INELASTIC ANALYSIS OF PIPING SYSTEMS: A BEAM-TYPE METHOD FOR CREEP AND PLASTICITY

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ABSTRACT

Since many years, piping systems are designed and calculated under elasticity assumptions, using a beam-type method. Thus, the analysis of large systems may be performed at a relatively low cost, using a finite element program. However such a method can not account for inelastic phenomena like plastic deformations or creep. The application of refined three-dimensional shell type method is possible for local components such as curved sections but leads to prohibitive costs for complete piping systems. Therefore simplified methods have been developped, based on a «global plasticity or creep model».

Following the conventional elastic approach, the pipe element is characterized by variables associated with the center line in the following way : generalized stresses are obtained by integration of local stresses giving way to hoop and tension stresses and to bending and torsional moments ; the conjugated strains are identified with uniform hoop and longitudinal strains and variations in neutral axis curvatuves. For plasticity problems, the yield surface is defined by a diagonal quadratic function in terms of the generalized stresses and work hardening parameters. By addition of the Hill's principle and a hardening rule, the formulation is similar to the one commonly used in finite element method.

Geometric non linearity due to important deformations of the cross section (often termed «ovalization») may be treated simultaneously with material non linearity. For this purpose the displacement normal to the pipe surface is represented by trigonometric series expansion, the coefficients of which are determined by minimizing the strain energy over the cross section.

The method presented is believed to be a simple economical and accurate tool, for dimensioning computations of large piping systems.

1. INTRODUCTION

The development of Liquid Metal Fast Breeder Reactors (LMFBR) projects as Phenix [1] and Superphenix [2], has led to higher safety requirements. This is specially the case for piping systems which must support internal pressure, high temperatures, cyclic loadings, and which are greatly influenced by local strain concentrations particularly in elbows. Therefore it is advisable to account for non elastic (plasticity, creep) behaviour of piping systems. Computations using available three dimensional finite elements programs lead to prohibitive costs (man power, computer run time) and must then be restricted to refined analysis of local components. This has given rise to economical simplified methods (enabling the design of complete piping systems). A review of these methods has been made recently by GRIFFIN [7]. The global method described in the following is based on a beam-type approach and limit load analyses ([3] to [6]). The yield surface is defined in terms of generalized stresses and work hardening parameters. When applying then common finite element techniques, accounting for both material and geometrical non linearities, yielded results are in the form of resultant forces over the cross section and displacements of the pipe neutral axis.

One of the drawbacks of the global method is that the results are global, it is to say are only known quantities relating to the cross section considered as a whole (bending moments, curvature variation, etc). This is not enough for practical applications, and strain repartition in the cross section is needed for complete analysis - **A method is presented here for the subsequent determination of local strains** by making assumptions on the form of transverse displacements.

Plasticity considerations have been emphasized because of their practical importance (e.g. : elastic follow up phenomena).

2. CLASSICAL PIPING SYSTEMS ANALYSIS

Current practice in piping systems analysis consists in using a beam type method : The piping system is broken down into a group of beams exhibiting elastic behavior.

The additional flexibility of curved sections such as elbows, due to cross section ovalization, is accounted for by means of flexibility coefficients. The methods for the determination of these coefficients have been investigated first by Karman and Marbec ([8] , [9]). Some basic considerations will be briefly recalled here for salse of ulterior developments.

Usual assumptions will be found in [11]. Notations are shown on fig. 1. A complete axisymetric bend is studied.

