Transactions, SMiRT-25
Charlotte, NC, USA, August 4-9, 2019
Division III

CALCULATION OF THE NATURAL FREQUENCIES OF A FLUID-FILLED PIPE SYSTEM WITH THE COUPLED CODES DYVRO/ROHR2: COMPARISON WITH AN ANALYTICAL SOLUTION AND VALIDATION

Thorsten Neuhaus[1]

[1] Technical Expert, TUEV NORD, Hamburg, Germany
(tneuhaus@tuev-nord.de)

ABSTRACT

The vibration of a pipe system can be caused by excitation of the acoustic modes as a result of pump-induced pressure pulsations or vortex shedding in fittings. For the investigation of these phenomena the knowledge about the natural frequencies of the pipe system is essential. In this context it is important to consider that the natural frequencies of the coupled system of fluid and structure commonly differ from the natural frequencies of the pure fluid system and of the structural pipe model in which the fluid is only taken into account as added mass. To obtain realistic natural frequencies of a pipe system filled with a fluid, the one-dimensional fluid code DYVRO was coupled with the code ROHR2STOSS that deals with dynamic analysis of complex piping and steel structures. Two code coupling concepts are described, the integral and the local coupling, considering a two-way interaction for both. For the first concept a verification was performed by comparison with an analytical solution for a simple case with a fluid column connected to a spring-mass system. Additionally, both coupling concepts were validated by means of the comparison with experimental data from literature that revealed strong fluid-structure interaction during a water hammer event. It is shown that the shift of the frequency of the pressure fluctuations comparing to the pure fluid case without fluid-structure interaction is captured by the coupled codes. Therefore, the coupled codes can be used for the assessment of dynamic pressure loads on safety related pipe systems.

INTRODUCTION

Fluid-structure interaction (FSI) in liquid-filled pipe systems is caused by three interaction mechanisms according to Wiggert (2001): friction coupling, Poisson coupling and junction coupling. Friction coupling is the mutual friction between the liquid and the axially vibrating pipe wall. Poisson coupling relates pressures in the liquid to a radial deformation of the pipe wall. While friction and Poisson coupling act along the entire pipe, the more important junction coupling acts wherever there is a branch or a change in area or flow direction.

When a pipe system is excited by a fluid transient, e.g. caused by valve operation or pump stop/start, or by ongoing pressure pulsations, e.g. caused by an operating pump or vortex shedding in fittings, all above-described FSI mechanisms have an influence. For vibrations resulting from an external excitation, e.g. earthquake or the effect of a machine standing on a resilient foundation on a pipe connected to or hanged from it, the above-described FSI mechanisms are also relevant. The extent of the fluid-structure interaction depends on the pipe wall material and thickness as well as on the pipe supports.

If the natural frequencies of a fluid-filled pipe system are important to know, e.g. for the assessment, whether resonance effects may occur caused by a periodic excitation, it must be considered that the natural frequencies of the coupled system of fluid and structure commonly differ from the natural frequencies of the pure fluid system and of the structural pipe model, in which the fluid is only taken into account as added mass. In this publication it is shown that the coupling of a one-dimensional fluid code with a structural code dealing with pipe systems is appropriate to capture the natural frequencies of the coupled system of fluid and structure.

FREQUENCY SHIFT IN THE COUPLED SYSTEM OF FLUID AND STRUCTURE

In Moody (1990) an analytical solution for the natural frequencies of a simple coupled system of fluid and structure was derived that is depicted in figure 1. It consists of a pipe with the internal cross-sectional area *A*, in which an internal movable solid plug with the mass *M* is mounted to a spring with the spring constant *K* at the right pipe end. A liquid column of the length *L*, the density ρ and the speed of sound *c* is adjacent to the other side of the plug. Within the liquid *p* denotes the pressure and *v* the fluid velocity, which are variable in the axial direction *x* and the time *t*. On the left boundary of the liquid a constant pressure p_0 is assumed.

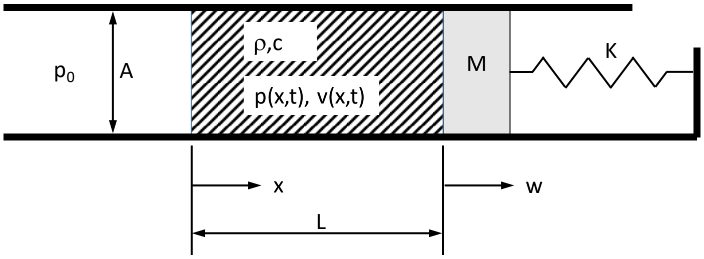

Figure 1. Simple coupled system of fluid and structure in Moody (1990).

