IMPROVED COUPLED EULER-LAGRANGE FINITE ELEMENT ANALYSIS OF THE FLUID-STRUCTURE DYNAMIC INTERACTION PROBLEM

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SUMMARY

Introduction. — Improved formulation are presented for the Finite Element Analysis of the linearized coupled structure-fluid with free surface interaction problem, during earthquake or flow induced vibrations. Eulerian coordinates are used for the compressible or incompressible fluid, and Lagrangian coordinates for the shell/solid containment and free surface waves. Assuming small displacements in the fluid, modes shapes and frequencies of the coupled system are obtained. The improvements reside in a systematic way of removing zero frequency modes by Lagrange multipliers, and the treatment of free surface waves in Lagrangian coordinates, resulting in a symmetric matrix equation system solvable with standard structural finite element programs. Examples are presented with PAM-AX3D, a finite element program for axisymmetric geometries and Fourier decomposition of non-axisymmetric earthquake loads.

Formulation. — Finite Element Analyses of the referred problem can be performed with methods of increasing complexity: simple added fluid masses, thin fluid (leaky) cylinder theory, Euler-Lagrange acoustic coupling. The first two methods are simple but incomplete. Euler-Lagrange coupling provides a precise formulation. However, published techniques lead to an unsymmetric matrix equation system, not solvable by most standard structural programs, unless simplifying assumptions are made, i.e.: neglecting the free surface waves, or the mass of all structural parts. To retain symmetry of the complete system in all cases, the following changes are proposed:
— the free surface waves are simulated by a massless Lagrange boundary line,
— for each independent fluid volume, an additional unknown is introduced (Lagrange multiplier) representing a generalized fluid pressure.

Finite element procedure. — The Lagrange zones with displacement unknowns, and the Eulerian zones with pressure unknowns are independently modelled, then coupled. Two procedures for solution can be followed :
— for incompressible fluid, the Eulerian pressure unknowns are eliminated. During this process Lagrange multipliers are generated and coupled to the displacement unknowns by stiffness matrices; a mass matrix is also generated and added to its Lagrange counterpart. A standard symmetric structural eigen-problem results, with added fluid mass and stiffness matrices.
— for compressible fluids, the Eulerian pressure unknowns are modified by introduction of the Lagrange multipliers. Similarly a symmetric matrix equation is generated.

Example problems. — Several interaction problems involving fluids vibrating in deformable containers are analyzed, to evaluate the proposed procedure. The Finite Element calculations, compared with sloshing experiments and an analytical solution, show excellent agreement.

Then a test problem for a nuclear vessel seismic analysis is studied. The axisymmetric steel vessel partially filled with liquid is analyzed for earthquake loading conditions for maximum sloshing waves and steel stresses, accounting for weight of internals linked to the shell. The first eigenvalues and vectors are computed, and the mode shapes and frequencies of the sloshing waves are shown.

Conclusion. — The proposed procedure provides an efficient and precise solution to the linearized free surface-fluid-structure dynamic interaction problem. In particular, it can account for weights of equipments fixed to the shell containment, or accomodate several independent fluid volumes as Eulerian superelements, still using standard structural finite element programs with symmetric equation solvers.

1. Introduction

During earthquake or flow induced vibrations, the interaction between shells, fluids and free surface waves is a complex phenomenon. In many cases, analytical solutions of a simplified version of the problem, or numerical solutions of approximate theories (i.e. added mass theory, thin fluid cylinders etc...) do not give acceptable results.

The complete linear differential equations which characterize the solid-fluid interaction (i.e. linearized Navier-Stokes equations for the fluid, and equations of motion for the solid) have been solved by Finite Element techniques (1,2). However, the resulting matrix equations are not symmetric, and cannot be easily incorporated into most programs used by the nuclear industry, which have symmetric equation solvers only. Symmetrisation procedures have been proposed (1 to 3) which do not retain all the important aspects of the problems, such as free surface (sloshing) waves, mass of the shell etc ...

This paper presents an improved Finite Element solution to the solid-fluid-free surface wave problem, which leads to a symmetric matrix equation, and still includes the basic phenomena : the usual linearized Navier-Stokes formulation for the fluid, the dynamic equilibrium for the solid and the motion of the free surface waves.

Two interaction problems involving fluids in deformable containers are shown to validate the proposed procedures.

