Theoretical Model of Creep Fracture under Tri-Axial State of Stress

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1 INTRODUCTION

Existing theories of creep damage and creep rupture for metals at elevated temperature subjected to the multi-axial state of stress (Hayhurst 1983; Murakami 1988) are formulated as a generalization of the classical Kachanov's theory. These theories were successfully used to describe the experimental results obtained for the uniaxial tension but because of some technical problems encountered in relevant experiments for the multi-axial state of stress their validity was checked for some selected cases of biaxial loading only. An attempt to furnish the tri-axial creep rupture data (Dyson et al. 1981) by employing the Bridgman notched bars (Bridgman 1952) did not give rise to a consistent analysis of the existing theories. More complete theoretical and experimental study of creep rupture for aluminium alloy and copper was performed (Hayhurst et al. 1986) for specially designed specimens subjected to a tri-axial tension. However, these results do not give the complete solution of the problem and that is why two directions of further research could be shown: (a) full experimental verification of existing theories by using experimental data available in the literature and (b) formulating new rational theory particularly if the results of the first step are not satisfactory.

This paper is a combination of both above mentioned approaches and includes the presentation of a modified theory of creep damage and creep fracture under tri-axial state of stress (Litewka 1989) together with its experimental verification by employing the available creep rupture experimental data (Dyson et al. 1981; Hayhurst et al. 1986). The preliminary results concerning this problem can be found in the previous paper by the authors of this note (Litewka et al. 1991).

2 CONSTITUTIVE EQUATIONS

The theoretical model used in this paper has been described elsewhere (Litewka 1989) together with its application to the uniaxial and biaxial states of stress. The creep rupture criterion that was developed within this constitutive model consists of the damage evolution equation and the failure criterion for a material with deteriorated internal structure. The details of the mathematical and physical background of those equations can be found in previous paper (Litewka 1989) that is why only final form of them will be shown here.

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The damage evolution equation has been assumed in the form of the tensor -valued function

$$\Omega = F\sigma^{*} \tag{1}$$

where Ω is a time derivative of the Murakami-Ohno tensorial damage measure (Murakami et al. 1981) and σ denotes a modified stress tensor whose compressive principal stresses are replaced by zeros, whereas tensile principal stress components are left unchanged. The scalar multiplier F has a form of scalar-valued function

$$F = \frac{C}{4E^2}\left\{\frac{1}{9}(1 - 2\nu)\mathrm{tr}^4\sigma + (1 + \nu)^2\mathrm{tr}^2S^2 + \frac{2}{3}(1 - 2\nu)(1 + \nu)\mathrm{tr}^2\sigma\mathrm{tr}S^2 + \right.$$
$$\left. + \left[\frac{2}{3}(1 - 2\nu)\mathrm{tr}^2\sigma + 2(1 + \nu)\mathrm{tr}S^2\right]\frac{D_1}{1 + D_1}\mathrm{tr}\sigma^2 D\right\} \tag{2}$$

where C is a temperature dependent coefficient, E and ν are Young's modulus and Poisson's ratio of the undamaged material, σ is the stress tensor and $S = \sigma - 1/3I\mathrm{tr}\sigma$ is the stress deviator where I stands for the unit tensor. The second rank symmetric damage effect tensor D that appears in (2) is associated with the damage tensor Ω by means of their principal values

$$D_i = \frac{\Omega_i}{1 - \Omega_i}, \qquad i = 1,2,3$$

To facilitate the practical application the damage evolution equation (1) has been written in an integrated form. The explicit form of this equation depends on the state of the stress and three different solutions of the differential equation (1) can be distinguished (Litewka et al. 1991). The most important solution written in the form

$$\frac{2b-a}{b\sqrt{\Delta}}\left(\arctan\frac{2b\Omega_1 - a}{\sqrt{\Delta}} - \arctan\frac{-a}{\sqrt{\Delta}}\right) - \frac{1}{2b}\ln\left|\frac{b}{a}\Omega_1^2 - \Omega_1 + 1\right| = k\sigma_1^5 t \tag{3}$$

for $\Delta = 4ab - a^2 > 0$ covers the wide range of various tri-axial states of the stress. Two another solutions discussed elsewhere (Litewka et al. 1991) corresponds to those states of stress where the compressive stresses predominate. The notation employed in equation (3) is as follows

$$\sqrt{a} = M = \frac{2(1+\nu)}{3}\left[(1 + m_1^2 + m_2^2) - (m_1 + m_1 m_2 + m_2)\right] + \frac{1-2\nu}{3}(1 + m_1 + m_2)^2$$