The angular frequency of the decoupled hydraulic system, i.e. the frequency of the pressure wave oscillations in the liquid column assuming a rigid pipe structure is:

$$\omega_{hyd} = \frac{2\pi c}{4L} \tag{1}$$

The angular frequency of the coupled system in case of an incompressible medium can be calculated by adding the mass of the fluid to the mass of the structure. This is a common approach in the calculation of pipe systems:

$$\omega_i = \sqrt{\frac{K}{M+\rho AL}} \tag{2}$$

According to Moody (1990) the angular frequencies ω_n of the coupled system shown in figure 1 in case of a compressible medium can be calculated with:

$$tan\left(\frac{\omega_n L}{c}\right) = \left[\left(1+\frac{\rho AL}{M}\right)\left(\frac{\omega_i L}{c}\right)^2 - \left(\frac{\omega_n L}{c}\right)^2\right]\left[\left(\frac{\rho AL}{M}\right)\left(\frac{\omega_n L}{c}\right)\right]^{-1} \tag{3}$$

The solutions for the different modes ω_n can iteratively be determined.

A parameter study was performed to investigate for which cases the first mode ω_n deviates from ω_i respectively ω_{hyd} most distinctly. For this purpose constant values were chosen for the liquid column length *L* (10 m), the speed of sound in the liquid *c* (1200 m/s), the density of the liquid ρ (1000 kg/m³) and the cross-sectional area *A* (0.007854 m² resulting from the assumption of a cylindrical pipe with an inner pipe diameter *d* of 0.1 m). For the parameter study the value for the spring constant *K* was varied. This was done for three different structural masses *M*, resulting in the correspondent ratios of structural and fluid mass listed in table 1.

Table 1: Ratios of fluid and structural mass for the parameter studies with variable spring constant.

case	M	$\rho AL/M$
1	30	2.62
2	78.5	1
3	200	0.39

The results of the parameter studies are shown in figure 2. It is obvious that the first mode ω_n deviates from ω_i respectively ω_{hyd} most distinctly, when ω_i and ω_{hyd} are similar. The effect becomes stronger with decreasing structural mass *M*.

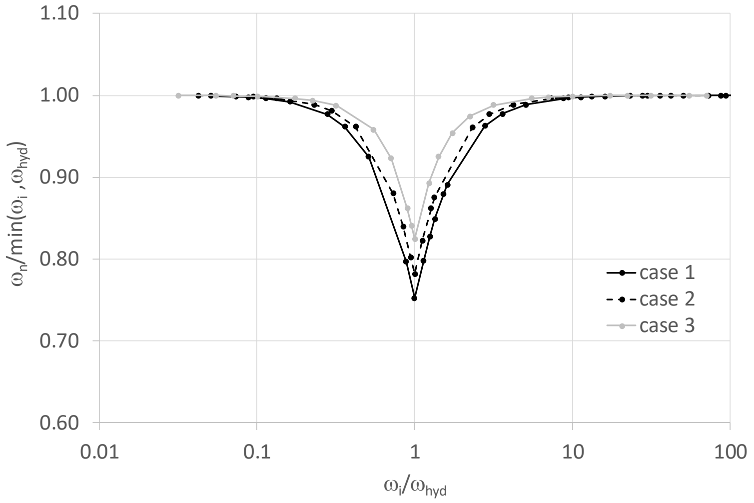

Figure 2. Results of the parameter studies regarding the modes of a coupled system of fluid and structure.

From this finding it can be concluded that also for real pipe systems the exclusive treatment of the structure respectively the fluid may lead to erroneous results regarding the natural frequencies of the coupled system. However, for complex pipe systems analytical solutions for the calculation of the natural frequencies ω_n are not available. Furthermore, in commercial available software for static and dynamic analysis of complex piping and steel structures the fluid is commonly treated as added mass so that only the frequencies ω_i for an incompressible fluid can be calculated, but not the more realistic frequencies ω_n for a coupled system of fluid and structure.

One promising strategy for the calculation of the frequencies ω_n for a complex pipe system in a real plant is the coupling of a fluid code with a structural code. The coupling strategy, a verification against the above-described Moody-case and a validation against an experiment are described in the following.

