18th International Conference on Structural Mechanics in Reactor Technology (SMiRT 18)
Beijing, China, August 7-12, 2005
SMiRT18-F01-9

APPLICATION OF A UNIFIED FATIGUE MODELLING TO SOME THERMOMECHANICAL FATIGUE PROBLEMS

K. Dang Van
LMS, Ecole Polytechnique, Palaiseau
France
Phone: 33169333373, Fax: 33169333026
E-mail: dangvan@lms.polytechnique.fr

H. Maitournam
LMS, Ecole Polytechnique, Palaiseau
France
Phone: 33169333346, Fax: 33169333026
E-mail: habibou@lms.polytechnique.fr

Z. Moumni
UME-MS ENSTA, Palaiseau
France
Phone: 33169319724, Fax: 33169319906
E-mail: moumni@ensta.fr

ABSTRACT

Fatigue under thermomechanical loadings is an important topic for nuclear industries. For instance, thermal fatigue cracking is observed in the mixing zones of the nuclear reactor. Classical computations using existing methods based on strain amplitude or fracture mechanics are not sufficiently predictive. In this paper an alternative approach is proposed based on a multiscale modelling thanks to shakedown hypothesis. Examples of predictive results are presented. Finally an application to the RHR problem is discussed.

Main ideas of the fatigue modelling:

Following an idea of Professor D. Drucker who wrote in 1963 “when applied to the microstructure there is a hope that the concept of endurance limit and shakedown are related, and that fatigue failure can be related to energy dissipated in idealized material when shakedown does not occur.” we have developed a theory of fatigue based on this concept which is different from classical fatigue approaches. Many predictive applications have been already done particularly for the automotive industry. Fatigue resistance of structures undergoing thermomechanical loadings in the high cycle regime as well as in the low cycle regime are calculated using this modelling. However, this fatigue theory is until now rarely used in nuclear engineering.

After recalling the main points of the theory, we shall present some relevant applications which were done in different industrial sectors.

We shall apply this modelling to the prediction of thermal cracking observed in the mixing zones of RHR.

1. INTRODUCTION

Fatigue under thermomechanical loadings is an important topic for nuclear industries. For instance, thermal fatigue cracking is observed in the mixing zones of the nuclear reactor as shown on the figure 1. Classical computations using existing methods based on strain amplitude or fracture mechanics are not sufficiently accurate and predictive. For instance application of recommendations contained in nuclear engineering practice fails to explain the formation of the observed cracks. In this paper an alternative approach is proposed based on a multiscale modelling and shakedown hypothesis. This “new” approach of fatigue (in fact it was first proposed in 1973 but it is ignored by the nuclear industry community) was first introduced by Dang Van for high cycle fatigue. Since, many industrial applications have been done for the design of mechanical industrial structures, particularly in the automotive industries. Example of predictive results are presented. Finally an application to the RHR problem is discussed.

Figure 1: Typical cracks observed in the mixing zone of RHR..

2. MAIN IDEAS OF THE FATIGUE MODELLING:

Professor D.C. Drucker who wrote in 1963 [1]”when applied to the microstructure there is a hope that the concept of endurance limit and shakedown are related, and that fatigue failure can be related to energy dissipated in idealized material when shakedown does not occur.”

Our proposal for fatigue modelling is based on this concept which is very different from classical fatigue approaches (stress, strain or fracture mechanic).

In what follows, we shall distinguish three scales:

The microscopic scale of dislocations

The mesoscopic scale of grains; σ,ε,ρ are respectively the mesoscopic stress, strain, and residual stress tensors;

The macroscopic scale representing phenomena at the scale of the engineering; Σ is the macroscopic stress tensor.

In a simplified analysis we could say that fatigue phenomena start generally with appearance of slip bands in grains which broaden progressively during the cycles. The proportion of grains in which slip bands develop increases with the applied load.

