The Departure Process of a Discrete-Time Finite Capacity System with Correlated Arrivals

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Abstract

Queueing network models have proven to be very useful in the analysis of communication systems. The departure process of a queue in a queueing network is of special interest because it is the arrival process to other queues in the network. For high speed networks, correlation and burstiness are very important factors for system performance and thus the smooth Bernoulli process is no longer a good assumption for an arrival process of the network traffic in a discrete-time system. We have investigated the Markov Modulated Bernoulli process (MMBP), which can adequately capture the properties of both burstiness and correlation, as a model for the arrival processes of high speed network traffic. In this paper, the departure process of a discrete-time finite capacity queue, which has an MMBP arrival process, is derived. First, the MMBP/Geo/1/K queue is studied by using a multi-dimensional Markov chain. The MMBP/D/1/K queue, which is a special case, is also investigated. The queue length distributions and blocking probabilities for both systems are derived. Furthermore, the exact solution for the departure process of the MMBP/Geo/1/K queue is investigated by using the knowledge of the queue length distribution. The generating function of the interdeparture interval time and the first four moments of this quantity are obtained. Finally, the departure process was fitted to a two-state MMBP by using a four moments matching technique.

1 Introduction

In queueing networks, the departure process of a queue is of special interest because it can be the arrival process to other queues. In this paper, the queue length distribution, the blocking probability, and the departure process of the MMBP/Geo/1/K queue are studied through a Markov chain analysis. Since the simulation results of the departure process of the MMBP/Geo/1/K queue indicate that the successive interdeparture times are correlated, the departure process is approximately characterized as a two-state Markov-Modulated Bernoulli process(MMBP) by using a four moments matching method. This matching technique was validated by comparisons against simulation data.

The particular motivation for studying the departure process of this discrete-time finite capacity queue stemmed from a study of Asynchronous Transfer Mode(ATM) networks. ATM is a high bandwidth, fast packet switching, and multiplexing technique for broadband ISDN [1]. In an ATM network, all information ranging from narrowband voice and data traffic to broadband video traffic is transmitted using a fixed size "cell". Normally, an ATM network consists of a number of nodes which interact each other. To evaluate the performance of this system, an approximation technique for analyzing large complex queueing system is used. That is, the queueing network that is a model of the network is decomposed into individual queues that are analyzed independently and then recombined. It is apparent that in order to analyze the queueing network by this method, the departure process of each queue in the network will be critical for solving the rest of the queues. Since most of the traffic that an ATM network supports is highly bursty and correlated, the Bernoulli Process, a traditional assumption for the network traffic, may lead us to erroneous conclusions [2]. Under this consideration, an MMBP, which can capture both the burstiness and correlation properties of traffic in high speed networks is adopted as a model for the traffic arrival process [3] [4] [5]. In this paper, we consider the MMBP/Geo/1/K queue and the MMBP/D/1/K queue, a special case of the MMBP/Geo/1/K queue, and we specifically determine the departure process of the MMBP/Geo/1/K queue. Recently, maximum likelihood estimation approach was used to estimate the parameters of the Markov-modulated Poisson process (MMPP) [6] [7]. In this paper, we will fit the departure process of the MMBP/Geo/1/K queue to a two-state MMBP by using the four moments match method.

This paper is organized as follows: In Section 2 we briefly describe the MMBP/Geo/1/K queue and we obtain the queue length distributions and blocking probabilities for the MMBP/Geo/1/K and MMBP/D/1/K queues by a Markov chain analysis. In Section 3 the derivation of the generating function of the interdeparture time distribution for the MMBP/Geo/1/K queue and the first four moments of the interdeparture time are presented. In Section 4 we report on the first four moments of the interarrival time distribution of an MMBP. The program "Interopt" [8] is then used to fit the departure process of the MMBP/Geo/1/K queue to a two-state MMBP. It is verified that our fitting approach provides a satisfactory accuracy by comparing the four moments of the interdeparture time against the moments of the interarrival time in the MMBP.

