

Free Vibration Analysis of FBR Vessels Partially Filled with Liquid

Junji Tani

Tohoku University, Sendai, Japan

Junichi Teraki

Daikin Company, Ltd., Sakai, Japan

ABSTRACT

A calculation code is developed to analyze the free vibration of partially liquid-filled shells of revolution by use of the finite element method. Iso-parametric thick shell-elements are used for the shell, and Isoparametric quadrilateral elements and line ones are used for the interior and surface of liquid, respectively. The liquid is assumed to be incompressible and inviscid. The initial axisymmetric deformation of the shell due to the static liquid pressure as well as the boundary condition on the free liquid surface are fully taken into consideration. The free vibration characteristics of fast breeder reactor vessels partially filled with liquid sodium are studied by means of this code. It is found that the natural frequency of the FBR vessel has the minimum value for the vibration mode with circumferential wave numbers $N=1$ or $6\sim 9$.

1. INTRODUCTION

The free vibration characteristics of fast breeder reactor vessels are one of the main factor in evaluating their safety against earthquake. Hence much research has been conducted on this subject (Brabant et al, 1985, Chiba, M., et al, 1984, Fujimoto et al, 1985, Gvildys, et al, 1895, Haroun et al, 1982, Ma, et al, 1985, 1987, Petruschke, 1985, Sakurai, 1985). However the whole aspect of the free vibration characteristics of pool-type FBR vessels does not seem to have been thoroughly explored.

In this paper, the free vibration of the FBR vessels partially filled with liquid sodium is analyzed by means of the finite element method. Isoparametric thick shell-elements are used for the shell of revolution, and isoparametric quadrilateral elements and line ones are used for the interior and surface of liquid, respectively. The sodium is assumed to be incompressible and inviscid. The initial axisymmetric deformation of the shell due to the static liquid pressure as well as the boundary condition on the free liquid surface are fully accounted in the analysis. The natural frequencies and modes are clarified in detail.

2. BASIC EQUATION AND BOUNDARY CONDITIONS

Consider the linear free vibration of a liquid-filled structure with fluid-structure interaction as shown in Fig. 1. In this figure, V_L and V_S are incompressible inviscid liquid and elastic solid, respectively, and S_u and S_f are the boundary along which the displacement and the free liquid surface are given, respectively. Further η and g are the displacement of liquid surface and gravitational acceleration, respectively.

We shall consider the small amplitude free vibration around the axisymmetric static deflection of the shell due to static liquid pressure. Assuming the liquid to be incompressible and inviscid, and the motion to be irrotational, we introduce the velocity potential ϕ which is to satisfy the following Laplace equation and boundary condition.

$$\nabla^2 \phi = 0 \quad : \quad V_L \quad (1)$$

The governing equation and boundary condition of the elastic solid are given by the following equations without body force.

$$\sigma_{0ij,j} + \sigma_{ij,j} = \rho_s \ddot{u}_i \quad : \quad V_s \quad (2)$$

$$u_{0i} + u_i = 0 \quad : \quad S_u \quad (3)$$

where σ_{0ij} and σ_{ij} are stress tensors which are produced by the deformations u_{0i} and u_i of the initial axisymmetric state and the small amplitude vibration, respectively, while ρ_s is the mass density of the elastic solid and dots stand for differentiation with respect to time. On the assumption that η is small, the following linearized free surface condition is obtained.

$$\eta = -\frac{1}{g}\dot{\phi}, \quad \dot{\eta} = \frac{\partial \phi}{\partial n_1} \quad : \quad S_f \quad (4), (5)$$

where n_1 is a unit normal vector in the external direction for the liquid. The continuity of the normal velocity and pressure on the contacting surface S_i between liquid and solid yields the following boundary conditions.

$$n_{1i}\dot{u}_i = \frac{\partial \phi}{\partial n_1}, \quad n_{si}(\sigma_{0ij} + \sigma_{ij})n_{sj} = \rho_l \dot{\phi} - P_s \quad : \quad S_i \quad (6), (7)$$

where n_{1i} and n_{si} are components of n_1 and n_s in the i direction, and n_s is a unit normal vector in the external direction for solid, while ρ_l is the mass density of liquid and P_s is a static liquid pressure given by the following equation.

$$P_s = \rho_l g h \quad (8)$$

where h is a depth in liquid.

