Finite element analysis with a special fracture linespring element

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I. INTRODUCTION

The concept of linespring has been extensively used on the past years to calculate the stress intensity factor of part through cracks in shell structures. Several authors obtained good results in the elastic range in particular J.C. Newman [1], F. Delale [2], Parks, King [4]. Its main advantage is to reduce a complete 3 dimensional finite element analysis (a discretization in the shell thickness is necessary to analyse correctly the discontinuous stress field at the crack tip neighbourhood) to a standard shell analysis (usign 3 or 4 nodes plate elements [4], [5], thus reducing computer costs and meshing intricacies.

The classical linespring concept is applied as follows :

- First, mesh the shell as if the part through crack were a full through crack.
- Second, bind pair of nodes that have same coordinates but are located on opposite edges of the crack with extensional and rotational stiffnesses representing the resistance of the remaining ligament of the part through crack.

So far, the computer implementation of these concepts were performed as mentionned above, however with a slight disadvantage : the user had to calculate for each crack geometry and for each pair of nodes the desired additionnal stiffnesses. Due to this rather cumbersome procedure, only few geometries have been correctly checked against solutions (various crack shapes and shell geometrics).

The aim of this paper is to present :

- a finite element formulation that has the advantage of computing at the element level the stiffness densities.
- the range of geometrical parameters for which this formulation yields accurate and reliable stress intensity factor for reactor engineering analysis.
- an improved element that takes care of locking problems occuring on non straight cracks.

2. FINITE ELEMENT FORMULATION

2.1 Bidimensionnal behavior

The finite element formulation is based on the analysis of a notched strip under tension or flexion.

The mechanical quantities of interest are the increase in a longitudinal extension or rotation due to the presence of the crack. The relation between these displacements and the applied forces is :

$$(1) \quad \begin{bmatrix} \delta_c \\ \theta_c \end{bmatrix} = \frac{2\,(1-\nu^2)}{E} \begin{pmatrix} \alpha_{mm} & \frac{6}{t}\,\alpha_{mF} \\ \frac{6}{t}\,\alpha_{mf} & \frac{36}{t^2}\,\alpha_{Ff} \end{pmatrix} \begin{pmatrix} N \\ M \end{pmatrix}$$

(a)

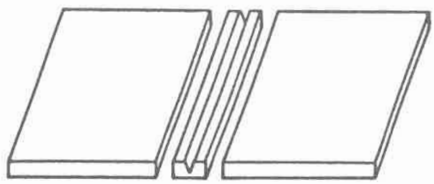
where ν, E are the Poisson's ratio and Young's modulus. t the plate thickness, δ_c and θ_c the extension and rotation due to the presence of the crack, N and M the applied force and moment.
α_{mm} α_{mf} and α_{ff} are dimensionless parameters related to a/t (ratio crack depth to plate thickness).

These parameters have been adjusted to fit the actual behavior of an equivalent bidimensionnal structure meshed with many elements at the crack tip. See Cheissoux [6] and King [3]. In fact, several techniques are available and yield consistant results.

2.2 Tridimensional behavior

One has now to generalize this bidimensional behavior to the one of a three dimensional shell.

An additional problem comes from the fact that the Linespring depth may vary along the element length.



(b)

This is taken care of by the following scheme for calculating the stiffness matrix and the energy :

$$(2) \quad E = \tfrac{1}{2} \int_0^L {}^t\Delta u \; \overline{R} \; \Delta u \; ds$$

where Δu are the 6 d.o.f. vector representing the displacements and rotation discontinuities accross the crack and $\overline{R}$ a 6 × 6 stiffness matrix that can be written as :

$$(3) \quad \overline{R} = \begin{bmatrix} P_1 & & \phi \\ & R & \\ \phi & & P_2 \end{bmatrix}$$

R is the inverse of the matrix of equation (1), and P_1 and P_2 are penalization matrices to force to 0 displacement and rotation discontinuities not pertaining to the opening of the crack. This penalization method has been used by Ebersolt [7] [8] and proved to be effective for three dimensional problems.

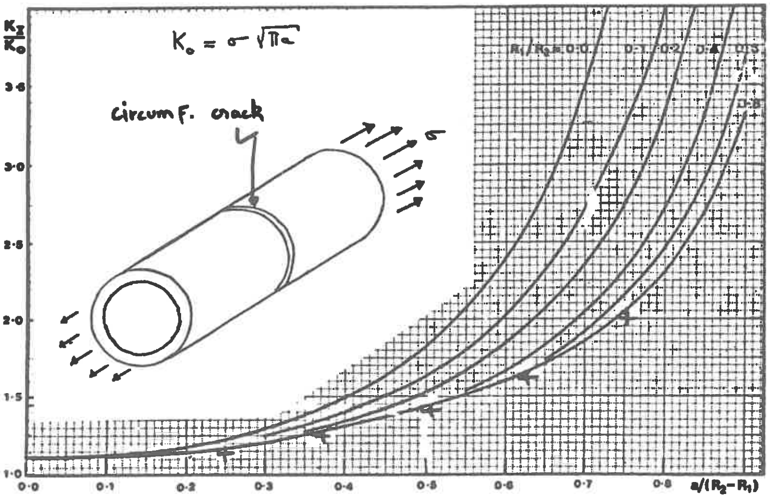
3. VALIDATION FOR ELASTIC ANALYSIS

In order to assess accuracy for ingeneer problems, several cases have been studied :

3.1 Cylindrical shell with circumferential crack with under tension

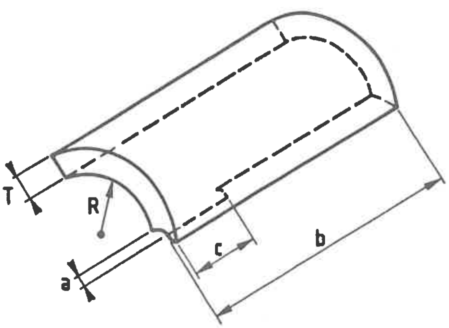
This case gives an example of accuracy of linespring element when it is located on curved shells.