2 The queue length distributions of the MMBP/Geo/1/K and MMBP/D/1/K queues

The m state MMBP is a doubly stochastic point process whose arrival phase process for each slot is governed by an m-state irreducible Markov chain [9] [10]. The dwell time at phase i of the arrival phase process is geometrically distributed. We further assume that if the n-th slot is in state i, (i = 1,2,...,m), an arrival occurs according to a Bernoulli process with rate λ_i. For simplicity we focus on a two state MMBP as shown in Figure 1. If the two state MMBP is in the state 1 (state 2) in the nth slot, in the (n+1)st time slot it will remain in the state 1 (state 2) with probability p (q), or it will change to the state 2 (state 1) with probability 1-p (1-q). In the state i, (i = 1, 2), arrivals occur in a Bernoulli fashion with rate λ_i. We assume that this process is the arrival process to a single server finite capacity

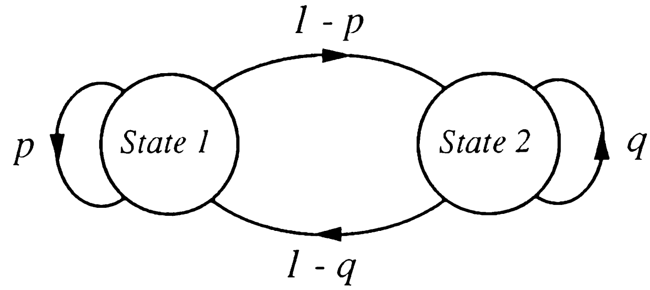

Figure 1: Markov Chain of Two State MMBP.

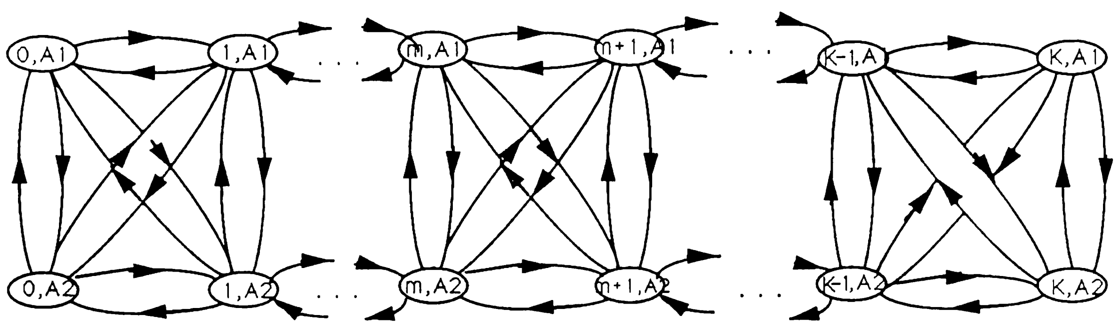

Figure 2: The Two dimensional Markov Chain of the MMBP/Geo/1/K Queue.

queue with geometric service time with rate σ.

In order to obtain the steady state queue length distribution $\pi_{j,1}$ and $\pi_{j,2}$ of this MMBP/Geo/1/K queue, where $\pi_{j,i}$ stands for the steady state probability of j customers in the queue and arrival embedded state i (i = 1,2), we observe the system at the slot points and construct the Markov chain [11]. In this Markov chain, shown in Figure 2, there are 2(K+1) states denoted by (m,A_1) and (m,A_2) (m=0,1,...,K), where (m,A_1) represents state 1 with m jobs in the system; (m,A_2) represents state 2 with m jobs in the system. Note that K represents the total number of jobs permitted in this system, i.e., in server and in queue. The state changes in the Markov chain can only be caused by:

