Some Exact Results on Closed Queueing Networks with Blocking

by

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ABSTRACT

We obtain equivalencies between closed queueing networks with respect to buffer capacities and number of customers in the network. Results that relate the throughput of closed queueing networks with blocking to the throughput of closed queueing networks with infinite buffers are also obtained.Exact efficient solutions are given for special cases.

Key words:equivalencies,throughput, blocking, closed queueing networks

1.INTRODUCTION

Closed queueing networks have proved to be useful in modelling computer systems, distributed systems, production systems and flexible manufacturing systems. Several efficient algorithms to calculate the performance measures of closed queueing networks with infinite buffers (CQN-I) have been reported in the literature. However, these algorithms are not applicable when limitations are imposed on buffer capacities.

In recent years, there has been a growing interest in the development of computational methods for the analysis of queueing networks with finite buffers. This is primarily due to a growing need to model actual systems which have finite capacity resources. An important feature of systems with finite buffers is that a server may become blocked when the capacity limitation of the destination queue is reached. Various blocking mechanisms have been considered in the literature so far. These blocking mechanisms arose out of various studies of real life systems. A discussion on these different blocking mechanisms can be found in Onvural and Perros [9].

CQN-I under certain restrictions have been shown to have product-form steady state queue length distributions. Efficient algorithms to calculate the performance measures of these networks have been developed based on this property. In general, closed queueing networks with finite buffers (CQN-B) could not be shown to have product form solutions. However, certain CQN-B have been reported in the literature as having product form solutions when: a) the routing matrix is reversible; b) the branching probability depends on the state of the originating node and the state of the destination node; c) the probability of blocking does not depend on the number of customers in the destination node but simply is constant; d) the service rate at each node is constant but there is zero probability that a queue is empty; (see, Kelly [6], Le Ny[8], Hordijk and Van Dijk [5]). Also, a CQN-B has always a product form solution if it consists of two nodes (see, Akyildiz[1], Gordon and Newell[3]. For a survey of two-node closed queueing networks with blocking see Perros [10].

Approximation algorithms for analyzing CQN-B consisting of more than two nodes have been proposed by Suri and Diehl[12], Yao and Buzacott[13], and Akyildiz[1,2].

In this paper, we obtain analytical results that enable us to understand better the behavior of CQN-B. Such results have not been obtained before for CQN-B under the blocking mechanism considered in this paper.

We will define the queueing network under study in the next section. Exact results that relate the throughput of CQN-B to the throughput of CQN-I and that equivalencies between CQN-B with respect to buffer capacities and number of customers in the network are given in Section 3. CQN-B with symmetric queues are investigated in Section 4.

2.THE NETWORK

We will consider closed queueing networks consisting of **N** nodes and K customers. Each node consists of a single queue served by a server with an exponentially distributed service time with rate μ_i , i=1,..,N. B_i is the capacity of node i including the service space in front of the server. A customer upon completion of its service at node i attempts to enter destination node j with probability p_{ij}, i=1,...,N; j=1,..,N. If at that moment, node j is full, the customer will be forced to wait in front of server i until a space would become available at node j. Server i remains blocked for this period of time, and it can not serve any other customer waiting in its queue. If more than one server is blocked by the same node, then these servers will get unblocked in a first-blocked-first-unblocked fashion.

Due to the blocking mechanism described above, and due to the fact that these N nodes are arbitrarily interconnected, it is possible that deadlocks may occur. For instance, assume that node i is blocked by node j. Now it is possible that a customer in node j may, upon completion of its service, choose to go to node i. If node i is at that time full, then deadlock will occur. In this paper, it is assumed that deadlocks are detected immediately and resolved by instantaneously exchanging blocking units This may violate the first-blocked-first-unblocked rule described above. For instance, let us assume that nodes i and k are blocked by node j in that order. That is, if a departure occurs from node j the blocked customer from node i will enter node j first. Now, let us assume that the departing unit from node j chooses node k as its destination, and, that node k is full at that moment. This causes a deadlock to occur, which is resolved by simultaneously exchanging the blocking units from nodes j and k. In view of this, the blocked customer from node k enters node j first while node i still remains blocked. Thus, the first- blocked-first-unblocked priority rule has been violated.

3.EXACT RESULTS

Let $n = \min_{i=1,..,N} \{B_i\}$. Clearly the number of customers in the network,K ,is such that $1 \leq K \leq \sum_{i=1}^{N} B_i$. For $1 < K \leq n$, blocking does not occur and hence the network has a product form solution. This product form solution can be obtained by treating the queueing network as if the queue at each node has an infinite capacity. In particular, let $r_i = e_i/\mu_i$ be the relative utilization of node i where e_i is the mean number of visits a customer makes to ith node and is given by:

$$e_i = \sum_{j=1}^{N} e_j p_{ji} \tag{1}$$

with $e_j = 1$ for some j Define $(i_1,..,i_j,..,i_N)$ to be the state of the closed queueing network with infinite buffer capacities where i_j is the number of customers at node j with $\sum_{j=1}^{N} i_j = K$. Let $\pi(i_1,..,i_N)$ be the steady state queue length distribution of such a network .Then, $\pi(.)$ is the solution of the global balance equations :

