QUASI-NONLINEAR DYNAMIC ANALYSIS

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ABSTRACT

The analysis of nuclear power plants involves many mechanical and structural systems exhibiting quasi-nonlinear behavior, such as contact or lift-off problems, gap-openings and -closings, and friction effects. Anchorage devices with higher compressive than tensile stiffness also belong to this category. Such systems are basically linear. Their nonlinearities are discrete (unlike those of large displacement problems) and are often limited to a rather small number of time steps of the response history.

In devising solution schemes for such nonlinear effects, no special penalties should be paid for either small nonlinearities occurring throughout the response, nor for large discrete nonlinearities which affect the stiffness only in a relatively small number of instances.

This paper describes a solution strategy that may be implemented effectively in linear structural analysis programs in order to expand their capabilities to cover quasi-nonlinear problems as well, while preserving the efficiency of the existing solution algorithms.

If the nonlinear effects are treated as corrections to the incremental load vectors, the number of operations is proportional to the number of iterations and the number of time steps during which the stiffness is different from the original stiffness. If the structure stiffness is redecomposed each time, when the stiffness changes, the apparent increase in effort can be reduced appreciably if the unknowns are numbered such that the degrees of freedom with nonlinear stiffnesses are numbered last, leaving the reduced stiffness of the modified system largely unaffected by the change. In most cases such a redecomposition will be more economical than iteration. Moreover, the numerical effort required in addition to a linear analysis is rather small for many practical applications. A one-dimensional bar or spring element with a tri-linear force-displacement curve is adequate to model a large number of the quasi-linear phenomena mentioned above.

1. Introduction

The analysis of nuclear power plants involves many mechanical and structural systems that exhibit quasi-nonlinear behavior, such as contact of lift-off problems, gap-openings and -closings, friction effects, etc. Anchorage devices with higher compressive than tensile stiffness also belong to this category. Such systems are basically linear. Their nonlinearities are discrete (unlike those of large-displacement problems) and are often limited to a rather small number of time steps of the dynamic response history.

The dynamic analysis of the nonlinear behavior of structures has received increased attention in recent years [1,2,3]. Methods that take advantage of the localized nature of certain nonlinear phenomena are comparatively scarce [4]. Nonlinearities such as those mentioned above are localized in space and time and therefore call for specialized solution techniques that fully utilize the efficiencies inherent in linear structural analysis and avoid penalties for both small nonlinearities occurring throughout the response, as well as large discrete nonlinearities which affect the structure stiffness only in a relatively small number of instances.

This paper describes a solution strategy that may be implemented effectively in currently available linear structuctural analysis programs such as SAP IV, in order to expand their capabilities to cover quasi-nonlinear problems as well, while preserving the efficiency of the existing numerical solution algorithms.

2. Nonlinear Dynamic Analysis

General nonlinear behavior of structures may arise either from large displacements or from material stress-strain laws such as the flat-top yield behavior of mild structural steel or the gradual stiffness decrease of concrete. During nonlinear excursions, the structure stiffness has be be continually updated, possibly at each time step. For such fully nonlinear dynamic analyses, efficient numerical solutions schemes have to be devised lest the computational effort become prohibitive [5].

Nonlinearities that are more or less discrete in space and time can be treated by two different techniques. The first one has been referred to as the force correction method [6] and can be described briefly as follows.

The incremental equations of motion can be written in matrix form,

$$\underline{M}\Delta\ddot{\underline{r}} + \underline{C}\Delta\dot{\underline{r}} + \underline{K}\Delta\underline{r} = \Delta\underline{P} \tag{1}$$

where $\underline{M}$, $\underline{C}$, and $\underline{K}$ are, respectively, the system mass, damping, and stiffness matrix. $\Delta\underline{r}$ is the incremental displacement vector and $\Delta\underline{P}$ the incremental load vector which may represent either a time-dependent forcing function or a gound motion induced inertia load vector, $-\underline{M}\Delta\ddot{\underline{r}}_g$. Depending on the particular numerical integration algorithm employed, the incremental velocity and acceleration vectors can in general be expressed as

$$\Delta\dot{\underline{r}} = \dot{\underline{r}}_{t+\Delta t} - \dot{\underline{r}}_t = C_1\Delta\underline{r} + C_2\dot{\underline{r}}_t + C_3\ddot{\underline{r}}_t \tag{2}$$

$$\Delta\ddot{\underline{r}} = \ddot{\underline{r}}_{t+\Delta t} - \ddot{\underline{r}}_t = C_4\Delta\underline{r} + C_5\dot{\underline{r}}_t + C_6\ddot{\underline{r}}_t \tag{3}$$

