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# BIFURCATION OF CUBIC NONLINEAR PARALLEL PLATE-TYPE STRUCTURE IN AXIAL FLOW

**Li Lu ***
*Department of Applied Mechanics and Engineering, Southwest Jiaotong University, Chengdu 610031, China*
Phone:028-87600328, Fax:028-87600797
E-mail: helloluli@sohu.com

**Yiren Yang**
*Department of Applied Mechanics and Engineering, Southwest Jiaotong University, Chengdu 610031, China*
Phone:028-87600328, Fax:028-87600797
E-mail: yangyir@sohu.com

## ABSTRACT

The Hopf bifurcation of plate-type beams with cubic nonlinear stiffness in axial flow was studied. By assuming that all the plates have the same deflections at any instant, the nonlinear model of plate-type beam in axial flow was established. The partial differential equation was turned into an ordinary differential equation by using Galerkin method. A new algebraic criterion of Hopf bifurcation was utilized to in our analysis. The results show that there's no Hopf bifurcation for simply supported plate-type beams while the cantilevered plate-type beams has. At last, the analytic expression of critical flow velocity of cantilevered plate-type beams in axial flow and the purely imaginary eigenvalues of the corresponding linear system were gotten.

**Keywords:** Hopf bifurcation, critical flow velocity, axial flow, parallel plate-type structure.

## 1 INTRODUCTION

The assemblies of parallel flat plates are used in the core design of some research and power reactors including the Engineering Test Reactor, the Materials Testing Reactors and Shipping Atomic Power Station (El-Wakil, 1962). One typical character of these plates is that there are narrow channels full of coolant between each two plates. And coolant flows to take away the redundant heat when system works. The length-to-width ratios of rectangular plates are usually as great as 10 to 1. In these structures, the coupling effect of coolant is usually strong when the structural components vibrate. The research on vibration and stability of parallel plate-type structure has been greatly concerned due to the importance in the design of plate-type fuel elements and the difficulty of such flow-induced problem. As early as 1960, Miller (1960) brought forward a method of calculating the critical velocities for collapse of Reactor parallel plate fuel assemblies. Huang (1995) established cantilevered plates model in axial flow to analyze snore of people. Guo and Paidoussis (2000) did some research on the stability of rectangular plates with free side-edges in two-dimensional inviscid channel flow based on Galerkin method and Fourier transformation.

Actually, to such parallel plate-type structure, the bearing between plates is often nonlinear which will cause more complex phenomenon. The aim of this paper is to found an appropriate nonlinear parallel plate-type model in axial flow and further to analyze the motions of the system. Due to the length-to-width ratios of rectangular plates, the plates are simplified to be beams. Two models were established according as different boundary conditions. The partial differential equation was turned into an ordinary differential equation by using Galerkin method. A new algebraic criterion of Hopf bifurcation (Zhang, 2000) was utilized in our analysis. The result shows that there's no Hopf bifurcation for simply supported plate-type beams while the cantilevered plate-type beams has in some region. Further more, the analytic expression of critical flow velocity of the cantilevered model, and the purely imaginary eigenvalues of the corresponding linear system were gotten.

## 2. AN ALGORITHM CRITERION FOR HOPF BIFURCATION

Consider the general nonlinear differential equation:

$$\dot{x} = f(x,\mu), x \in R^n, \mu \in R. \quad \text{(Eq. 1)}$$

By solving equation $f(x,\mu)=0$, one can get the isolated equilibrium point of system (1), written as $x = x_0(\mu)$. To be general, suppose the equilibrium point is the coordinate origin, the Jacobian matrix of system (1) in point $x=0$ can be written as:

$$J(\mu) = D_x(0,\mu). \quad \text{(Eq. 2)}$$

Further, solve the characteristic equation of Jacobian matrix $\det(J(\mu)-\lambda I)=0$, and rewrite it to be the form:

$$\lambda^n + A_1(\mu)\lambda^{n-1} + A_2(\mu)\lambda^{n-2} + \cdots + A_{n-1}(\mu)\lambda + A_n(\mu) = 0. \quad \text{(Eq. 3)}$$

**Lemma 1.** (Zhang, 2000) Equation (3) has a pair of purely imaginary eigenvalues and the other n-2 eigenvalues have negative real parts if and only if the following two conditions were satisfied:

(C1) $A_i > 0 (i = 1,2,\cdots,n)$,

(C2) $\Delta_i > 0 (i = n-3, n-5, \cdots)$, $\quad \Delta_{n-1} = 0$,

here $\Delta_i$ represents the Hurwitz determinant of equation (3).