$$\pi(i_1,..,i_N) \sum_{j=1}^{N} \sum_{k=1}^{N} p_{jk} \delta_j \mu_j = \sum_{j=1}^{N} \sum_{k=1}^{N} p_{jk} \delta_k \mu_j \pi(i_1,..,i_j+1,..,i_k \quad 1,..,i_N) \tag{2}$$

$$\sum_{(i_1,..,i_N) \in Z} \pi(i_1,..,i_N) = 1$$

where

$$\delta_j = \begin{cases} 1 & \text{if } i_j > 0 \\ 0 & otherwise \end{cases}$$

and Z is the set of all feasible states.

The queue length distribution of the network defined in Section 2 for $1 \le K \le n$ is:

$$\pi(i_1,...,i_N) = G_K^{-1} \prod_{j=1}^{N} x_j^{i_j} \qquad (3)$$

where G_K^{-1} is a normalizing constant, chosen so that the distribution sums to unity (see, Gordon and Newell[3]).

When $K \ge n+1$ blocking occurs. In this case product form solutions are, in general, not available. In the following theorem, we prove that the CQN-B with K=n+1 customers has a product form solution.

THEOREM 1:Let us consider a CQN-B, as described in Section 2, with buffer capacities B_i. If the number of customers in the network, K, is equal to n+1 where $n = \min_{i=1,..,N} \{B_i\}$, then the network has a product form solution.

Proof: Let $(i_1, i_2, ... i_N)$ be the state of the CQN-B where i_j is the number of customers at node j with $\sum_{j=1}^{N} i_j = n+1$, $i_j \le B_j$, j=1,..,N. Also, let $(0,0,...,i_l=1,0,..,i_j=B_j+l,0,..,0)$ indicate the state of the network, where $i_j = B_j + l$ denotes that node l is blocked by node j. Note that, since K=n+1, there can be at most one node blocked at any time. The steady state queue length distribution of CQN-B, $p(i_1,...,i_N)$, is the solution of the following system:

$$\begin{cases} p(i_1,..,i_N)\sum_{j=1}^{N}\sum_{k=1}^{N} p_{jk}\delta_j\mu_j = \sum_{j=1}^{N}\sum_{k=1}^{N} p_{jk}\delta_k\mu_j p(i_1,...,i_j+1,...,i_k-1,...,i_N), \quad (i_1,\ldots,i_N)\in Z^1 \\ p(i_1,..,i_l,..,i_N)p_{lj}\mu_l = p(0,...,i_l,..,i_j=B_j+l,...,0)\mu_j, \quad (0,..,i_l...,B_j+l,..,0)\in Z^2 \\ \sum_{(i_1,...,i_N)\in Z} p(i_1,...,i_N)=1 \end{cases} \quad (4)$$

where Z^1 and Z^2 is the set of all feasible states in which no node is blocked and in which a node is blocked respectively. $Z = Z^1 \cup Z^2$ is the set of all feasible states. Let,

$$p(i_1,..,i_N) = \begin{cases} \pi(i_1,..,i_N) & \text{if } i_j \le B_j,\ j=1,...,N \\ \dfrac{p_{lj}e_l}{e_j}\pi(0,..,i_j=B_j+1,0,..,0) & \text{if } i_l=1 \text{ and } i_j=B_j+l \end{cases} \quad (5)$$

where e_j is the mean number of visits a customer makes to node j and $\pi(.)$ is the solution given by (3) obtained by assuming that CQN-B has infinite buffers and K=n+1. Clearly, $\sum_{(i_1,...,i_N)\in Z} p(i_1,..,i_N)=1$. By substituting these expressions for $p(.)$ into (4) ,it can be easily verified that the balance equations are satisfied.

Intuitively, this result can be explained as follows. Let queue j be such that B_j=n. When some queue l is blocked by queue j, all the customers in the system except the blocked customer are in queue j. The blocked customer occupies the space in front of the server l. Hence, during the blocking period this service space in front of server l behaves like an additional buffer capacity for node j.

COROLLARY 1:In CQN-B, if nodes with B_i=K-1 are replaced by nodes with B_i=K while keeping all other parameters fixed, then the new network will be equivalent to the original network in the sense that they will have the same rate matrix, after the states in which a node is blocked by a node with B_i=K-1 are combined into one state.