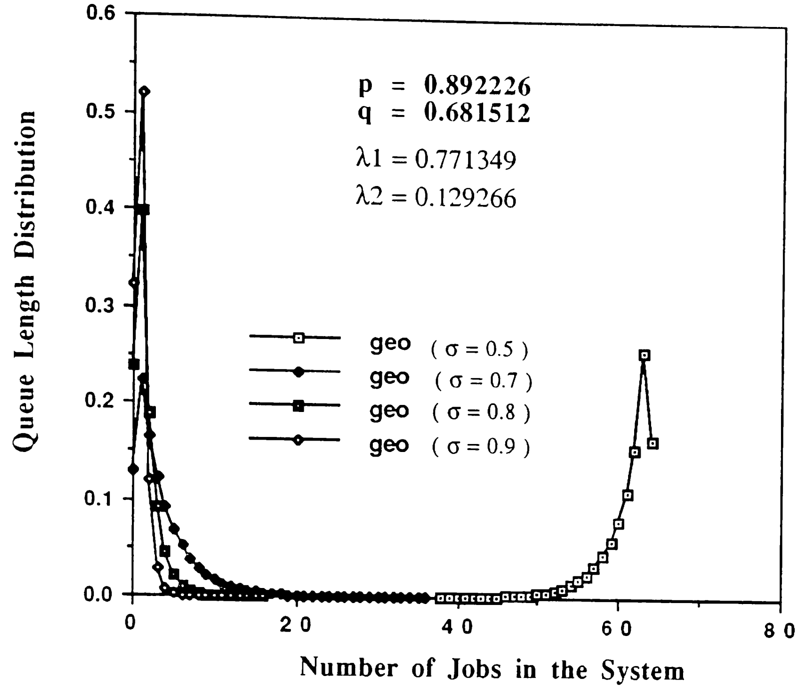

Figure 3: MMBP/Geo/1/K Queue Length Distribution w.r.t. Different Service Rate σ .

1) a state change between state 1 and state 2; this can only happen at the beginning of the current slot, or,

2) an arrival; this can only happen after the potential state change point between state 1 and state 2, or,

3) a departure, this can only happen at the end of the slot point. Note that the potential arrival at the current slot cannot be served until the beginning of the next slot.

By solving the Global Balance Equations of the Markov chain, the queue length distribution of the MMBP/Geo/1/K queue is obtained. Figure 3 shows the queue length distribution of the MMBP/Geo/1/K queue for the given parameters p, q, λ_1, and λ_2. As a special case, when the system service time is constant, i.e., σ =1, the MMBP/Geo/1/K queue can be

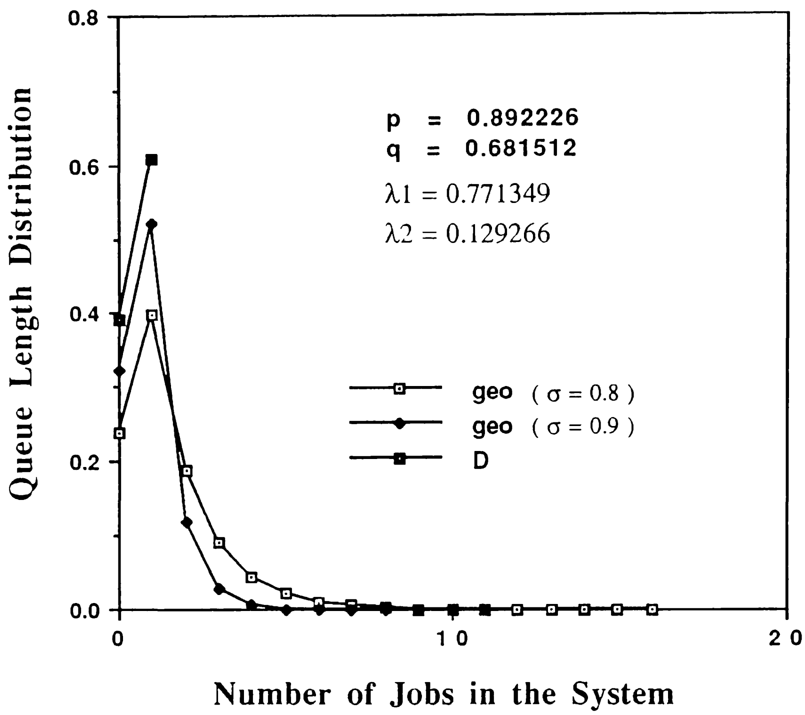

Figure 4: MMBP/D/1/K Queue Length Distribution Compared to MMBP/Geo/1/K System w.r.t. Different Service Rate σ.

reduced to an MMBP/D/1/K queue. The result for the queue length distribution of the MMBP/D/1/K queue is presented in Figure 4.

From the knowledge above, the blocking probability of the MMBP/Geo/1/K queue can be derived as follows:

$$Prob(block) = B_K[A_1] + B_K[A_2],$$

where $B_K[A_i]$ represents the probability that the arrival sees K customers in the system and in the current slot the state of the Markov chain in the arrival process is i (i = 1, 2).

$$B_K[A_1] = \frac{(2-p-q)\lambda_1}{(1-q)\lambda_1 + (1-p)\lambda_2}[\pi_{K,1}p + \pi_{K,2}(1-q)],$$

$$B_K[A_2] = \frac{(2-p-q)\lambda_2}{(1-q)\lambda_1 + (1-p)\lambda_2}[\pi_{K,1}(1-p) + \pi_{K,2}q].$$