**Lemma 2.** (Zhang, 2000) If equation (3) has a pair of purely imaginary eigenvalues written as $\pm i\omega$ and the other n-2 eigenvalues have negative real parts, then one can get the purely imaginary eigenvalues by:

$$\omega^2 = \frac{\Delta_{n-3}}{\Delta_{n-2}} A_n. \quad \text{(Eq. 4)}$$

**Lemma 3.** (Zhang, 2000) If all eigenvalues of equation (3) have negative parts at $\mu = \mu_0$, and if the following two conditions were satisfied:

(S1) Suppose $\Delta_{n-3}(\mu_c) > 0$ holds, here $\mu_c = \min_{\mu}\{|\mu-\mu_0| : \Delta_{n-1}(\mu) = 0\}$, then equation (3) has a pair of purely imaginary eigenvalues and the other n-2 eigenvalues have negative real parts at $\mu = \mu_c$.

Suppose *U* and *V* be the left and right eigenvector corresponding to eigenvalue $i\omega$, and they satisfy the normalization *UV=1*. And if the following inequality is satisfied:

(S2) $\mathrm{Re}(UPV) \neq 0, here\ P = \left.\frac{dJ(\mu)}{d\mu}\right|_{\mu=\mu_c}$.

Then the system undergoes Hopf bifurcation at $\mu = \mu_c$.

## 3 HOPF BIFURCATIONS OF PLATE-TYPE BEAMS IN AXIAL FLOW

The parallel plates are placed in a flexible rectangular pipe shown in figure 1. Five plates are arranged in the pipe and the gaps of narrow channels are the same. The plates are very thin and the gaps between plates are usually very small. Since the length-to-width ratios of plates are great enough, the plates can be simplified to be beams.

Figure 1(a) gives the whole model, figure 1(b) shows the simply-supported beams with cubic nonlinear stiffness in the middle of beam and figure 1(c) the cantilevered beams with cubic nonlinear stiffness at the right hand. The length of each plate is $l$ and the initial velocity of coolant water is *U*.

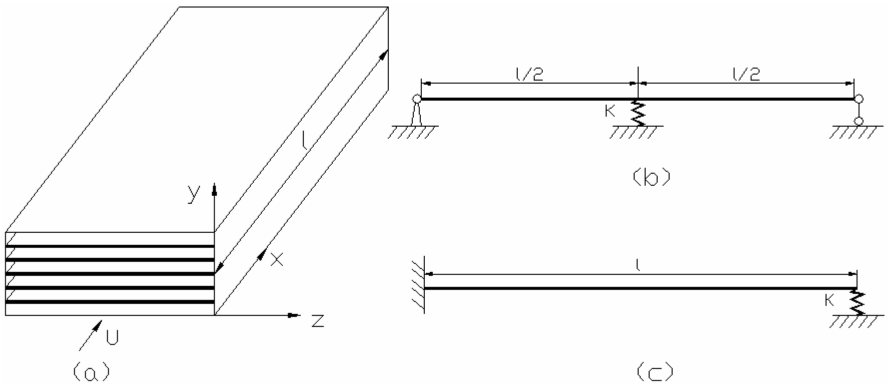

*Fig. 1 Sketch map of parallel plates in flexible rectangular pipe*

By assuming that all the plates have the same deflections at any instant, and the plates are simplified to be beams considering of the length-to-width ratios of rectangular plates. The governing equation of such flow-induced structure can be written as the following (Guo, 1993):

$$EI\frac{\partial^4 y}{\partial x^4}+M_d U^2\frac{\partial^2 y}{\partial x^2}+2M_d U\frac{\partial^2 y}{\partial x\partial t}+(M+M_d)\frac{\partial^2 y}{\partial t^2}=0\,, \qquad \text{(Eq. 5)}$$

in which, *EI* is the flexural rigidity of plate-type beam; $y=y(x,t)$ is the transversal displacement of structure; *M* is the mass of beam per length; $M_d$ is the fluid mass per length and *U* the velocity of fluid.