The application of a constant moment along the bend causes a curvature variation χ and radial and tangential displacements v and w in the cross section. The method consists in applying a Rayleigh - Ritz method by expanding w and v in Fourrier series, the coefficient of which are found by an energy minimization technique. More precisely, the strain energy is written :

$$U = \frac{1}{2} \iint (\epsilon_\varphi^2 + \epsilon_\theta^2 + 2\nu\epsilon_\varphi \epsilon_\theta) \, dS$$

where ϵ_φ and ϵ_θ are the strains in longitudinal and meridional directions respectively. The latter is reckoned from the meridional curvature variation χ_θ :

$$\epsilon_\theta = z \chi_\theta$$

Due to symmetry considerations and inextensibility hypothesis ($w = - \frac{dv}{d\theta}$), expansions of w and v are taken as :

$$W(\theta) = \sum_{j=1}^{p} aj \cos 2j\theta \qquad \text{and } V = \sum_{j=1}^{p} - \frac{aj}{2j} \sin 2j\theta$$

By expressing the strain - displacement relations and integrating in θ, the strain energy can be written as a quadratic function of the j unknown parameters aj, the coefficients of which depend on χ and on the characteristic parameter of the bend : $\lambda = eR/r^2$. The ratio r/R has been neglected if compared with unity ; separate studies have revealed that this hypothesis can be well admitted for usual values of λ.

Under the applied in plane bending moment M, the total energy of the bend is :

$$W = U(aj, \chi) - M\chi$$

The W minimization with regards to aj and χ leads to a (p + 1) dimensional algebraic system :

$$\frac{\partial U}{\partial a_j} = 0 \qquad j = 1 \text{ to } p \qquad \text{and} \qquad \frac{\partial U}{\partial \chi} = M$$

The last equation yields the famous bend flexibility factor, once the aj coefficients have been calculated.

3. GLOBAL METHOD APPLIED TO PIPING SYSTEMS

The first idea of the method applied to piping systems is to use a beam type method, enabling the reduction of a three dimensional solid to a uni-dimensional one defined by the neutral axis of the pipe. Therefore it seems natural to characterize the state of stress in the pipe by the stress resultants over the cross section : N, T, Mt, Mb depicted on fig. 1. For pipes, the pressure is an additional parameter while shear forces may be neglected.

The second idea is to stick to the formulation of inelastic problems as used in many 3D and shell finite element programs, enabling the use of uniaxial tensile strength curves. More precisely, a yield surface is expressed in terms of stress dimensional parameters (termed «generalized stresses»).

$$F\left(\frac{N}{2\pi re}, \frac{pr}{e}, \frac{Mb}{\pi r^2 e}, \frac{Mt}{\pi r^2 e}\right) = 0$$

It is then possible to derive the corresponding energy conjugate strains, which are :

$$\epsilon_N = \frac{\Delta l}{l} \qquad \epsilon_j = \frac{\Delta r}{r} \qquad \epsilon_b = \frac{r\,Xb}{2} \qquad \epsilon_t = \frac{r\,Xt}{2}$$

The application of classical plasticity and creep algorithms is then made possible by addition of the Hill's principle and a hardening rule.

The last step consists in choosing a suitable function F for each component. For sake of simplicity this is achieved by defining the equivalent stress as a diagonal quadratic function in terms of the generalized stresses and some work hardening parameters βi :

$$\sigma^* = \sqrt{\sum_i \alpha i^2 (\sigma i - \beta i)^2}$$

The αi coefficients depend on the geometry of the pipe element and are determined by means of a separate limit analysis of the various components. Special attention has been given to curved pipes because of their influence on the overall behavior of the piping system.

An example of comparison with experimental results and three dimensional computation is given by the analysis of the elasto-plastic behavior of a 180^0 bend under cyclic in-plane bending [12]. This thin walled U bend has been tested at room temperature at CEA/DEMT. Two straight parts were welded on the elbow in order to apply cyclic opening and closing moments. Computations were carried out with the TEDEL finite element program of the CEASEMT system [19].

The geometry, mesh and tensile strength curve are pictured on fig. 2 to 4. An imposed displacement was prescribed to the assembly and the corresponding force was measured (fig. 5) Results of simplified method calculations and experiments are displayed on fig. 6. They were also compared with three-dimensional shell type computations (fig. 7) with the TRICO program. In both cases, agreement is excellent. Some other examples in plasticity and creep have been treated for benchmark problems, which may be found in [12].

