Collapse of thick tubes subjected to external pressure: an interpretation of present code requirements

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ABSTRACT

Recent proposals for innovative nuclear power plant design consider steam generator tubes pressurized from outside. If sized according to presently accepted procedures, such as *ASME Boiler & Pressure Vessel* Code (Section III), the resulting thickness is extremely high. This entails a major contribution of the thermal conduction through the wall thickness to the thermal resistance in the heat exchange process between primary and secondary fluids.

ASME code seems excessively conservative for thick tubes. In fact, a more liberal design procedure, Code Case 2286-1, was accepted recently by *ASME* and can be used in lieu of Section VIII requirements. Its application as an alternative to Section III rules presently is under study, but the results it provides suggest that for the steam generator tubes under consideration a significant thickness reduction can be achieved without jeopardizing safety.

This paper aims at a rational assessment of the collapse behavior of thick tubes, based on the interpretation of computational results by means of a comparatively simple formula. This also permits a better understanding of code requirements.

INTRODUCTION

The collapse behavior of cylindrical shells pressurized from outside is examined. Attention is focused on long tubes of considerable thickness, as those envisaged for some innovative nuclear power plant proposals (e.g., the IRIS project [1]). Their collapse is dominated by yielding but, due to the decreasing nature of the post-collapse evolution, the effects of interaction with instability are likely to be significant enough to demand consideration. At present, no quantitative assessment of such effects is available, because tubes so thick have been studied little if at all.

Plasticity-instability interaction is activated by imperfections and to assess their influence on the collapse pressure a systematic numerical study is undertaken. Computations produce a meaningful measure of the failure level, explicitly accounting for interaction effects, and it is proposed that the allowable pressure be determined on this basis, by introducing a suitable safety factor. Its value is chosen so that results reproduce the *ASME Boiler & Pressure Vessel* code recommendations [2] for medium-thick tubes. Such tubes have been studied extensively [3-5] and it is assumed that codes consider the proper safety factor in this range. When the same safety margin is applied to thick tubes, the resulting allowable pressure is significantly higher than the value suggested by the code, which apparently reacts to the present lack of knowledge by assuming an extremely conservative attitude. The effectiveness of the procedure is enhanced by developing a comparatively simple formula, which reproduces with good accuracy the numerical results for long tubes of different materials and of different diameter to thickness ratios, ranging from the very thick cylinders that are the objective of this study up to comparatively slender ones, which do not require additional investigation since both sound theoretical results and reliable design formulas are available.

The results obtained are thought to be useful under at least three respects: first, they produce an alternative, rational design procedure; secondly, they provide an insight on the collapse behavior of tubes in a thickness range so far overlooked; finally, they permit the identification of the imperfections that most significantly affect the overall strength and that must be included in a numerical model when a complete non-linear numerical analysis is required.

SUMMARY OF ASME SIZING PROCEDURE

A cylindrical shell of nominal circular shape, with outer diameter D and wall thickness t, is subjected to an external pressure p. The shell is long enough for end effects to be disregarded. The material is isotropic, elastic-perfectly plastic and governed by von Mises criterion. E and ν are its elastic constants (Young modulus and Poisson ratio, respectively) and σ_0 denotes the tensile yield strength.

In the theoretical situation of a perfect tube, the limit pressure is given by the smallest among the elastic buckling and the plastic collapse values. If computed under thin shell assumptions, these values are

$$p_E = 2\frac{E}{1-\nu^2}\left(\frac{t}{D}\right)^3 \qquad p_0 = 2\sigma_0\frac{t}{D} \tag{1}$$

As an alternative, the theoretical limit can be expressed in terms of the hoop stress *S*, for thin tubes related to pressure through the equation

$$S = \frac{1}{2}p\frac{D}{t} \tag{2}$$

As the tube gets thick, stress variations over the thickness acquire significance and Eq. (2) only defines an average value which, however, remains a meaningful piece of information.

In real tubes imperfections are present, causing an interaction between plasticity and instability, which makes the failure pressure to be smaller than the theoretical values. Such a decay is significant when p_0 and p_E are not too different from each other, while interaction effects diminish as the ratio p_0/p_E departs from one. In fact, as it was discussed in [6], *ASME* Section III rules define a *reference failure pressure* p_C through the relation

$$p_C = 4B\frac{t}{D} \tag{3}$$

where *factor B* is a stress depending on tube geometry and material properties. Its value is read on graphs referring to the material under consideration at the given design temperature. Comparison with Eq. (2) shows that $2B$ can be interpreted as the reference failure hoop strain.

