Application of the Probabilistic Fracture Mechanics to the Multiple Crack Problem and Inelastic Behavior Problem

N. HANDA
Toshiba Corporation, Tokyo, Japan
T. UNO
Toshiba Corporation, Yokohama, Japan

ABSTRACT

The paper presents the results of application of the probabilistic fracture mechanics to the reliability evaluation of structure with multiple closely located cracks and of structure showing inelastic behavior at elevated temperatures, using the "PROOF" code[1].

For the problem of more than one cracks, a probabilistic technique was developed to simulate events with coalescence of these cracks. For the problem of inelastic behavior at elevated temperature, a technique to enable simulation of the creep model and stress hysteresis at elevated temperatures.

1 APPLICATION TO STRUCTURE WITH MULTIPLE CLOSELY LOCATED CRACKS

1.1 Probabilistic evaluation technique

The probability, $P_\ell{}^t$ for fracture in mode ℓ (e.g. small leakage, large leakage, fracture) at time t is given by following equation:

$$P_\ell{}^t = \sum_{\nu=1}^{\infty} \gamma_\nu P_\nu{}^t{}_\ell$$

where, γ_ν is the probability for the presence of ν initial cracks, which is usually approximated by a Poisson's distribution as follows:

$$\gamma_\nu = \frac{\lambda^\nu}{\nu!} e^{-\lambda}$$

where λ is the mean number of initial cracks. $P_{\nu\ell}{}^t$ is the fracture probability in condition with ν initial cracks., which is approximated by the Monte Carlo method as follows:

$$P_\nu{}^t{}_\ell \simeq \sum_{i_1 \cdots i_\nu} \frac{q_{i1\cdots i_\nu}}{N_{i1\cdots i_\nu}} \sum_{i_1\cdots i_\nu}^{N_{i1\cdots i}} f^{\,t}_{\nu\ell}(\alpha^1_{i1n} \cdots \alpha^\nu_{i\nu n}\ ,\ \gamma^1_{i1n} \cdots \gamma^\nu_{i\nu n})$$

where, $(\alpha^1_{i1n}\ ,\ \gamma^1_{i1n})$, $\cdots$, $(\alpha^\nu_{i\nu n}\ ,\ \gamma^\nu_{i\nu n})$, are initial form of the 1st, $\cdots\nu$ cracks (realized values by the Monte Carlo method), belonging to the $i_1, \ldots i_\nu$ cell, respectively. $f_\nu t_\ell\ (\alpha^1\cdots\alpha^\nu, \gamma^1\cdots\gamma^\nu)$ is a parameter for the fracture defined by following equations:

$$f^{\,t}_{\nu\ell} = \begin{cases} 1: & \text{A fracture in mode I occurred on any of cracks till time t in a structure having } \nu \text{ cracks with shapes represented by } (\alpha^1, \gamma^1), \cdots (\alpha^\nu, \gamma^\nu) \text{ respectively.} \\ 0: & \text{The above crack did not occur at any cracks,} \end{cases}$$

$q_{i_1\cdots i_\nu}$ shows a probability for the realized value of (α, γ) to belong to the ith cell by the Monte Carlo method. In the above, $i_1\cdots i_\nu$ show the numbers of permutaions for allocation of cracks to M cells that divide the (α, γ) space.

1.2 Coalescence of cracks

The coalescence of cracks is evaluated on one-dimensional coordinate system for the simplicity. It is assumed that N_L cracks from $k_{\ell L}$ th to $k_{\ell L+N_L-1}$ th in the L group integrated. Then, the form and center coordinate of the integrated crack are given as follows:

1) Depth of coalescenced crack

$$a_{kL} = \max\,(a_{k\ell 1}, \cdots, a_{k\ell 1+N_L-1})$$

2) Length of coalescenced crack

$$b_{kL} = \sum_{m=\ell_L}^{\ell_L+N_L-1} b_{km}$$

3) Coordinates of the center of coalescenced crack

$$X_{kL} = \frac{1}{2}(X_{K\ell L} - b_{k\ell L} + X_{k\ell_L+N_L-1} + b_{k\ell_L+N_L-1})$$

Then, the number of cracks changes from its initial value, N to $(N - N_L + 1)$.