The nonlinear restriction is considered to be cubic nonlinear stiffness. To simply supported beam and cantilevered beam, the nonlinear force has different form:

$$f=(K_1 y+K_2 y^3)\delta(x-l/2)\,,\quad f=(K_1 y+K_2 y^3)\delta(x-l)\,, \qquad \text{(Eq. 6)}$$

here, $\delta(\,)$ represents Dirac function and $K_1$, $K_2$ the linear and cubic nonlinear stiff coefficient of stiffness separately.

Combining Eq. 5 and Eq. 6, the dynamic equation of the nonlinear system has the form:

$$EI\frac{\partial^4 y}{\partial x^4}+M_d U^2\frac{\partial^2 y}{\partial x^2}+2M_d U\frac{\partial^2 y}{\partial x\partial t}+(M+M_d)\frac{\partial^2 y}{\partial t^2}+f=0\,. \qquad \text{(Eq. 7)}$$

Defining the following non-dimensional variables and parameters:

$$w=\frac{y}{l},\xi=\frac{x}{l},\beta=\frac{M_d}{M_d+M},u=\left(\frac{M_d}{EI}\right)^{0.5}Ul,\tau=\left(\frac{EI}{M_d+M}\right)^{0.5}\frac{t}{l^2},k_1=\frac{K_1 l^3}{EI},k_2=\frac{K_2 l^5}{EI}\,,$$

Eq. 7 can be rewritten as:

For simply-supported beam:

$$\frac{\partial^4 w}{\partial \xi^4}+u^2\frac{\partial^2 w}{\partial \xi^2}+2\sqrt{\beta}u\frac{\partial^2 w}{\partial \xi\partial \tau}+\frac{\partial^2 w}{\partial \tau^2}+(k_0 w+k_1 w^3)\delta(\xi-1/2)=0\,. \qquad \text{(Eq. 8-a)}$$

For cantilevered beam:

$$\frac{\partial^4 w}{\partial \xi^4}+u^2\frac{\partial^2 w}{\partial \xi^2}+2\sqrt{\beta}u\frac{\partial^2 w}{\partial \xi\partial \tau}+\frac{\partial^2 w}{\partial \tau^2}+(k_1 w+k_2 w^3)\delta(\xi-1)=0\,. \qquad \text{(Eq. 8-b)}$$

Here Eq. 8 is an infinite dimension partial differential equation and usually, it is difficult to be solved directly. So Galerkin method is utilized to disperse the equation.

Suppose:

$$w(\xi,\tau)=\sum_{i=1}^{n}\eta_i(\tau)\varphi_i(\xi), \qquad \text{(Eq. 9)}$$

here, $\eta_i(\tau)$ are the generalized coordinates and $\phi_i(\xi)$ the eigenvectors of beam undertaking harmonic vibration in air.

## 3.1 Hopf analysis of simply- supported beams

Choose the orthonormal natural modes of simply-supported beams doing bending vibration in air:

$\phi_i(\xi)=A\sin(i\pi\xi), i=1,2,\cdots,n\,, A=\sqrt{2}\,.$

Putting Eq. 9 into Eq. 8-a, both sides of equation are multiplied by $\phi_j$ and doing integral along $\xi$ from *0* to *1*, noticing the eigenvectors being orthogonal, the partial differential equation Eq. 8 is turned into a group of ordinary differential equations of independent variable $\tau$ :

$$\ddot{\eta}_i + \omega_i^2\eta_i - u^2\omega_i\eta_i + 2\sqrt{\beta}u\sum_{\substack{j=1\\ j\neq i}}^{n}\left[\frac{2ij}{i^2-j^2} - \frac{j\cos(i+j)\pi}{i+j} - \frac{j\cos(i-j)\pi}{i-j}\right]\dot{\eta}_j$$
$$+ k_1 A\sin(i\pi)\sum_{j=1}^{n}\eta_j\phi_j(\frac{1}{2}) + k_2 A\sin(i\pi)\left[\sum_{j=1}^{n}\eta_j\phi_j(\frac{1}{2})\right]^3 = 0, \qquad i = 1,2,\cdots,n \quad . \qquad \text{(Eq. 10)}$$

Here $\omega_i = (i\pi)^2, i = 1,2,\cdots,n$ are the non-dimensional natural frequencies of simply-supported beam doing bending vibration in air.

First, suppose the system has two degrees of freedom, that is $n = 2$ .

And let $q_1 = \eta_1, q_2 = \dot{\eta}_1, q_3 = \eta_2, q_4 = \dot{\eta}_2,$ we have:

$$\begin{cases} \dot{q}_1 = q_2 \\ \dot{q}_2 = a_1 q_1 + a_2 q_4 - f_1 \\ \dot{q}_3 = q_4 \\ \dot{q}_4 = b_1 q_3 + b_2 q_2 - f_2 \end{cases} . \qquad \text{(Eq. 11)}$$

Where, $\begin{cases} a_1(u) = -(\omega_1^2 - \omega_1 u^2 + 2k_1), a_2(u) = 2\sqrt{\beta}u \times 8/3, f_1 = 4k_2\eta_1^3 \\ b_1(u) = -(\omega_2^2 - \omega_2 u^2), b_2(u) = 2\sqrt{\beta}u \times (-8/3), f_2 = 0 \end{cases}$ .

From Eq. 11 it is easy to see that point $(q_1, q_2, q_3, q_4) = (0,0,0,0)$ is the equilibrium point of Eq. 11. Further, from Eq. 11 we can get the Jacobian matrix of the system at this equilibrium:

$$J(0,\mu) = J(0,u) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_1 & 0 & 0 & a_2 \\ 0 & 0 & 0 & 1 \\ 0 & b_2 & b_1 & 0 \end{bmatrix} . \qquad \text{(Eq. 12)}$$

By operation of $(J - \lambda I) = 0$ , one can get the characteristic equation:

$$\lambda^4 + (-a_1 - a_2 b_2)\lambda^2 + a_1 b_1 = 0 . \qquad \text{(Eq. 13)}$$

Compared Eq. 13 with lemma 1, we know that, to the system in this paper, there is:

$$A_1(\mu) = A_3(\mu) = 0 . \qquad \text{(Eq. 14)}$$

That is to say, Eq. 14 can't satisfy (C1) of lemma 1. That is also to say, the situation that to be a pair of purely imaginary eigenvalues and all other eigenvalues having negative real parts can't occur to Eq. 13. So when it is described by two degrees of freedom, the system can't bifurcate with Hopf.

Then, suppose the system has *n* degrees of freedom, by calculating we get the characteristic equation:

$$\begin{vmatrix} -\lambda & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ a_{21} & -\lambda & a_{23} & a_{24} & a_{25} & \cdots & a_{2n-1} & a_{2n} \\ 0 & 0 & -\lambda & 1 & 0 & \cdots & 0 & 0 \\ a_{41} & a_{42} & a_{43} & -\lambda & a_{44} & \cdots & a_{4n-1} & a_{4n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & -\lambda & 1 \\ a_{n1} & a_{n2} & a_{n3} & a_{n4} & a_{n5} & \cdots & a_{nn-1} & -\lambda \end{vmatrix} = 0 . \qquad \text{(Eq. 15)}$$

Where parameters $a_{ij}(i=2,4,\cdots,n;\ j=1,2,\cdots,n)$ are calculated by Eq. 10.