4. DETERMINATION OF LOCAL STRAINS

The following method for determination of local strains is based on a non linear elasticity approach. It is well known that deformation type plasticity can be studied on this assumption. This method has been developped by SPENCE [15] for computation of flexibility factors in case of creep.

It may be used for determination of strain repartition in the cross section.

The finite element algorithms expressed in terms of global variables yields the variation in curvature χ. Therefore, it is possible to calculate a posteriori the strains at any point of the cross section by the following procedure : Starting from the expansions of w and v in fourier series given in section II, adimensional coefficients $\alpha j = \frac{aj}{rR\chi}$ are introduced.

Therefore, strains can be calculated at any point of the cross section, for example in case of in-plane bending by the following well known formula :

$$\epsilon\varphi(\theta, z) = r\chi\cos\theta + \frac{1}{R}(w\cos\theta - v\sin\theta)$$

$$\epsilon\theta(\theta, z) = z\chi_\theta(\theta) = -\frac{z}{r^2}\left(w + \frac{d^2 w}{d\theta^2}\right)$$

Replacing v and w, they can be put in the form :

$$\epsilon\varphi(\theta, z) = r\chi\, f(\theta, \alpha j, \lambda)$$

$$\epsilon\theta(\theta, z) = z\chi\frac{R}{r}\, g(\theta, \alpha j, \lambda)$$

Practical applications require the previous determination of the αj coefficients - For linear elasticity problems, they are found, as indicated in parag. 2, by solving a linear algebraic system :

$$U = \frac{1}{2}\frac{EI}{1-\nu^2} F(\alpha j, \lambda)\chi^2 \quad \text{gives} \quad \frac{\partial F}{\partial \alpha j} = 0$$

For plasticity and creep problems, following the Spence's approach ([14], [15]) the strain energy to minimize is reckoned assuming a n - power non linear elasticity or stationary creep low - Thus for plasticity :

$$\epsilon = A\sigma^n \text{, which gives :}$$

$$U = \frac{n}{n+1} A^{-\frac{1}{n}}\left(\frac{4}{3}\right)^{\frac{n+1}{2n}} \int_0^{2\pi}\int_{-\frac{e}{2}}^{+\frac{e}{2}} \left(\epsilon\varphi^2 + z^2\chi_\theta^2 + z\epsilon\varphi\chi_\theta\right)^{\frac{n+1}{2n}} r d\theta dz$$

This expression is classicaly [14] approximated by the following :

$$U \simeq \frac{n}{n+1} A^{-\frac{1}{n}}\left(\frac{4}{3}\right)^{\frac{n+1}{2n}} e\int_0^{2\pi} \left[\epsilon\varphi^2 + \left(\frac{n}{2n+1}\right)^{\frac{2n}{n+1}}\frac{e^2}{4}\chi_\theta^2\right]^{\frac{n+1}{2n}} r d\theta$$

Accounting for above strains expressions, the strain energy can be written :

$$U = \frac{n}{n+1} A^{-\frac{1}{n}} \left(\frac{4}{3}\right)^{\frac{n+1}{2n}} \quad {}^{\frac{n+1}{2n}} \; |r\chi|^{\frac{n+1}{n}}$$

where : $D = D(\alpha j, n, \lambda)$

Therefore the U - minimization provides a set of m non linear equations :

$$\frac{\partial D}{\partial \alpha_j} = 0$$

From the estimated value of n relating to given material characteristics, and for a known bend parameter λ, these equations supply the αj coefficients giving way to the determination of local strains.

For examples, above equations were solved by a common non linear programming technique, making use of linear elasticity coefficients as initial values. Fourier expansions have been limited to two terms ; results have been computed with a program accounting for any number of terms.

Figures 8 and 9 reproduce the variation of coefficients α_1 and α_2 with n and λ. It may be noticed that the n parameter does not greatly influence the shape of the cross section deformation .

