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SEISMIC RESPONSE OF UNANCHORED LIQUID STORAGE TANKS

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ABSTRACT

The objectives of this paper are to highlight the principal effects of base uplifting on the seismic response of ground supported cylindrical steel tanks that are unanchored at their base. The system is analyzed approximately by a procedure which is highly efficient and is believed to represent adequately the essential aspects of the problem. The resistance to rocking of the base plate is evaluated with due consideration to non-linearities associated with variation in the base contact area, membrane action, and yielding in the base plate. The subject matter is presented in physically motivated terms with minimum reference to the underlying mathematics.

1 INTRODUCTION

Since a pioneering contribution made in 1949 [1], the dynamic response of ground-excited, liquid-filled tanks has been a subject of numerous analytical studies. The initial studies [1-3] were carried out on the assumption that the tank is rigid and fully anchored to a rigid base. Subsequent studies [4-7] considered the flexibility of the tank wall and demonstrated its significant influence on the magnitude and characteristics of the resulting response. The latest studies [8,9] have been concerned with the effects of ground flexibility and of the associated coupling between the vibrating tank-liquid system, its foundation and the supporting medium. These studies have provided practical means of analysis and design of fully anchored tanks subjected to earthquakes.

In practice, however, complete anchorage is not economical or may not be warranted for certain class of tanks; as a result, many existing tanks are unanchored and may uplift during ground shaking. Base uplifting completely changes the dynamic characteristics such as the stiffness and energy dissipation capacity of the system. The dynamic response of the system also becomes highly non-linear. Studies [10,11] of the performance of uplifting tanks during past earthquakes have revealed that such systems are prone to extensive damage due to: (i) buckling of the tank wall, caused by large compressive stresses; (ii) rupture at the plate-shell junction, caused by excessive plastic yielding; and (iii) failure of the piping connections to the wall that are incapable of absorbing large base uplifts. In contrast to the response of fully anchored tanks, which is reasonably well understood, the seismic response of unanchored systems has attracted researchers attention only in the recent years and requires additional research.

Past analytical studies on this subject varied in complexity. In the simpler analyses [13,14] the hydrodynamic forces were computed from the analysis of a corresponding fully anchored tank and the uplifting resistance of the base plate was computed from the small-deflection analysis of a prismatic beam. Similar analysis is also the basis of the guidelines in the American Petroleum Institute Standard [22] for the design of welded steel tanks for oil storage. In subsequent analyses highly complex modeling techniques using finite difference energy [15,16], Ritz energy [17,18] and finite element [19] methods were used to represent the base plate, the shell and the liquid in the tank. Whereas, the simpler analyses lacked the refinement needed to adequately represent the important aspects of the problem, the complex analyses precluded an easy interpretation of the results and an efficient solution of the dynamic response problem.

The objectives of this paper are: (i) to present a simple but rational method for the analysis of uplifting tanks; and (ii) to highlight the principal effects of base uplifting on the seismic response of such tanks. The response quantities that are examined are the hydrodynamic pressures, base uplift, plastic rotation at the plate-shell junction and maximum compressive stress in the tank wall. The system parameters varied are the intensity of ground shaking and the thicknesses of the base plate and the tank wall.

2 SYSTEM CONSIDERED AND MODEL USED

The system considered is a cylindrical steel tank of radius R, filled to height H, with a liquid of mass density ρ_l. The tank, assumed resting on a rigid base, is excited by a unidirectional component of horizontal ground motion which induces rocking ψ in its wall and partial uplifting in its flexible base, as shown in Fig. 1. Cylindrical coordinates (z, r and ϕ) with origin at the center of the base plate are used to specify the position of points in the tank.

The hydrodynamic action in a tank can be divided into two components: (i) the impulsive action which is due to the liquid moving in unison with the tank wall; and (ii) the convective action which is due to liquid moving in sloshing modes near the free surface. The impulsive modes usually have smaller periods of vibration, whereas, the convective modes have much longer periods. The contribution of convective action to the response is generally small and can be ignored. In most cases the fundamental impulsive mode is all that is needed to obtain reasonable results.