3. METHOD OF SOLUTION

On the basis of the preceding basic equations and boundary conditions, the problem is solved by means of the finite element method.

Considering equations (5) and (6) and applying the Galerkin method to equations (1) and (4) of liquid, we obtain the following equations in the matrix form.

$$[H]\{\phi\} = [P_1]\{\eta\} + [P_2]\{\delta_s\}, \quad g[K_1]\{\eta\} = -[P_1]^T\{\dot{\phi}\} \quad (9), (10)$$

where $[H]$, $[P_1]$, $[P_2]$ and $[K_1]$ are the matrices which are composed of shape functions of liquid, liquid surface and elastic solid, while $\{\delta_s\}$ is a displacement vector of elastic solid.

Next, considering equations (3) and (7), applying the Galerkin method to equation (2) and assuming that the initial deformation due to static liquid pressure is small, we obtain the governing equation of the small amplitude vibration and the initial deformation of elastic solid as follows.

$$[M_s]\{\delta_s\} + [K_s]\{\delta_s\} = -\rho_l [P_2]^T\{\phi\} \quad (11)$$

$$[K_s^L]\{\delta_{s0}\} = \{Q_1\} \quad (12)$$

where the mass matrix $[M_s]$ is composed of the shape function of elastic solid and the stiffness matrix $[K_s]$ is given by

$$[K_s] = [K_s^L] + [K_s^G] \quad (13)$$

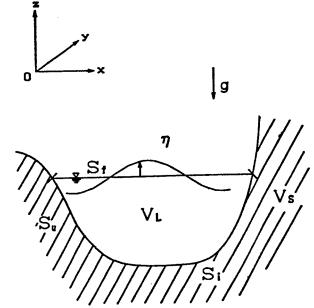


Fig. 1 A Fluid-Coupled Structure

Table 1 Material Properties

Shell (Stainless Steel)	
Young's Modulus (Pa)	2.0×10^{11}
Poisson's Ratio	0.3
Density (kg/m ³)	7.8×10^3
Liquid (Solidum)	
Density (kg/m ³)	0.97×10^3

The initial deflection due to static liquid pressure is examined for the various values of the filling ratio (L/H) and the results are shown in Fig. 4. From this figure it can be seen that the deflection increases with the filling ratio and that the deflection of the torispherical vessel is about 3 times larger than that of the semispherical one. It is to be noted that the deflection changes remarkably at a joint part of two shells.

Next, the effect of the filling ratio on the natural frequency and mode are calculated and shown in Figs. 5 through 10. Figures 5, 7 and 9 show the relations between the natural frequency and the circumferential wave number N. In these figures, m indicates the axial mode number. Figures 5 and 6 show the case without liquid for reference. Figure 9 shows the effect of the initial deflection on the natural frequency of the torispherical vessel. In Figs. 8 and 10, $x = \eta_{\max} / \delta_{s \max}$ indicates the ratio of the maximum displacements of the liquid surface to that of the shell wall. The following features can be observed on the natural frequency and mode. The natural frequencies of the torispherical and semispherical vessels have a minimum value in the vicinity of $N=6\sim 9$ or at $N=1$, depending on the filling ratio, which leads to be the necessity of the analysis for the various values of the circumferential wave number. The natural frequencies of both the vessels decrease with the increase in the filling ratio. The natural frequency of the torispherical vessel is lower than that of the semispherical one, for the vessels with the geometrical configuration and material properties considered here. The vibration mode is dominated generally by the deflections of the spherical and cylindrical shell-parts in the cases with the small and large values of N, respectively. The initial deflection due to the static liquid pressure has only a little effect on the natural frequency of the torispherical vessel.

