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Study on Application of CQC Method to Seismic Design (Proposal of Revised CQC Method)

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ABSTRACT

The CQC (Complete Quadratic Combination) method is well-known as an accurate response spectrum method especially for structures with closely spaced natural frequencies. For secondary systems such as piping and equipments mounted on a structure, however, the CQC method does not always give accurate responses by comparing with direct time integration results. This error may be derived from the application limit of the cross-correlation coefficients which are obtained on the assumption of a white-noise input, to a narrow-band process as excitation input for the case of secondary system. The purpose of this paper is to present an improved CQC technique which can get more accurate results for secondary systems with closely spaced frequencies compared with conventional CQC method. The proposed technique is evaluated through comparisons of results by presented technique with simulation results.

1. INTRODUCTION

Recently, CQC technique which is one of the response spectrum methods attracts attention as a seismic response analysis with good accuracy to the structural system which has the closely spaced natural frequencies. However, the cross-correlated coefficient which is an important parameter of the CQC method, by which the correlation between modal responses is described, is derived under the assumption of the white noise input. This means that the accuracy of the seismic response analysis to a secondary system such as piping/equipment structure is not good because of its narrow-band excitation.

Therefore, a revised CQC method is newly developed in order to presume accurately the response of secondary system with closely spaced natural frequencies.

2. PROBLEM OF ORDINARY CQC METHOD

2.1 Outline of CQC Method

The response of arbitrary point k in the multi degree-of-freedom linear structure, $Q_k(t)$, is shown as follows by modal analysis technique:

$$Q_k(t)=\sum_i \Psi_{ki}x_i \equiv \sum_i Q_{ki}(t) \tag{1}$$

in which Ψ_{ki} is the product of i-th mode ϕ_{ki} and the participation factor β_i, and x_i is i-th response to an earthquake excitation. Under the assumption of system linearity and the input being a stationary Gaussian process, the response as the stationary random process can be derived theoretically. That is, m-th spectral moment λ_m of the response $Q_k(t)$ (all subscripts k are omitted for simplification after this) is shown as follows:

$$\lambda_m=\sum_i \sum_j \Psi_i \Psi_j \rho_{m,ij}\sqrt{\lambda_{m,ii}\lambda_{m,jj}} \tag{2}$$

$$\rho_{m,ij}=\lambda_{m,ij}/\sqrt{\lambda_{m,ii}\lambda_{m,jj}} \tag{3}$$

in which $\lambda_{m,ij}$ is cross spectral moment. It is noted that the amount of $\rho_{m,ij}$ in Eq.3 decreases rapidly when there becomes some difference between ω_i and ω_j, saying that it is not the closely spaced eigenvalue problem. Especially, these characteristics are remarkable for the structure with small damping. This $\rho_{m,ij}$ can be calculated if the power spectral density of the input is given. Moreover, the analytical solution is obtained easily for the white noise input as a special case. Kiureghian has derived the approximated expression used easily at the design level by applying the first-order approximation technique to this analytical solution (1). The expression for m=0 is shown as follows:

$$\rho_{0,ij}=\frac{2\sqrt{h_i h_j}\left\{(\omega_i+\omega_j)^2(h_i+h_j)+(\omega_i^2-\omega_j^2)(h_i-h_j)\right\}}{4(\omega_i-\omega_j)^2+(\omega_i+\omega_j)^2(h_i+h_j)^2} \tag{4}$$

in which h_i and ω_i represent i-th modal damping and natural circular frequency, respectively. It is confirmed by Kiureghian (1) to be able to apply Eq.4 to a narrow-band input, though Eq.4 is the approximated expression to the white noise input.