Proof: Without loss of generality, in CQN-B-1, let node j be such that B_j=K-1. Let CQN-B-2 be identical to CQN-B-1, except that B_j=K. The only difference between these two networks is that in CQN-B-1, node j may cause blocking of a node while node j in CQN-B-2 will not cause any blocking. Let $P^{B_j=K}(.)$,$P^{B_j=K-1}(.)$ be the steady state probability distributions of CQN-B-2 and CQN-B-1 respectively. Then if node j is not blocking any node, we have

$$P^{B_j=K}(.)=P^{B_j=K-1}(.)$$

Otherwise, if node j is blocking some node l, we have from Theorem 1:

$$P^{B_j=K-1}(0,\ldots,i_l,\ldots,B_j+l,0,\ldots,0) = \frac{e_l p_{lj}}{e_j} P^{B_j=K}(0,\ldots,B_j,\ldots,0)$$

Furthermore,

$$\sum_{l=1}^{N} P^{B_j=K-1}(0,\ldots,i_l,\ldots,B_j+l,0,\ldots,0) = \sum_{l=1}^{N} \frac{e_l p_{lj}}{e_j} P^{B_j=K}(0,\ldots,B_j,\ldots,0)$$

$$= P^{B_j=K}(0,\ldots,B_j,\ldots,0)$$

Thus, under the above aggregation, the two queueing networks have the same rate matrix. Note that, in the above proof we assume that $p_{jj}=0$. Also, we give the proof assuming that there is only one node with $B_j=$K. If $p_{jj}>0$ then, we have

$$P^{B_j=K-1}(0,..,i_l,..,B_j+l,0,..,0) = \frac{e_l p_{lj}}{e_j(1-p_{jj})} P^{B_j=K}(0,..,B_j,..,0)$$

If more than one node has the buffer capacity K-1, then the above discussion should be repeated for all those nodes.

Below, we will summarize some of the well known results about the throughput of CQN-I. Let $\beta_i(K)$ be the throughput of node i and $\beta(K)$ be the throughput of the network with K customers in it. Then:

$$\beta(K)=\frac{K}{\sum_{m=1}^{K}\sum_{j=1}^{m-1}\beta(K-j)\sum_{i=1}^{N}x_i^m} \tag{6}$$

where $x_i=e_i/\mu_i$ is the relative utilization of node i, and e_i is the mean number of visits a customer makes to ith node.

Furthermore,

$$\beta_i(K)=\beta(K)\ e_i \qquad i=1,..,N \tag{7}$$

and $\beta(K)$ is monotonically increasing on K and bounded from above. This upper bound is given by:

$$u= \min_{i=1,..,N} \{p_{ij}\ \mu_i,\ j=1,..,N;\ j\neq i\} \tag{8}$$

For CQN-B, let $\lambda_i(K)$ and $\lambda(K)$ be the throughput of node i and the throughput of the network respectively assuming K customers . Then we have,

$$\lambda_i(K)=(1-P_i^K(0)-P_i^K(b))\mu_i \tag{9}$$

where $P_i^K(0)$ and $P_i^K(b)$ are the probabilities that node i is empty and blocked respectively given there are K customers in the network. Also,

$$\lambda_i(K)=\lambda(K)\, e_i, \quad i=1,..,N, \qquad (10)$$

where e_i is given by (1).

Clearly, $\lambda_i(K)$ depends on the parameters of the network. In figures 3 to 6, we give four examples of $\lambda_3(K)$ as K changes from 1 to $M(=\sum_{i=1}^{N} B_i)$ for the networks shown in figures 1 and 2.

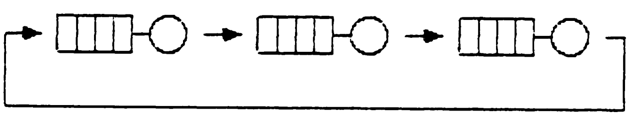


Figure 1

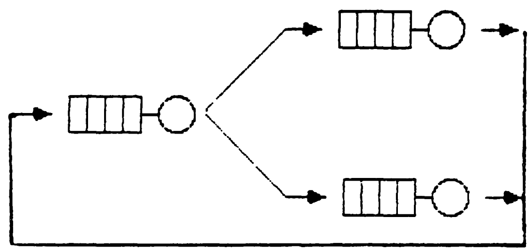


Figure 2

We note that in figures 3 to 6, $\lambda_3(K)$ increases as K increases until it reaches a maximum value, λ^*, for some K^*, $K^* \epsilon \{L:\lambda_3(L) \geq \lambda_3(i), i=1,..,M\}$ where the set can be a singleton or can have more than one element. For $K>K^*$, $\lambda_3(K)$ is non-increasing on K. The behavior of these graphs can be explained as follows: $P_3^K(0)$ decreases, as K increases, until it reaches 0 at $K=M-B_3+1$. This value of K is such that the number of holes (i.e. free spaces) in the queueing network, M-K, is equal to B_3-1. That is, in all states queue 3 will contain at least one customer. For $M-B_3+1\leq K \leq M$, $P_3^K(0)=0$. $P_3^K(b)=0$ for $1\leq K\leq B_1$ and non-decreasing beyond B_1. Hence, $P_3^K(0)+p_3^K(b)$ is non-increasing in the interval