The hydrodynamic pressures induced in impulsive action are due to the motion of the tank wall as well as the rocking motion of the base plate. For a fixed-base tank the contribution of the base plate motion to the hydrodynamic action is usually small [20]. For an uplifting tank this contribution is even smaller because in this case only a small portion of the base plate actually experiences the motion.

The following assumptions are employed in carrying out the analysis: (i) the tank-liquid system responds in its fundamental impulsive mode; (ii) the convective component of response is negligible; and (iii) the hydrodynamic pressures are induced by the translation and rocking motions of the tank wall only. The validity of these assumptions is discussed in detail elsewhere [20]. Under these assumptions, the uplifting tank-liquid system may be represented by the model shown in Fig. 2, in which m is the mass associated with fundamental impulsive mode and $\bar{h}$ is the height at which the resultant of hydrodynamic forces on the tank wall may be assumed to be acting for a corresponding fully anchored tank [9]. The rotational spring at the base represents the resistance of the base plate to rocking.

The moment-rotation, M–ψ, relationship for the rotational spring is obtained by the application of the uplifting plate model [20] making due provisions for the non-linearities associated with large-deflections, material yielding and the variation of the base pressure in time and location. In this model, the tank bottom is represented by eight semi-infinite beams, each of uniform width $b = 2\pi R/8$.

3 METHOD OF SOLUTION

The equations of motion for the system are formulated by reference to the model shown in Fig. 2. Equilibrium of forces on the mass m requires that,

$$m\ddot{u}_o + c(\dot{u}_o - \dot{\psi}\bar{h}) + k(u_o - \psi\bar{h}) = -m\ddot{x}_g(t) \tag{1}$$

whereas equilibrium of moments at a section just above the rotational spring requires that,

$$\left[c(\dot{u}_o - \dot{\psi}\bar{h}) + k(u_o - \psi\bar{h})\right]\bar{h} = M(\psi) \tag{2}$$

where u_o = the overall displacement of the mass relative to the horizontal displacement of the base; ψ = the base rotation; $\ddot{x}_g(t)$ = the horizontal ground acceleration at any time t; c = the damping coefficient; k = the stiffness of the superstructure; and M = the moment in the spring, which is a function of base rotation, ψ.

Because the M–ψ relationship is nonlinear, Eqs. 1 and 2 are solved incrementally, assuming a linear relationship between the moment increment ΔM, and rotation increment $\Delta\psi$, i.e.,

$$\Delta M = K_\psi \Delta\psi \tag{3}$$

where K_ψ = the instantaneous value of the rotational spring stiffness.

The solution of Eqs. 1 and 2 is obtained by the linear acceleration method [21] in which the acceleration is assumed to vary linearly over each time increment Δt. A detailed description on the solution procedure may be found elsewhere [20].

In addition to the values of u_o and ψ, of interest in the solution of Eqs. 1 and 2 are the values of the superstructure deformation, u and the pseudoacceleration, $A(t)$. These are obtained from:

$$u = u_o - \psi\bar{h} \tag{4}$$

and

$$A(t) = \frac{k}{m}u(t) \tag{5}$$

On substituting $A(t)$ into the following expressions, one obtains hydrodynamic pressures, p in the tank [9]:

$$p(z,\phi,t) = \alpha(z)\,R\,\rho_l\,A(t)\,\cos\phi \tag{6}$$

$$p(r,\phi,t) \approx \alpha(r)\,R\,\rho_l\,A(t)\,\cos\phi \tag{7}$$

in which $\alpha(z)$, $\alpha(r)$ = the dimensionless functions that define the height-wise and radial distribution of hydrodynamic pressures, respectively [9].