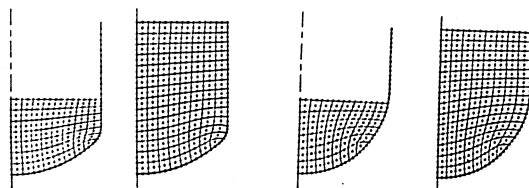
5. CONCLUSION

The free vibration characteristics of the pool-type FBR vessels partially filled with liquid sodium are analyzed for two kinds of the vessels with torispherical and semispherical bottoms by means of the finite element method. The major conclusions drawn from this study are as follows :

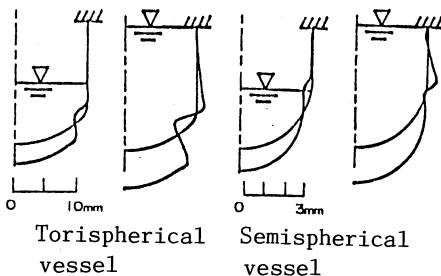
- (1) The natural frequencies of both the vessels have a minimum value in the vicinity of the circumferential wave number $N=6\sim 9$ or at $N=1$, depending on the filling ratio of liquid.
- (2) The natural frequencies of both the vessels decrease with the increase in the filling ratio.
- (3) The initial deflection due to the static liquid pressure changes remarkably at a joint part of the cylindrical and torispherical shells, but has no effect on the natural frequency.

REFERENCES

- [1] Brabant, F., Gantenbein, F. and Gibert, R. J. (1985). 3-D Seismic Analysis of Pool Type LMFBR Structures with Fluid-Structure Interaction, Proc. 8th Inter Conf. on S. M. I. R. T., Brussels, EK1/1, pp.303-308.



Torispherical vessel Semispherical vessel
Fig. 3 Meshes of FEM



Torispherical vessel Semispherical vessel
Fig. 4 Initial Deflection

In this equation, $[K_S^L]$ is the linear stiffness matrix, where $[K_{S0}^O]$ is the stiffness matrix of initial stress due to the static liquid pressure and obtained from the initial deformation of equation (12).

Substituting $\{\phi\}$ obtained from equation (9) into equations (10) and (11), we finally obtain the governing equations of the liquid surface and elastic solid.

$$[M]\{\ddot{\delta}\} + [K]\{\delta\} = \{0\} \quad (14)$$

where

$$[M] = \begin{Bmatrix} [M_{11}], [M_{12}] \\ [M_{21}], [M_{22}] \end{Bmatrix}$$

$$[M_{11}] = \rho_1 [P_1]^T [H]^{-1} [P_1] \quad , \quad [M_{12}] = \rho_1 [P_1]^T [H]^{-1} [P_2]$$

$$[M_{21}] = \rho_1 [P_2]^T [H]^{-1} [P_1] \quad , \quad [M_{22}] = [M_S] + \rho_1 [P_2]^T [H]^{-1} [P_2] \quad (15)$$

$$[K] = \begin{Bmatrix} \rho_1 g [K_1], [0] \\ [0], [K_S] \end{Bmatrix} \quad , \quad \{\delta\} = \begin{Bmatrix} \{\eta\} \\ \{\delta_S\} \end{Bmatrix}$$

Here we shall consider the free vibration of partially liquid-filled shells of revolution. In this case, unknown variables can be developed and separated in the revolution direction with Fourier series as follows.

$$\phi(r, \theta, z; t) = \sum_{N=0}^{\infty} \phi(r, z; t) \cos N\theta \quad , \quad \eta(r, \theta, z; t) = \sum_{N=0}^{\infty} \eta(r, z; t) \cos N\theta$$

$$u(r, \theta, z; t) = \sum_{N=0}^{\infty} u(r, z; t) \cos N\theta \quad , \quad v(r, \theta, z; t) = \sum_{N=1}^{\infty} v(r, z; t) \sin N\theta \quad (16)$$

$$W(r, \theta, z; t) = \sum_{N=0}^{\infty} W(r, z; t) \cos N\theta$$

Using these equations (16), we can obtain finally the isolated equation of motion for each value of N .

$$[M_N]\{\ddot{\delta}_N\} + [K_N]\{\delta_N\} = \{0\} \quad (17)$$

Axisymmetric isoparametric thick shell-elements are used for the shell, and isoparametric quadrilateral elements and line ones are used for the interior and surface of liquid, respectively.