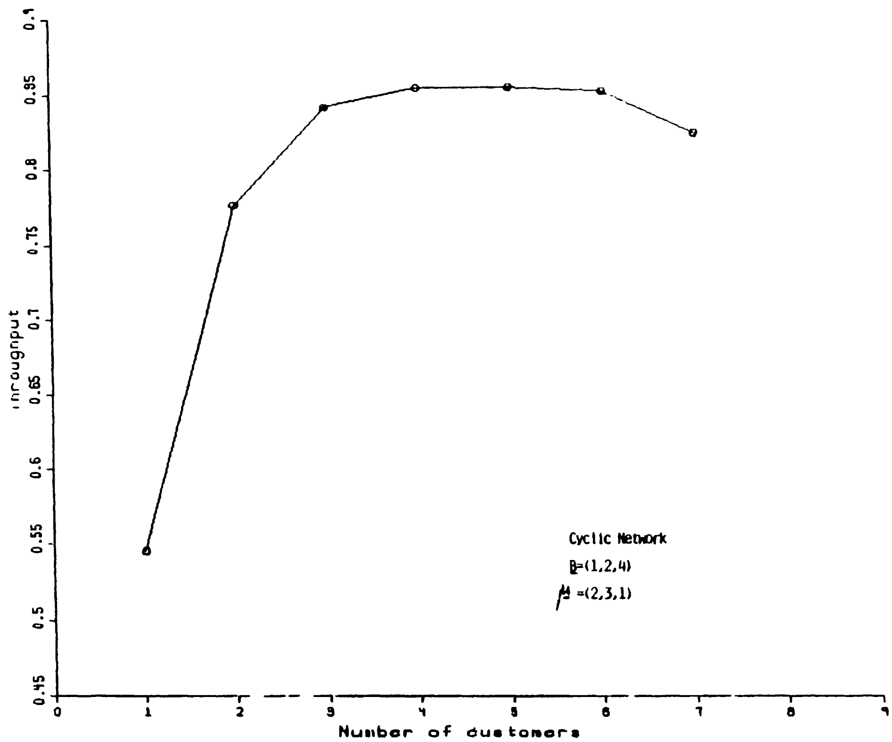
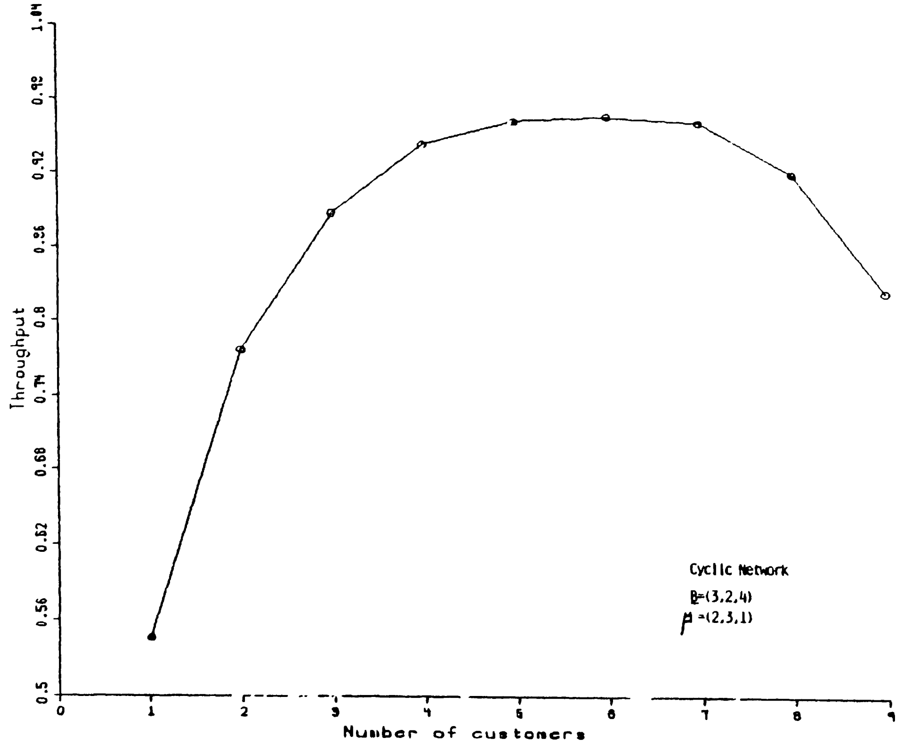