$$k = C/4E^2; \qquad b = 2M(1 + n_1 m_1^2 + n_2 m_2^2); \qquad m_1 = \sigma_2/\sigma_1; \qquad m_2 = \sigma_3/\sigma_1$$

$$n_1 = \Omega_2/\Omega_1 = \begin{cases} m_1 & \text{for } 0 \le m_1 < 1 \\ 0 & \text{for } m_1 < 0 \end{cases}; \qquad n_2 = \Omega_3/\Omega_1 = \begin{cases} m_2 & \text{for } 0 \le m_2 < 1 \\ 0 & \text{for } m_2 < 0 \end{cases}$$

According to the theoretical model used in this paper creep rupture of the material occurs when damage amount, represented by Ω or D growing in the material subjected to the given state of stress $\underset{\sim}{\sigma}$, satisfies the failure criterion

$$C_1 \mathrm{tr}^2 \underset{\sim}{\sigma} + C_2 \mathrm{tr} \underset{\sim}{S}^2 + C_3 \mathrm{tr} \underset{\sim}{\sigma}^2 \underset{\sim}{D} - \sigma_u^2 = 0 \tag{4}$$

where σ_u is an ultimate strength of undamaged material and C_1, C_2 and C_3 are temperature and damage dependent material constants which are determinated from three simple tests like an uniaxial tension in two mutually perpendicular directions of the material structure symmetry and for an equal biaxial tension. However, it was found that the results concerning the ultimate strengths determinated in these tests and presented in previous papers (Litewka 1989, Litewka et al. 1991) are sufficient and accurate for the biaxial state of stress only and for the tri-axial state of stress some modifications in the calculation of C_1, C_2 and C_3 should be introduced. Finally the following set of equations obtained from(4)

$$\begin{aligned}
C_1 + \frac{2}{3} C_2 + D_1 C_3 &= [1 + (1.164 n_1 - 1)\Omega_1 - 2.235 n_1 \Omega_1^2 + 1.071 n_1 \Omega_1^3]^{-2} \\
C_1 + \frac{2}{3} C_2 + D_2 C_3 &= [1 + (1.164 n_1 - 1)\Omega_2 - 2.235 n_1 \Omega_2^2 + 1.071 n_1 \Omega_2^3]^{-2} \\
4C_1 + \frac{2}{3} C_2 + (D_1 + D_2) C_3 &= [1 + (0.858 n_1 - 0.84)\Omega_1 - \\
&\quad - (2.049 n_1 + 0.16)\Omega_1^2 + 1.191 n_1 \Omega_1^3]^{-2}
\end{aligned} \tag{5}$$

was used to calculate the constants C_1, C_2 and C_3.

3 EXPERIMENTAL VERIFICATION

3.1 Bridgman notched bars

The experimental and theoretical analysis (Dyson et al. 1981; Hayhurst et al. 1977) show that relatively uniform tri-axial state of stress can be obtained in the minimum cross section at the notch throat of the Bridgman notched bar (Bridgman 1952) with a geometry shown in Fig. 1. Creep rupture experimental data for such specimens made of Nimonic 80A and tested under various loading at the temperature of 1023K (Dyson et al. 1981) are also shown in Fig. 1. Solid line in this figure represents the theoretical results obtained from creep rupture criterion proposed here and expressed by means of the equations (3), (4) and (5). This curve was obtained for the ratio of the principal stresses $m_1 = m_2 = \sigma_2/\sigma_1 = \sigma_3/\sigma_1 = 0.457$ (Hayhurst et al. 1977) and material constants shown in Table 1. Broken lines shown in Fig. 1 represents the theoretical curves for uniaxial tension and pure shear. For these states of the stress the agreement of the theory and experiment is almost perfect but for a tri-axial tension the discrepancies are considerable particularly for shorter time to rupture. This can be explained by the dependence of the stress distribution on time and also by the strong nonhomogeneity of the stress distribution along the axis Z of the specimen with approximately homogeneous state of stress limited rather to thin layer at the notch throat (Hayhurst et al. 1977). This could slow-down the damage growth in the minimum cross section what results in longer experimentally measured rupture time. Better approximation represented in Fig. 1 by means of chain line is obtained from this theoretical model for Bridgman effective stress $\sigma_{ef} = 0.72\sigma_a$ where σ_a is the net section stress at notch throat.