1.3 Application to piping with multiple cracks

The reliability of a piping is calculated based on the approach described in Section 1.2. The analytical conditions are shown in Table 1. Conditions for coalescence of cracks are evaluation using the polar coordinates as follows:

1) When the kth and (k+1)th cracks integrate $(k+1<\nu)$:

$$\theta_{Lk} - \theta_{Rk+1} < \theta_\varepsilon$$

2) When the 1st and ν th cracks integrate:

$$\theta_{R1} - \theta_{L\nu} + 2\pi < \theta_\varepsilon$$

where, θ_{Lk} and θ_{Rk+1} indicate the left end point of the kth crack and the right end point of (k+1)th crack as viewed from the center coordinates of the crack of the kth crack, respectively. θ_ε is a constant to determine the coalescence.

A replica exists which shows the changes in position and shape of the crack along the crack's growth as shown in Fig. 1. The figure shows a replica which indicates that, two of the five initial cracks on a weld coalescenced at 13 years after plant start up and another crack integrated at 28 years plant start up, which lead to unstable fracture. When cracks integrates with each other, a crack with greater length and depth are formed, which causes increased stress intensity factor and acceleration of the crack propagation. As a result, both the leakage and fracture probabilities non-linearly increases to give values of $P_{\nu\ell}$ (fracture probability with ν initial cracks) as follows:

Failure probability	Mode	1	2	3	4	5
$P_{\nu\ell}$	Leakage	8×10^{-4}	5×10^{-3}	6×10^{-3}	1×10^{-2}	1×10^{-2}
$P_{\nu\ell}$	Fracture	~0	4×10^{-9}	6×10^{-9}	1×10^{-8}	2×10^{-8}

With this result, the piping fractures only when cracks coalescence. The non-linear increase of the leakage probability for piping is evaluated since it is conservatively treated on the one-dimensional coordinates. The final fracture probability is given by the product with the probability of presence of cracks γ_ν. The presence probability depends upon the volume of weld line and the density of cracks. Assuming 400 cm^2 and $1.0\times10^{-3}/cm^2$ for these values, the probability for existence of a crack is 0.4, leading to a condition with higher possibility for no crack in existence. The failure probability at the final period of the plant life in these conditions is 6×10^{-4} for leakage and 1×10^{-9} for fracture, giving lower leakage probability and higher fracture probability for fracture than for assumed single crack in existence. These results indicate that they depend on the presence density of initial cracks and the volume of welded portions with possible cracks, requiring careful consideration on them.

2 APPLICATION TO STRUCTURES WITH INELASTIC BEHAVIOR

2.1 Creep crack propagation model

The creep crack propagation is often taken as a serious problem in the crack propagation analysis for structures with high thermal stress at elevated temperatures. In these cases, evaluation of steady state load which accelerates the creep is most significant for the crack propagation analysis. The creep crack propagation is evaluated by following equations:

$$\begin{cases} \dfrac{da}{dt} = C_\circ (C^*)^m \\[2ex] C^* = A_1 \circ \mathrm{Func}\,(Jp) \\ \overset{\circ}{\varepsilon} = A_1 \circ \sigma^n \end{cases}$$

where, C is a coefficient obtained by reevaluation of the creep crack propagation speed using C^* [2], $\mathrm{Func}(J_p)$ is the J-integral for the surface crack[3], A_1 is a coefficient to give the creep propagation speed with application of Norton rule. and n is the power in Rangberg Osgood rule.

2.2 Steady state stress

Denoting the plastic strain at a time as $\varepsilon_0{}^P$, plastic strain with given stress increment or total strain increment ε^P is given using the configuration equation in the single axis stress field as follows:

$$
\begin{cases}
H' = \beta|\varepsilon^P|^{\gamma} \\
d\sigma = \dfrac{EH'}{E+H'}\, d\varepsilon \\
d\varepsilon^P = \dfrac{1}{H'}\, d\sigma \\
\varepsilon^P = \varepsilon_0{}^P + d\varepsilon^P \\
\qquad \text{where} \quad \beta = \dfrac{1}{nA^{\frac{1}{n}}} \qquad \gamma = \dfrac{1}{n} - 1 \qquad \varepsilon^P = A\sigma \operatorname{sgn}\sigma\sigma^n
\end{cases}
$$