Substitution of Eqs. (2) and (3) into (1) leads to

$$\underline{K}^*\Delta\underline{r} = \Delta\underline{P}^* \tag{4}$$

where

$$\underline{K}^* = \underline{K} + C_4\underline{M} + C_1\underline{C} \tag{5}$$

$$\Delta\underline{P}^* = \Delta\underline{P} - \underline{M}(C_5\dot{\underline{r}}_t + C_6\ddot{\underline{r}}_t) - \underline{C}(C_2\dot{\underline{r}}_t + C_3\ddot{\underline{r}}_t) \tag{6}$$

The physical interpretation of Eq. (4) is that the problem has been linearized over the small time step Δt and converted to an equivalent static problem which is readily solved for the incremental displacement vector $\Delta\underline{r}$. In a linear problem, $\underline{K}$, $\underline{C}$, and $\underline{M}$ remain constant and $\underline{K}^*$ needs to be decomposed only once. For each time step the incremental load vector $\Delta\underline{P}$ has to be formed and reduced, followed by backsubstitution to complete the solution for $\Delta\underline{r}$ [5].

If the structure stiffness $\underline{K}$ changes at some time t, then Eq. (4) may be written as

$$(\underline{K}^* + \Delta\underline{K})\Delta\underline{r} = \Delta\underline{P}^* \tag{7}$$

or

$$\underline{K}^*\Delta\underline{r} = \Delta\underline{P}^* - \Delta\underline{K}\,\Delta\underline{r} \tag{8}$$

which is readily solved by iteration. As in a linear analysis, $\underline{K}^*$ needs to be decomposed only once. Even though $\Delta\underline{r}$ may represent a large number of degrees of freedom, the vector $\Delta\underline{K}\Delta\underline{r}$ contains only n non-zero terms, if n is the number of freedoms with modified stiffness coefficients.

The additional numerical effort involved in the solution of Eq. (8) as compared to Eq. (4) is rather small. Still, the iterations are required for each time step during which the structure stiffness is different from the initial stiffness, even though $\underline{K}^*$ may be constant for a rather large number of subsequent time steps. For example, in a pipe whip problem, there may be very few time steps during which the structure stiffness actually changes. In such a case it may be more advantageous to redecompose the modified system matrix $(\underline{K}^* + \Delta\underline{K})$. The efficiency of this method can be increased considerably if the unknowns are numbered appropriately, Fig. 1. Any rows or columns associated with degrees of freedom numbered prior to those with modified stiffness coefficients will remain unaffected during and after the redecomposition. Partitioning the stiffness into those freedoms that are and are not affected by nonlinearities requires prior knowledge of where these nonlinearities are likely to occur. But this prior knowledge can be assumed in the case of the quasi-nonlinear effects considered here.

3. Selection of Solution Strategy

In a general purpose computer program it is desirable to automate the analysis as much as possible. However, the program should be flexible enough to allow for the user's experience and insight into the nature of the analysis problem. Such an insight might be advantageous in choosing the analysis method--iteration or redecomposition.

Let N_c be the number of time steps during which the stiffness coefficients for at least one degree of freedom change, and N_d the number of time steps during which the structure stiffness is different from the initial stiffness. The solution of the problem by iteration requires approximately

$$N_1 = 2\ m\ N\ M\ N_d$$

operations, where m is the average number of iterations required for convergence, N is the total number of freedoms, and M the half-bandwidth of the structure stiffness. The solution by redecomposing the stiffness at each change requires approximately

$$N_2 = \frac{1}{2} N M^2 N_c$$

operations. The ratio

$$R = \frac{N_1}{N_2} = \frac{4mN_d}{MN_c}$$

is illustrated graphically in Fig. 2. For example, in a pipe whip problem in which the gap is closed only once for the duration of 10 time steps, we have $N_c = 2$, $N_d = 10$, so that for $m = 5$ and $M = 50$, $R = (4)(5)(10)/(50)(2) = 2$ in favor of redecomposition.

In refining the above comparison, the sparse nature of $\underline{\Delta K}$ has to be recognized in assessing the iteration method. If n is the number of modified stiffness coefficients, the number of operations needed in addition to those for a linear analysis step (load vector reduction and backsubstitution) can be reduced to approximately

$$N_1 = m(n + NM)N_d \approx m\ N\ M\ N_d$$

provided the affected freedoms are numbered last so that the reduction effort can be neglected compared with backsubstitution. The numerical effort for redecomposition can be reduced even more if the unknowns are numbered appropriately, requiring only about

$$N_2 = \frac{1}{2} p^3 N_c$$

operations, where p is the number of affected freedoms, i.e., $n = p(p + 1)/2$. The ratio becomes now

$$R = \frac{N_1}{N_2} = \frac{2mNM}{p^3} \frac{N_d}{N_c}$$

Redecomposition is obviously more efficient unless a relatively large number of stiffness coefficients are subject to change. For large systems with stiffness matrices that do not fit into the core, the peripheral processing requirements place an additional burden on iteration relative to redecomposition. In either case, compared with the 2 N M operations needed for one time step of linear analysis, the additional effort due to quasi-nonlinear effects is rather small.