Results are depicted in figure 1a, referring to the material presently considered for IRIS steam generator tubes (Nickel-Chromium-Iron alloy UNS N06690 or *INCONEL* 690) at the design temperature of 345°C. Figure 1b shows the same results in terms of the average hoop stress Eq. (2) (this value being less sensitive to the *D/t* ratio, the picture is easier to interpret). Black solid lines plot the reference failure values, comparing them to the theoretical limits, the smallest among the values Eqs. (1) (dashed). As expected, the maximum decay occurs when $p_0 = p_E$ ($D/t \approx 30$ in our case). However, the picture shows a somehow anomalous feature: p_C becomes equal to p_E already for moderately thin tubes (D/t = 40), but it coincides with p_0 only for very thick ones (D/t = 5.34). It appears that interaction effects are considered more severe for thick than for thin tubes.

The *allowable working pressure* p_a is computed by dividing the value Eq. (3) by a safety factor, which is assumed as equal to 3 for all $D/t \geq 10$, with some decrease only for thicker tubes (the resulting values for the case under consideration are plotted as gray lines in figure 1). The introduction of such a high value is justified in the interval $40 \leq D/t \leq$ 60-70, where the reference failure pressure is taken as equal to the theoretical value p_E, but significant interaction effects are still expected. However, essentially the same safety factor is used for thick tubes, which are less sensitive to interaction; using in this range the same safety margin as for the most severe situation leads to extremely conservative designs.

COMPUTATIONAL ASSESSMENT OF THE FAILURE PRESSURE

The discussion above suggests that some shortcoming are present in *ASME* Section III design procedure. Reason is that, when defining the reference failure pressure Eq. (3), the role of imperfections is not considered explicitly. Among possible imperfections, the most detrimental consequences are induced by *out of roundness* or "*ovality*", which is defined by the dimensionless parameter

$$W = \frac{D_{\max} - D_{\min}}{D} \tag{4}$$

and can be taken as representative of all imperfections of geometric (e.g., non constant thickness) or mechanical (e.g., initial stresses) nature. The quantities entering Eq. (4) are depicted in figure 2.

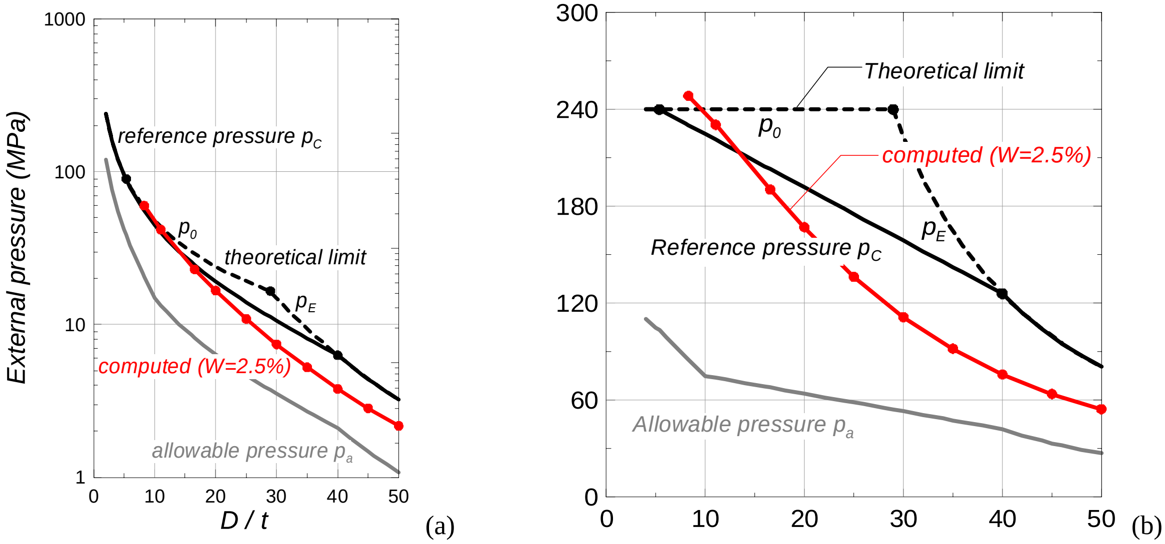

Fig. 1 Tube design according to ASME Section III rules

To assess quantitatively the role of imperfections, a systematic numerical study is undertaken. Computations consist of elastic-plastic, large displacement finite element analyses up to collapse, which are performed under the following assumptions:

1. The numerical model consists of solid (3-D) finite elements and the mesh is refined enough to ensure that convergence is attained.
2. Cross sections remain plane, i.e. strains in the axial direction are uniform, but possibly different from zero (*generalized plane strain*). Their value is dictated by the condition that axial stresses vanish on the average.
3. The material is elastic-perfectly plastic, governed by von Mises condition. The yield strength σ_0 is taken as the value measured at the 0.2% offset strain level.
4. Possible imperfections consist of an initial ovality, measured by the dimensionless parameter W, Eq. (4). Values up to 5% are examined.

Assumption 1 requires rather refined meshes but, because of assumption 2, only one slice of the tube needs to be analyzed and the size of the problem remains fairly small. This not only reduces the computational burden, but also permits a far better accuracy and, hence, improves the reliability of results. In first computations 20-node brick elements, with transverse displacements constrained to have the same value, were used, with a mesh involving 5 of them over the wall thickness and 60 over half circumference. 8+2-node *generalized plane strain* elements were also employed: these are quadratic Serendipity plane elements with two additional degrees of freedom controlling transverse displacements, which are given the same value throughout; in this case the mesh involves up to 12 elements over the thickness. The ABAQUS code is employed for computations.

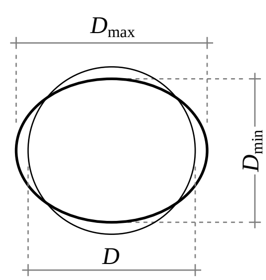

Fig. 2 Ovality

The adequacy of the model and of the assumptions on which is based was carefully tested, with fully satisfactory results (see [7] for details), and a systematic numerical study was undertaken. A first set of analyses (partial results have already been published in [8]) refers to *INCONEL* 690 tubes. To permit comparison with *ASME* Section III sizing procedure, material properties were defined consistently with the definition of *factor B*, which, for the design temperature under consideration, establishes

$$E = 183\ GPa \qquad \sigma_0 = 240\ MPa \tag{5}$$

while for the Poisson ratio the value indicated by the producer (ν = 0.289) was assumed. Different D/t ratios, ranging from the value D/t = 8.27 that *ASME* code requires for the IRIS steam generator tube bundles to D/t = 50 have been considered. This range also includes moderately thick and comparatively slender tubes, out of the direct interest of this study. Nevertheless, they are considered because they belong to a well explored range (values of D/t = 20-40 are representative of current oil industry applications), which makes the interpretation of results somewhat easier.

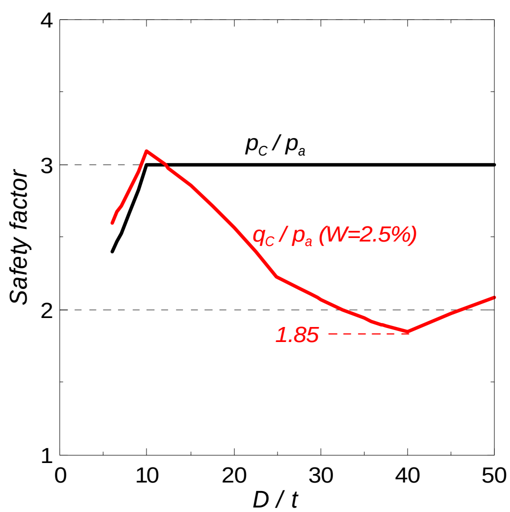

Fig. 3 Comparison among safety factors

The collapse pressures computed on this basis are indicated as q_C. For comparison purposes, the values corresponding to W = 2.5% are meaningful, since this is more or less the maximum value admitted by Section III rules for the diameters of interest to the IRIS steam generator tube bundles. The results obtained are depicted in figure 1 (red lines). Comparison with *ASME* reference failure pressure shows that for $D/t \geq 20$ the computed collapse load is significantly lower than p_C, indicating that interaction between plasticity and instability, induced by imperfections, causes a strong decay in load bearing capacity. On the other hand, interaction has no apparent effect when the tube is thick.

