Evaluation of Modal Combination Methods in Response Spectrum Analysis of Structures with Closely Spaced Modes

L.E. Malik, A.F. Kabir, S. Bolourchi
URS/Blume Engineers, 130 Jessie Str., San Francisco, California 94105, U.S.A.

ABSTRACT

The square-root-of-the-sum-of-squares (SRSS), double algebraic sum (DAS), and double absolute sum (DABS) modal combination methods are evaluated. These methods are used to compute dynamic responses of a large complex building with many closely spaced coupled and uncoupled modes. These responses are then compared to the average response of three time-history modal superposition (T-H) analyses. Deviation of responses using different modal combinations from the average time-history response are compared statistically. It is shown that for structures with closely spaced modes the SRSS method estimates responses which are poorly correlated to those calculated by the T-H method, the DABS significantly overestimates the responses, while the DAS estimates responses reasonably close to those from T-H method.

1. INTRODUCTION

Response spectrum analysis method is commonly used for determination of seismic responses in nuclear power plant structures. In such analyses, modal responses are generally combined by square-root-sum-of-the-square method (SRSS) if the modes are well separated. For closely spaced modes, other methods of combining modal responses are available, among which are the double algebraic sum (DAS), the double absolute sum (DABS), and the grouping methods.

Several studies [1,2] have been reported in the recent literature which investigate the accuracy of the different modal combination methods in calculating the responses of structures. These studies, however, consider simple structural systems. In the study reported herein, the responses from a large mathematical model of a nuclear power plant turbine building are examined to evaluate differences due to the use of various modal combination schemes. The model, because of its size and complex structural system, has a very wide range of different dynamic behaviors. Hence, the results of this study could be considered equivalent to a series of analyses of many different structures.

2. DESCRIPTION OF MODAL RESPONSE COMBINATION METHODS

The SRSS method is based on the assumption that the modal vibrations are statistically independent, that is, the vibration of any mode is not correlated to that of any other mode. The DAS method includes the correlation between the responses in different modes assuming that the earthquake excitation is a sample of a broad frequency-band stationary Gaussian process and the modal responses of the structure are also stationary processes [3]. The total maximum response, R, is given by the expected mean as:

$$R = \left[\sum_{k=1}^{M} \sum_{s=1}^{M} R_k \, R_s \, \varepsilon_{ks} \right]^{1/2} \tag{1}$$

where R_k and R_s are the maximum responses in the kth and sth modes, respectively, and modal correlation factor

$$\varepsilon_{ks} = \left[1 + \left(\frac{(\omega_k' - \omega_s')}{(\beta_k' \, \omega_k + \beta_s' \, \omega_s)} \right)^2 \right]^{-1} \tag{2}$$

in which: $\omega_k' = \omega_k \, (1 - \beta_k^2)^{1/2}$, and $\beta_k' = \beta_k + 2/t_d\omega_k$; where ω_k and β_k are the modal frequency and the damping ratio in the kth mode, respectively, and t_d is the duration of the earthquake.

In equation (1), ε_{ks} expresses the coupling between the different modal responses. It accounts for the mutual reinforcement and/or cancellation of modes with closely spaced frequencies. Elements of the ε_{ks} matrix can assume values ranging from zero to one depending primarily on the proximity of the structural frequencies. It is clear from equation (1) that DAS method reduces to the SRSS method for structures with well separated modes, while for very closely spaced modes the method converges to algebraic sum combination.

In the DABS method, the components of equation (1) are added on an absolute basis rather than algebraically. There is no theoretical basis for this except that it is more conservative than the DAS method and is one of the methods recommended by Nuclear Regulatory Commission [4].

3. BUILDING LAYOUT AND FINITE ELEMENT MODEL DESCRIPTION

The building analyzed is rectangular in plan, approximately 400 ft long in the longitudinal direction, 140 ft wide in the transverse direction, and 108 ft high. It has four working floor levels with a large opening for the turbine pedestal which is structurally isolated from the building. The building has a complex lateral force resisting system with a stiff concrete substructure and a more flexible steel superstructure above.

A detailed finite element model is developed to rigorously model the complex lateral and longitudinal force-resisting system of the building. The building model has a total of 492 nodes, 439 beam elements, 264 plane stress elements, 269 truss elements, and 30 generalized uniaxial elements. The model is fixed at base and has 7% of critical damping for all modes. Figure 1 presents the building layout and some details of the finite element model.