In the classical high-cycle fatigue regime (HCF), in general no irreversible deformation,(i.e. plastic or viscous), is detected at the macroscopic level. The material behavior seems to be purely elastic and as a consequence, the use of stress or strain at this engineering scale are equivalent. In practice stress is often preferred to strain. However at a mesoscopic level, plasticity occurs in certain number of grains and generates a heterogenous plastic strain. Only misoriented crystals undergo plastic slip corresponding to a heterogeneous distribution of microcracks. The initiation of the first visible crack, at the macroscopic scale represents a large part of the fatigue life.

The low-cycle fatigue regime (LCF) implies significant macroscopic deformations conducting at this level to irreversible deformations. At the mesoscopic level, the metal grains are subjected to plastic deformation in a more homogeneous manner than in HCF regime. The first microcracks in the persistent slip bands appear quite early in the life of the structure. The strain and the plastic strain are no more related to the stress through a simple relation since, as it is very well known, it depends also on the loading path.

In both LCF and HCF, damage phenomena occur in the grains, and therefore the use of mesoscopic fields seems to be relevant for studying fatigue phenomena. However, it is well known(cf. the theory of polycrystalline aggregates) that the mesoscopic parameters differs from the macroscopic parameters (i.e. classical engineering parameters). For instance, the relation between and Σ is:

$$\sigma = A\Sigma + \rho \tag{1}$$

where ρ is the local residual stress and **A** is the elastic stress localization tensor. **A** is the identity tensor if local and macroscopic elastic moduli are supposed to be similar. This relation shows that it is in general incorrect to use the macroscopic stress Σ for characterizing phenomena which occur at the grain scale since the local stress is not proportional to Σ and does not include information about ρ. Moreover, ρ is related to the plastic deformations (local and mesoscopic) which depends on the loading path.

As fatigue is caused by irreversible phenomena let us compare the dissipated energy at both mesoscopic and macroscopic scales. It is well known (see for instance ref.1) that the total macroscopic work rate $\Sigma \dot{E}_p$ is the mean value $\sigma \dot{\varepsilon}_p$ of the local total work rate. However, the equality between the mesoscopic and macroscopic energy rate does not hold for plastic dissipation as proven by Professor H.D. Bui and recalled also in [2]. The difference between macroscopic plastic dissipation and mean value of mesoscopic plastic dissipation decreases with increasing plastic strain, as the plastic heterogeneity from grain to grain decreases. This also justifies why macroscopic plastic deformation is a reasonable approximation in LCF.

The evaluation of the local mesoscopic fields from the macroscopic ones is in general a difficult task since the material is locally heterogeneous and has to be considered as a structure when submitted to complex loading histories. Depending on the loading characteristics one can accept different simplifying assumptions which will permit a solution of the problem. The multiscale approaches in fatigue which is proposed are precisely based on the use of mesoscopic parameters instead of engineering macroscopic quantities. In order to derive a unified theory of fatigue, we suppose that the elastic shakedown occurs at the level of the microstructure as well as at the macroscopic one.

Thanks to these assumptions we can use shakedown theorems to calculate local parameters which will be used in prediction of fatigue occurrence on structures.

Under cyclic loadings, an elastoplastic mechanical structure may have three possible asymptotic responses after a certain numbers of cycles (which could be infinite):

-elastic shakedown which corresponds to stabilization on a pure elastic response;

-elastoplastic shakedown when this response is stabilized on an elastoplastic cycle;

-ratchet when their is no possible stabilization.

The static theorem of Melan gives a sufficient condition for elastic shakedown for a structure made of elastic perfectly plastic material. It can be stated as followed,:

If there exist a time θ and a fixed (i.e. independent of time t) self equilibrated stress field R(x) and a security coefficient m such that $\forall$ point x of the structure and t>θ, $g(m(\Sigma el(x,t)) + R(x)) < k^2$ the structure will shakedown elastically.

Σel is the stress response of the structure under the same external loading, but under the assumption that the constitutive material has a pure elastic behavior.