4. NUMERICAL RESULTS

The validity of the calculation code proposed here has been confirmed for the free vibration analysis of cylindrical shells partially filled with liquid (Tani et al, 1988). In this paper, varying the filling rate of liquid, numerical calculations are carried out for two kinds of the pool-type fast breeder reactor vessels which consist of the semispherical and torispherical shells jointed to the cylindrical shells. These geometrical configuration and material properties are shown in Fig. 2 and Table 1. Figure 3 shows the meshes of the finite element method used here, by which the practically accurate solutions are obtained.

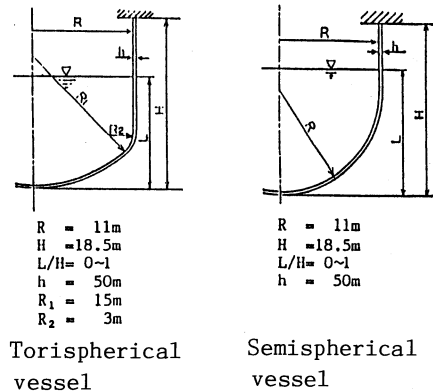
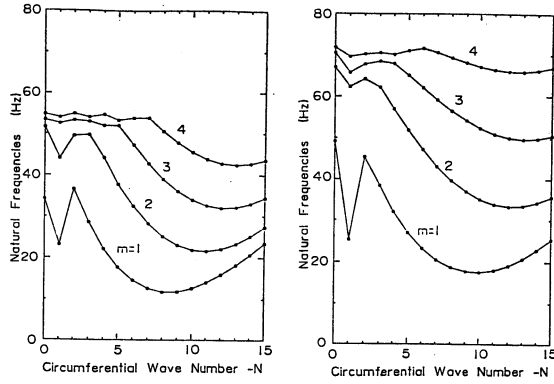
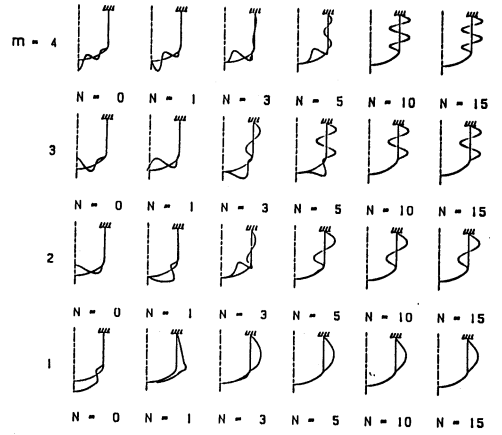


Fig. 2 Models of FBR Vessel

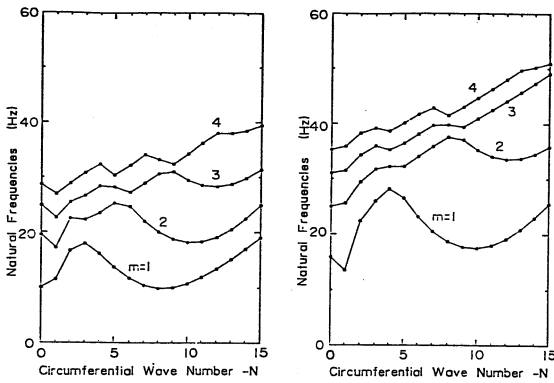


Torispherical vessel Semispherical vessel

Fig. 5 Natural Frequencies (L/H=0)