PRACTICAL FORMULATION

Given data : e, r, R, n, p, χ (see previous nomenclature)

Step 1 : $\left. \begin{array}{l} n \\ \lambda = \frac{eR}{r^2} \end{array} \right\} \Rightarrow$ αj from known curves (ex fig. 8 and 9) (j = 1 to p)

Step 2 : $$\epsilon\varphi = r\chi \left[\cos\theta + \sum_{j=1}^{p} \alpha j \left(\cos 2j\theta \cos\theta + \frac{1}{2j} \sin 2j \sin\theta \right) \right]$$

$$\epsilon\theta = z \frac{R}{r} \chi \left[\sum_{j=1}^{p} \alpha j \, (4j^2 - 1) \cos 2j\theta \right]$$

Such a formulation has been included in the TEDEL routine (for piping system analysis) of the CEASEMT system of structural analysis by F. E. M. [4] [19]. It is not only used for the appraisal of plastic strains but also for large displacement analysis (variation of modules of inertia) especially in pipe whiping problems.

5. CONCLUSION

Plastic and creep analysis of complete piping systems can be performed with sufficient accuracy and at a low cost using the global method presented above. Additional information concerning the local strain repartition in the cross section may be gained by post - processing the global displacements.

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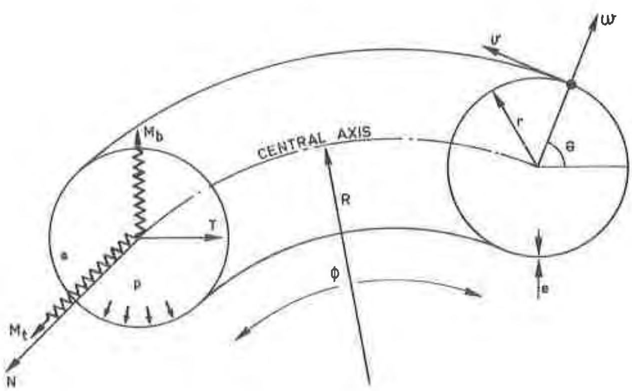