4 NUMERICAL EXAMPLE

Also examined in a previous study [23], the tank analyzed is of 20 feet radius and is filled with water to its full height of 37 feet. The shell thickness varies from 0.38 inch for the bottom course to 0.19 inch for the top course. A uniform base plate thickness of $h = 0.19$ inch is assumed. The material properties of the tank are: Young's modulus of elasticity, $E = 29\times10^3$ ksi; yield stress, $\sigma_y = 36$ ksi; and Poisson's ratio, $\nu = 0.3$. The unit weights of the water and tank material are: $\gamma_l = \rho_l g = 62.4$ pcf and $\gamma = \rho g = 490$ pcf. The weight of the tank wall, $W = 44$ kips. The dynamic properties for the first impulsive mode of vibration of the fixed-base system are obtained [9] using a value of $H/R = 1.85$ and an equivalent uniform shell thickness of $h_s = 0.23$ inch. These properties are: $mg = 2{,}012$ kips, $\bar{h} = 214$ inch and f = the fixed base frequency of the system=6.43 Hz. Modal damping is assumed to be 2% of its critical value.

The response of the tank is obtained for the N–S component of the 1940 El Centro earthquake ground motion record (shown in Fig. 3) scaled to peak ground acceleration $\ddot{x}_g = 0.15$g.

The top two curves in Fig. 4 represent the pseudoacceleration time histories of the tank-liquid system for fully anchored and unanchored conditions of support. It may be recalled that the hydrodynamic pressures are directly proportional to the pseudoaccelerations (Eqs. 6 and 7). Please note that as compared to the response history of anchored tank the corresponding history for unanchored tank (i) has significantly longer periods of oscillation and significantly smaller peak value; and (ii) decays at a slower rate at the end of the excitation.

The first trend may be explained by noting that the unanchored system is the 'softer' system with a much longer natural period than the anchored system. The increase in period may in turn explain the reduction in the pseudoacceleration. The small rate at which the response amplitudes, following the end of the excitation, decay is an indication of small system damping. This is due primarily to the reduced deformations and velocities in the superstructure which make its viscous damping mechanism nearly ineffective. This fact is further emphasized by the histories of deformations shown at the bottom of Fig. 4 which reveal that after uplifting nearly all the deformation is absorbed by the base plate and the superstructure (tank-liquid system) experiences a more or less a rigid-body motion. The additional damping due to the hysteresis action of the base plate is found to be small due to the highly 'pinched' nature of the M–ψ diagram of the rotational spring which is shown in Fig. 5. The overturning moment M in this diagram is normalized with respect to $W_l R$ (W_l=weight of liquid in the tank and R is its radius), and the base rotation is expressed in degrees.

Critical responses of the unanchored system are the base uplift, the plastic rotation at plate-shell junction, and the axial compressive stress in the tank wall. The maximum values of these quantities occur at points 1 and 2 on the tank (see Fig. 1). The response histories of these quantities are shown in Fig. 6. The maximum base uplift of 2.6 inch for the tank calls for suitable flexibility in the design of any piping that may be attached to the tank wall. The plastic rotation, because of its cyclic variation, is a potential source of fatigue damage at the plate-shell junction. For the mild-steel plate used in this tank, the maximum plastic angle of 14 degrees is not considered to be unduly high. The maximum compressive stress in the tank wall was evaluated by assuming that after uplifting the contact between the wall and the foundation is confined to an arc of central angle 40 degrees and that the stress distribution over the contact length is of triangular shape with its peak at the center of the contact length. The histories of maximum compressive stress in the tank wall are shown at the bottom of Fig. 6 for the unanchored and fully anchored conditions of response. As expected base uplifting results in an increased compressive stress in the tank wall. The maximum compressive stress of 8.15 ksi for the unanchored system is marginally greater than the value of 7.5 ksi allowed by API Standard 650 [22] in its guidelines against buckling failure.

4.1 Effects of varying ground motion intensity

The base excitations of different intensities were obtained by scaling the El Centro record to different values of peak ground acceleration, $\ddot{x}_g$. The maximum responses obtained for different intensities are presented in Table 1 and selected histories of pseudoacceleration response are shown in Fig. 7. As expected, an increase in intensity results in a 'softer' system (with longer effective period, $\tilde{T}$) and smaller amplification ratio, $A/\ddot{x}_g$. The responses that are found to change most significantly with intensity are the maximum base uplift, w_{max} and plastic rotation, $\Delta\theta_{max}$. The responses that change at a slow rate are the pseudoacceleration, A , and the maximum compressive stress in the tank wall, σ_{max}.