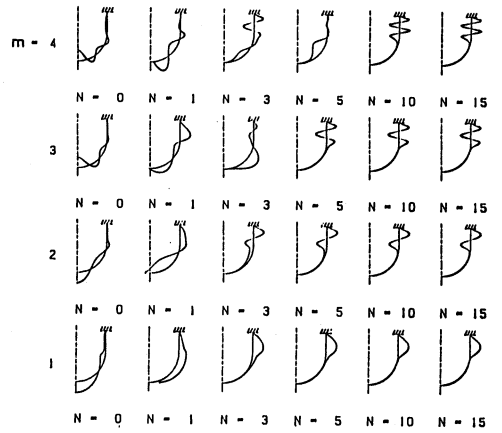


Torispherical vessel



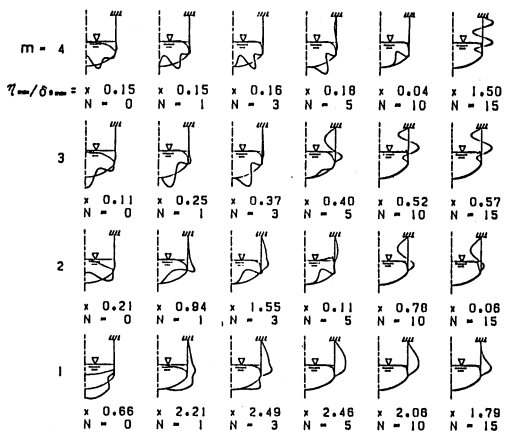
Torispherical vessel Semispherical vessel

Fig. 7 Natural Frequencies (L/H=0.5)

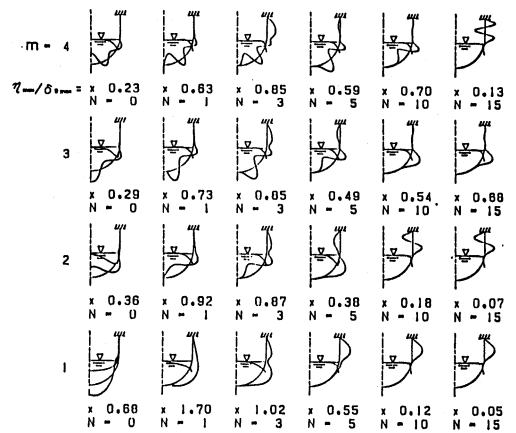


Semispherical vessel

Fig. 6 Vibration Modes (L/H=0)



Torispherical vessel



Semispherical vessel

Fig. 8 Vibration Modes (L/H=0.5)

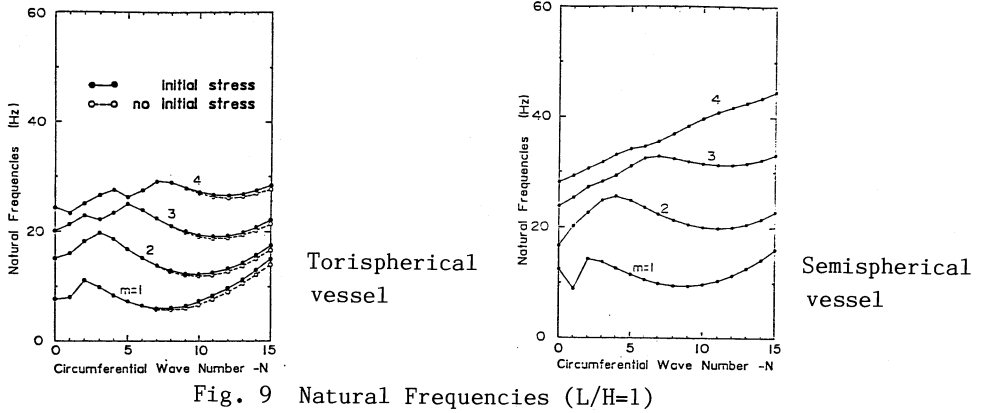


Fig. 9 Natural Frequencies (L/H=1)