CODE COUPLING OF DYVRO WITH ROHR2STOSS

TUEV NORD has coupled the in-house fluid code DYVRO mod. 3 with the code ROHR2STOSS to account for the fluid-structure interaction, i.e. the exchange of fluid forces and accelerations, in pipe systems. A description of DYVRO can be found in Neuhaus et al. (2009). With this approach the FSI mechanisms junction coupling and friction coupling can be modelled. The FSI effect of widening of the pipe wall at pressure increase due to the pipe elasticity is only modelled within the fluid code by adaption of the pressure wave propagation velocity according to Wylie and Streeter (1993). This is a simplified approach to account for the FSI mechanism Poisson coupling. The heat transfer between fluid and structure is not considered by the coupling of both codes.

ROHR2STOSS is an alternative dynamic module in the program system ROHR2. It deals with the dynamic analysis of complex piping and steel structures using direct integration in the time-domain. Its task is the assessment of temporal varying loads such as those induced by fluid hammer in pipe structures. Linear as well as non-linear boundary conditions can be considered. The possibility to include non-linear elements allows the design of new elements, which cannot be analysed by modal time history calculations of the ROHR2 standard calculation process.

DYVRO Mod. 3 is a fluid program for the calculation of transient, one- or two-phase flow processes in piping systems. It solves the system of partial differential equations consisting of six conservation equations for mass, momentum and energy for two phases using a finite volume method, optionally with a combined momentum balance for liquid and gas. In DYVRO second-order accurate spatial schemes with flux limiters based on the Godunov method are available for the calculation of highly transient processes with steep gradients ("shock-capturing methods") without computing unphysical oscillations. Depending on the desired modelling accuracy, different phase change terms can be applied to consider thermodynamic or mechanical equilibrium or non-equilibrium. Several material databases (e.g. water/steam) are implemented in DYVRO, further material databases can be integrated easily. In DYVRO numerous hydraulic components are modelled, such as pipelines, a wide variety of valves and pumps, containers or accumulators. For swing check valves, spring-loaded lift check valves, safety valves, media-controlled valves, centrifugal pumps and rotary displacement pumps, the interaction between moving parts of pumps and valves with the fluid may be taken into account by solving the corresponding equations of motion.

The two-way coupling of DYVRO with ROHR2STOSS was realized by TUEV NORD in collaboration with the company SIGMA who is the code developer of ROHR2STOSS. For this purpose a module for inter-process communication (IPC) was developed, i.e. a “named pipe”. At each time step the fluid loads calculated by DYVRO are transferred to ROHR2STOSS and the resulting displacements are transmitted to DYVRO. The scheme of this concept is shown in figure 3.

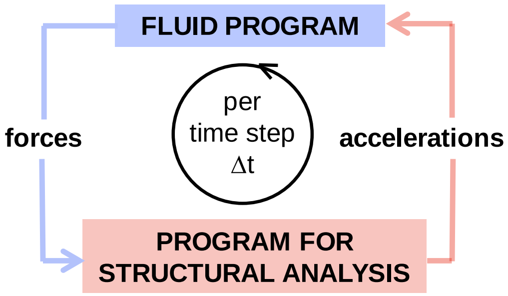

Figure 3. Concept for the coupling of DYVRO and ROHR2STOSS to account for FSI.

Two concepts of coupling between DYVRO and ROHR2STOSS were developed. With the first one the integral fluid force on a pipe segment (e.g. between two bends) is calculated by the fluid code and transferred to the structural code. The resulting structural movement acts in the fluid code on the entire fluid within this pipe segment as an acceleration. With the second approach the forces are calculated by the fluid code locally on each bend, each hydraulic component, change of flow area etc. and the movement of the pipe structure acts locally on the fluid. With the local coupling concept, the longitudinal deformation of a pipe segment can be captured, in the integral coupling concept an axially rigid pipe is assumed. The two concepts of integral and local coupling are depicted in figure 4.