By eliminating $d\sigma$, ε^P, we obtain.

$$H' - \beta|\varepsilon_0{}^P + \frac{EH'}{E+H'}\, d\varepsilon|^{\gamma} = 0$$

This is a non-linear equation for H', which can be solved by the Newton method as follows:

Putting

$$F(H') = H' - \beta|\varepsilon_0{}^P + \frac{EH'}{E+H'}\, d\varepsilon|^{\gamma}$$

and make a repetition as follows:

$$H'^{n+1} = H'^{n} - \frac{F(H'^{n})}{F'(H'^{n})}$$

When H' is calculated from the above, a step of plastic calculation is performed as follows:

$$
\begin{cases}
d\sigma = \dfrac{EH'}{E+H'}\, d\varepsilon \\
d\varepsilon^P = \dfrac{d\sigma}{H'} \\
\varepsilon^P = \varepsilon^P + d\varepsilon^P
\end{cases}
$$

The effect of displacement is given by

$$d\alpha = c d\varepsilon^P$$

where, α is the center of yielded surface and C is a constant.

By introduction of a step to solve the non-linear equation of H' by the Newton method, the configuration equation can be solved with quick conversion, enabling it to be applied to the probabilistic fracture mechanics which requires more than ten thousand simulation operations.

2.3 Application to portions near the surface level of FBR

The analytical conditions are shown in Table 2. Fig. 2 shows the stress cycle obtained by the elastic analysis, which follows the Poisson's process. Changes in the stress hysteresis and yield stress obtained by the analysis are shown in Fig. 3, in which the hysteresis is given for 6 years after plant start up. Cumulative leakage probability is given in Fig. 4. Those for assumption of constant application of steady state stress at 5kg/mm^2 and 11.7 kg/cm^2 which accelerate the creep are also given in Fig. 3.

3 CONCLUSION

Multiple cracks often occur at close locations in actual welded potions. In the paper, the author extends the sampling scheme by the Monte Carlo method and introduced simple crack coalescence model to reflect this situation to the probabilistic fracture mechanics.

Calculation of stress which accelerates creep at steady state is most significant for analysis of structures with inelastic behavior at elevated temperatures. The author successfully calculated the fracture probability by incorporating a simulation method of the configuration equation at elevated temperature into the probabilistic fracture mechanics model in this paper.

4. REFERENCES

1) S. Hattori, N. Handa "Probabilistic Fracture Evaluation of a Fast Breeder Reactor Cover" Nuc. Technology 79 (1987) 228
2) N. Handa "Application of Probabilistic Fracture Mechanics" 8th Japanese Symposium on the Structural Reliability of Nuclear Components (1991) (in Japanese)
3) V. Kumar, M. D. German "Elastic Plastic Fracture Analysis of Through-Wall and Surface Flaws in Cylinders" EPR1 NP-5596 (1988)

Table 1　Analysis conditions of a pipe with multiple cracks

MATERIAL	SUS 316
DIMENSIONS	32 B/10 MM
FREQUENCY OF LOAD	POISSON PROCESS MEAN 10/YEAR
INITIAL CRACK POSITION	UNIFORM DISTRIBUTION

Table 2　Analysis conditions of a high temperature structure with large thermal stress

MATERIAL	SUS 316
CALCULATED LOAD BY ELASTIC ANALYSIS	START UP : $+35\ kg/mm^2 \rightarrow -10\ kg/mm^2$ SHUT DOWN : $-25\ kg/mm^2$ SCRAM : $+15$ kg/mm2 $\rightarrow -35$ kg/mm2
FREQUENCY OF LOAD	START UP : 1/YEAR SHUT DOWN : 1/YEAR SCRAM : 10/YEAR START UP AFTER SCRAM : 10/YEAR

