Dynamic Behaviour of Raft and Pile Foundations Tests and Computational Models: Part 5 — 3-D Dynamic Analysis of Groups of Piles and Comparisons with Experiments

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A method of analyzing the dynamic response of pile foundations in the frequency domain is proposed in this paper. The method is based on the successive use of boundary integral procedure and substructuring. A Fourier decomposition of the boundary integrals with respect to the angular component of each pile is performed in order to reduce the size of the problem and the stiffnesses of the piles are computed using finite element method. The piles are assumed to be connected to a rigid foundation and the final solution is obtained via a relaxation technique. Finally, the technique is applied to a real case and a comparison between numerical and experimental results is presented showing the importance of pile to pile interaction. Only homogeneous properties have been treated but the method can easily be extended to the heterogeneous case.

1. Introduction

Very recently much interest has been directed towards the dynamic analysis of pile foundations either under sismic or foundation input. There exists in the literature much information on surface foundations but when we come to embedded foundations several approximations leading to empirically chosen correcting factors are always introduced. If this way of analyzing may be justified for lightly embedded foundations, it does not seem to be correct for piles which are typically deeply embedded light foundations.

Several authors have thus developed original methods to analyze pile foundations. The first type of approach follows the work of Novak [8]- [12] and is based mainly on analytical calculations. Some interesting results are presented by this author for either a floating or end bearing pile submitted to vertical, horizontal or torsional loadings. The most interesting part is connected with the possibility of taking account of a layered soil. The second type of approach uses the finite element method Kausel [5] , Kuhlemeyer [6] - [7], Wolf [13] . Some inherent difficulties are often associated with this method as for dealing with infinite media and a consistant transmitting boundary must be incorporated in order that stationary modes linked to the finite size of the mesh are eliminated from the analysis [5]. When several piles are dealt with, some sort of substructuring must be used because the problem is threedimensional and the bandwidth of the matrix would be too large. Wolf [14] has presented a recent study where it is possible to analyze several piles in a layered medium resting on a bedrock at finite depth. The pile to pile interaction is incorporated in the theoretical study but, to our knowledge, no numerical results have been yet presented. In our opinion, the main shortcoming seems to be fact that a bedrock of finite depth must be present because no consistent absorbing boundary has been yet developed for the bottom of the mesh and it is well known that simple viscous boundaries give erroneous results.

The last way of attacking the problem is due to Luco [15] using integral equations and the Green function for a half space or even a layered medium. This technique is both elegant and efficient and, although it seems to have been implemented only for surface foundations, it could be developed for pile as well, using substructure technique presented below.

In the following sections we shall present an original method based also on integral equations but using the Green function for the whole space. Several piles can be dealt with and pile-soil-pile interaction is incorporated. The sollicitations may be threedimensional and either with seismic or foundation input. To facilitate the presentation, we shall first develop the technique for one pile, then for several piles. Finallly we shall present some comparisons with experimental results obtained at the site of Nice on small scale structures.

2. The boundary element for one pile

In section 3, we shall analyze the technique for a given number of piles. However one of the first ingredients is a technique for analyzing one pile either in an elastic half space or in a layered medium. As we recalled, the infinite boundaries of these domains led us to use the technique of integral equation and boundary elements. Given the axisymetric geometry of one pile it is interesting to develop a technique where the loads and displacements are not axisymetric. We will thus use a Fourier analysis with respect to the angular direction θ.

Let Ω be the exterior domain of the pile, Γ the pile boundary and eventually the free surface; let $u_i(x)$, $t_i(x)$ be the component of the displacement and of the stress vector respectively on the boundary . Finally let $U_{ij}(x, y;\omega)$, $T_{ij}(x, y;\omega)$ be the simple and double layer potentials of the Helmholtz equation. Then the integral equation relating u_i and t_i on Γ is :

$$\frac{u_i}{2}(y)=\int_\Gamma \left\{U_{ij}(x,y;\omega) - T_{ij}(x,y;\omega)\, u_j(x)\right\} d\gamma_x \qquad (1)$$

The kernels U_{ij}, T_{ij} may be found for instance in Eringen [2] in the three-dimensional case.