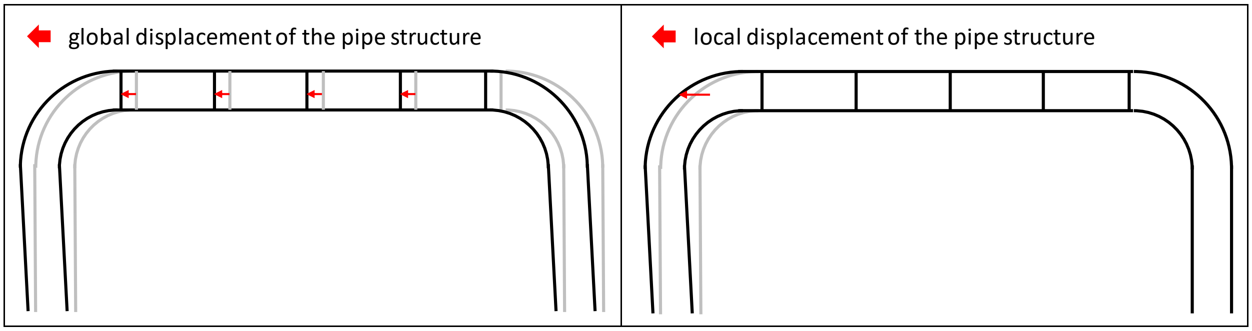

Figure 4. Left: Integral coupling, the structural movement acts on all finite volumes representing the pipe segment in the fluid code. It is considered by means of an additional acceleration term (equation 8).
Right: Local coupling, the structural movement affects only the finite volumes in the fluid code at the pipe bends. It is considered by means of additional terms (equations 10 and 11).
Grey colour: Initial position of the pipe segment, black colour: displaced/deformed pipe segment.

The advantage of the integral coupling concept is that the fluid forces are zero for steady-state conditions in the fluid system. This means that after the determination of a stationary solution in the fluid code the coupled calculation can directly be started by the initiation of the fluid transient. In the local coupling concept the local fluid forces are usually not zero, even at steady-state conditions. As a result, the coupled calculation always starts with an excitation of the structural model initiated by the local fluid forces, although a fluid transient is not initiated. Therefore, the fluid transient may not be started, until steady-state conditions of both, the fluid and the structure, are attained. Both concepts are described in more detail in the following.

Integral Coupling

The resultant force acting on a container by its fluid inventory can be calculated with two different approaches. The first one, which is also used in the American standard ANS (1988), is based on Newton's second law, which implies that all volume and surface forces exerted on the fluid is balanced by the change in momentum of the fluid. The momentum equation for the fluid in a control volume V is

$$\int_V \frac{\partial}{\partial t}(\rho\vec{v})\,dV = \int_V \vec{g}\rho\,dV - \vec{F} - \int_{A,fluid} p\vec{n}\,dA - \int_{A,fluid} \rho\vec{v}\vec{v}\cdot\vec{n}\,dA \qquad (4)$$

In equation 4 the term $\vec{F}$ represents all surface forces at the control volume boundary, which are exchanged between fluid and structure. This is equivalent to the resultant force acting on the container by the fluid. *A,fluid* is the control volume boundary, where no surface forces act on the structure, but on fluid that is located outside of the control volume. $\vec{g}$ is the acceleration due to gravity, $\vec{n}$ the normal vector. For the axial force on a pipe segment between two 90° bends with an inclination of 0° using $\dot{m}$ as the mass flow rate in axial direction in the pipe equation 4 leads to:

$$F = -\frac{\partial}{\partial t}\int_0^L \dot{m}\,dx \qquad (5)$$

The second approach integrates the forces directly at the interface between fluid and inner pipe wall:

$$\vec{F} = \int_{A,structure} p\vec{n}\,dA + \int_{A,structure} \vec{\tau}\,dA \tag{6}$$

$\vec{\tau}$ is the shear stress vector tangentially to the pipe wall. In the German standard VDI (2004) equation 6 is converted for a pipe segment between two 90° bends for the case that the pressure is computed by a one-dimensional approach. Because a code, that solves the set of equations along the pipe axis, is not able to account for a change in the flow direction, the terms $\dot{m}_i v_i$ for the impact pressure must be added:

$$F = (p_2 - p_1)A + \dot{m}_2 v_2 - \dot{m}_1 v_1 + F_{friction} \tag{7}$$

The indices 1 and 2 refer to the beginning and the end of the pipe segment. The equations 5 and 7 lead exactly to the same results in case of a pipe segment with an inclination of 0°.

With the concept of integral coupling it is assumed that the pipe segments are insignificantly deformed in axial direction. The velocity *w* of the pipe structure that is calculated by ROHR2STOSS is considered in the equations of the fluid code by an additional acceleration term. In equation 8 this concept is demonstrated exemplarily for the equation on motion of the liquid/gas mixture. The structural acceleration is $\partial w/\partial t$, the coefficient of friction between pipe wall and fluid is λ.