It should be noted that the building model, because of its size and complex structural system, has a very wide range of different dynamic behaviors. There are 208 significant modes up to a frequency of 33 Hz. No single mode dominates the overall dynamic behavior. The superstructure is dominated by localized behavior which accounts for most of the 208 modes with a small number of modes describing the dynamic response of one localized area. The stiff concrete substructure has more of a global behavior defined by a smaller set of modes. Of interest is the coupling of many longitudinal modes with transverse modes producing a relatively large set of torsional modes.

4. ANALYSIS METHODOLOGY

Seismic responses of the turbine building are calculated using response spectrum modal superposition method with modal responses combined on SRSS, DAS, and DABS bases. The analyses are done for a broad-band design spectrum applied as input motion in the transverse direction. The same responses are also calculated using the time-history modal superposition method. Three statistically independent time histories, whose spectra closely matched the design spectrum, are chosen as input in the transverse direction.

For each time-history analysis, the maximum value of each response quantity is retained. The average of these values from the three time-history analyses are chosen as base responses. The difference between base responses and responses from the SRSS, DAS, and DABS response spectrum methods normalized to base response are called error coefficients. Error coefficients for member stresses and member forces are calculated for a large sample from every element type used in the model. The error coefficient is negative or positive when the maximum response from a particular modal combination is lower or higher than maximum base response from time history analyses.

5. DISCUSSION OF RESULTS

Responses from the three input time histories were found to converge very closely. Error coefficients were calculated for all nodal maximum relative displacement and maximum member stress responses for the three input time histories. The maximum mean and standard deviations of these error coefficients are 3% and 0.6%, respectively.

The error coefficients for various responses from the three modal combination methods were also calculated. Table 1 gives the mean and standard deviations of the error coefficients for many of the significant responses calculated on the basis of the three modal combination methods. The maximum sample mean of the error coefficient for the DAS, SRSS, and DABS methods is 13%, 40%, and 225%, respectively, and the maximum standard deviation for the threee methods is 8%, 127%, and 498%, respectively. Figure 2 gives histograms of the error coefficients for axial stress in steel truss members and shear stress in concrete floor diaphragms for the three modal combination methods.

The results show that the DAS method yielded the closest results to those from a time history analysis, as reflected by the small values of the mean and standard deviation of the error function. The SRSS method followed, with mean and standard deviation error functions several times higher than those for the DAS. The DABS consistently overestimated the responses by a large margin.

The sample standard deviation of the error coefficients given in Table 1 and the histograms of Figure 2 give further insight into the performance of the three modal combination methods. The DAS method consistently gave low standard deviations, and the histograms show a fairly narrow distribution of the error coefficients about the mean. The SRSS and DABS methods both yield wide distributions of the error coefficients about the mean, with the DABS clearly biased towards positive error coefficients, while the SRSS seems to have a more symmetrical distribution. This seems to indicate that the DAS method is expected to yield responses that are on the average very close to those from a time history analysis. Furthermore, the maximum deviation in individual members is also expected to be small.

The DABS responses are expected to be consistently higher than those from a time history analysis, with the increases in response being quite substantial in some individual members. The SRSS method seems to give inconsistent results, with mean error coefficients of approximately -40% to +40% when compared with time history responses, and significantly larger error margins, both positive and negative, in individual member responses.

A close examination of the individual responses also shows that the error coefficients for longitudinal responses (orthogonal to direction of ground motion) seem to increase for the SRSS and DABS methods when compared with coefficients for transverse responses but remain essentially within the same order of magnitude for the DAS method. This trend can be seen in Table 1 when comparing transverse and longitudinal relative displacements. The reason is that the closely spaced modes contribute more significantly to the total longitudinal responses than to transverse responses. This seems to indicate that the DAS method is more stable when the contribution of closely spaced modes in the response increase, whereas the other two methods yield error coefficients which increase with the increase in the contribution of closely spaced modes.

6. CONCLUSIONS

Dynamic responses to seismic motions were calculated for a large building model with a large number of coupled and closely spaced modes. Responses calculated on the basis of response spectrum method with DAS, SRSS, and DABS modal combination procedures show that for calculating responses of structures with closely spaced modes:

a. The DAS method closely approximates maximum responses calculated on the basis of a time history procedure. Furthermore, the error function is narrowly distributed about its mean.

b. The DABS method greatly exaggerates the maximum responses and the error function is skewed sharply towards positive error functions (overestimates of response) with a wide band distribution. Any merit derived from the fact that responses from this method are always expected to be conservative has to be assessed in light of the excessive overestimation of the responses and its impact on design.

c. The SRSS method inconsistently estimates maximum responses with error function having a mean ±40% and a wide distribution about the mean. The results from this method are the least desirable, as it is difficult to asses the expected error in any given structure or response parameter.

d. The DAS method appears to be stable in that the error coefficient does not seem to be sensitive to the contribution of closely spaced modes to the total response. The SRSS and DABS methods have increasing error functions as the contribution of closely spaced modes to the total response increases.