From Eq. 15, one can see that the elements on the cross are $-\lambda$ instead of such as $-\lambda+c$ (here c may be one constant or a function of non-dimensional velocity). Spread the determinant one can see that there is not the item about $\lambda^{n-1}$. That is, corresponding to Eq. 3, there is $A_1(\mu)=0$. From Lemma 1 and Lemma 3 we see, the hopf bifurcation will not happen to this system.

### 3.2 Hopf analysis of cantilevered beams

Choose the natural modes of cantilevered beams doing bending vibration in air:

$$\varphi_i = ch\alpha_i\xi - \cos\alpha_i\xi - r_i(sh\alpha_i\xi - \sin\alpha_i\xi),\ r_i = (\sin\alpha_i - sh\alpha_i)/(\cos\alpha_i + ch\alpha_i),$$
$$\alpha_1 = 1.875, \alpha_2 = 4.694, \alpha_i = (i-0.5)\pi \quad (i \ge 3).$$

To be simple, here we take *n=2*. Putting Eq. 9 into Eq. 8-b and noticing the eigenvectors being orthogonal, the partial differential equation Eq. 8 is turned into a group of ordinary differential equations of independent variable $\tau$ (Ni, 2001):

$$\begin{cases} a_1\eta_1 + a_2\eta_2 + a_3\dot{\eta}_1 + a_4\dot{\eta}_2 - \ddot{\eta}_1 + f_1 = 0 \\ b_1\eta_1 + b_2\eta_2 + b_3\dot{\eta}_1 + b_4\dot{\eta}_2 - \ddot{\eta}_2 + f_2 = 0 \end{cases}. \quad \text{(Eq. 16)}$$

Here,

$$a_1 = -({\alpha_1}^4 + u^2c_{11} + k_1g_{11}), a_2 = -(u^2c_{12} + k_1g_{12}), a_3 = -2\sqrt{\beta}ud_{11}, a_4 = -2\sqrt{\beta}ud_{12}$$
$$b_1 = -(u^2c_{21} + k_1g_{21}), b_2 = -({\alpha_2}^4 + u^2c_{22} + k_1g_{22}), b_3 = -2\sqrt{\beta}ud_{21}, b_4 = -2\sqrt{\beta}ud_{22}$$
$$f_1 = -k_2\varphi_1(1)[\varphi_1(1)\eta_1 + \varphi_2(1)\eta_2]^3, \qquad f_2 = f_1\varphi_2(1)/\varphi_1(1)$$
$$d_{ij} = \begin{cases} 4/[(\alpha_i/\alpha_j)^2 + (-1)^{i+j}] & i=j \\ 2 & i \ne j \end{cases}, \quad c_{ij} = \begin{cases} 4(\alpha_j r_j - \alpha_i r_i)/[(-1)^{i+j} - (\alpha_i/\alpha_j)^2] & i \ne j \\ \alpha_i r_i(2-\alpha_i r_i) & i=j \end{cases}.$$
$$g_{ij} = \begin{cases} \varphi_i(1)\varphi_j(1) & i \ne j \\ \varphi_i^2(1) & i=j \end{cases} \qquad (i,j=1,2)$$

Let $q_1=\eta_1, q_2=\dot{\eta}_1; q_3=\eta_2, q_4=\dot{\eta}_2$, Eq. 16 can be written into the formal form:

$$\begin{cases} \dot{q}_1 = q_2 \\ \dot{q}_2 = a_1q_1 + a_2q_3 + a_3q_2 + a_4q_4 + f_1 \\ \dot{q}_3 = q_4 \\ \dot{q}_4 = b_1q_1 + b_2q_3 + b_3q_2 + b_4q_4 + f_2 \end{cases}. \quad \text{(Eq. 17)}$$

From Eq. 17 it is easy to see that point $(q_1,q_2,q_3,q_4)=(0,0,0,0)$ is the equilibrium point of Eq. 17. Further, from Eq. 17 we can get the Jacobian matrix of the system at this equilibrium:

$$J(0,\mu) = J(0,u) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_1 & a_3 & a_2 & a_4 \\ 0 & 0 & 0 & 1 \\ b_1 & b_3 & b_2 & b_4 \end{bmatrix}. \quad \text{(Eq. 18)}$$