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = -g\,sin\beta - \frac{\lambda}{2d}v|v| - \frac{\partial w}{\partial t} \tag{8}$$

Local Coupling

With the local coupling concept the force terms in equation 7 are split into individual terms. For example in equation 9 the fluid force action on a pipe bend in the axial direction of one adjacent pipe segment is given:

$$F = (p - p_{amb})A + \dot{m}v \tag{9}$$

p_{amb} is the ambient pressure outside the pipe. The velocity *w* of the pipe structure that is calculated by ROHR2STOSS is considered in the equations of the fluid code by several additional terms. In the equations 10 and 11 this is shown exemplarily for the continuity equation and the equation of motion for the liquid/gas mixture.

$$\frac{\partial p}{\partial t} + (v - w)\frac{\partial p}{\partial x} + c^2\rho\frac{\partial(v-w)}{\partial x} = 0 \tag{10}$$

$$\frac{\partial v}{\partial t} + (v - w)\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = -g\,sin\beta - \frac{\lambda}{2d}v|v| \tag{11}$$

The pipe velocity *w* affects the equations by the relative velocity $v - w$ between pipe structure and fluid.

VERIFICATION AGAINST THE MOODY CASE

The simplified Moody case that is described above was chosen for the verification of the coupling between DYVRO and ROHR2STOSS. On this occasion, the integral coupling concept was applied. The same values were used for the density ρ (1000 kg/m³) and speed of sound *c* (1200 m/s) of the liquid, the length of the liquid column *L* (10 m) and pipe diameter *d* (0.1 m). For the structural mass *M* and the spring constant *K* values of 200 kg and 10000 kN/m respectively were chosen, damping and friction were disabled in the codes. By applying the equations 1 to 3 the following results were obtained by the analytical approach derived by Moody: ω_{hyd} = 188.50 1/s (30 Hz), ω_i = 188.48 1/s (30.156 Hz) and for the frequencies of the coupled system ω_n = 155.38 1/s (24.73 Hz) for the first mode and ω_n = 260.61 1/s (41.48 Hz) for the second mode. With these values the frequencies ω_i (assuming an incompressible fluid) and ω_n (assuming a compressible fluid) differ significantly.