4.2 Effects of varying base plate and wall thickness

In the analysis of the uplifting plate [20], the thickness of both base plate and tank wall were found to have significant effect on the uplifting resistance. Solutions of dynamic analysis for different values of base plate thickness are presented in Table 2. Please note that an increase in base plate thickness causes a significant reduction in the effective period of the unanchored system, which in turn effects the pseudoacceleration and the compressive stress accordingly.

The effect of increasing wall thickness is three-fold: (i) it increases the frequency, f of the fixed-base system [9]; (ii) it increases the uplifting resistance of the base plate [20]; and finally (iii) it increases the weight of the tank wall. The results for different values of wall thickness are presented in Table 3. Please note that effective periods becomes smaller as wall thickness increases and base uplift, plastic rotation and compressive stress reduce significantly.

5 CONCLUSIONS

An unanchored tank after uplifting responds in a highly flexible manner with much reduced hydrodynamic action. For the tank examined here nearly all deformation after uplifting was due to the base rotation, with the result that the tank wall experienced a more or less rigid-body motion. An unanchored tank's response is characterized by significant values of base uplift, plastic rotation, and compressive stress in the tank wall. A significant loss in the viscous damping occurs as a result of base uplifting. Additional damping due to hysteresis action in the base plate is quite small. For a tank of given overall dimensions, an increase either in base plate or wall thickness significantly reduces the uplifting response.

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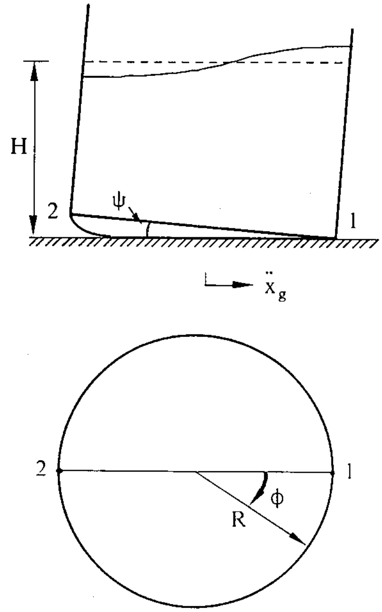