3.2 Equal tri-axial tension

The specimen with the geometry optimized to achieve relatively uniform state of stress and loaded in three mutually perpendicular directions supplied the experimental data for an equal tri-axial tension (Hayhurst et al. 1986). These experiments were done by employing two tri-axially loaded specimens made of aluminium alloy and copper (one specimen for each material). The rupture time determined experimentally together with the relevant uniaxial creep rupture data used to calculate the material constant shown in Table 1 are represented for both materials in Figs 2 and 3. As is seen from these figures the theoretical curves obtained from equations (3), (4) and (5) fall

Table 1. Material constants

Material	Temperature K	σ_u MPa	k $1/MPa^5 h$	ν
Nimonic 80A	1023	650	$3.03 \cdot 10^{-15}$	0.3
Aluminium alloy	423	325	$7.07 \cdot 10^{-16}$	0.3
Copper	573	48	$6.02 \cdot 10^{-11}$	0.35

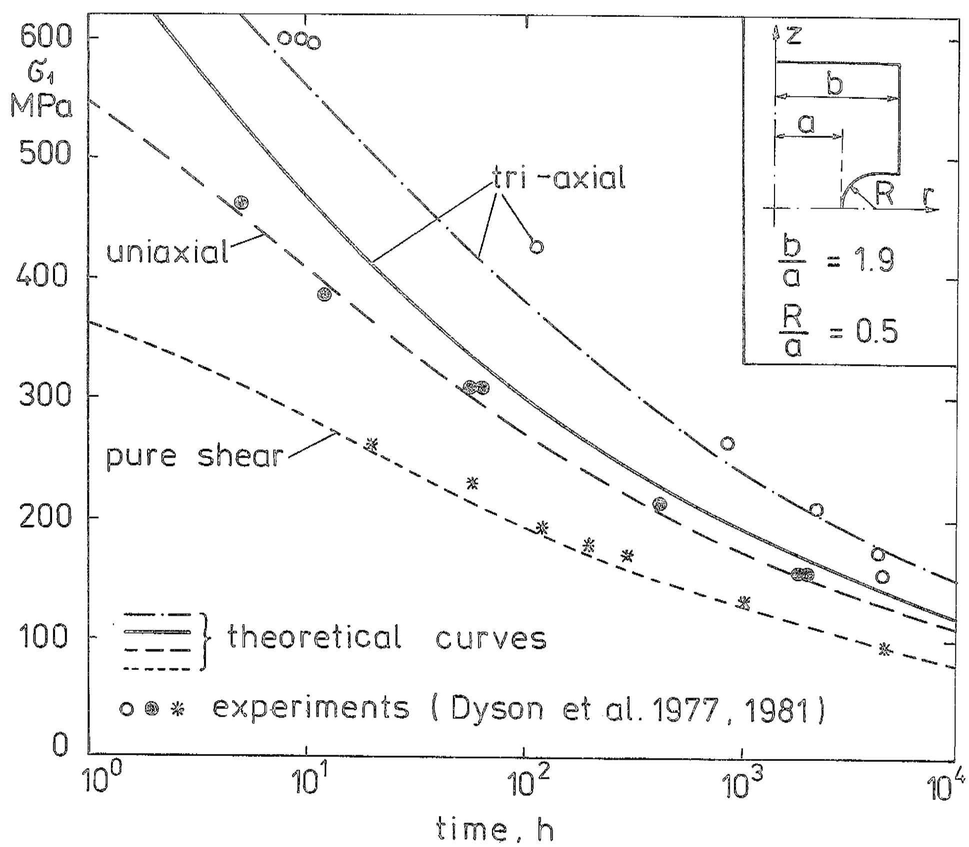

Fig. 1 Maximum principal stress versus rupture time for Nimonic 80A at temperature of 1023K.