Using a Fourier development of $u_i(y) = u_i(r, \theta, z)$ and $t_i(y)$ it is possible to write an integral equation for each Fourier mode and it can be shown that the integration on Γ in the above equation decouples all the modes so that we arrive at the following equation for the mode 0 (for instance) :

$$\frac{u_k^\circ(r,z)}{2}=\int_\Gamma \left\{u_j^\circ(r,z)\,\Sigma_{jk}(r,0,z,r,0,z)- t_j^\circ(r,z)\,\Gamma_{jk}(r,0,z,r,0,z)\right\} d\gamma \qquad (2)$$

where Σ_{jk} , Γ_{jk} may be computed from U_{ij} , T_{ij}.

Similar equations may be obtained for other modes. Then a discretization using constant displacement and stress boundary elements is performed following now standard procedures (Jawson [4]). In this manner a linear relationship is retained between the stress and displacement vector on the boundary which may be considered as giving the dynamic stiffness of the surrounding soil.

We shall present simple bench mark cases :

- Infinite circular pile : let it be a unit amplitude of a given mode, we compute the soil reaction for the following cases (i) vertical stiffness (ii) horizontal stiffness (iii) rocking stiffness (iv) torsional stiffness.

On figure (1) we compare the analytical results (Novak [12]) with our method. A general good agreement may be seen in each case.

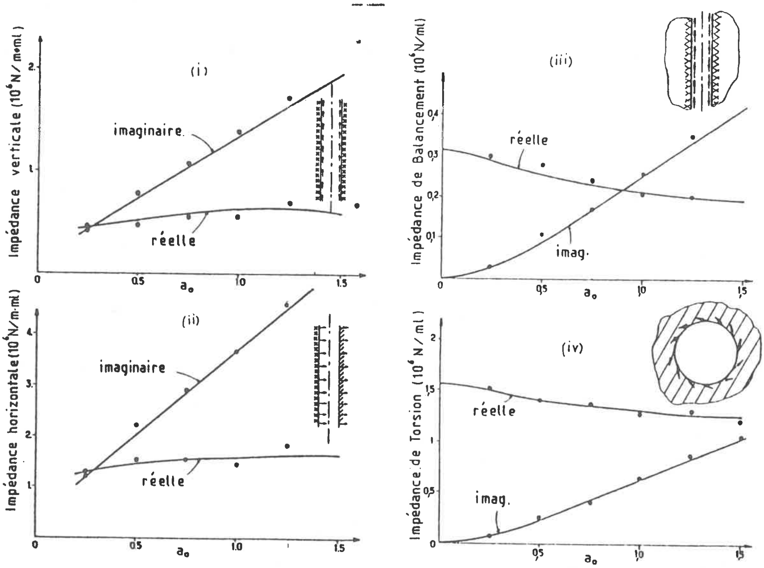
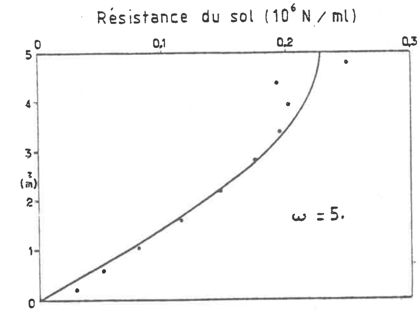
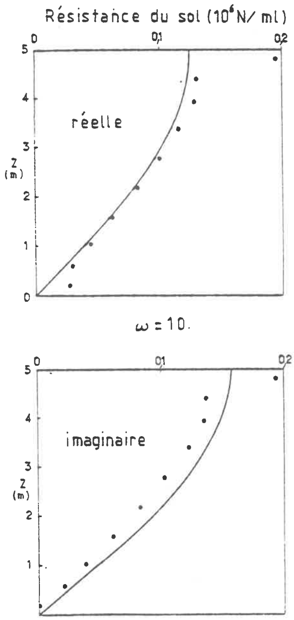
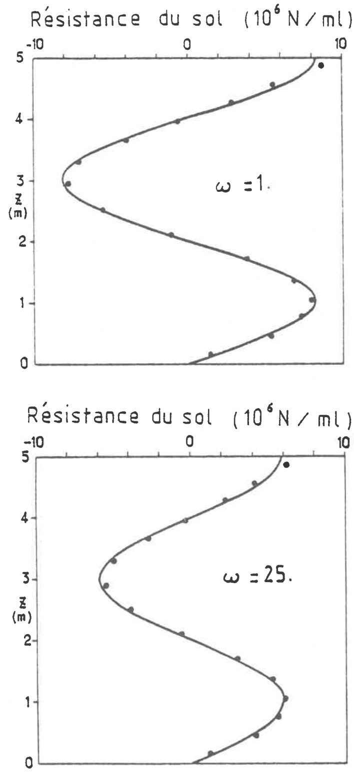