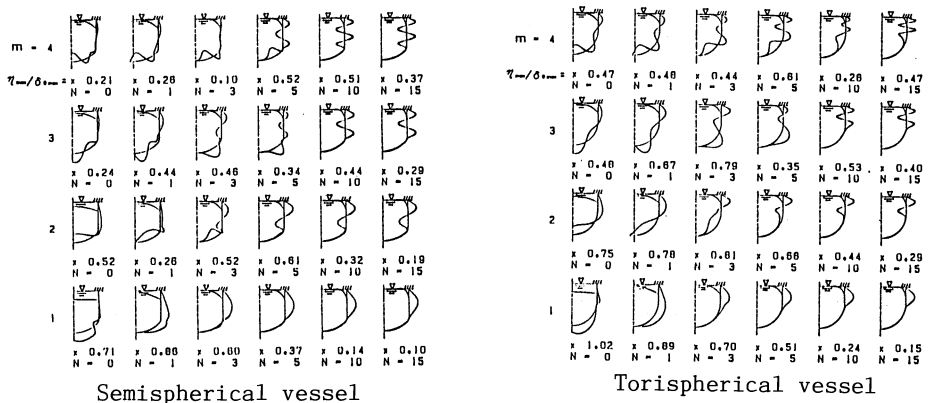


Fig. 10 Vibration Modes (L/H=1)

- [2] Chiba, M., Yamaki, N. and Tani, J. (1984). Free Vibration of a Coamped-Free Circular Cylindrical Shell Partially Filled with Liquid-Part I: Theoretical Analysis, Thin-Walled Structures, Vol. 2, pp.265-284. PartII : Numerical Results, *ibid*, Vol. 2, pp.307-324. Part III : Experimental Results, *ibid*, Vol. 3, pp.1-14.
- [3] Fujimoto, S., Yamamoto, S., Shimizu, H., Murakami T., Sakurai, A., Kurihara, C. and Mashiko, Y. (1985). Experimental and Analytical Study on Seismic Design of a Large Pool Type LMFBR in Japan, *ibid*, Ek1/4, pp.315-320.
- [4] Gvildys, J., Ma, D. C. and Chang, Y. W. (1985). Seismic Response of LMFBR Tanks with Imperfections, *ibid*, Ek1/9, pp.339-344.
- [5] Haroun, M. A. and Housner, G. W. (1982). Complications in Free Vibration Analysis of Tanks, Proc. ASCE, Vol. 108, No.EM-5, pp.801-818.
- [6] Ma, D. C. (1985). Seismic Analysis of a Large LMFBR with Fluid-Structure Interactions, *ibid*, Ek1/3, pp.309-314.
- [7] Ma, D. C. (1985). Seismic Interactions Between Primary Tank and Core Barrel, *ibid*, Ek1/6, pp.327-332.
- [8] Ma, D. C. and Gvildys, J. (1987). Seismic Analysis of a 400-MWe Pool Type Fast Reactor, Proc. 9th Inter. Conf. on S. M. I. R. T., Lausanne, Ek-10, pp.493-498.
- [9] Petruschke, W. (1985). Seismic Analysis of the Reactor Vessel for a Fast Reactor, Including Fluid-Structure Interacion, *ibid*, Ek1/5, pp.321-326.
- [10] Sakurai, A., Kurihara, C., Nalagawa, M., Kawashima, H. and Madokoro, M. (1985) Fluid-Coupled Vibration Analysis of Reduced Models of Pool-Type LMFBR, *ibid*, Ek1/8, pp.333-338.
- [11] Tani, J. and Teraki, J. (1988). Free Vibration Analysis of Partially Liquid-Filled Shells of Revolution, Trans. Japan Soc. Mech. Eng. (Ser. C), (in Japanese), Vol.54, pp. 73-79.