Fig. 3:Throughput vs # of customers-



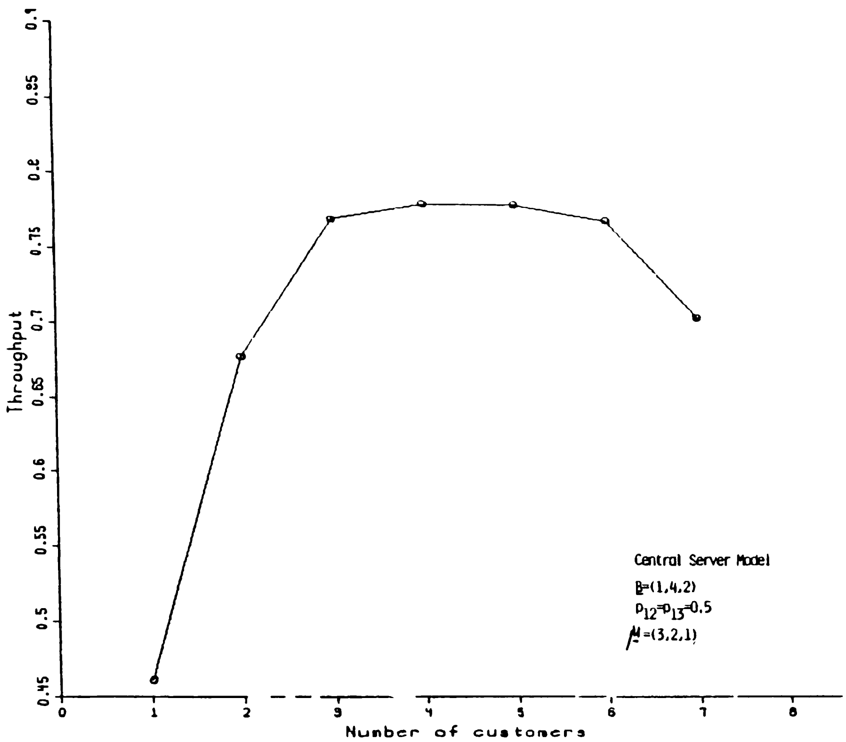


Fig. 5:Throughput vs # of customers

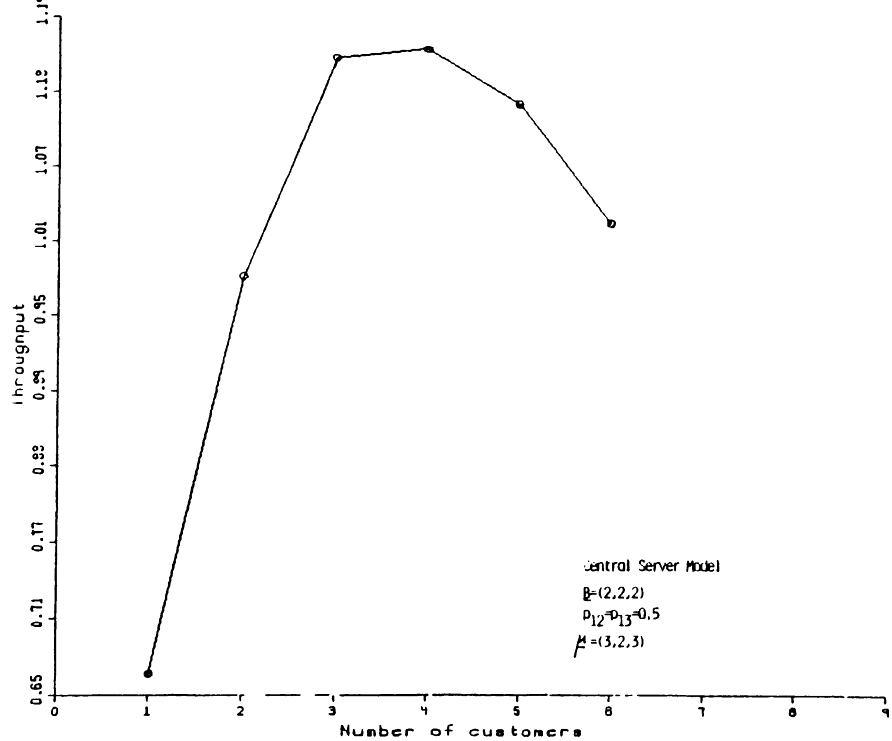


Fig. 6: Throughput vs # of customers

$[1,B_1]$ and non-decreasing in $[M-B_3+1,M]$. It is not clear what happens in the interval $[B_1+1,M-B_3]$. Empirically, we have observed that there is a point $K^* \in [B_1+1,M-B_3]$ where $P_3^K(0)+P_3^K(b)$ continues to be non-increasing in $[B_1+1,K^*]$ and non-decreasing in $[K^*+1,M-B_3]$.

In the derivations given below, we will use the following two properties without any proof.

Property 1:$P_i^K(0)$ decreases as K increases until it reaches zero.

Property 2:For the range of values that $P_i^K(b)\neq 0$, $P_i^K(b)$ increases as K increases

THEOREM 2: Let $\lambda^* = \max_K\{\lambda(K)\}$, $n = \min_{i=1,\ldots,N}\{B_i\}$ and $M = \sum_{i=1}^{N} B_i$. Then

$$\beta(n+1) \leq \lambda^* \leq \beta(M - \max_{i=1,\ldots,N}\{B_i\}+1)$$

Proof: For $1 \leq K \leq n+1$, the network has product form solution. Hence, $\beta(1)<\beta(2)<\ldots<\beta(n+1)$,and therefore, $\lambda^* \geq \beta(n+1)$. We first note that $\lambda_i(K)e_i = \lambda(K)$. Also, λ_i is non-decreasing in the interval $[M-B_i+1,M]$. Now, let queue j be such that $B_j = \max_{i=1,\ldots,N} B_i$. Then, $M-B_j+1 \leq M-B_i+1$, $i=1,\ldots,N$. Thus, $M-B_j+1$ is the first point past which we can show that $\lambda(K)$ is non-increasing. Seeing that in general $\lambda(K) \leq \beta(K)$, $K=1,\ldots,M$, we have that $\lambda^* \leq \lambda(M-B_j+1) \leq \beta(M-B_j+1)$, or that $\lambda^* \leq \beta(M-B_j+1)$.