By operation of $(J-\lambda I)=0$, one can get the characteristic equation:

$$\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0. \quad \text{(Eq. 19)}$$

Here,

 

$$\begin{aligned}
A_1 &= 2\sqrt{\beta}u(d_{11}+d_{22}) \\
A_2 &= \alpha_1^4+\alpha_2^4+u^2[c_{11}+c_{22}+4\beta(d_{11}d_{22}-d_{12}d_{21})]+k_1(g_{11}+g_{22}) \\
A_3 &= 2\sqrt{\beta}u^3(d_{11}c_{22}-d_{21}c_{12}+d_{22}c_{11}-d_{12}c_{21}) \\
&\quad +2\sqrt{\beta}u[(d_{11}\alpha_2^4+d_{22}\alpha_1^4)+k_1(d_{11}g_{22}-d_{21}g_{12}+d_{22}g_{11}-d_{12}g_{21})] \\
A_4 &= \alpha_1^4\alpha_2^4+u^4[c_{11}\alpha_2^4+c_{22}\alpha_1^4+k_1(c_{11}g_{22}-c_{21}g_{12}+c_{22}g_{11}-c_{12}g_{21})] \\
&\quad +k_1(g_{11}\alpha_2^4+g_{22}\alpha_1^4)+k_1^2(g_{11}g_{22}-g_{12}g_{21})
\end{aligned} \quad . \qquad \text{(Eq. 20)}$$

Validating Eq. 20 we know that when $u > 0$, if $A_1 > 0, A_2 > 0, A_4 > 0$ holds, and when $k_1 < 48.967$, the inequality $A_3 > 0$ is always satisfied no matter how velocity changes.

Let

$$B = 2\sqrt{\beta}\,. \qquad \text{(Eq. 21)}$$

By calculating we get the Hurwitz determinant of Eq. 19:

$$\Delta_1 = A_1 > 0\,,\ \ \Delta_3 = u^2B^2(H_1u^4 + H_2u^2 + H_3)\,, \qquad \text{(Eq. 22)}$$

where,

$H_1 = -623.87 + 918.29B^2, H_2 = 12886 + 1202.4k_1 + 30326B^2,\ H_3 = 895380 + 0.00018305k_1$.

Discussion 1: From Eq. 22 we know, to let $\Delta_3 = 0$ hold, there must be $u = 0$ or $H_1 < 0$. But it is easy to see here $u = 0$ is not the critical velocity we want. Since by Lemma 1, the velocity must greater than zero if (C1) is satisfied. So we can say, to let $\Delta_3 = 0$ hold, there must be $H_1 < 0$. That is, when coefficient $\beta$ satisfies the inequality $0 < \beta < 0.1698$, the Hurwitz determinant $\Delta_{n-1}$ may be zero, and the system may have a pair of purely imaginary eigenvalues and all other eigenvalues are complex numbers.

Discussion 2: When coefficient $\beta$ satisfies the inequality $\beta \geq 0.1698$, the Hurwitz determinant $\Delta_{n-1}$ is always greater than zero. It shows that when $k_1 < 48.967$ holds, if $\beta \geq 0.1698$, however the non-domensional velocity changes, all the eigenvalues of the system have negative real parts and the system will never undergo Hopf bifurcation.

Now we know, if $k_1 < 48.967$ and $0 < \beta < 0.1698$ are satisfied, Eq.19 may have a pair of purely imaginary eigenvalues while the other n-2 eigenvalues having negative real parts.