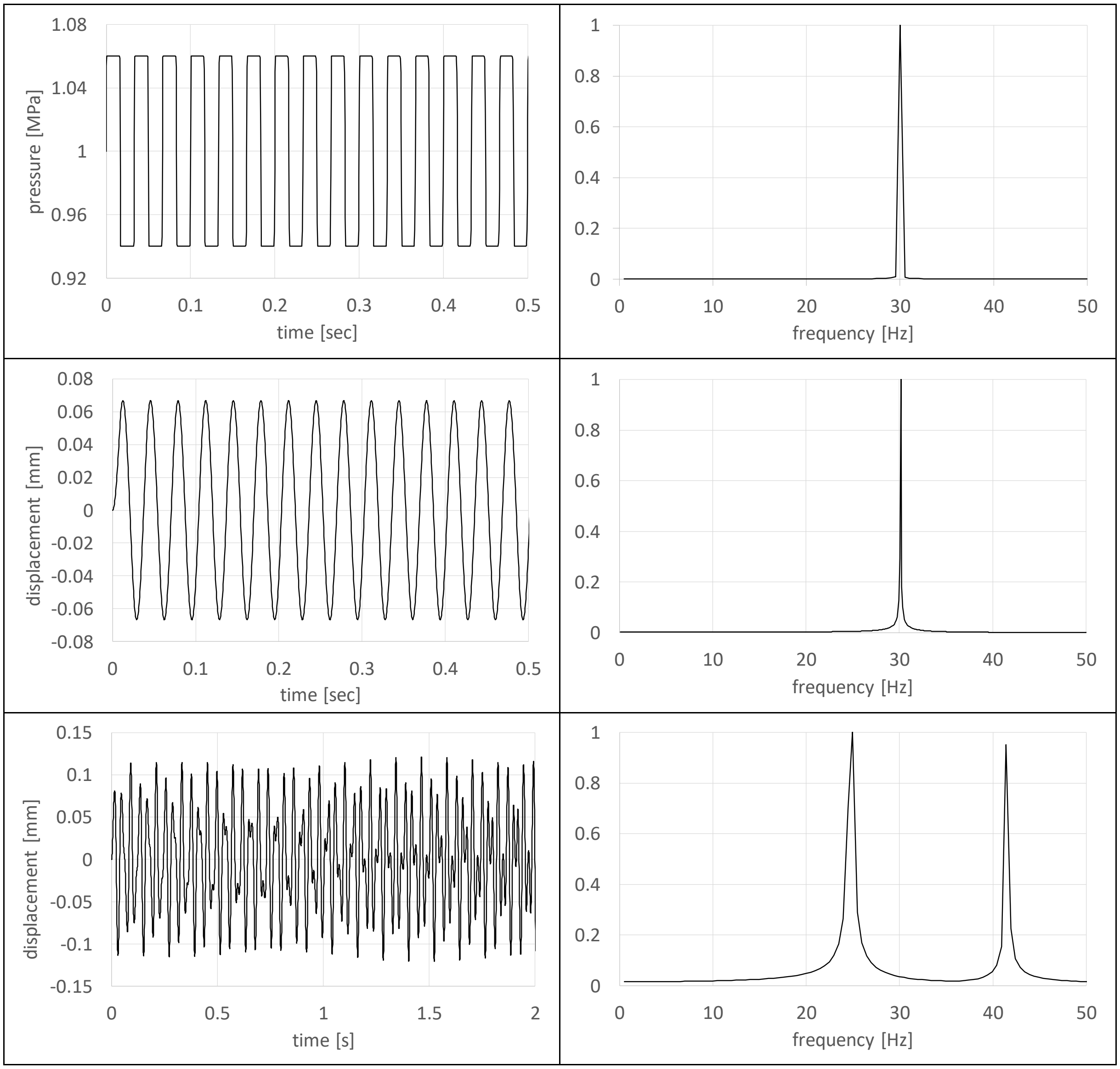

Figure 5. First line: pressure wave oscillations in the fluid system assuming a rigid structure (detail), second line: oscillations of the structural model assuming an incompressible fluid (detail), third line: oscillations of the coupled model assuming a compressible fluid (detail).

In figure 5 the computation results by DYVRO and ROHR2STOSS are shown in the time and frequency domains for three scenarios. In the first scenario, only the propagation of the pressure waves is calculated assuming a non-deformable and non-movable pipe structure. The initial pressure is 1 MPa and the initial fluid velocity 0.05 m/s. The pressure wave is initiated by an abrupt stop of the liquid flow at the right pipe end. The result is the typical rectangular-shaped pressure wave curve with an amplitude of 0.06 MPa that can be calculated by the Joukowsky equation ($\Delta p = \Delta v \cdot \rho \cdot c = 0.05\ \mathrm{m/s} \cdot 1000\ \mathrm{kg/m^3} \cdot 1200\ \mathrm{m/s}$)

and a period t_{hyd} of 0.0333 s (t_{hyd} = 4·L/c). The result of the Fourier analysis (based on 60 periods with ~4500 data pairs) gives a frequency of 30.00 Hz. This shows that the hydraulic frequency ω_{hyd} according to equation 1 is exactly captured by DYVRO.

In the second scenario it was checked, whether the frequency ω_i of the structural model can be captured by ROHR2STOSS. For this purpose, the fluid mass is added to the structural mass in ROHR2STOSS. The structural model is excited by one single impulse due to a pressure wave calculated by the fluid code applying a one-way coupling from the fluid to the structural code. A special boundary condition in the fluid code at the left pipe end makes it possible that the initial pressure wave, that is generated by a sudden stop of the fluid flow at the right pipe end, is not reflected, so that - after the first pressure wave passed the pipe - the fluid system is at rest. The Fourier analysis of the time-dependent displacement (300 periods with ~50.000 data pairs) gives a frequency of 30.144 Hz. This shows that the structural frequency ω_i is captured by ROHR2STOSS with a deviation of 0.04 % compared to the analytical solution.

In the third scenario the two-way coupling is applied. Therefore, the added fluid mass in ROHR2STOSS must be removed, because the fluid mass is now considered in the fluid code. The Fourier analysis of the time-dependent displacement (scenario time of 10 sec with ~50.000 data pairs) gives a frequency of ω_n = 24.738 Hz for the first mode and ω_n = 41.51 Hz for the second mode. This is a deviation of 0.03 % for the first mode and 0.07 % for the second mode comparing to the analytical solution by Moody. The frequencies ω_i and ω_{hyd} are not present anymore in the coupled system. It can be concluded that the verification was successful.

VALIDATION AGAINST THE DELFT EXPERIMENT

Laboratory tests on a large scale regarding the fluid-structure interaction in pipe systems were conducted by the research group in Delft and described by Wiggert (2001). The FSI test rig depicted in figure 6 consisted of seven straight pipe sections connected by 90° bends. The total length of the hydraulic system was 77.5 m. The wire suspended steel pipes had an internal diameter of 109 mm with 3 mm wall thickness and they were filled with water. The structural system was highly flexible and easily excited by closure of the fast-acting valve at the fixed point H. The bends B and G were restrained in lateral directions only. At the bend E an adjustable spring was located. However, for the validation an experiment was chosen without this spring. The initial pressure was 0.58 MPa at a temperature of 20 °C and the initial velocity 0.3 m/s.