COROLLARY 2:Let K^* be such that $\lambda^* = \lambda(K^*)$. Then,

$$n+1 \leq K^* \leq M - \max_{i=1,\ldots,N}\{B_i\}+1$$

COROLLARY 3:Let K^* be defined as in Corollary 2. Then

$$\max_{i=1,\dots,N} \{ \min_{j=1,\dots,N} \{B_j \text{ such that } p_{ij} \neq 0\}\} \leq K^*.$$

Proof: Consider any node i. Let $K'_i = \min_{i=1,\dots,N} \{B_j : \text{s.t. } p_{ij} \neq 0\}$, that is for node i K'_i is the smallest downstream buffer capacity. For $K \leq K'_i$, node i can not get blocked and therefore $P_i(b)=0$. In view of this , throughput should increase as K increases from 1 to K'_i. Now, seeing that this is true for all nodes, we have that $K^* \geq \max_{i=1,\dots,N} K_i$.

THEOREM 3:Let $M = \sum_{i=1}^{N} B_i$, $B^* = \max_{i=1,\dots,N} \{B_i\}$

and $S=\{L: M-B^*+1 \leq L \leq B^*+1\}$. If $2B^* \geq M$ (i.e. S is not empty) then the network with K customers in it has the same steady state queue length distribution for all $K \in S$.

Proof:Without loss of generality let node j be of capacity B^*. Consider all nodes other than node j. Then, there is a state in which all nodes are full, another state in which all nodes are empty and states with all combinations in between. Hence, the transitions between these nodes are independent of K. For a state $(i_1,..,i_N)$, we have $i_j = K - \sum_{\substack{l=1 \\ l \neq j}}^{N} b_l$ where $b_l = \begin{cases} i_l & \text{if } i_l < B_l \\ B_l & \text{if } i_l \geq B_l \end{cases}$

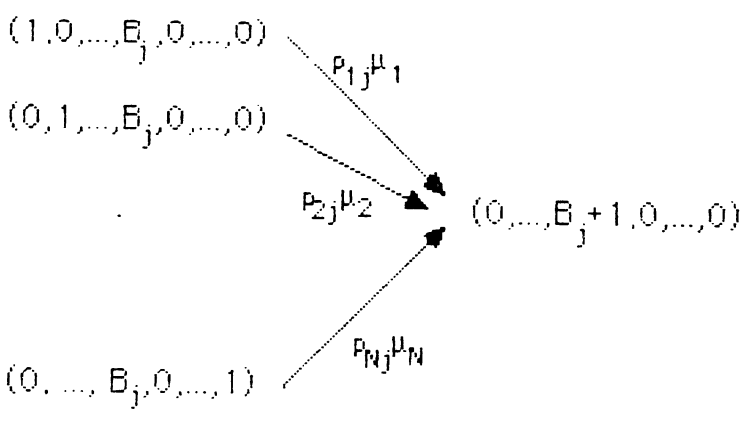
(Note that $i_l > B_l$ denotes that some node(s) are blocked by node l).

For $M-B^*+1 \leq K \leq B^*$ node j can not block any node in the network and can not be empty. Hence, transitions into and out of node j are independent of K. Therefore,for $M-B^*+1 \leq K \leq B^*$, we have the same rate matrix. That is, for K and K' such that $M-B' \leq K < K' \leq B'+1$ we have

$P^K(i_1,...,i_N)=P^{K^*}(i_1,...,i_j+K^*-K,...,i_N)$ where $P^K(.)$ and $P^{K^*}(.)$ are the steady state queue length distributions with K and K^* customers in the network respectively.

To complete the proof, we need to show that the networks with K= B^* and K=B^*+1 customers are also equivalent. However, this is immediate from Corollary 1. Hence, the networks with K ϵ S customers have the same steady state queue length distributions. As it was in the case of Corollary 1, both networks will have the same rate matrix after all the states where node j blocks some node i are aggregated into one state.

For a better understanding of the application of Corollary 1 in the above proof note that we only need to consider node j with K=B^* and K=B^*+1 customers in the network. Let $Z_j=\{i:p_{ij} \neq 0\}$ be the set of nodes that can get blocked by node j. Furthermore, when K=B^*, Z_j is empty and when K=B^*+1, there can be at most one node blocked by node j at a time. As before, let $(0,..,i_l=1,..,i_j=B_j+l,..,0)$ denote that node l is blocked by node j, and collect all these states into one state, say $(0,..,B_j+1,..,0)$ with the rates into it as follows:



The rate out of state $(0,..,B_j+1,0,..,0)$ is μ_j which is the same rate out of state $(0,..,i_l,...B_j+l,..0)$ for $l\epsilon Z_j$. Then, the networks with B^* and B^*+1 customers in it have the same rate matrix after all states where node j is blocking some node l, $l\epsilon Z_j$, are combined into a macro state with rates given above. Furthermore, $P^{B_j+1}(0,...,i_l=B_j+1,0,...0) = P^{B_j}(0,..,B_j,0,...,0)$, and $P(0,...,i_l,...B_j+l,0,...0)$ are readily available from Corollary 1.