Let $\Delta_3 = 0$, we can get the non-dimensional critical velocity when system have a pair of purely imaginary eigenvalues and all other eigenvalues have negative real parts, written as:

$$u_c^2 = \frac{-H_2 - \sqrt{H_4}}{2H_1}\ . \qquad \text{(Eq. 23)}$$

Here, $$\begin{aligned} H_4 &= 2.4004\times10^9 + 3.0988\times10^7 k_1 + 1.4459\times10^6 k_1^2 - 2.5073\times10^9 B^2 \\ &\quad + 7.2929\times10^7 B^2 k_1 + 9.1967\times10^8 B^4 \end{aligned}\ .$$

From Lemma 2 we have:

$$\omega^2 = \frac{114.34u_c^4 + (2909 - 59.408k_1)u_c^2 + 7965.5k_1 + 24001}{(-5.5224 + 30.457B^2)u_c^2 + 32k_1 + 995.68}\,. \qquad \text{(Eq. 24)}$$

Suppose vectors *U* and *V* are the left and right eigenvectors corresponding to eigenvalue $i\omega$ separately, by calculating, we have:

$$\begin{aligned} U &= h\{1,[1+b_3(-a_1+i\omega)/b_1]/(-a_3+i\omega),u_3,(-a_1+i\omega)/b_1\} \\ V &= \{(-a_2-i\omega a_4)/(a_1+\omega^2+i\omega a_3), i\omega(-a_2-i\omega a_4)/(a_1+\omega^2+i\omega a_3),1,i\omega\}^T \end{aligned}, \qquad \text{(Eq. 25)}$$

here,

$$
\begin{cases}
u_3 = [a_2 + a_2 b_3(-a_1 + i\omega)/b_1 + b_2(-a_1 + i\omega)(-a_2 + i\omega)/b_1]/[i\omega(-a_3 + i\omega)] \\
h = (h_1 + h_2 + h_3) + i\omega(h_4 + h_5 + h_6) \\
h_1 = a_4\omega^2(-a_3 - \omega^2 b_3/b_1) + \omega^2 a_2(2 - a_1 b_3/b_1) \\
h_2 = [a_2(1 - a_1 b_3/b_1) + b_2(a_1 a_3 - \omega^2)/b_1](a_1 + \omega^2) - \omega^2 a_3[a_2 b_3 - b_2(a_1 + a_3)]/b_1 \\
h_3 = \omega^2[(\omega^2 - a_1 a_3)(a_1 + \omega^2) - \omega^2 a_3(a_1 + a_3)]/b_1 \\
h_4 = a_4\omega^2(2 - a_1 b_3/b_1) + a_2(a_3 + \omega^2 b_3/b_1) \\
h_5 = a_3[a_2(1 - a_1 b_3/b_1) + b_2(a_1 a_3 - \omega^2)/b_1] + (a_1 + \omega^2)[a_2 b_3 - b_2(a_1 + a_3)]/b_1 \\
h_6 = \omega^2[(a_1 + a_3)(a_1 + \omega^2) + a_3(\omega^2 - a_1 a_3)]/b_1
\end{cases}
.
$$

In Eq. 25 the coefficients have the feature $a_i = a_i(u_c), b_i = b_i(u_c), \quad i = 1,2,3,4$.

To be simple, let parameter $\beta = 0.1, k_1 = 10.0$, by calculating, we get:

$$\mathrm{Re}(U(u_c)P(u_c)V(u_c)) = 0.58 \neq 0. \qquad \text{(Eq. 26)}$$

From Eq. 26 and combining with Lemma 3, we can judge that a Hopf bifurcation occurs at $u_c$. That is, a unique limit cycle bifurcates from the origin.

## 4 CONCLUSIONS

The Hopf bifurcation of plate-type beams with cubic nonlinear stiffness in axial flow was studied in this paper. By assuming that all the plates have the same deflections at any instant, there are two main conclusions:

(1) There's no Hopf bifurcation for simply-supported plate-type beams while the cantilevered plate-type beams has.

(2) Whether the cantilevered plate-type beams undergoes Hopf bifurcation or not are mainly dependent on two parameters, they are $\beta$ and $k_1$. If the inequalities $k_1 < 48.967$ and $0 < \beta < 0.1698$ are satisfied, the system has Hopf bifurcation. Then, the analytic expression of critical flow velocity of cantilevered plate-type beams in axial flow and the purely imaginary eigenvalues of the corresponding linear system were gotten.

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