4. Nonlinear Element

In order to model a large number of quasi-nonlinear phenomena with a minimum of program modification, a one-dimensional spring or bar element is recommended, characterized by an arbitrary tri-linear force-displacement relationship. Fig. 3 illustrates some of the possible applications. Such an element can be used to model an actual structural component connecting two active degrees of freedom (such as a cable connecting two structural nodes), or as a boundary element affecting only a single degree of freedom.

5. Program Implementation

The implementation of the quasi-nonlinear capability into a general purpose program requires the following steps.

1. Establish an indicator array for all degrees of freedom affected by nonlinearities.
2. Renumber the structure to minimize the bandwidth, subject to the constraint that all affected nodes are numbered last or almost last.
3. For each time step, as in a linear analysis, form the incremental load vector $\underline{\Delta P}^*$.
4. Solve Eq. (4) for the incremental displacements $\underline{\Delta r}$.

5. Compute all nonlinear spring forces and update the indicator array if the element response changes branches of the force-displacement curve.
6. For each nonlinear element with an updated flag in the indicator array, add the incremental element stiffness into the $\underline{K}^*$ matrix.
7. Redecompose the affected rows and columns of $\underline{K}^*$ to be used in the next time step.

For a single nonlinear element, the actual increase in numerical effort constitutes only a very small amount, depending on the size of the problem. The larger the system, the less significant is the additional effort. Only for large numbers of nonlinear elements, such as are needed to analyze the lift-off of a containment mat, will the stiffness portion to be redecomposed increase fast, and also the bandwidth will increase as a result of the constraints imposed on the minimization algorithm.

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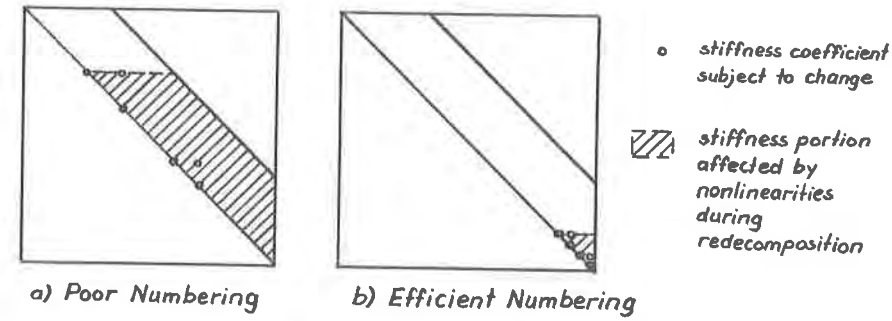

Fig. 1 Effect of Numbering of Unknowns on Reduced Stiffness

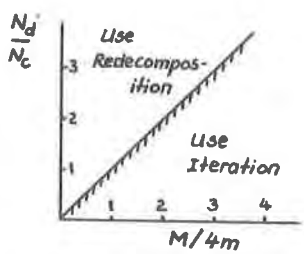

Fig. 2 Redecomposition vs Iteration

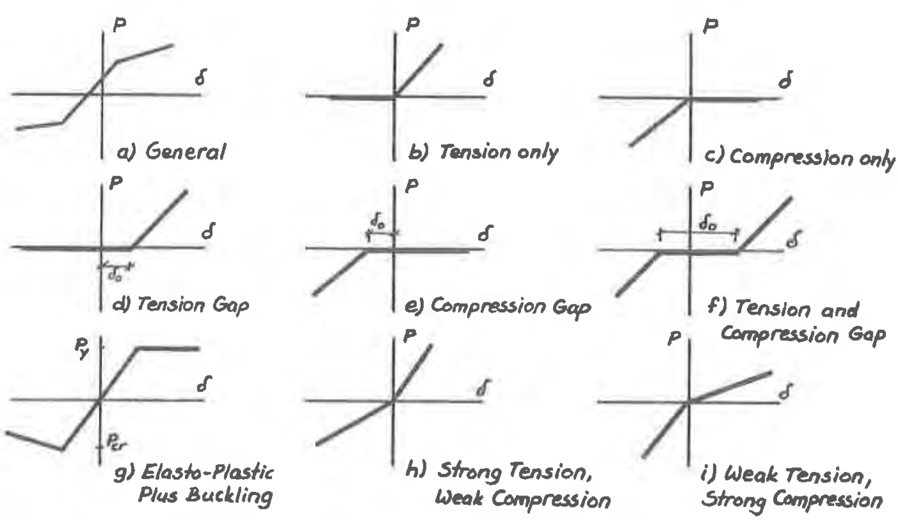

Fig. 3 Applications of Nonlinear Element Model