(This formulation due to W. Koiter differs slightly from the original formulation of Melan)

Demonstration and discussions of this theorem was given in a famous paper of Professor W. Koiter [3]. In this paper, Koiter draw our attention on the fact that this theorem and its proof do not say anything about the magnitude of plastic deformation which may occur before the structure reaches its shakedown state. It is clear that too large plastic deformation gives a solution, which has no physical meaning. But, Professor Koiter added that if “the total amount of plastic work performed in the loading is accepted as suitable criterion for assessing the overall deformation, boundedness of the overall deformation may be proved if the structure has a safety factor m>1 with respect to shakedown.”

We do not reproduce more detail of this discussion; we shall retain the condition that total plastic work must be bounded to ensure acceptable bounds on plastic deformation.

Melan’s theorem was extended by different authors to account for more realistic material behavior. In particular, generalization to elastoplastic material combining linear kinematic and isotropic hardening by Mandel et al. [4] and more recently by Q.S. Nguyen [5] for a class of material called generalised standard material which contains all classical strain hardening effects (isotropic, kinematic or combined isotropic and kinematic hardening). using an other formalism which is particularly interesting. However, these theorems are difficult to apply, because the fixed stress field R(x) must be self equilibrated, a condition which is not easy to fulfil.

It is why Mandel et al. gives an other proposal, which is a necessary condition of elastic shakedown. This last condition can be summarized as followed (Mandel-Halphen-Zarka [4]).

Shakedown occurs if it exists a fixed stress tensor Σ^* (not necessarily self equilibrated) such that

$$\forall t>\theta, g(\Sigma el(x,t)-\Sigma^*)-K^*_2(P_{eq})\leq 0 \tag{3}$$

The isotropic hardening parameter K is supposed to be an increasing function of equivalent plastic strain P_{eq} beyond some limit K*. $K^*_2(P_{eq})$ is the maximum acceptable value of the yield radius.

Thus, at the shakedown limit, Σ^* is the center of the of the smallest hypersphere surrounding the local loading path $\Sigma el(x,t)$, the radius of which is $K^*(P_{eq})$.

This theorem will be used in the proposed fatigue approach discussed hereafter.

Let us return to the Melan Koiter sufficient shakedown condition. Koiter's reasoning can be also extended to strain hardening material in the framework of the generalized standard material theory as introduced by Q.S. Nguyen et al. and recalled in [5]. (Most of the classical metallic material belong to this class). The boundedness of the dissipation (which corresponds to plastic work plus work induced by generalized strain hardening parameters) ensures that the plastic deformation as well as strain hardening parameters are bounded. (see ref.5)

We propose to apply the shakedown theory to the microstructure in order to derive a unified fatigue model valid for structural applications [6]. The main assumptions of this model is the following:

1- near the fatigue limit but below it, elastic shakedown takes place at all scales of material description, at the macroscopic scale as well as at the mesoscopic scale. In particular the local plastic dissipation must be bounded.

2- If the loading history is such that elastic shakedown is not possible, then the local admissible dissipation is bounded, This bound corresponds to fatigue initiation energy. The number of cycle necessary to dissipated this energy corresponds to the initiation period.

These assumptions are exactly those formulated by D.C. Drucker in 1963 [1]. However at that time, shakedown theory was limited to Melan Koiter theorem which only apply to elastic perfectly plastic material.

.

3. APPLICATION TO FATIGUE LIMIT CRITERION.

In the high cycle regime, only few misoriented grains (relative to the loading) undergo plastic deformation in localized slip bands. Under the fatigue limit, the dissipation is bounded so that the dissipation per cycle decreases and after a while becomes negligible. It is therefore difficult to evaluate it directly. In the high cycle regime an approach based on the apparent stabilised mesoscopic stress is preferred: the proposed polycyclic multiaxial fatigue resistance criterion is a combination of mesoscopic shear $\tau(t)$ and the concomitant hydrostatic pressure $p_H(t)$; more precisely

$$\tau(t) + a\, p_H(t) - b > 0 \tag{4}$$

then fatigue will occur. The two coefficients a and b are material constants that can be determined by two simple types of fatigue experiments; b for instance corresponds to the fatigue limit in simple shear.