ACKNOWLEDGMENT

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APPENDIX - REFERENCES

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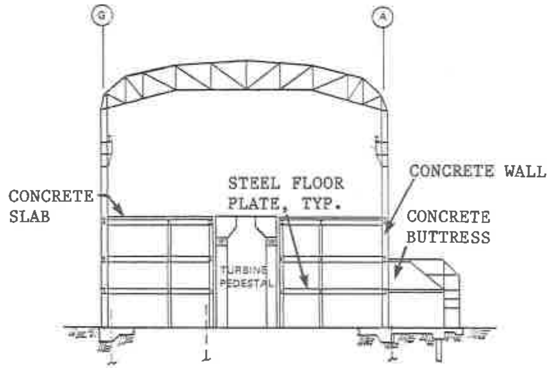
[3] Rosenblueth, E., and Elorduy, J., "Responses of Linear Systems to Certain Transient Disturbances," *Proceedings of Fourth World Conference on Earthquake Engineering*, Vol. I, Santiago, Chile, 1969.

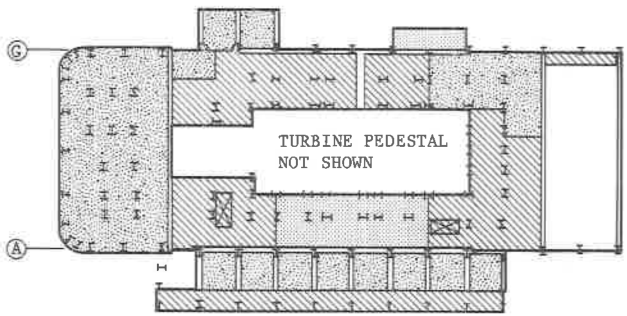
[4] United States Nuclear Regulatory Commission, "Regulatory Guide 1.92," Rev. 1, 1976.

TABLE 1 - Statistics of Error Coefficients for Responses From the Three Modal Combination Methods

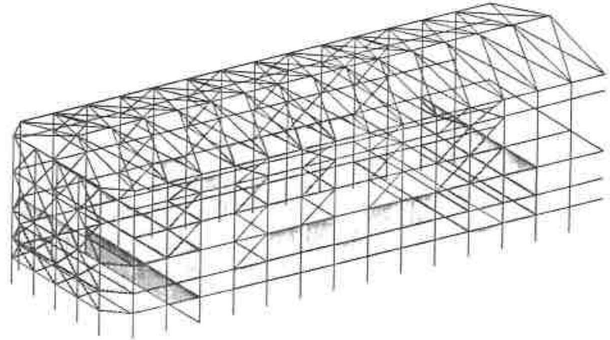
Parameter	Sample Size	Sample Mean			Sample Standard Deviation		
		DAS	SRSS	DABS	DAS	SRSS	DABS
All Relative Displacements	864	-.03	.09	1.21	.01	.76	4.98
Transverse Relative Displacements	432	-.06	-.21	.17	.08	.23	.25
Longitudinal Relative Displacements	430	.01	.40	2.25	.07	1.13	2.78
Axial Stresses in Truss Members	252	-.01	.35	1.93	.07	1.27	3.69
Moments in Beams	189	-.02	.05	1.12	.06	.25	.75
Axial Force in Columns	201	-.03	.04	.93	.06	.53	1.81
Shear Stress in Floor Diaphragms	172	-.01	-.06	.87	.07	.39	.86
Shear Stress in Transverse Walls	48	-.13	-.37	.02	.04	.09	.08
Shear Stress in Longitudinal Walls	43	.00	.01	1.45	.05	.24	.79