FIG. 1

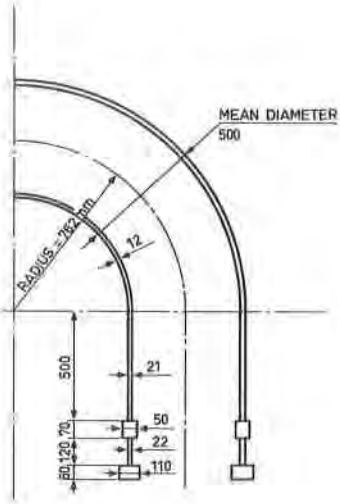

FIG 2 _ GEOMETRY

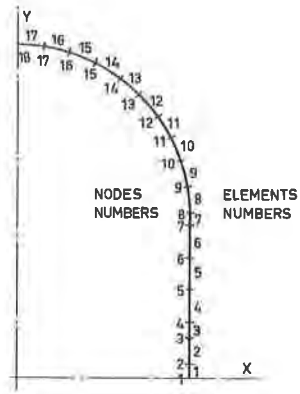

FIG. 3 _ MESH

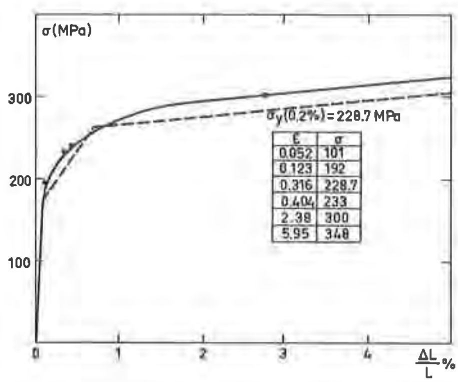

FIG. 4 _ TENSILE STRENGTH CURVE

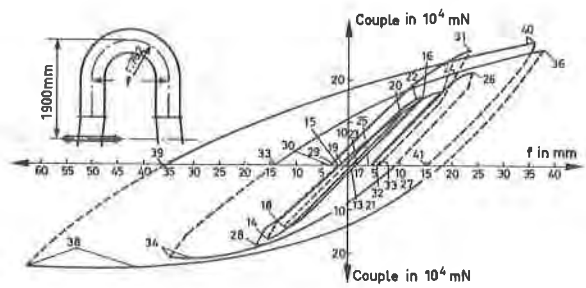

FIG. 5 _ MEASURED LOADING

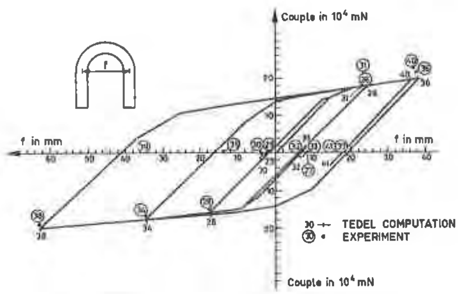

FIG. 6 _ DEFLECTION VARYING WITH THE COUPLE

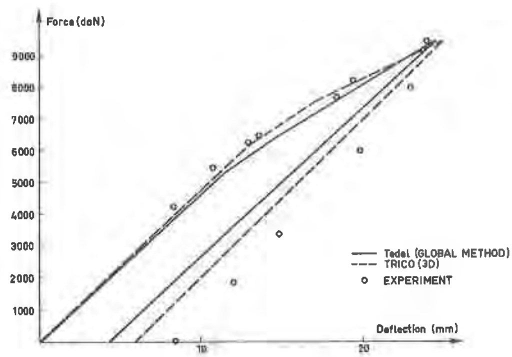

FIG 7 _ COMPARISON GLOBAL - 3D COMPUTATIONS

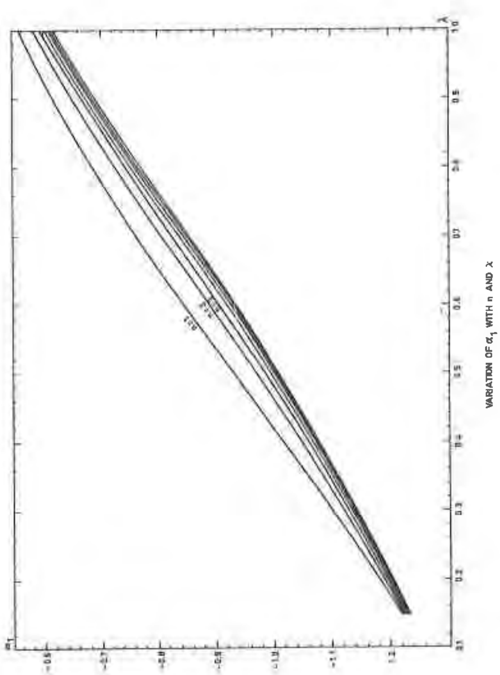

FIG. 8

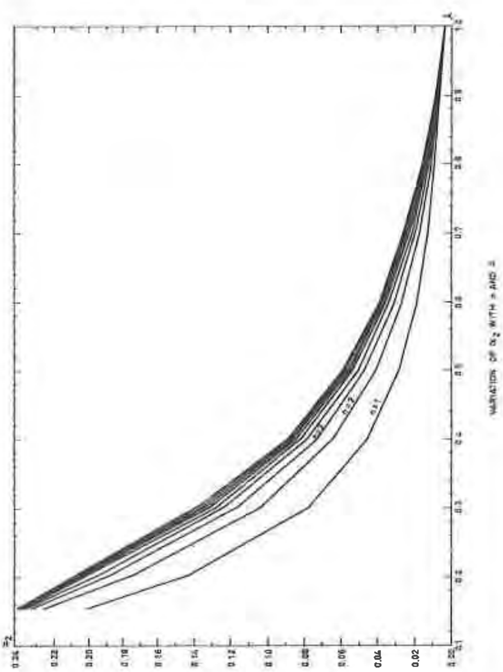

FIG 9