General application of this criterion requires

-first to evaluate the mesoscopic stress tensor knowing the macroscopic stress cycle; this can be done under the assumption of elastic shakedown near the fatigue limit by constructing the smallest hypersphere surrounding the macroscopic loading path. Details of this construction is given in [2];

-second one must consider the plane on which the set ($\tau(t)$,$p_H(t)$) is a “maximum" relative to the criterion. This computation can be done as following: the maximum local shear at any time t is given by

$$\tau(t) = Tresca(\sigma(t)) = \underset{I,J}{Max}\left|\sigma_i(t) - \sigma_J(t)\right| \tag{5}$$

The stresses $\sigma_I(t)$, $\sigma_J(t)$ are the principal stresses at time t. The quantity that quantifies the danger of fatigue occurrence is defined by:

$$d = \underset{t}{Max} \frac{\tau(t)}{b - a\, p_H(t)} \qquad (6)$$

d is calculated over a period, and the maximum is to be taken over the cycle.

It is frequent in the applications in high cycle fatigue (elastic regime) to use *the concept of local equivalent stress* for a life duration N_i defined by:

$$\tau_{0,i} = \tau + a_i p_H. \qquad (7)$$

For the fatigue limit $\tau_{0,i}$ corresponds to material constant b, but is different from b in general because $\tau_{0,i}$ and a_i depend of N_i. if a_i (slope of the fatigue line in $\tau - p_H$ diagram) depends weakly on N_i, taking $a_i \approx a$, it is possible to define the local equivalent stress by

$$\tau_0 = \tau + a p_H. \qquad (8)$$

Very often τ and p_H are maximum at the same time, so that is sufficient in some applications to plot τ max versus p_{Hmax} .

4. APPLICATION TO LOW CYCLE FATIGUE

On the contrary of previously, in the low cycle regime plastic deformation is more homogeneous. This type of fatigue is very much studied since the pioneering work of Manson and Coffin. In order to fit experimental results, they proposed to use the amplitude of

plastic strain as relevant parameter. These tests were uniaxial and strain controlled, and there are many indications that in that case, the stress is related to the plastic strain amplitude in the stabilised state so that the plastic dissipation can be considered as function of strain amplitude.

However in case of more complex cycles stress state, there is no such relation, since it is also well known that in plasticity, the response depends closely of the loading path and of the constitutive equations. Generalisation to 3D formulation of elastoplastic cyclic curve is a convenient way to do, but which is not justified by any theoretical background. In view of plastic fatigue applications, many elastoplastic or elastoviscoplastic constitutive equations were proposed in the eighties (cf. J.L. Chaboche, J. Lemaitre, Mroz...). and summarised for instance in [8]. By numerical computations, it is then possible to evaluate plastic strain amplitude or plastic dissipated energy, but let us notice that for general cyclic loading paths, there is (on the contrary of uniaxial loading mentioned previously) no evident relation between those two quantities. Criteria based on plastic strain amplitude are then not equivalent to those based on plastic dissipation. The following question arises: what is the “good” parameter in plastic fatigue from practical and from theoretical point of view? From previous discussion, we prefer to use a criterion based on a limitation of the local dissipation, since this feature ensures that the corresponding deformation is also bounded which is a natural necessary condition for no rupture [6]. Let us recall that this condition is necessary to ensure the existence of a elastic shakedown state as stated by static shakedown theorems. Moreover, dissipation is easy to calculate, without any ambiguity, and presents many advantages particularly in problems involving thermomechanical loadings which are very frequent and important in mechanical industries (engines, power plants…). Consider for instance the case of an exhaust manifold which is submitted to gas pressure and temperature varying on a wide range as summarised on figure. Since this structure is clamped on the engine body, thermomechanical stresses arise inducing anelastic deformations and low cycle fatigue and even creep fatigue. For such a problem, the approaches deriving from classical L.C.F. are not efficient , since the stress varies with the temperature, for a given plastic strain. The use of a criterion based on a bound on dissipated energy, first identified on laboratory test specimens (isothermal strain controlled LCF tests and thermal fatigue tests on clamped specimens), then applied to the industrial structure for the prediction on the fatigue life (locus of crack and life duration) give very good results. This methodology is now successfully applied for the design of structures submitted to thermomechanical loadings like cylinder head in cast aluminium alloys of modern diesel engines.