2. Formulation

Finite Element analyses of the referred problem can be performed with methods of increasing complexity : simple added fluid masses, thin fluid (/leaky) cylinder theory, Euler-Lagrange acoustic coupling. The first two methods are simple, but incomplete. Euler-Lagrange coupling provides a precise formulation. However the standard solution technique leads to unsymmetric matrix systems unless simplifying assumptions are made. Irons's method of symmetrisation (1) assumes that the pressure is given at some point (ex : zero pressure at the free surface without wave). Hsiung's method (2) assumes either a massless container or no surface wave. Morand and Ohayon's algorithm presented in (3) does not yet include free surface waves.

In order to retain symmetry of the complete system in all cases, the following changes are proposed :

- the free surface waves are simulated by a massless Lagrange boundary line
- for each independant fluid volume, an additional unknown is introduced (in analogy with a lagrange multiplier) representing a generalized fluid pressure.

3. Finite Element Procedure

3.1. Description of solid and fluid media

In this procedure, the fluid is described in an eulerien mesh with pressure unknowns p at fixed points, while the solid and the free surface waves are represented by a Lagrangian mesh which follows the motion defined by the displacement unknowns u.

The behavior of the fluid can be described by the linearized Navier-Stokes equations, where the nonlinear convection terms (nonlinear terms in the eulerian expression for $\frac{dv}{dt}$) have been dropped , and by the boundary conditions shown on fig. 2

$$\nabla^2 p - \frac{\ddot{p}}{c^2} = 0 \qquad (1a)$$

$$\frac{\partial p}{\partial n} = -\rho \ddot{u}_n - \rho g_n \quad \text{or} \quad p_s - p = -\rho \ddot{u}_n u - \rho g_n u_n \qquad (1b)$$

where the symbols are described in the appendix and on the figures.

Then the usual Finite Element matrix equations for the fluid and the solid are

$$Q\ddot{p} + Hp = -\rho L\ddot{u} + \rho G \quad (2) \qquad M\ddot{u} + Ku = F \qquad (3)$$

where M, K, F are the usual mass, stiffness and load matrices for structural analysis.

3.2. Coupling between solid and fluid (Euler/Lagrange coupling)

The effect of the solid and the free surface wave on the fluid are included in (1b) and consequently in (2). The pressure of the fluid on the solid or shell container can be approximated by $p_s = p$. However, at the free surface wave-fluid interface, the linear gravity term must be included in the value of the pressure, to simulate the free surface wave motion :

$$p_s = p - \rho g_n u_n \qquad (4)$$

The $\rho g_n u_n$ term in (4) adds a stiffness term K^{*} to the structural dynamic equation. Finally, the unsymmetric matrix equation, yielding the dynamic increase of pressure p and of displacement u is :

$$\begin{vmatrix} Q & \rho L \\ 0 & M \end{vmatrix} \begin{vmatrix} \ddot{p} \\ \ddot{u} \end{vmatrix} + \begin{vmatrix} H & 0 \\ -L^T & K+K^{*} \end{vmatrix} \begin{vmatrix} p \\ u \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix} \qquad (5)$$

3.3. λ symmetrisation

Several symmetrisation procedures, which correspond to various simplifying assumptions are discussed in ref.4. Here, Lagrange multiplier λ-symmetrisation procedures, will be briefly discussed.

It can be seen from the analytical expression of H (see ref.4), that this matrix is singular. As matrix H needs to be inverted, a change of basis for the pressure unknowns is useful : for each of the independant fluid volume V_i (fig. 1), the pressure vector p (n x 1) can be defined as a function of a generalized pressure λ(1 x 1) and a new pressure vector p ((n-1)x1) with a transformation matrix R(nx(n-1)). For instance, λ can be the pressure at fluid nodal point n° 1 and $\tilde{p}$ can be, for each of the remaining nodes, the difference p-λ. Then

$$p = R\tilde{p} + J\lambda \quad \text{with} \quad J = (1,1,1,\dots,1_n)^T \qquad 6)$$

Procedure 1 : Derivation of a fluid superelement for incompressible fluid analysis

In this case Q=0, and the first line of (5) can be transformed with (6) into two equations :

$$p = -\rho R(R^T HR)^{-1} R^T L\ddot{u} + J\lambda \quad (7a) \qquad J^T L\ddot{u} = 0 \quad \text{or} \quad J^T Lu = 0 \qquad (7b)$$