Thus,

$$Prob(block) = \frac{(2-p-q)}{(1-q)\lambda_1 + (1-p)\lambda_2}[(p\lambda_1 + (1-p)\lambda_2)\pi_{K,1} + (q\lambda_2 + (1-q)\lambda_1)\pi_{K,2}].$$

3 The departure process of MMBP/Geo/1/K queue

Having developed a method to solve the queue length distributions of the MMBP/Geo/1/K queue, in this section we will use some of the results in section 2 to obtain the generating function of the interdeparture time and the first four moments of interdeparture time.

Denote by the random variable d the interdeparture time of the MMBP/Geo/1/K queue, and by j the number of jobs the current departure leaves in the system. We have

$$d = \begin{cases} t_s & \text{if } j \geq 1 \\ t_s + R_{A_1} & \text{if } j = 0 \text{ and next slot is in state 1} \\ t_s + R_{A_2} & \text{if } j = 0 \text{ and next slot is in state 2} \end{cases} \tag{1}$$

where

R_{A_1} is the time interval from the current departure which leaves the buffer empty to the next arrival plus one more slot since the new arrival will not be served until the next slot, and the next slot immediately after the current departure is in state 1;

R_{A_2} is the time interval from the current departure which leaves the buffer empty to the next arrival plus one more slot, and the next slot immediately after the current departure is in state 2;

and t_s is the service time.

Let $d_0[A_1]$ ($d_0[A_2]$) be the probability that the departing job leaves the buffer empty and the next slot is in the state 1 (2); $d_0[A_1]$ and $d_0[A_2]$ are given as follows

$$d_0[A_1] = \frac{(p^2(1-\lambda_1) + (1-p)(1-q)(1-\lambda_2))\pi_{1,1} + (1-q)(p(1-\lambda_1) + q(1-\lambda_2))\pi_{1,2}}{1-\pi_0} \quad (2)$$

$$d_0[A_2] = \frac{(1-p)[p(1-\lambda_1) + (1-\lambda_2)q]\pi_{1,1} + [(1-q)(1-\lambda_1)(1-p) + q^2(1-\lambda_2)]\pi_{1,2}}{1-\pi_0} \quad (3)$$

$$1 - d_0[A_1] - d_0[A_2] = 1 - \frac{((1-\lambda_1)p + (1-p)(1-\lambda_2))\pi_{1,1} + ((1-\lambda_1)(1-q) + (1-\lambda_2)q)\pi_{1,2}}{1-\pi_0} \quad (4)$$

where, $\pi_{1,1}$ is the probability that there is one job in the buffer and the state is 1, $\pi_{1,2}$ is the probability that there is one job in the buffer and the state is 2, π_0 is the probability that the buffer is empty. $\pi_{1,1}$, $\pi_{1,2}$ and π_0 can of course be obtained from the queue length analysis in the previous section. We note that

$$R_{A_1} = \begin{cases} 1 & , w.\ prob.\ \lambda_1 \\ 1 + R_{A_1} & , w.\ prob.\ (1-\lambda_1)p \\ 1 + R_{A_2} & , w.\ prob.\ (1-\lambda_1)(1-p) \end{cases} \quad (5)$$

and

$$R_{A_2} = \begin{cases} 1 & , w.\ prob.\ \lambda_2 \\ 1 + R_{A_1} & , w.\ prob.\ (1-\lambda_2)(1-q) \\ 1 + R_{A_2} & , w.\ prob.\ (1-\lambda_2)q. \end{cases} \quad (6)$$

It is found that

$$E[z^d] \;=\; E[z^{t_s}]\ (\ 1 - d_0[A_1] - d_0[A_2] + E[z^{R_{A_1}}]d_0[A_1] + E[z^{R_{A_2}}]d_0[A_2]\ ), \quad (7)$$

where

$$E[z^{R_{A_1}}] \;=\; z(\lambda_1 + E[z^{R_{A_1}}](1-\lambda_1)p + E[z^{R_{A_2}}](1-\lambda_1)(1-p)), \quad (8)$$

$$E[z^{R_{A_2}}] \;=\; z(\lambda_2 + E[z^{R_{A_1}}](1-\lambda_2)(1-q) + E[z^{R_{A_2}}](1-\lambda_2)q), \quad (9)$$

and

$$E[z^{t_s}] \;=\; \frac{z\sigma}{1 - z(1-\sigma)}. \quad (10)$$

By solving the equations (7)-(10), the generating function of the interdeparture time distribution, $E[z^d]$, is obtained.