CYLINDER IN TRACTION CIRCUMFERENTIAL CRACK

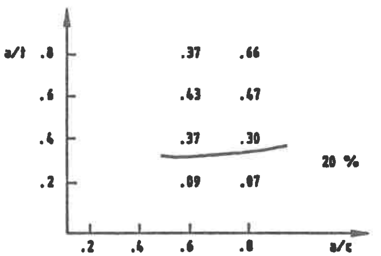


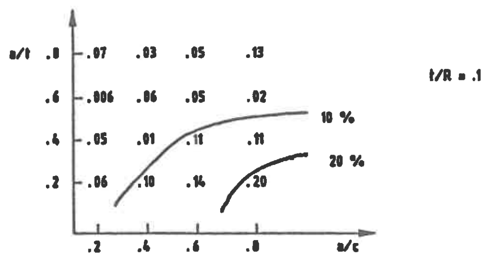
(+ results given by the linespring)

3.2 Cylindrical shell in internal pressure with a longitudinal elliptical crack.

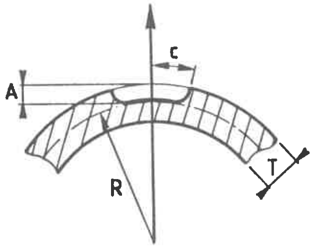


For differents values of the parameter a, b, t, the accuracy the calculated stress intensity factor is displayed on the following figures.

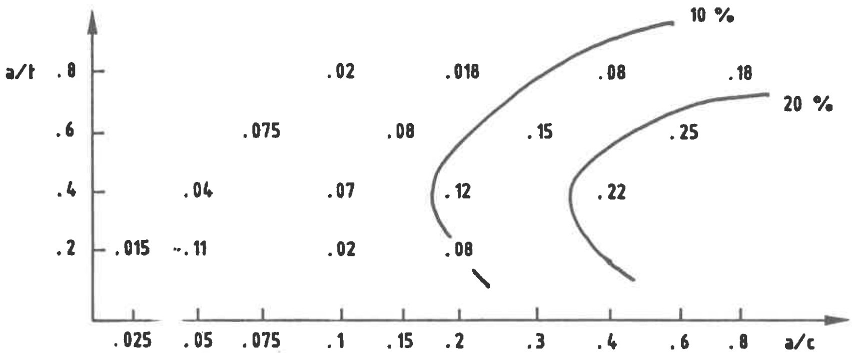




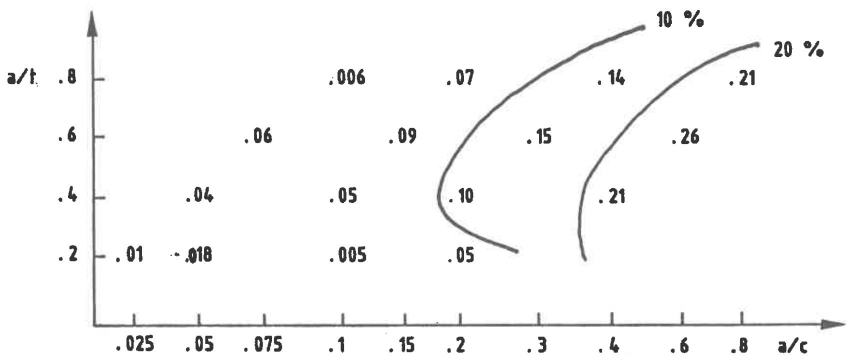
3.3 Cylindrical shell in traction with an elliptical circumferential cracks (external or internal)



For different values of the parameters a, c, t, the accuracy of the calculated stress intensity factor is displayed on the following figures :



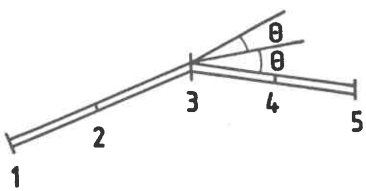
External cracks



Internal crack

4. AN IMPROVED FINITE ELEMENT FOR CURVED CRACKS

For specific geometries (displayed on the next figure) the linespring element exhibits a locking behavior. To improve the formulation,



a special under integration procedure has been deviced : the penalization part of stiffness matrix is underintegrated, whereas the coefficients related to the opening of the crack is normally integrated. This procedure is summerized as follows : we split the matrix of equation (3) in 2 parts :

$$R_1 = \begin{bmatrix} P_1 & \phi & \phi \\ \phi & \phi & \phi \\ \phi & \phi & P_2 \end{bmatrix} \qquad R_2 = \begin{bmatrix} \phi & \phi & \phi \\ \phi & R & \phi \\ \phi & \phi & \phi \end{bmatrix}$$

and we integrate the matrix R_1 with one point and the matrice R_2 with three points :

$$K = \int_{1pt} R_1 \, ds \times \int_{2pt} R_2 \, ds$$

The scheme proved to be quite effective for solving the case of non rectilinear cracks and has been used effectively in an industrial study involving a bended pipe with a crack located at a point where stresses are the highest.

5. CONCLUSION

The linespring element proved to be accurate for many simple tests where other analytical solutions are available. In the case of complex structures where the elements are not rectilinear, an improved formulation has been deviced and show also success.

Acknowledgement

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