Figure 3 plots as functions of D/t the ratios p_C/p_a (the *ASME* safety factor) and q_C/p_a. The latter (red line) is the margin that *ASME* allowable pressure p_a exhibits with respect to q_C, i.e. the safety factor p_a would entail if q_C were the reference failure pressure. Such a safety factor is only 1.85 for $D/t \approx 40$ and less than 2.2 in the entire interval $30 \leq D/t \leq 50$, indicating that, at least in this range, a significant portion of the *ASME* value of 3 is meant at contrasting ovality. On the other hand, the red line is above the black one when $D/t < 15$: for thick tubes the consequences of ovality are so small that they turn out to be more than compensated by other effects, such as stress redistribution, which are included in computations but *ASME* Code does not consider when establishing the reference pressure. Using in this range the same safety factor as for thinner tubes appears at least questionable.

Among possible imperfections, ovality has the most detrimental consequences and the computed failure pressure q_C, explicitly accounting for it, seems a reasonable choice for the reference value. Since a significant part of uncertainties is considered in this definition, the safety factor with respect to it can be smaller than *ASME* Section III requirements. A value of 2 seems adequate, since this is more or less the margin Section III allowable pressure p_a has with respect to q_C for medium and moderately thin tubes. This is a well explored thickness range and it is reasonable to assume that the code considers the proper safety factor. On this basis, we propose an alternative definition for the allowable pressure, denoted by q_a and expressed as

$$q_a = \frac{1}{2} q_C \tag{6}$$

Figure 4 compares the definition above (red line) with *ASME* allowable pressure (black line). Because of the choice for the safety factor, results are similar for $D/t \geq 30$, but for thick tubes Eq. (6) permits significant thickness saving. Such a thickness reduction does not jeopardize safety. In fact, a more liberal design procedure has been accepted recently by ASME (Code Case 2286-1 [9]). Such a procedure provides alternative rules for determining the allowable external pressure for cylinders (and other components) under compressive stresses, to be used in lieu of Section VIII requirements. At present, its application as an alternative to Section III rules is under study, but the results it provides are meaningful for comparison purposes. The allowable pressure obtained on this basis are also shown in figure 4 (gray curve): it appears that the value Eq. (6) is more conservative than Code Case predictions.

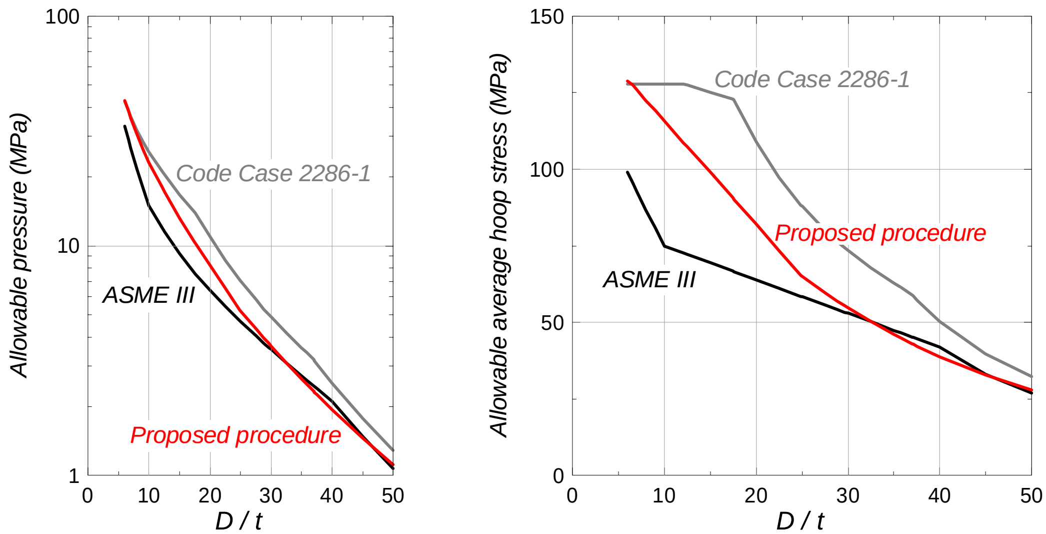

Fig. 4 Comparison among allowable working pressures

A PROPOSAL FOR A POSSIBLE DESIGN FORMULA

Equation (6) can be proposed as a definition for the allowable pressure, but the requirement that q_C be computed numerically considerably limits the applicability of the procedure. An attempt at reproducing computed results by means of a comparatively simple formula was advanced in [8]. The procedure requires the preliminary computation of the following quantities:

1. theoretical failure pressures

$$q_E = 2\frac{E}{1-\nu^2}\frac{1}{\frac{D}{t}\left(\frac{D}{t}-1\right)^2} \qquad q_0 = 2\sigma_0\frac{t}{D}\left(1+\frac{1}{2}\frac{t}{D}\right) \tag{7}$$

2. ovality parameter

$$Z = \frac{\sqrt{3}}{4}\left(2\frac{D}{t}+1\right)W \tag{8}$$

3. upper and lower bounds

$$q_U = \frac{q_0}{\sqrt{1+Z^2}} \qquad q_L = \frac{1}{2}\left[q_0 + q_E\left(1+Z\right) - \sqrt{\left(q_0 + q_E\left(1+Z\right)\right)^2 - 4q_0 q_E}\right] \tag{9}$$

where Eqs. (7) have the same meaning as Eqs. (1), except that they are computed by accounting for stress redistribution over the tube thickness (the first expression is well known [10], the second was established in [11]). The values defined by Eqs. (9) bound the collapse pressure in the thick and medium thick tube range; as cylinders get thin, the second expression looses its bounding nature and becomes a good approximation.

The collapse pressure q_C can be expressed on the basis of Eqs. (9) by writing

$$q_C = \mu q_U + \left(1-\mu\right) q_L, \quad 0 \le \mu \le 1 \tag{10}$$

The problem that remains open is the definition of the slenderness dependent coefficient μ (in [8] a very crude proposal was made, which only established the feasibility of the procedure). Once the numerical values of q_C are computed, it is possible to express the value μ should assume to reproduce computational results through the relation

$$\mu = \frac{q_C - q_L}{q_U - q_L} \tag{11}$$

Eq. (11) can be used to define an expression for μ consistent with computations.

Coefficient μ, however, depends on slenderness and, if the resulting formula has to apply to different materials, this cannot be simply taken as equal to the *D/t* ratio. A more meaningful measure is the ratio q_0/q_E between the theoretical collapse and buckling pressures of the round tube. Because of Eqs. (7) this ratio can be written as

$$\frac{q_0}{q_E} = \frac{\sigma_0}{E}\left(1-\nu^2\right)\left(\frac{D^2}{t^2} - \frac{3}{2}\frac{D}{t} + \frac{1}{2}\frac{t}{D}\right) \tag{12}$$

showing that slenderness also depends on material properties through the dimensionless parameter

$$\kappa = \frac{1}{1-\nu^2}\frac{E}{\sigma_0} \tag{13}$$

It seems reasonable to infer that coefficient μ can be defined independently of material properties by expressing it as function of q_0/q_E . To support this conjecture, the results obtained for *INCONEL* 690 with the material properties so far considered have been supplemented by additional computations, that considered different properties for the same material, as listed in Table 1

Table 1. Material properties considered for INCONEL 690 at T=345°C

material	E (GPa)	σ_0 (MPa)	ν	κ	notes
1	183	240	0.289	832	From curve for *factor B*
2	195	267	0.289	797	Given by the producer
3	183	213	0.289	937	Required by CC 2286-1

In addition, tubes made of different metals were also analyzed, with the properties summarized in Table 2. These materials can be divided into two groups: steel and copper alloys have a value of κ higher than *INCONEL*, but comparable; on the contrary, the yield strengths of aluminum and titanium alloys are great in comparison to their Young moduli, which entails significantly lower values for κ.