Now, we will focus on the first four derivatives of the generating function of the interdeparture time. For simplicity, let us introduce the compact and obvious notations $D[z] = E[z^d]$, $R_{A_1}[z] = E[z^{R_{A_1}}]$, $R_{A_2}[z] = E[z^{R_{A_2}}]$, and $T_s[z] = E[z^{t_s}]$.

The four derivatives of the generating function D[z] can be calculated as follows

$$D^{(1)}[1] = T_s^{(1)}[1] + a \tag{11}$$

$$D^{(2)}[1] = T_s^{(2)}[1] + 2T_s^{(1)}[1]a + b \tag{12}$$

$$D^{(3)}[1] = T_s^{(3)}[1] + 3T_s^{(2)}[1]a + 3T_s^{(3)}[1]b + c \tag{13}$$

$$D^{(4)}[1] = T_s^{(4)}[1] + 4T_s^{(3)}[1]a + 6T_s^{(2)}[1]b + 4T_s^{(1)}[1]c + e \tag{14}$$

where

$$a = d_0[A_1]R_{A_1}^{(1)}[1] + d_0[A_2]R_{A_2}^{(1)}[1] \tag{15}$$

$$b = d_0[A_1]R_{A_1}^{(2)}[1] + d_0[A_2]R_{A_2}^{(2)}[1] \tag{16}$$

$$c = d_0[A_1]R_{A_1}^{(3)}[1] + d_0[A_2]R_{A_2}^{(3)}[1] \tag{17}$$

$$e = d_0[A_1]R_{A_1}^{(4)}[1] + d_0[A_2]R_{A_2}^{(4)}[1] \tag{18}$$

$$T_s^{(1)}[1] = \frac{1}{\sigma} \tag{19}$$

$$T_s^{(2)}[1] = \frac{2(1-\sigma)}{\sigma^2} \tag{20}$$

$$T_s^{(3)}[1] = \frac{6(1-\sigma)^2}{\sigma^3} \tag{21}$$

$$T_s^{(4)}[1] = \frac{24(1-\sigma)^3}{\sigma^4}. \tag{22}$$

$R_A^{(i)}[1]$ and $R_I^{(i)}[1]$, (i = 1,2,3,4), can be derived by the following computation:

First, rewrite equations (8)(9) in matrix form as

$$(I - zW)\begin{bmatrix} R_{A_1}[z] \\ R_{A_2}[z] \end{bmatrix} = z\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \tag{23}$$

where

$$W = \begin{bmatrix} (1-\lambda_1)p & (1-\lambda_1)(1-p) \\ (1-\lambda_2)(1-q) & (1-\lambda_2)q \end{bmatrix}. \tag{24}$$

Differentiating equation (23) and taking z=1, we have

$$\begin{bmatrix} R^{(k)}_{A_1}[1] \\ R^{(k)}_{A_2}[1] \end{bmatrix} = k\,!(I-W)^{-k}W^{k-1}e \tag{25}$$

where $e = [1,1]^T$, and $k = 1, 2, 3, \ldots$. By using (11)-(25), the first four moments of the interdeparture time are obtained.

The departure process of the MMBP/Geo/1/K queue has been found through simulation when p = 0.892226, q = 0.681512, λ_1 = 0.771349, λ_2 = 0.129266, σ = 0.8, and K = 64. The simulation result indicates that the successive interdeparture times are correlated, and the mean and C^2 of the interdeparture time is 1.67133 and 0.40454, respectively. C^2 is the squared coefficient of variation defined as the variance divided by the mean squared. Based on the equations (11)-(25), the analytical results for the mean and C^2 related to the given parameters are 1.67243 and 0.40508, respectively.

4 Matching the departure process to an MMBP

In this section, we characterize the departure process of the MMBP/Geo/1/K queue as an MMBP. Since simulation result reported on in the previous section indicates that the successive interdeparture times are correlated, neither a Bernoulli process nor an IBP is a reasonable candidate for the departure process. Instead we have selected the Markov Modulated Bernoulli process (MMBP), which can successfully capture both the burstiness and correlation properties, as a model for the departure process of the MMBP/Geo/1/K queue.