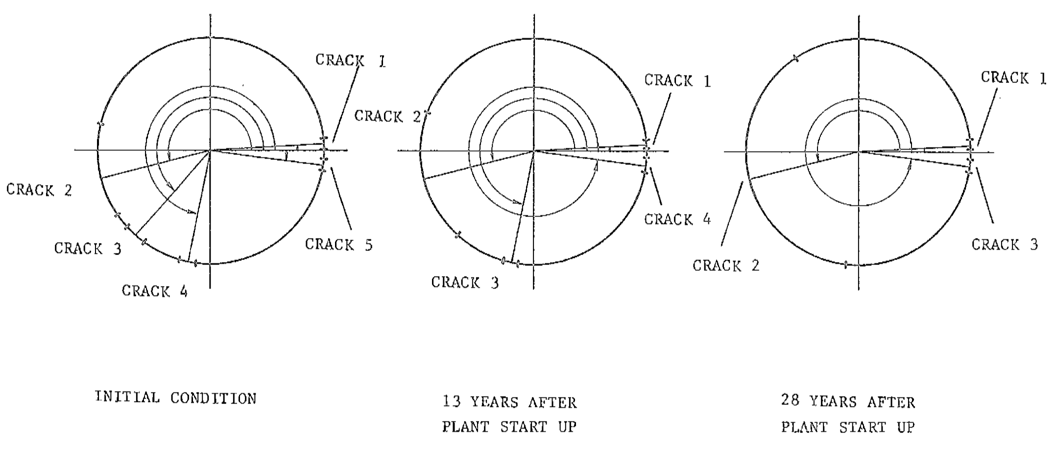

Fig. 1　Crack coalescence behavior shown in one of replica

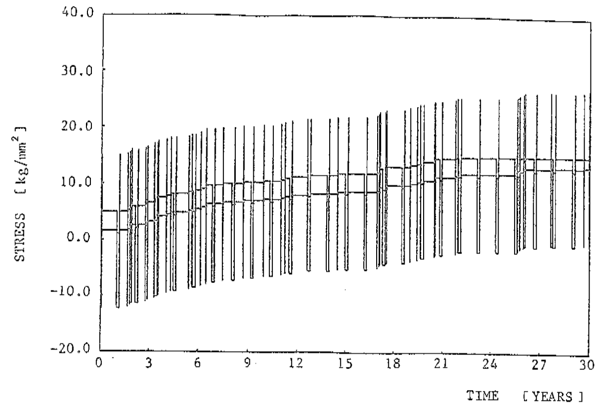

Fig. 2 Load history calculated by elastic analysis

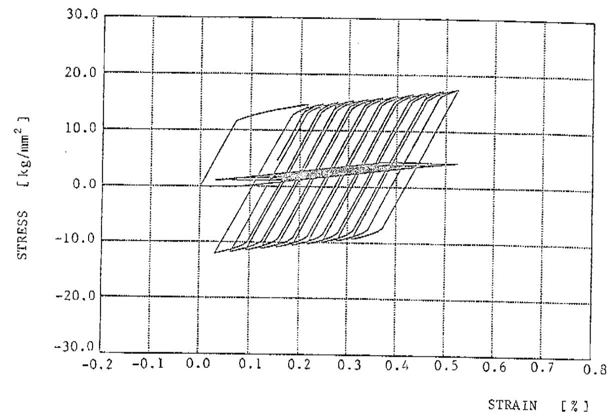

Fig. 3 Stress-Strain curve by six years after plant start up

CASE 1	INELASTIC ANALYSIS
CASE 2	σ=11.7 kg/mm^2
CASE 3	σ=5 kg/mm^2

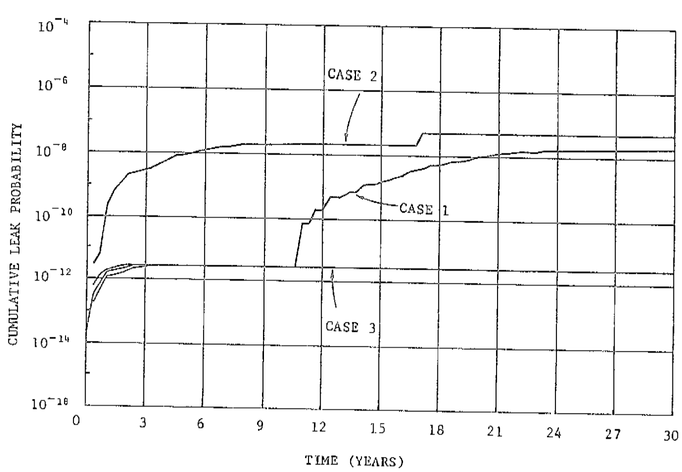

Fig. 4 Cumulative leak probabilities as a function of time