COROLLARY 4:If there exists a j such that $2B_j \geq$M then increasing the buffer capacity of node j will not change the value of the maximum throughput.

Proof: We first note the trivial fact that if there is a node j with buffer capacity $B_j \geq M/2$ then this node has the largest buffer, seeing that no other node can have a buffer capacity $\geq M/2$. Now, let B_j be increased to B'_j Let.

$M' = \sum_{i=1}^{N} B_i + B'_j$, and let S and S' be the sets defined in theorem 3, when the buffer capacity of node j is B_j and B'_j respectively. Clearly S$\cup S'$. Using theorem 3, for K$\epsilon S'$ the queueing network has the same rate matrix whether node j has the buffer capacity B_j or B'_j. Therefore, increasing the buffer capacity of node j will only result in more points in the set S and hence will not change the value of the maximum throughput.

THEOREM 4:Let μ_i, p_{ij} be the parameters of two closed queueing networks with buffer capacities B_i and C_i, i=1,..,N; j=1,..,N. Let l be an integer such that $0 \leq l \leq \min\{ \min_{i=1,...,N} B_i - 1, \min_{i=1,...,N} C_i - 1\}$, then the two networks with $\sum_{i=1}^{N} B_i - l$

and $\sum_{i=1}^{N} C_i - l$ customers respectively have the same rate matrix.

Proof: Note that, in both networks, each node contains at least one customer and that the two networks have the same state space. Let $d_i = B_i - C_i$, i=1,..,N. Then a state $(i_1,...,i_N)$ of the network with buffer capacities B_i is equivalent to the state $(i_1-d_1,...,i_N-d_N)$ of the network with buffer capacities C_i. Equivalency here is used to state that both states have the same transition rates into and out of corresponding equivalent states.

COROLLARY 5:Let M be the total capacity of the network. Then, $\lambda(M)$ is independent of buffer capacities B_i, i=1,...,N.

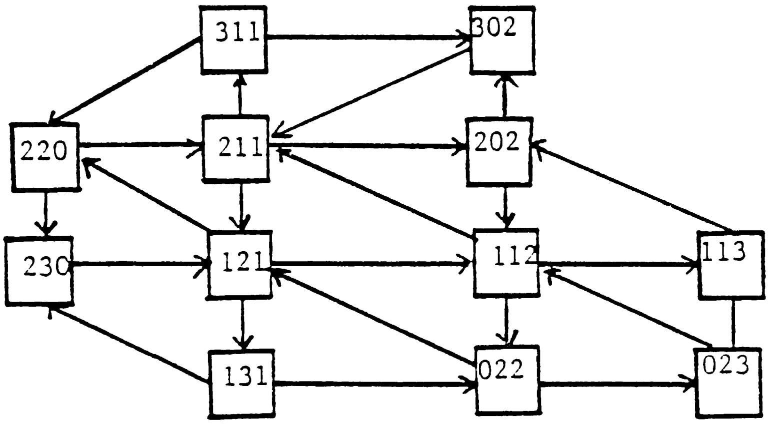
COROLLARY 6:For all K> $\sum_{j=1}^{N} B_j - \max_{i=1,...,N} B_i + 1$, the CQN-B with buffer capacities B_i and K customers has the same rate matrix as the network with buffer capacities C_i and K+l customers if $C_j = B_j + l$ for some j and $C_i = B_i$ for i=1,..,N and i≠ j.

4.SPECIAL CASES

In this section, we will study three special cases involving symmetric queues. For each case, we give an algorithm to calculate the steady state queue length distribution.

4.1.Cyclic Networks-Symmetric Queues

A cyclic network is a closed queueing network consisting of queues in tandem as shown in Figure 2. Assume that $B_i = B < \infty$ and $\mu_i = \mu$, i=1,..,N. Then, 1≤K≤NB. The algorithm given in this section utilizes an aggregate state space obtained from the original state space of the network after it is reduced by a factor of N. For presentation purposes, consider a cyclic network with B=2, K=4 and N=3. The state space has the following structure with all transition rates equal to μ.



Solving the system numerically (see Perros, Nilsson and Liu [11]), we have:

$$P(2,2,0)=P(0,2,2)=P(2,0,2)=0.071429$$

$$P(2,3,0)=P(0,2,3)=P(3,0,2)=0.11905$$

$$P(2,1,1)=P(1,2,1)=P(1,1,2)=0.095238$$

$$P(3,1,1)=P(1,3,1)=P(1,1,3)=0.047619$$

This result is not surprising seeing that nodes are indistinguishable. In view of this, let us define the following classes, where a state is a member of a class if that state has the same steady state probability as all the other states in the same class.