Fig. 1 Tank-Liquid System Considered

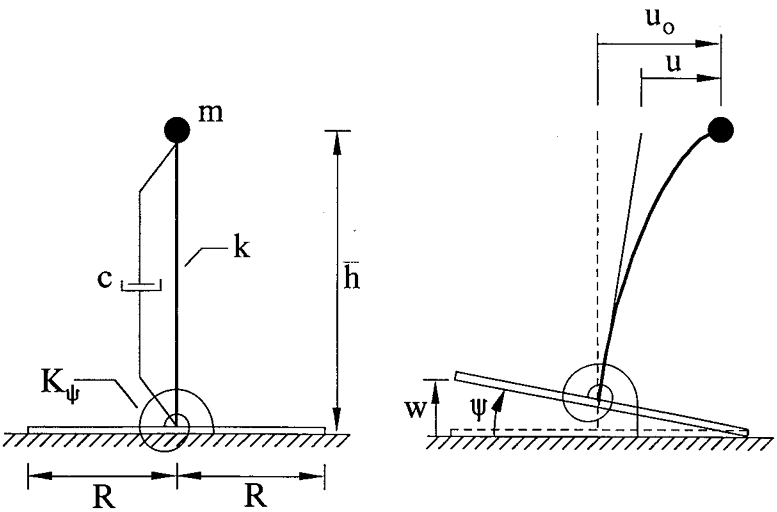

Fig. 2 Model of Unanchored Tank-Liquid System

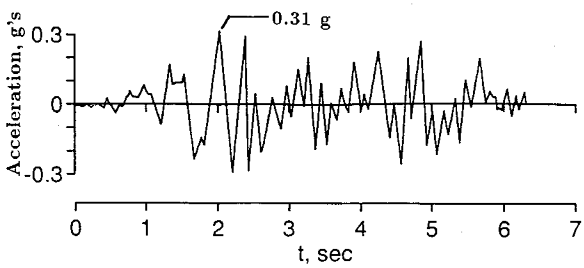

Fig. 3 N-S Component of 1940 El Centro Earthquake Ground Motion Record

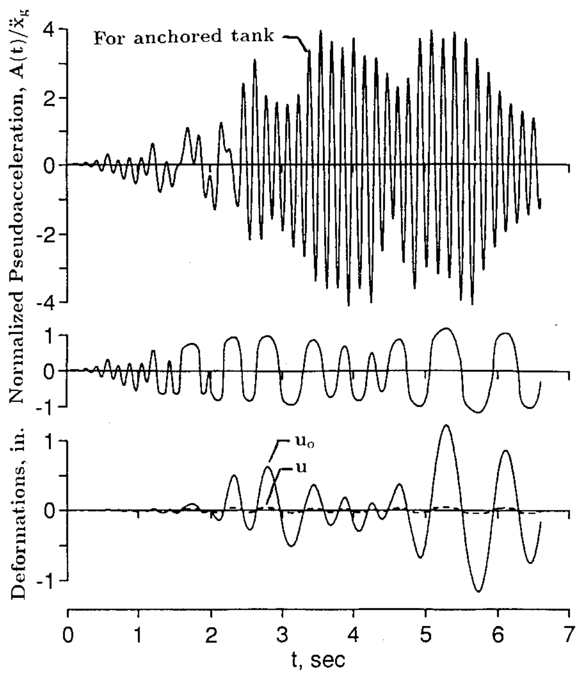

Fig. 4 Pseudoacceleration and Deformation Histories for Tank Subjected to Scaled El Centro Record with $\ddot{x}_g$ =0.15g (Unless otherwise noted, the responses are for unanchored condition)

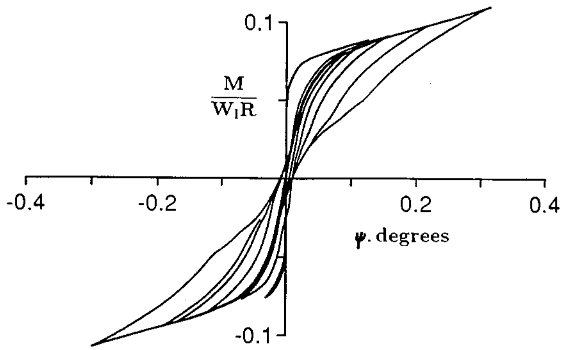

Fig. 5 Moment-Rotation Diagram at Base of Unanchored Tank

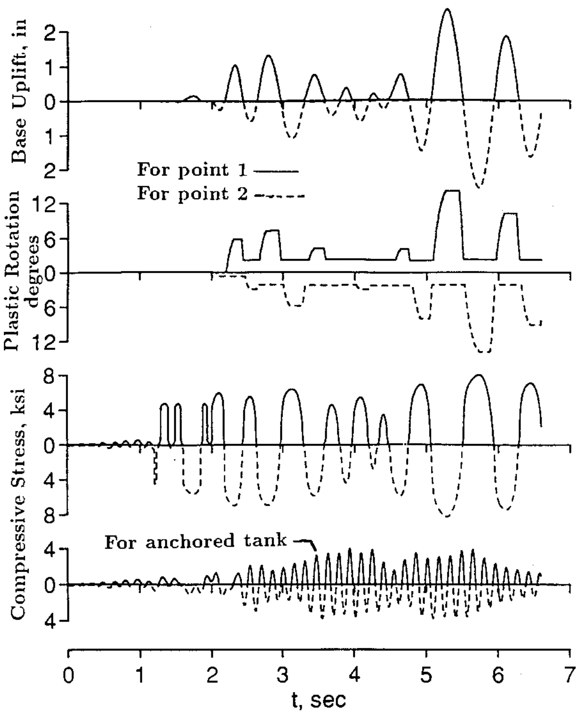

Fig. 6 Critical Responses of Unanchored Tank Subjected to Scaled El Centro Record with $\ddot{x}_g$=0.15g (Unless otherwise noted, the responses are for unanchored condition)