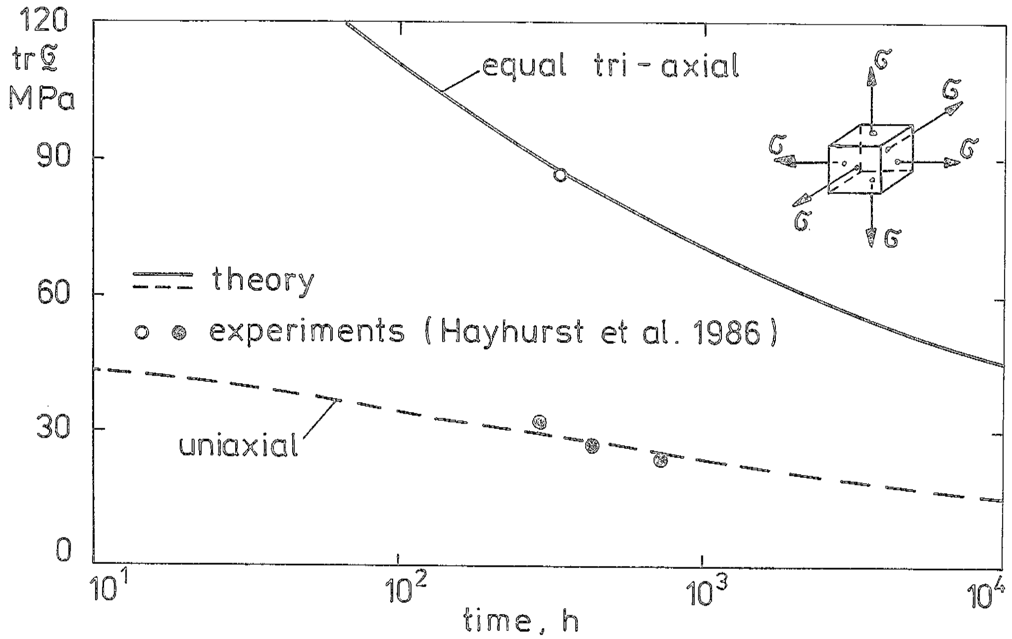

Fig. 2 First stress invariant versus rupture time for copper at temperature of 573K.

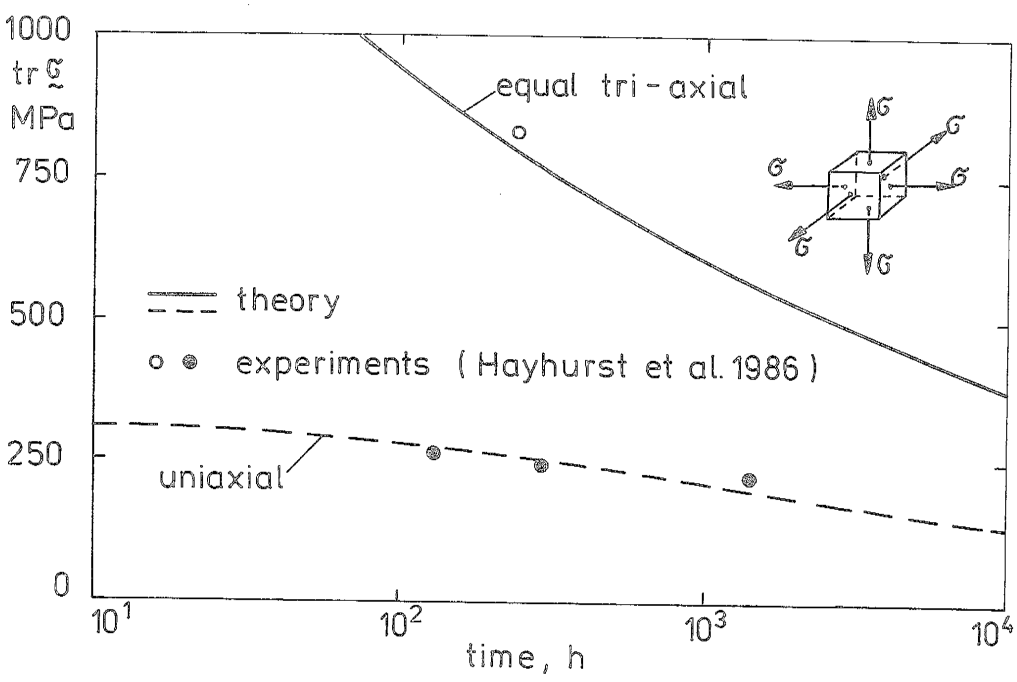

Fig. 3 First stress invariant versus rupture time for aluminium at temperature of 423K.

very close to the appropriate experimental results. However, having only one experimental point for each material it is difficult to draw the final conclusion concerning the validity of the theory in the case of equal tri-axial tension.

4 CONCLUSIONS

Comparison of theoretical prediction with the results of the experiments for two cases of a tri-axial state of stress show that the theory proposed gives satisfactory results of a creep rupture time. Particularly good agreement of the theoretical and experimental results was obtained for most rigorous test of an equal tri-axial tension. Higher discrepancies observed in the case of a tri-axial state of stress produced in Bridgman notched bar is a result of a stress redistribution and strong nonhomogeneity of the state of stress in the longitudinal direction of the specimen. The results presented in this paper are encouraging but the full experimental verification of the theory proposed could be done if more tri-axial experimental data are available.

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