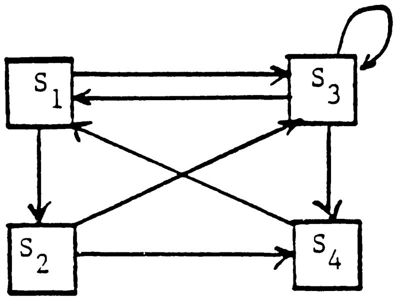
$$S_1=\{(2,2,0),(0,2,2),(2,0,2)\}$$

$$S_2=\{(2,3,0),(0,2,3),(3,0,2)\}$$

$$S_3=\{(2,1,1),(1,2,1),(1,1,2)\}$$

$$S_4=\{(3,1,1),(1,3,1),(1,1,3)\}$$

Then, we have the following state space structure for these equivalence classes with all transition rates equal to μ.



Solving this system numerically, we have:

$P(S_1)$=0.214287; $P(S_2)$=0.35715; $P(S_3)$=0.28571; $P(S_4)$=0.142857

Furthermore, $P(S_i) = \sum_{(i_1,i_2,i_3)\in S_i} P(i_1,i_2,i_3)$, i=1,..,4. Hence, to solve the original network, we can form the equivalence classes ,S_i, create the rate matrix for these classes and solve the system. Then we can obtain the queue length distribution of the original network. The following algorithm summarizes this procedure.

ALGORITHM

1.Generate the equivalence classes, S_i, and set up the rate matrix.

2.Solve the system to obtain $P(S_i)$.

3.Calculate the normalizing constant, G_K, for the original network as follows:

$$G_K = \sum_{i=1}^{S} R_i P(S_i)$$

where S is the number of equivalence classes and R_i is the number of states in equivalence class i.

4.$P(i_1,...,i_N) = G_K^{-1} P(S_i)$ where $(i_1,...,i_N) \in S_i$

The equivalence class of a state $(i_1,\ldots,i_N)$, S_i, can be found with the following algorithm:

```
procedure equiliv((i_1,...,i_N));
  for k:=2 to N do
    begin
      i:=0;
      for l:=k to N do
        begin
          i:=i+1; R(i):=i_l
        end;
      j:=1;
      for l:=i+1 to N do
        begin
          R(l):=i_j; j:=j+1
        end;
      S_i:=S_i+(R(1),...,R(N))
    end;
```

4.2.Central Server Model-Symmetric Queues

A central server model is shown in Figure 7. Each node has a single server, buffer capacity B_i, and service times are exponentially distributed with rate μ_i. p_{1i} is the probability that a customer upon completion of its service at node 1 attempts to go to node i, i=2,...,N.

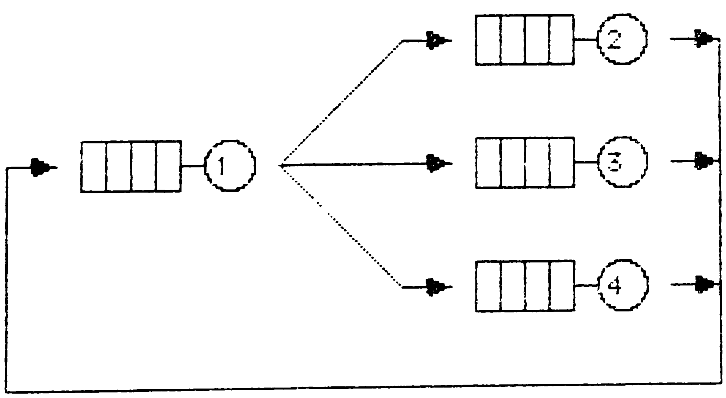


Fig.7:Central server model with symmetric queues

Let us assume that B_i=B $< \infty$ and $\mu_i=\mu$,i=2...,N. Note that, in general, $\mu_1 \neq \mu$ and $B_1 \neq B$. Furthermore, let $p_{1i} = 1/(N-1)$, i=2,..,N. Then nodes 2 through N are indistinguishable and hence the algorithm given for the case discussed in 4.1 can be applied to nodes 2 through N.

4.3:Merge Configuration-Symmetric Queues

Let us assume that ,in figure 7, $B_1 < \infty$ and $B_i = \infty$, and $p_{1i} = p = 1/(N-1)$, i=2,...,N. Then, the network has the local balance property if the service rates are modified as follows:

$$\mu_i = \begin{cases} \mu_i & i = 2,..,N \\ \mu_1 & i = 1 \text{ and } \textit{no node is blocked} \\ \dfrac{\mu_1}{p} & i = 1 \text{ and } \textit{some node(s) are blocked} \end{cases}$$

Hence, we have:

$$p\mu_1 P(i_1,...,i_j,...,i_N) = \mu_j \; P(i_1 - 1,..,i_j + 1,..,i_N) \quad \text{if } \textit{no node is blocked}$$

$$\mu_1 P(i_1 +^*,...,i_N = \mu_j P(i_1,...,i_N) \qquad \text{if } \textit{some node(s) are blocked.}$$

where $i_1 +^*$ denotes that node 1 is blocking a node.

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