(7b) is a constraint equation imposing a zero volume change condition. Then the symmetric matrix equation can be written

$$\begin{vmatrix} M+\tilde{M} & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} \ddot{u} \\ \lambda \end{vmatrix} + \begin{vmatrix} K+K^{*} & A \\ A^T & 0 \end{vmatrix} \begin{vmatrix} u \\ \lambda \end{vmatrix} = 0 \qquad (8)$$

with $\tilde{M} = \rho L^T R(R^T HR)^{-1} R^T L$ and $A^T = -J^T L$ (9)

Procedure 2 : Symmetrisation in the general case of compressible fluids

A generalisation of Iron's method is now introduced. Inverting the first line of (5) yields : $p = R(R^T HR)^{-1} R^T(\rho L\ddot{u}+Q\ddot{p})+J\lambda$ which can be substitued into the second row. Then, the constraint equation $J^T(\rho Lu+Qp)=0$ is generated, and the final symmetric equation is produced :

$$\begin{vmatrix} Q & QJ & 0 \\ J^TQ & 0 & \rho J^TL \\ 0 & \rho L^TJ & \rho(K+K^*) \end{vmatrix} \begin{vmatrix} p \\ -\lambda \\ u \end{vmatrix} + \begin{vmatrix} Q\overline{H}^{-1}Q & 0 & \rho Q\overline{H}^{-1}L \\ 0 & 0 & 0 \\ \rho L^T\overline{H}^{-1}Q & 0 & \rho(\rho L^T\overline{H}^{-1}L+M) \end{vmatrix} \begin{vmatrix} \ddot{p} \\ -\ddot{\lambda} \\ \ddot{u} \end{vmatrix} = 0 \qquad (10)$$

with $\overline{H}^{-1} = R(R^THR)^{-1}R^T$

4. Example Problems

Some of the hereabove formulations have been included in a special version of the program PAM AX3D a general Finite Element program for axisymmetric structures submitted to three dimensional loads by Fourier series decomposition . Following are test problems used for validation of the program.

First, various experiments corresponding to horizontal earthquake (harmonic nb. 1) where backcalculated (fig.3). In this case the free surface was simulated according to Zienkiecwicz's formulation (1). The agreement between the different results is excellent.

Second, a test problem for a Nuclear Vessel seismic analysis is studied : an axisymmetric deformable steel vessel, partially filled with an incompressible liquid, is analysed for the first sloshing modes, which are shown on fig. 4. In this case, the "structural" modes are not coupled with the sloshing modes. For higher modes, the structural modes are coupled with the fluid modes, and the proposed procedures permit the coupled analysis, with the mass of the structure (shell, internals, core) and the free surface waves, using still symmetric matrices.

5. Conclusion

The proposed procedure provides an efficient and precise solution to the linearized free surface-fluid-structure dynamic interaction problem. In particular, it can account for weights of equipments fixed to the shell containment, or accomodate several independant fluid volumes as eulerien super elements, still using symmetric equation solvers.

References

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Appendix

A	Incompressibility stiffness matrix
c	Velocity of sound in fluid
F	Force matrix
G	Gravity matrix
g	Gravity acceleration
g_n	Componant of gravity normal to a boundary
H	Fluid Matrix
$\bar{H}$	Fluid matrix after singularity removal
J	Column of 1 $J^T = (1,1,\ldots,1)$
K	Solid stiffness matrix
K^*	Stiffness matrix due to the Lagrange free surface
L	Coupling matrix
M	Mass matrix for solid
$\tilde{M}$	Mass matrix for the fluid super element
n	Index for normal
n	Number of dimensions
$\tilde{p}$	Modified basis for fluid pressure
p_s	Fluid pressure on solid
p	Fluid pressure of fixed eulerien point
Q	Fluid matrix
R	Matrix for change of basis
S	Surface of the solid and free surface
T	Index for transpose
u	Displacement of the Lagrange mesh
$\ddot{u}$	Acceleration of the Lagrange mesh
$\ddot{u}_n$	Acceleration normal to the fluid boundary
V, V_i	Volume of fluid
ρ	Mass per unit volume
λ	Generalized fluid pressure