4.1 Markov Modulated Bernoulli process

Assume that the four parameters in a two state MMBP are $\sigma_{12}, \sigma_{21}, \alpha_1$, and α_2, corresponding to p, q, λ_1, and λ_2, respectively as defined above. Let us denote by Q the transition probability matrix of a two state MMBP.

$$Q = \begin{bmatrix} 1-\sigma_{12} & \sigma_{12} \\ \sigma_{21} & 1-\sigma_{21} \end{bmatrix}$$

We use the notation

$$\lambda = (\alpha_1 \ , \ \alpha_2)^T$$

$$\Lambda = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix}.$$

The steady-state vector of the Markov chain π is such that

$$\pi \ = \ \pi Q. \tag{26}$$

Let

$$p_i = Prob(\textit{the arrival comes from state } i \mid \textit{there is an arrival})$$

$$i = 1, \ 2,$$

we have

$$\begin{aligned} P &= (p_1, \ p_2) \\ &= (\frac{\pi_1\alpha_1}{\pi_1\alpha_1 + \pi_2\alpha_2}, \ \frac{\pi_2\alpha_2}{\pi_1\alpha_1 + \pi_2\alpha_2}). \end{aligned} \tag{27}$$

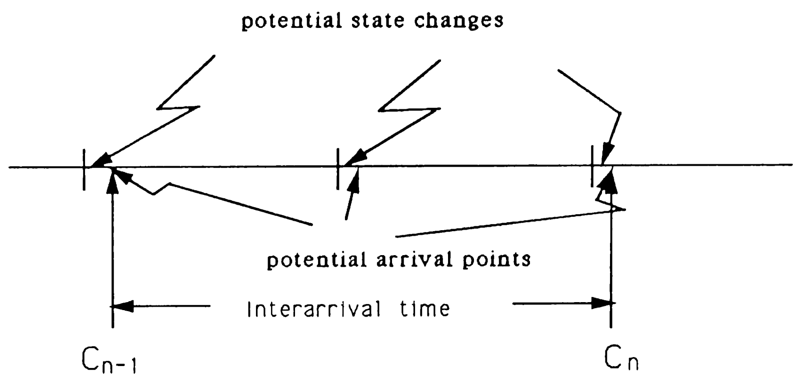

Figure 5: Interarrival Time .

4.2 Fitting the departure process of MMBP/Geo/1/K queue to an MMBP

From the description of an MMBP in the previous subsection, we know that the four parameters σ_{12}, σ_{21}, α_1 and α_2 must be known in order to get a model of an MMBP. Our approach is to match the first four moments of the interdeparture time for the MMBP/Geo/1/K queue against the four moments of the interarrival time of an MMBP. By using the program "Interopt", these four parameters can be found from the four equations related to the four moments. The program "Interopt" uses a simulated annealing approach to extract unknown parameter values from nonlinear equations.

For completeness we introduce in this section a condensed derivation of the MMBP interarrival time distribution. Let c_{n-1} and c_n be the (n-1)-st and n-th arrival, respectively, see Figure 5, and $t_{n,i}$ be the time interval from any slot point to the next arrival given the state in the current slot is i (i = 1, 2). Assume, as above, that the potential state switch point is at the beginning of each slot, and a potential arrival point is at the beginning of each slot immediately after the potential state switch point.

It follows that

$$t_{n,1} = \begin{cases} 1, & w.\ prob.\ (1-\sigma_{12})\alpha_1 + \sigma_{12}\alpha_2 \\ 1+t_{n,1}, & w.\ prob.\ (1-\sigma_{12})(1-\alpha_1) \\ 1+t_{n,2}, & w.\ prob.\ \sigma_{12}(1-\alpha_2), \end{cases} \tag{28}$$

and similarly

$$t_{n,2} = \begin{cases} 1, & w.\ prob.\ (1-\sigma_{21})\alpha_2 + \sigma_{21}\alpha_1 \\ 1+t_{n,1}, & w.\ prob.\ \sigma_{21}(1-\alpha_1) \\ 1+t_{n,2}, & w.\ prob.\ (1-\sigma_{21})(1-\alpha_2). \end{cases} \tag{29}$$

Let $A_1[z]$ and $A_2[z]$ denote the z-transforms of $t_{n,1}$ and $t_{n,2}$, respectively.