It necessitates to use a global approach including a strategy (from experimental and modelling point of view) in order to

To derive representative thermomechanical constitutive equations,

To identify the limit value of dissipated energy corresponding to the life duration.

This approach is detailed in different papers [7,8].

5. APPLICATION TO THE RHR PROBLEM

Cracks are observed in the mixing zones of the nuclear reactor (RHR or residual heat removal zone) as shown on the figure 1. Classical computations using existing methods based on strain amplitude or fracture mechanics are not sufficiently accurate and predictive. For instance application of recommendations contained in nuclear engineering practice fails to explain the formation of the observed cracks. It is the reason why French Electricity Company EDF is interested in the explanation of such cracking phenomenon. Thermo-hydraulic computations where performed by EDF which show that fluctuating temperatures can be observed on the inner surface of the tubes in the vicinity of the mixing zone. As a consequence variables stresses are induced which can initiate fatigue cracking. Thermomechanical stresses cycles are computed and analysed by the presented fatigue criterion. The material fatigue characteristics are given by EDF which permit to determine the fatigue material line corresponding to the fatigue limit threshold represented on the figure 2-b. The loading paths of different points of the tube near the mixing zone show that fatigue limit predicted by our method is violated. On the figure 3 the map of risk of crack initiation is shown.

Because of the importance of this problem, other investigations where recently undertaken by CEA and successfully interpreted using the bound on energy concept by our colleagues S. Amiable and A. Constantinescu. In this work which will be published in a near future, the global approach as described above is used. [9]