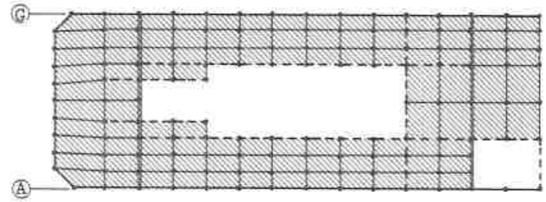
a. Typical Elevation



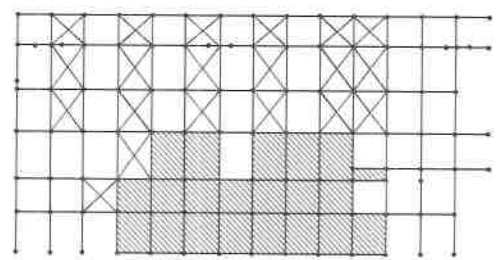
b. Plan at Elevation 19 ft



c. Three-dimensional Schematic View

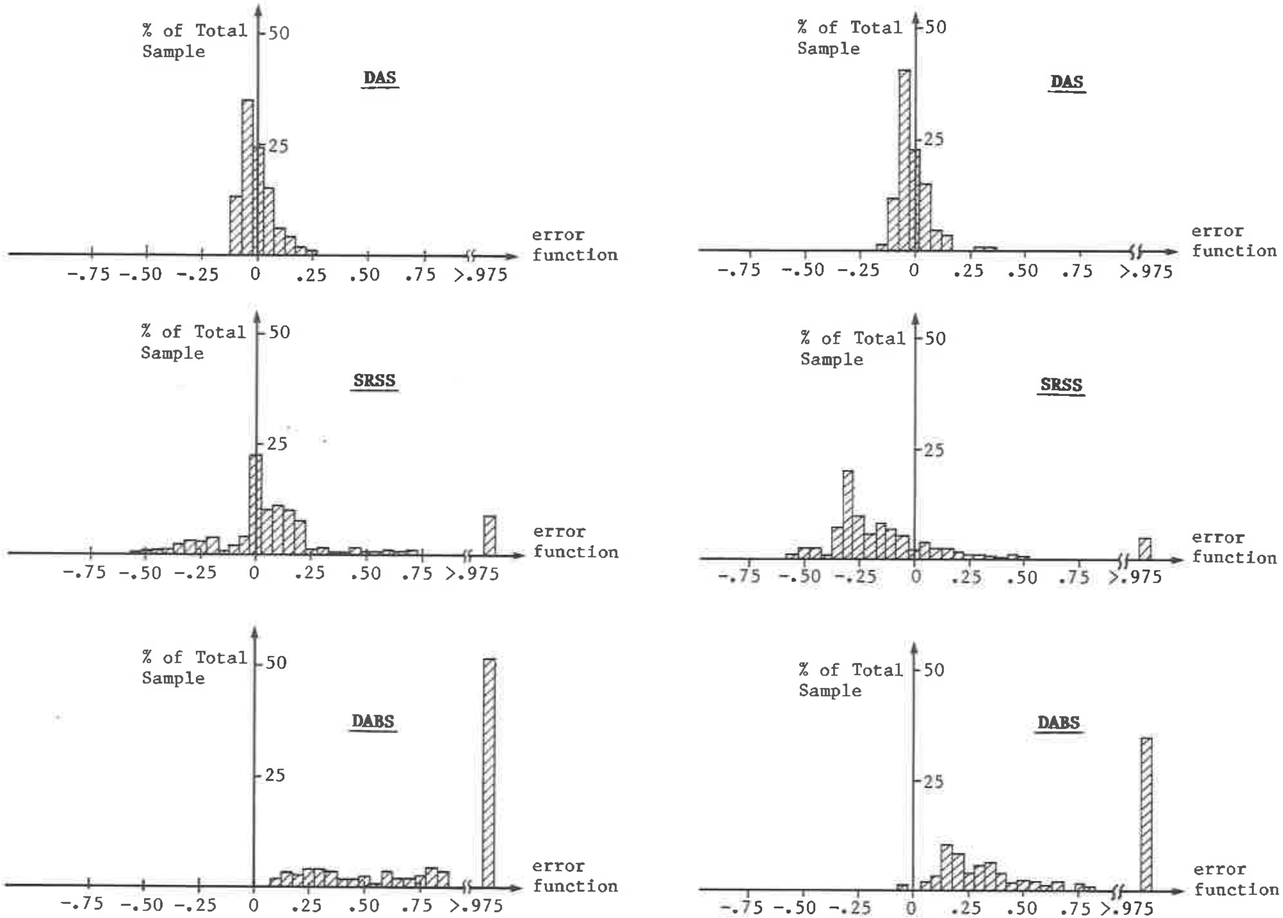


d. Plan at Elevation 55 ft



e. Elevation Along Line A

Figure 1 Building Layout and Finite Element Model



a. AXIAL STRESS IN STEEL TRUSS MEMBERS

b. SHEAR STRESSES IN CONCRETE FLOOR DIAPHRAGMS

Figure 2 Histogram of Error Functions