Trivially it is found that

$$A_1[z] = z((1-\sigma_{12})\alpha_1 + \sigma_{12}\alpha_2) + (1-\sigma_{12})(1-\alpha_1)A_1[z] + \sigma_{12}(1-\alpha_2)A_2[z] \tag{30}$$

$$A_2[z] = z((1-\sigma_{21})\alpha_2 + \sigma_{21}\alpha_1) + \sigma_{21}(1-\alpha_1)A_1[z] + (1-\sigma_{21})(1-\alpha_2). \tag{31}$$

A[z], the z-transform of the unconditional interarrival time is given as

$$A[z] = P \begin{bmatrix} A_1[z] \\ A_2[z] \end{bmatrix} \tag{32}$$

The k:th derivative of A[z] evaluated at z=1 is

$$A^{(k)}[1] = P \begin{bmatrix} A_1^{(k)}[1] \\ A_2^{(k)}[1] \end{bmatrix} \tag{33}$$

where, $A_1^{(k)}[1]$ and $A_2^{(k)}[1]$ can be found by the following calculations:

Equations (30)(31) can be written as

$$[I - zF] \begin{bmatrix} A_1[z] \\ A_2[z] \end{bmatrix} = z \begin{bmatrix} (1-\sigma_{12})\alpha_1 + \sigma_{12}\alpha_2 \\ (1-\sigma_{21})\alpha_2 + \sigma_{21}\alpha_1 \end{bmatrix}, \tag{34}$$

where

$$F = \begin{bmatrix} (1-\sigma_{12})(1-\alpha_1) & \sigma_{12}(1-\alpha_2) \\ \sigma_{21}(1-\alpha_1) & (1-\sigma_{21})(1-\alpha_2) \end{bmatrix}.$$

By differentiating equation (34) k times and putting z = 1, we get

$$\begin{bmatrix} A_1^{(k)}[1] \\ A_2^{(k)}[1] \end{bmatrix} = k\,!(I-F)^{-k}F^{k-1}e \tag{35}$$

where $e = [1,1]^T$, and $k = 1, 2, 3, \ldots$. Thus, the first four moments of the interarrival time can be obtained. Matching these four moments against the first four moments of the interdeparture time derived in the previous section, we have four equations from which the four parameters $\sigma_{12}, \sigma_{21}, \alpha_1$ and α_2 can be obtained efficiently, by using the program "Interopt".

Using the same group of parameters p, q, λ_1, λ_2 and K as we used in the previous section, the first four moments of the interdeparture time of the MMBP/Geo/1/K queue are found to be 1.67243, 2.25761, 4.63837 and 13.1338. By using the matching approach above, the four parameters σ_{12}, σ_{21}, α_1 and α_2 are approximately obtained as 0.540463, 0.248660, 0.365463 and 0.714913, respectively. Thus the departure process of the MMBP/Geo/1/K queue is characterized by an MMBP with $\sigma_{12} = 0.540463$, $\sigma_{21} = 0.248660$, $\alpha_1 = 0.365463$ and $\alpha_2 = 0.714913$.

In order to test the accuracy, the first four moments of the MMBP can be obtained by using the values of $\sigma_{12}, \sigma_{21}, \alpha_1$, α_2 and equations (33) and (35). They are 1.65344, 2.21795, 4.65732 and 13.1226, respectively, and the absolute errors for the four moments are 0.01899, 0.03966, 0.01895, and 0.0112. This example consequently indicates a good accuracy in our approach.

5 Conclusion

Queueing networks are very useful models for investigating the performance of communication systems. A simple way to study a queueing network is to approximately decompose it into individual queues that are analyzed independently and then recombined. The departure process of a queue becomes a critical issue since the departure process can be the arrival process of other queues. In this paper, the queue length distributions, blocking probabilities, and departure processes of the MMBP/Geo/1/K and MMBP/D/1/K queues are studied by a Markov chain analysis. Since the simulation results indicates that the interdeparture times are correlated, a Markov-Modulated Bernoulli process(MMBP), which can capture both burstiness and correlation properties of traffic in high speed network, may be a good approximation for modeling the departure process. A four moments fitting approach is used to match the departure process of the MMBP/Geo/1/K queue to a two state MMBP. It is verified through examples that our fitting approach provides a satisfactory accuracy by comparing the four moments of interdeparture time against the moments of interarrival time in the MMBP.

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