Fig. 1 : Infinite cylinder test. (points calculated)

- End bearing pile in an horizontal layer of finite depth. Novak [9] has already studied this problem and presented an approximate analytical solution using a simplifying assumption on the coupling between the displacements in one direction and the forces in the other direction. On the figure (2) we compare our results with those obtained by Novak for the first horizontal mode, for two different frequencies. An excellent agreement is to be seen except at the top where the free surface boundary condition cannot be accounted for by Novak's technique. It is interesting to note the occurrence of travelling waves past the natural frequency of the layer. Finally for the third horizontal mode a similar agreement is obtained (Fig. 3).







'ig.2: Resistance of soil to laterally loaded pile (1st mode of layer)

Fig.3:Resistance of soil to laterally loaded pile (3rd mode of layer)

(Calculated points)

3. Development of the method for several piles

Let a group of vertical piles-floating or end bearing-be given in either a homogeneous half space or layer of finite depth. To simplify the presentation, we shall assume that the group of piles is submitted to harmonic head-forces.

Let Ω_j , Γ_j be the domain and boundary of pile number j.
The variational formulation of the equations of each pile gives :

$$\int_{\Omega_j} (-\rho_p \cdot \omega^2)\, u_j \cdot v \, dx + \int_{\Omega_j} \sigma(u_j) \cdot \varepsilon(v) \, dx - \int_{\Gamma_j} t_j \cdot v \, d\gamma = 0 \qquad (3)$$
$$u_j = \bar{u}_j \quad \text{on } S_j$$

where S_j is the top surface of the pile, ρ_p the density, ω the circular frequency, u_j the displacements, $\sigma(u)$, $\varepsilon(u)$ the stresses and the deformations respectively, t_j the stress vector representing the forces applied by the soil on the pile and $\bar{u}_j$ is the top displacement; v is a field of virtual displacements. The formulation (3) will ultimately be discretized by finite elements. As the whole problem is linear we can write :

$$t_j = \sum_{k=1}^{N} A_{jk} \cdot u_k \qquad (4)$$

where N is the number of piles and u_k is the displacement of the pile number k at the soil pile boundary. A_{jk} is the impedance of the half-space minus the piles. Then if eq. (4) is reported into eq. (3), we get :

$$\int_{\Omega_j} (-\rho_p\, \omega^2)\, u_j \cdot v \, dx + \int_{\Omega_j} \sigma(u_j) \cdot \varepsilon(v) \, dx = \int_{\Gamma_j} \left(\sum_{k=1}^{N} A_{jk}\, u_k \right) \cdot v \, d\gamma \qquad (5)$$

we can see classically that the impedance of the soil contributes to the dynamic stiffness of the piles.
Now if the superstructure is assumed to be rigid, the displacements $\bar{u}_j$ at the top of each pile are depending on six parameters only describing the rigid body movement of the upper structure. In this mamer, if a given harmonic displacement U is acting on the upper-structure, we way compute u_j for each pile, then solve the linearly coupled system (5) for u_j, compute the stresses t_j at the top of each pile and the resulting forces and moments F acting on the upper-structure.A linear relationship will be obtained between U and F giving rise to the dynamic stiffness matrix of the ensemble of the piles and the soil, the piles being connected through a rigid upperstructure.

Now, two problems must be solved in order to reach our goal : (i) compute the impedance of the soil A_{ji} (ii) solve the highly coupled system (5).

(i) Computation of the impedance of the soil.

The computation of the impedance of either a half-space or a layer where several cavities have been inserted is not easy task. Ideally, if we able to compute analytically the Green function of this domain, the A_{ij} would readily come out. Due to the current lack of such an analytical expression, we shall rather try to build numerically such a Green function using the technique of integral equations.