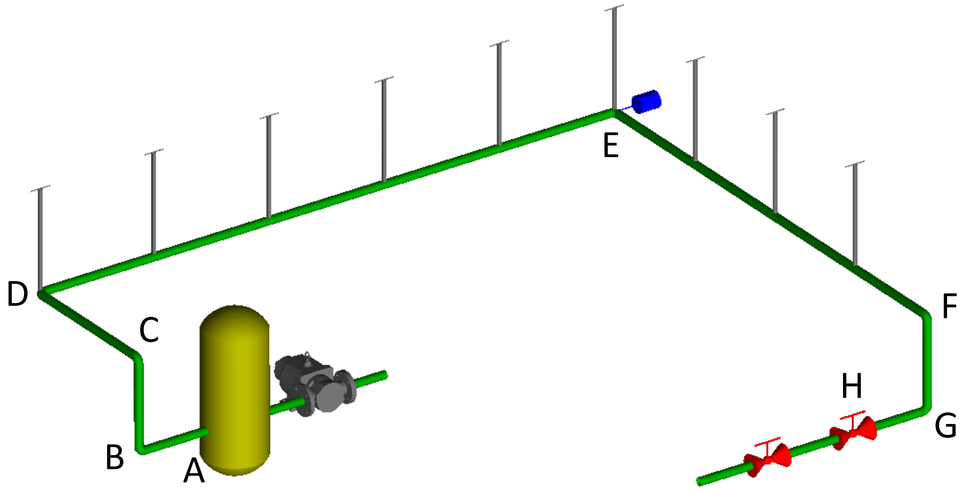

Figure 6. Configuration of the test rig at the research group in Delft.

The validation of the coupled codes DYVRO and ROHR2STOSS was performed against the Delft experiment by applying the integral as well as the local coupling concept. The measured and calculated time-dependent pressure is shown in figure 7. The pressure history is entirely different from the classical water hammer rectangular-shaped curve without FSI. By considering the fluid-structure interaction the decrease of the pressure gradients and the secondary waves can be captured. The calculation results obtained by the integral and local coupling concept differ only slightly. This is an indication that the longitudinal deformation of the pipe segments is not particularly significant in this experiment. The computational time was only 8 seconds on a standard PC for 310 finite volumes in the fluid code and a time step size of 0.19 msec. The additional amount of work for a user carrying out a coupled simulation comparing to stand-alone simulations of the fluid system and the pipe structure respectively is insignificant.

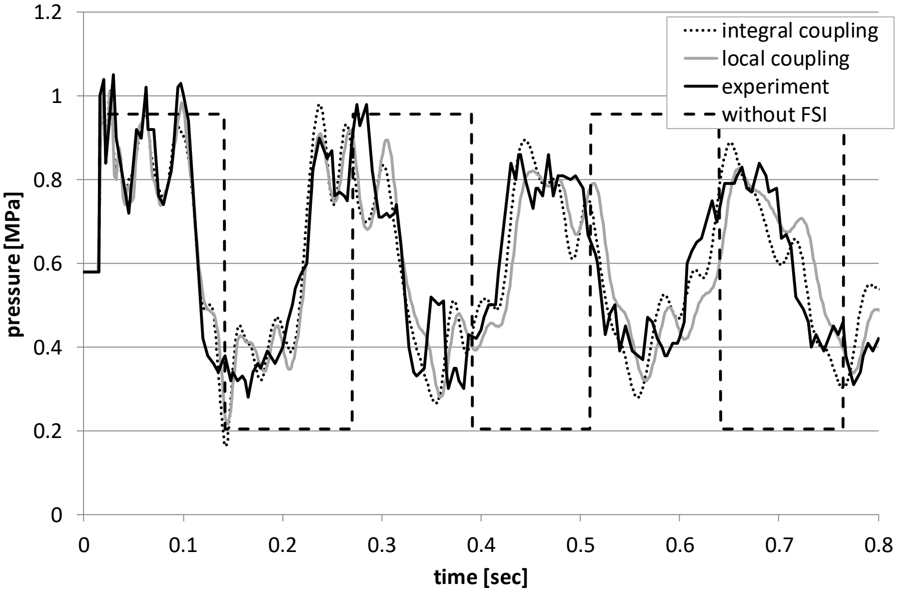

Figure 7. Measured and calculated pressure time histories at the valve for the Delft experiment.

