

$$U_1 = -\frac{C}{\omega_c^2} (1 - \cos \omega_c t) \quad \text{(A26)}$$

Equation (A26), the uncoupled solution for U_1 is identical to the coupled solution in Equation (A17).

References

[A1] ASME Task Group on Dynamic Analysis - Meeting Minutes, Attachment D, November 1, 1973, New York, New York.

[A2] PRZEMIENIECKI, J. S., Theory of Matrix Structural Analysis, McGraw-Hill (1968), Chapter 13, Section 7.

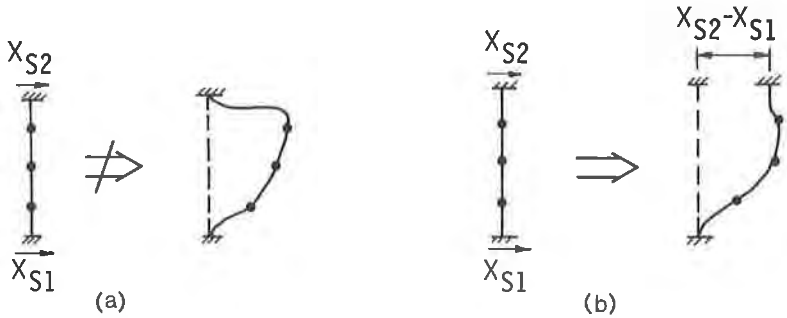

Fig. A1: Deformed structural configuration

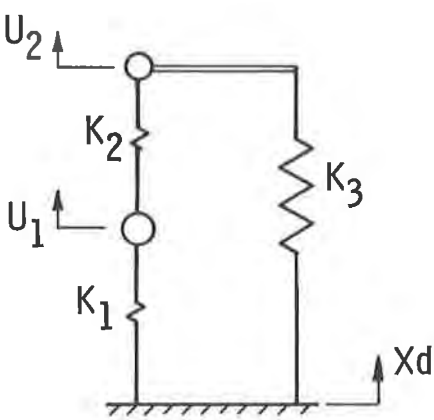

Fig. A2: Coupled model

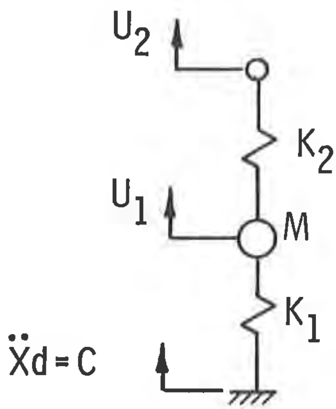

Fig. A3: Uncoupled equipment model