Let t_{ext} be a surface density of stresses applied at the soil pile boundary and u_{ext} the corresponding displacement on the same boundary. These two quantities are related through the following integral equation :

$$\frac{u_{ext}}{2} = \int_{\Gamma} \left\{ - U \cdot t_{ext} - T \cdot u_{ext} \right\} d\gamma \qquad (6)$$

with $\Gamma = \bigcup_j \Gamma_j \cup \Gamma_s$ (Γ_s : half-space boundary)
The relationship above gives of course directly the A_{ij} terms. However, using boundary elements on each pile would be very costly and of too large a size.Furthermore, in order

to reduce the size of the problem we would like to split the displacement of each pile in each Fourier mode with respect to the local cylindrical system of coordinates but the problem (6) has not a cylindrical geometry. So we proceed in the following way :

At the soil pile interface, we introduce a density of stresses, which gives a displacement field on the piles, which satisfies:

$$u = \int_{(\bigcup_j \Gamma_j)} - U \cdot t \, d\gamma - \int_{\Gamma_s} T \cdot u \, d\gamma \qquad (7)$$

Using boundary element this equation can easily be splitted into a finite number of problems with cylindrical symetry in the geometry, but non axisymetrical forces. During this step, the method presented in the preceding section is applied.

Now the density t corresponds to a stress discontinuity

$$t = t_{ext} - t_{int} \qquad (8)$$

But for each pile j we may write the following interior problem :

$$\frac{u_{int}}{2} = \int_{\Gamma_j} \left\{ U \cdot t_{int} - T \cdot u_{int} \right\} d\gamma_j \qquad (9)$$

with the normal on Γ_j always oriented from the pile towards the soil. Now, if we recall that we are looking for the relationship between t_{ext} and u_{ext} , from (8) we get :

$$t_{ext} = t + t_{int}$$

with (10)

$$u_{ext} = u + u_{int}$$

From (7) we get :

$$u_{ext} = \int_{(\bigcup_j \Gamma_j)} -U \cdot (t_{ext} - t_{int}) \, d\gamma - \int_{\Gamma_s} T \cdot u_{ext} \, d\gamma \qquad (11)$$

But from (9), we may compute t_{int} (u_{ext}) so that :

$$u_{ext} = - \int_{(\bigcup_j \Gamma_j)} U \cdot t_{ext} \, d\gamma + \int_{(\bigcup_j \Gamma_j)} U \cdot t_{int}(u_{ext}) \, d\gamma - \int_{\Gamma_s} T \cdot u_{ext} \, d\gamma \qquad (12)$$

In this manner, we obtain a functional relationship between u_{ext}, t_{ext} using only two axisymetrical subproblems which may readily be solved with the technique developed in section 2 .

(ii) Solution of the system (5)

As the system (5) is not sparse due to the interaction between the piles, it is impossible to solve it by a direct method of elimination. Using the following iterative scheme we were able to get the solution in only a few iterations.

$n = 1,2,....$

$j = 1,2,....,N$

$$\int_{\Omega_j} -(\rho_p \omega^2) \, u_j^{n+1} \cdot v \, dx + \int_{\Omega_j} \sigma(u_j^{n+1}) \, \varepsilon(v) \, dx - \int_{\Gamma_j} \Big(\sum_{k=1}^{N} A_{jk} \, \tilde{u}_k^{n+1} \Big) \cdot v \, d\gamma = 0 \qquad (13)$$

with

$$\tilde{u}_k^{n+1} = \begin{cases} u_k^n & \text{si } k > j \\ u_k^{n+1} & \text{si } k \leqslant j \end{cases}$$

4. Real case applications.

Under the leadership of Electricite de France , medium scale structures resting on pile foundations have been submitted to different loading conditions. More particularly we have studied harmonic sollicitations on a 5 m high, 4 m diameter structure supported by ten meters long concrete piles of radius 0.45 m each, for which we refer to the two preceding papers.

4.1. Vertical impedance of the group of piles.

As was said above, the initial value of the displacements is the displacements of each pile as if it were standing alone for a unit vertical force. The soil is considered as a homogeneous half-space with properties the mean values of the soil parameters along the pile. On the underneath table three cases are compared (a) the piles are assumed to be independant (b) the piles interaction is accounted for without modal interaction (c) full interaction is accounted for.