Fourier analyses of the pressure histories were performed and the corresponding results are listed in table 2. The measured and calculated frequencies are in good agreement and differ considerably from the natural frequency of pressure wave oscillations without FSI with a value of 4.1 Hz. The natural frequencies below 50 Hz of the structural model with water as added mass (assumption of incompressible fluid) are 0.2 Hz, 0.6 Hz, 0.9 Hz, 3 Hz, 7.3 Hz, 12.2 Hz, 13.7 Hz 16.3 Hz, 30 Hz, 39.9 Hz and 46.4 Hz. They differ significantly from the frequencies of the experimental data.

Table 2: Frequencies of the pressure histories shown in figure 7.

natural frequencies	experiment	integral coupling	local coupling
1	4.9	4.9	4.9
2	14.9	14.6	14.1
3	23.7	24.1	24.3
4	27.4	28.7	28.6
5	36.6	37.2	36.6
6	47.1	45.6	45.4

CONCLUSIONS

The one-dimensional fluid code DYVRO was coupled with the code ROHR2STOSS that deals with dynamic analysis of complex piping and steel structures to obtain the natural frequencies of a pipe system filled with a fluid. Two code coupling concepts are described, the integral and the local coupling, considering for both a two-way interaction. The first concept was verified by comparison with an analytical solution for a simple case concerning a fluid column connected to a spring-mass system. Thus, since an analytical solution for the calculation of the natural frequencies of a more complex fluid-filled pipe system is not available in general, the code coupling strategy described in this publication proved to be a viable option.

The integral and the local coupling concept were validated by means of the comparison with experimental data from literature that revealed strong fluid-structure interaction during a water hammer event. The results of the experiment and the computations are in good agreement. However, the frequencies of the structural model with water as added mass as well as the natural frequency of pressure wave oscillations without FSI differ significantly from the frequencies of the experimental data. This shows the general necessity of a realistic, more detailed modelling strategy, like the code coupling concept, if the natural frequencies of a fluid-filled pipe system shall be determined precisely.

The local coupling concept is the more realistic one, because in the model the fluid forces are effective at the positions, where they actually act in the real pipe system. In contrast, by the integral coupling concept one fluid force is calculated for an entire pipe segment. This force is usually placed in the middle of the pipe segment in the structural code at a position, where only the friction coupling and Poisson coupling is active in a real pipe system, but not the more important junction coupling. For scenarios, in which an axial deformation of the pipe segments is not relevant, the integral coupling concept is sufficient, as shown by the validation with the Delft experiment. For very long pipe segments and relevant axial elongation, the results obtained by the integral coupling concept may become erroneous and the local coupling concept should be applied.

In this publication it is shown that the frequencies of a coupled fluid-structure pipe system can be captured by the coupled codes DYVRO and ROHR2STOSS. Therefore, the coupled codes can be used for the determination of the natural frequencies of a fluid-filled safety related pipe system and the assessment of dynamic pressure loads on the system. The coupled codes were applied successfully in several industrial projects by TUEV NORD. It is not necessarily required to apply more detailed analyses, like e.g. three-dimensional CFD or FEM analyses, for pipe systems.

REFERENCES

ANS (1988), American Nuclear Society: *Design Basis for Protection of Light Water Nuclear Power Plants against the Effects of Postulated Pipe Rupture*, ANSI/ANS-58.2-1988.

Moody, F. J. (1990). *Introduction to Unsteady Thermofluid Mechanics, 1st ed.*, Wiley-Interscience.

Neuhaus, T., Schaffrath, A., Jerinić, D. (2009). ”The Pressure Surge Computer Code DYVRO mod. 3: Modelling and Validation”, *The 13th International Topical Meeting on Nuclear Reactor Thermal Hydraulics (NURETH-13),* Kanazawa City, Japan, September 27-October 2, 2009.

VDI (2004), Verein deutscher Ingenieure: *Vibrations in Piping Systems*, VDI-standard 3842, June 2004.

Wiggert, D. C. and Tijsseling, A. S. (2001). “Fluid Transients and Fluid-Structure Interaction in Flexible Liquid-Filled Pipe Systems”, *Appl Mech Rev,* Vol 54, No 5, September 2001, pages 455-481.

Wylie, E. B. and Streeter, V. L. (1993). *Fluid Transients in Systems*, Prentice Hall.