Table 2. Material properties for other metals considered – room temperature

material	identification	E (GPa)	σ_0 (MPa)	ν	κ
Stainless steel	AISI 316L	200	200	0.31	1106
Copper alloy	CuZn15	120	140	0.35	977
Aluminum alloy	EN AW-6061	70	240	0.35	332
Titanium alloy	Ti6A14V	110	830	0.34	150

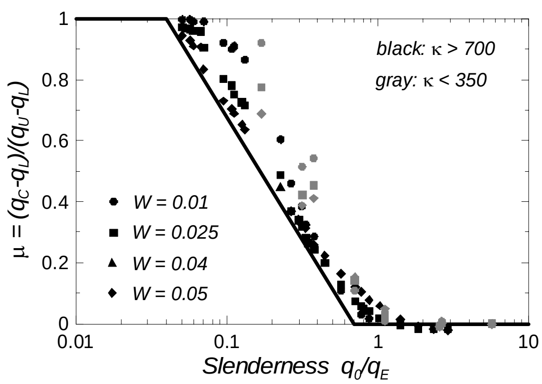

Fig. 5 Coefficient μ , Eq. (11), from computed cases

The results of all the computations performed are summarized in figure 5. For simplicity, materials are divided into two groups, corresponding to κ ranging from about 700 to more that 1100 (black symbols) and to $\kappa < 350$ (gray symbols), respectively. The picture shows a promising continuous trend; in particular, the lowest values roughly belong to a straight line in the graph, indicating that a safe estimate for μ could be expressed by means of a logarithmic equation. In particular, the expression

$$\mu = \begin{cases} 1 & \text{if} \quad q_0 / q_E < 0.04 \\ 0.35 \ln\left(q_E / q_0\right) - 0.125 & \text{if} \quad 0.04 \le q_0 / q_E \le 0.7 \\ 0 & \text{if} \quad q_0 / q_E > 0.7 \end{cases} \tag{14}$$

corresponds to the solid line in the figure. Results appear always good when $\kappa > 700$; the materials usually employed for tubes or pipes under external pressure belong to this group and the formula provides accurate estimates of their collapse pressure. Materials in the second group find limited applications in this context, but they are of engineering significance and cannot be ignored by an approach aiming at generality. For such materials, the approximation is not equally good, but the formula appears to be useful at least for preliminary design purposes. Observe that μ simply measures the relative weight of the upper and lower bounds in the estimate of the collapse pressure and even significant errors on its value entail much smaller errors on the value of q_C, Eq. (10). In fact, even for titanium alloy Ti6A14V ($\kappa = 150$) errors are less than 10% and on the safe side.

CONCLUDING REMARKS

Comparison among the allowable pressure pictures in figure 4 permits some consideration. The first point worth remarking is that Section III and Code Case curves have similar trends for $D/t > 30$, but the former requires a higher safety factor (about 1.2-1.4 times bigger) than the latter. The two codes seem to give the same interpretation to the critical situation, even if the safety requirements of Section III are more severe. For thicker tubes, on the contrary, qualitative differences show up: the Code Case assumes that instability effectively plays no role for $D/t < 17$, while Section III considers its effects significant for any $D/t > 10$ and non marginal even for thicker tubes.

The results provided by the proposed procedure reproduce Code Case recommendations from $D/t \geq 17$ up to thin tube values, except that for the value assumed for the safety factor. This suggests that the proposed procedure agrees with Code Case 2286-1 as for the interpretation of failure at different slenderness, but retains the same safety margin as Section III regulations. For $D/t < 17$ the two trends become different and the relevant safety margins approach each other. Part of this discrepancy is due to stress redistribution, which computations (and the formula reproducing their results) include but the Code Case does not consider; also, the difference in the values of the yield strength (213 MPa for the Code Case, instead of the 240 MPa used in the present approach) justifies part of it.

These, however, are only partial explanations and it is felt that most of the difference rests on the present lack of knowledge on the collapse behavior of thick tubes. Actually, by explicit statement of the authors [12, 13], the Code Case is based mainly on experimental evidence, often referred to monitoring of existing tubes. But very stocky tubes have found so far little, if any, application, so that their collapse pressure hardly can be assessed precisely on experimental ground. Therefore, the interpretation of thick tube behavior given in the present study can be considered as a new contribution to the field. The fact that the results obtained are in line with Code Case predictions in the slenderness range in which extensive experimental evidence is available can be regarded as an experimental support to the proposal.

A final remark refers to the effort demanded by the proposed procedure. Perhaps, the operations to be performed are slightly more involved than those of *ASME* Section III rules, but they are fully comparable to those required by sizing according to Code Case 2286-1.

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