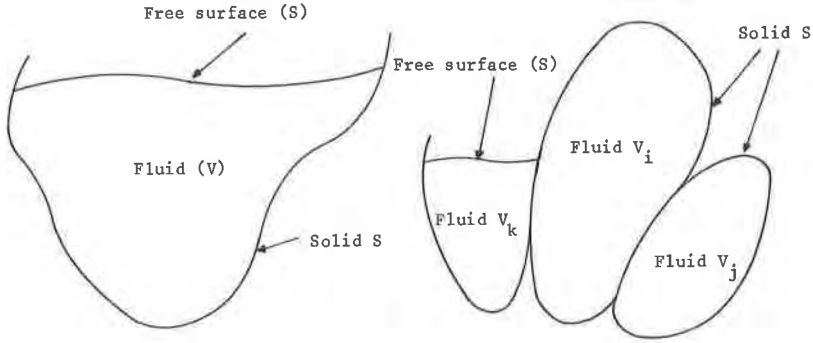


FIGURE 1

SOLID FLUID FREE SURFACE PROBLEMS

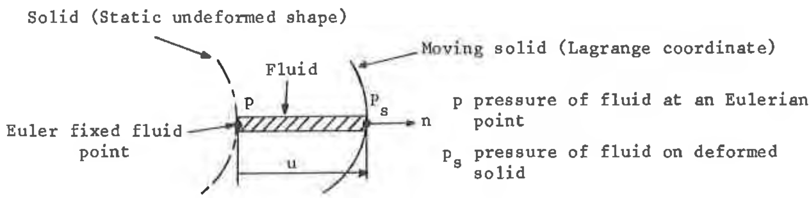
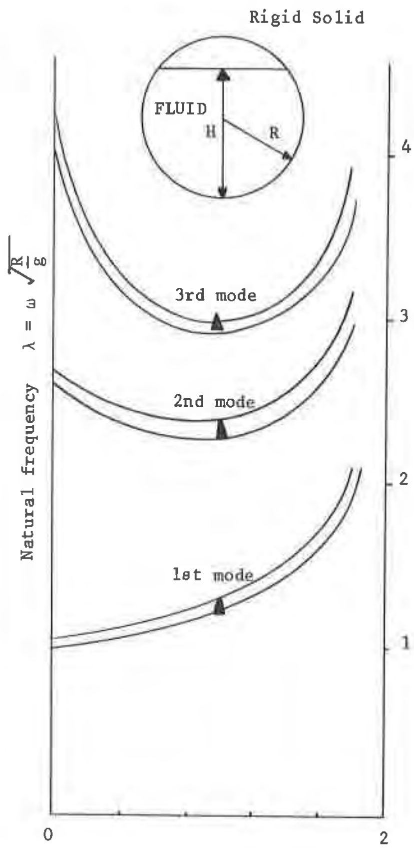


FIGURE 2

SOLID FLUID BOUNDARY CONDITION
(Valid also for free surface)



after Hsiung and Weingarten

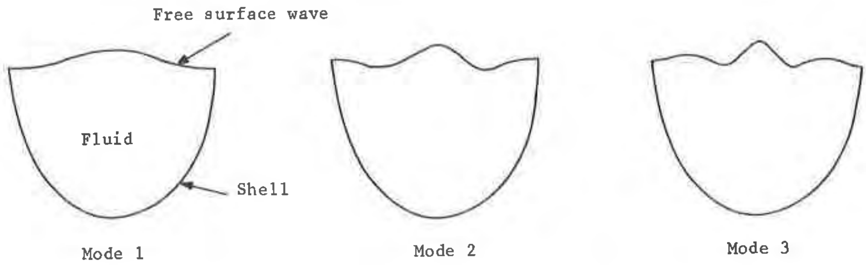
Experimentals Results

▲ PAM AX3D

FIGURE 3

Comparison of the performances of various theories and PAM AX3D with experiments

Non axisymmetric modes of harmonic 1 for horizontal earthquake



	Frequencies CPS	Mode 1	Mode 2	Mode 3
1	Guyan[*]	.46	.62	.75
2	Hsiung[**] Weingarten	.44	.63	.82
3	PAM AX3D[***]	.44	.62	.79

Material Properties :

Shell : Radius of 200 inch ; thickness of .1 inch ;
Young's modulus $E = 10^7$ psi ; Poisson's ratio .3
Mass density 2.59×10^{-4} lbs x sec^2/in^4
Fluid : mass density 1.06×10^{-4} lbs x sec^2/in^4

FIGURE 4

COUPLED SHELL-INCOMPRESSIBLE FLUID-FREE SURFACE WAVE
DYNAMIC INTERACTION AXISYMMETRIC MODE SHAPE AND FREQUENCIES
(Harmonic Zero)

[*] Finite Element Method with special formulation for fluid (5)
[**] F.E.M. without mass for the shell in order to uncouple the equations (2)
[***] F.E.M. with Lagrange line for free surface wave, direct invertion of H
For cases 2 and 3 : 57 fluid elements and 11 shell elements