K_v	N = 10Hz	N = 14Hz	N = 17Hz	N = 20Hz
(a)	$(6.45+i2.39)10^9$	$(6.92+i3.02)\ 10^9$	$(7.13+i3.47)10^9$	$(7.22+i3.69)\ 10^9$
(b)	$(2.89+i2.45)10^9$	$(3.0+i3.34)\ 10^9$	$(2.95+i4.09)10^9$	$(2.95+i4.52)\ 10^9$
(c)	$(3.04+i2.6)\ 10^9$	$(3.25+i3.62)\ 10^9$	$(3.25+i4.45)10^9$	$(3.24+i4.98)\ 10^9$

Table 1

It can be seen that the interaction is quite important for the range of frequencies considered and that the modal interaction may be neglected although its influence increases with the frequency about 5% for 10 Hz, 10% for 20 Hz.

4.2. Vertical response of the structure.

From the impedance Kv, the response of the structure of mass M submitted to a harmonic force of amplitude F_z can be computed

The consequences of the two assumptions (a) and (c) are illustrated on figure (4) where we can show that the interaction between the piles must be taken into account to fit the experimental curves. On the next figure (5), we compare the response of the structure resting either on piles or directly on the soil. It is shown that the response is highgly decreased when the piles are present and a resonance phenomenon is avoided in the vicinity of N = 13 Hz.

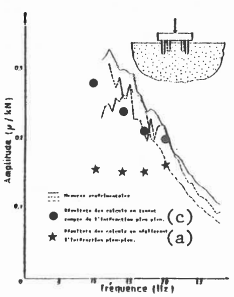


Fig.4: Vertical response spectrum of a 4 piles foundation.

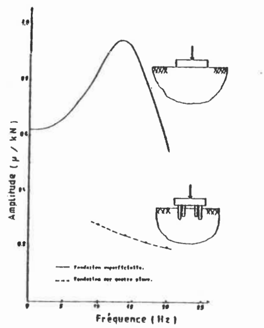


Fig 5: Compared response between pile and surface foundations.

4.3. Horizontal impedance of the group of piles.

The same iterative procedure has been used starting from the response of a single pile. It turned out that the interaction between the piles was at a very low level and we decided to neglect it in the following results. We also noticed that the horizontal displacements along the piles vanish very rapidly with the depth and that the horizontal impedance was almost independant of the frequency.

4.4. Rocking impedance of the group of piles.

The rocking impedance of a group of piles must be computed even in the case of horizontal loading on the structure due to the coupling between these two modes. The impedance varies somewhat with the frequency (especially the imaginary part) and it will be, with the mass term the principal factor contributing to the structural response.

4.5. Horizontal response of the top of the structure.

Using the original parameters of the soil we did not succeed to fit the computed results to the experimental values. Keeping the same value of the rocking impedance we modified the horizontal impedance in order to get a better agreement and the results are shown on figure (6).

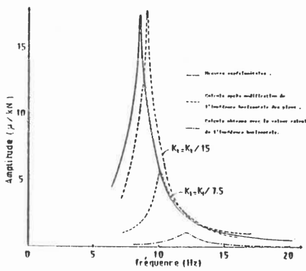


Fig.6: Sliding response spectrum of a 4 piles foundation.

The discrepancy between the numerical and experimental values is supposed to be due to the very skinny profile of the horizontal displacement : the soil properties are varying very erratically in the first few meters and the half space hypothesis is certainly highly idealized in this case.

5. Conclusions.

A method has been presented where the interaction between a group of piles may be fully accounted for, for either a half space or a layer of finite depth. It uses simultaneously integral equations and boundary elements for the evaluation of the soil impedance at the pile-soil interface, finite elements for the piles themselves and a rigid body assumption for the structure.

It has been shown on real case application that for vertical harmonic loading, the pile interaction is predominant and a general good agreement has been obtained for the numerical and experimental values of the amplitude. However some inconsistancy was produced for the horizontal mode and it is advanced that in this case the half-space hypothesis is questionable. However the method is not limited to a homogeneous half space or layer and research effort is now directed towards the development of the method in the non-homogeneous case.

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