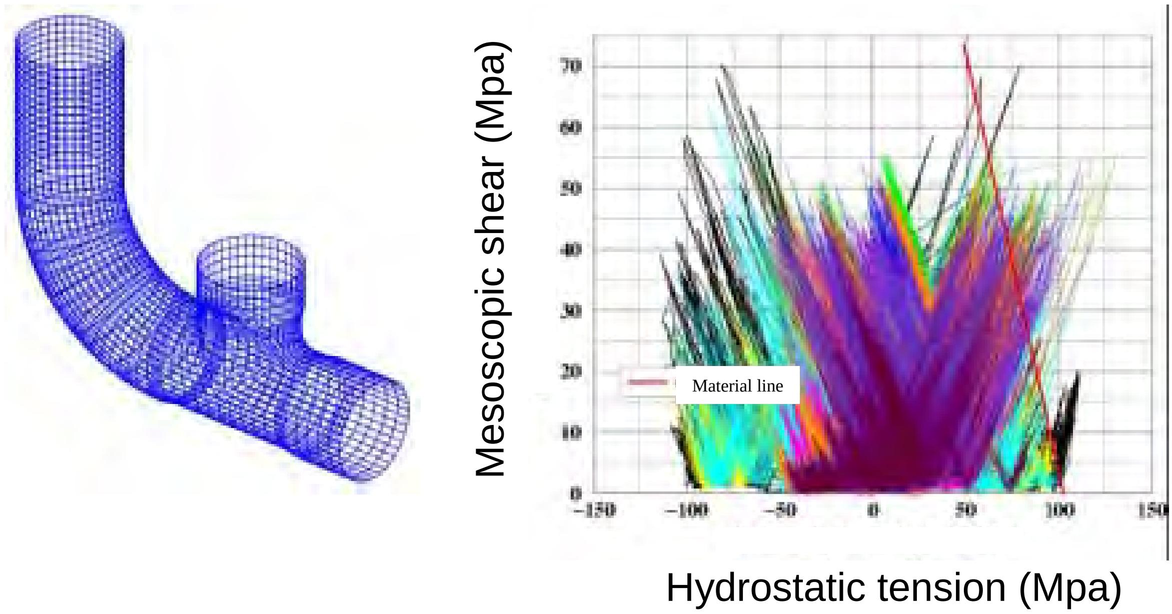

Fig 2-a: meshing of the RHR zone; Fig 2-b: Prediction of fatigue behaviour for different points in the RHR zone; the corresponding loading path intersect the fatigue limit threshold.

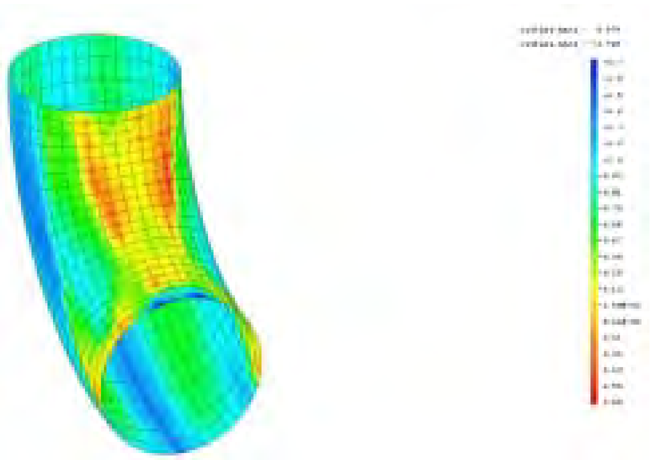

Fig 3: Prediction of the risk of crack initiation .

REFERENCES

[1] D.C. Drucker, On the macroscopic theory of inelastic stress-strain-time temperature behaviour, Advances in Materials Research in the NATO Nations (AGAR Dograph 62), Pergamon press, pp.641-651, 1963.

[2] K. Dang Van, Fatigue analysis by the multiscale approach, in K.dang Van and I.V. Papadopoulos (Eds), High Cycle Metal Fatigue, From Theory to Applications, C.I.S.M. Courses and Lectures N° 392, Springer, 1999.

[3] W.T. Koiter, "General Theorems for Elastic-Plastic Solids". *In Progress in Solid Mechanics*, eds. Sneddon, J. N. and Hill, R., **1**, North-Holland, Amsterdam,pp.165-221, 1960.

.

[3]-Mandel, J. Halphen B., Zarka, J. (1977), Adaptation d'une Structure Elastoplastique à Ecrouissage Cinématique, *Mech. Res. Comm.*, 4, 309-314.

[5] Nguyen, Q.S., Stability and Nonlinear Solid Mechanics, J. Wiley & Sons, 2000.

[6] A. Constantinescu, K. Dang Van, H.Maitournam, A unified approach for high and low cycle fatigue based on shakedown concepts, *Fat. Fract. Engng. Mat. Struct.*, 26, 1-8, 2003.

[7] E. Charkaluk, A. Bignonnet, A. Constantinescu, K. Dang Van, Fatigue Design of Structures under Thermomechanical Loadings, *Fat. Fract. Engng Mat. Struct.*, 25 (12), 1199-1206, 2002.

[8] A. Constantinescu, E. Charkaluk, G. Lederer, L. Verger, A computional approach to thermomechanical fatigue, *Int. J; of Fatigue*, 28 january 2004.

[9] S. Amiable, S. Chapuliot, A. Constantinescu, A. Fissolo, A computational lifetime prediction of thermal shock experiment, to be published in *Fat. Fract. Engng Mat. Struct.*