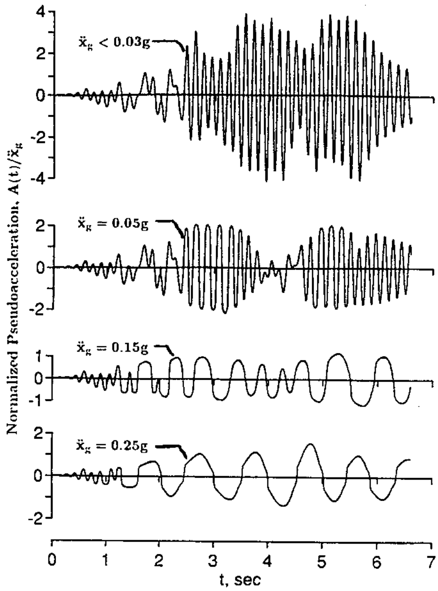

Fig. 7 Pseudoacceleration Histories for Unanchored Tank Subjected to Different Intensities of El Centro Record

$\ddot{x}_g$ g's	$\frac{A}{\ddot{x}_g}$	A g's	σ_{max} ksi	w_{max} in.	$\Delta\theta_{max}$ degrees	$\tilde{T}_{max}$ sec
0.02	4.13	0.08	0.59	0.00	0.00	0.16
0.03	3.67	0.11	5.39	0.11	0.00	0.26
0.04	2.57	0.10	5.08	0.07	0.00	0.23
0.05	2.23	0.11	5.45	0.12	0.00	0.27
0.06	2.02	0.12	5.89	0.23	0.46	0.33
0.08	1.53	0.12	5.96	0.25	0.66	0.33
0.10	1.51	0.15	7.02	1.51	8.34	0.71
0.12	1.55	0.19	8.57	3.00	15.66	0.95
0.15	1.17	0.18	8.15	2.62	14.09	0.84
0.18	1.27	0.23	10.12	4.47	21.45	0.96
0.20	1.23	0.25	10.81	5.05	23.24	0.98
0.25*	1.59*	0.40*	16.38*	9.52*	32.76*	1.06*

*Base uplifting in this case is so large that the suitability of the model is questionable. Moreover, the response of the system is considered to be unacceptably high.

Table 1 Maximum Responses of Unanchored Tank Subjected to Different Intensities of El Centro Record

h in.	$\frac{A}{\ddot{x}_g}$	σ_{max} ksi	w_{max} in.	$\Delta\theta_{max}$ degrees	$\tilde{T}_{max}$ sec
0.13	1.32	8.62	4.04	23.83	0.98
0.16	1.92	8.15	3.16	18.58	0.72
0.19	1.17	8.15	2.62	14.09	0.84
0.29	1.64	11.14	3.73	13.21	0.84
0.38	1.28	9.08	0.40	0.00	0.35

Table 2 Maximum Responses of Unanchored Tank with Different Base Plate Thicknesses Subjected to Scaled El Centro Record with $\ddot{x}_g = 0.15g$

h_s in.	h_e† in.	f cps	W kips	$\frac{A}{\ddot{x}_g}$	σ_{max} ksi	w_{max} in.	$\Delta\theta_{max}$ degrees	$\tilde{T}_{max}$ sec
0.19	0.12	4.52	22	1.32	17.40	5.09	27.96	1.11
0.29	0.17	5.41	33	1.02	9.37	2.06	11.57	0.81
0.38	0.23	6.43	44	1.17	8.15	2.62	14.09	0.84
0.50	0.29	7.38	55	1.49	8.03	3.69	17.43	0.88
0.57	0.35	7.51	66	1.57	7.09	3.62	16.27	0.85

†h_e = effective wall thickness used in the computation of f

Table 3 Maximum Responses of Unanchored Tank with Different Wall Thickness Subjected to Scaled El Centro Record with $\ddot{x}_g = 0.15g$