Though the mean square values of the response are obtained from Eqs.2 and 4, some information on peak statistics of the response, in addition, is needed to proceed to the response spectrum analysis. Kiureghian has derived the peak factor P from the extremal distribution function proposed by Vanmarcke (2), and consequently, mean value $\overline{Q_T}$ concerning the maximum response Q_T during the time T (that is, expected value on the response spectrum) is obtained as follows:

$$\overline{Q_T}=\sqrt{\sum_i \sum_j (P^2/P_iP_j)\rho_{0,ij}\overline{Q_i}\,\overline{Q_j}} \tag{5}$$

By the way, (P^2/P_iP_j) in Eq.5 should obtain $\lambda_{m,ij}$(m=0,1,2) and therefore becomes complex considerably, then it is not convenient as the brief formulation used on the design side. Eventually, the following expression is approximately proposed by Kiureghian.

$$(P^2/P_iP_j)\approx 1.0 \tag{6}$$

This is the ordinary CQC method which is widespread now.

2.2 Problem of CQC Method

Eq.6, which is very important assumption in the ordinary CQC method, is quantitatively examined.

It is necessary to obtain the cross spectral moments as the first stage in order to estimate (P^2/P_iP_j) quantitatively. When the input has the following Kanai-Tajimi spectrum $G_F(\omega)$ (3), those spectral moments are obtained by Kiureghian (1).

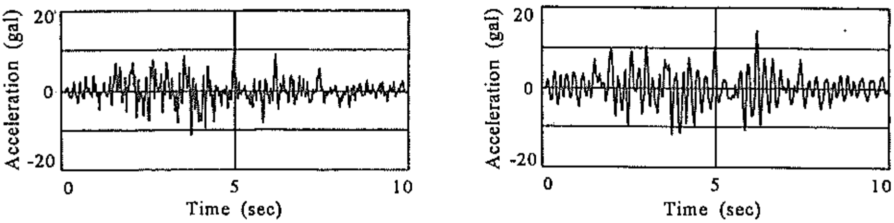
$$G_F(\omega) = G_0 \frac{\omega_g^{\ 4} + 4\zeta_g^{\ 2}\omega_g^{\ 2}\omega^2}{(\omega_g^{\ 2} - \omega^2)^2 + 4\zeta_g^{\ 2}\omega_g^{\ 2}\omega^2} \tag{7}$$

Then, by invoking Eq.7 and using semi-empirical formula to the peak factor given by Kiureghian (4), (P^2/P_iP_j) (this value is named by modified coefficient in this paper and expressed as R_{ij} after this) becomes the following:

$$(\frac{P^2}{P_iP_j}) = \frac{\left\{\sqrt{2l_n\nu_{et}T} + \frac{0.5772}{\sqrt{2l_n\nu_{et}T}}\right\}^2}{\left\{\sqrt{2l_n\nu_{ei}T} + \frac{0.5772}{\sqrt{2l_n\nu_{ei}T}}\right\}\left\{\sqrt{2l_n\nu_{ej}T} + \frac{0.5772}{\sqrt{2l_n\nu_{ej}T}}\right\}} \tag{8}$$

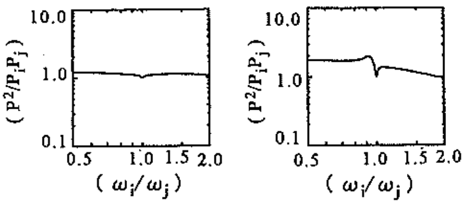
$$\nu_{eI} = (1.63\delta_I^{\ 0.45} - 0.38)\nu_I, \delta_I = \sqrt{1 - \frac{\lambda_{1,II}^2}{\lambda_{0,II}\lambda_{2,II}}}, \nu_I = \frac{1}{\pi}\sqrt{\frac{\lambda_{2,II}}{\lambda_{0,II}}} \ ; for\ I = t,i,j \tag{9}$$
$$\lambda_{m,tt} = \Psi_i^{\ 2}\lambda_{m,ii} + \Psi_j^{\ 2}\lambda_{m,jj} + \Psi_i\Psi_j(\lambda_{m,ij} + \lambda_{m,ji}) \quad ; \quad for\ \ m = 0,1,2$$

Modified coefficient R_{ij} is concretely estimated to the response due to wide or narrow band random process as seismic input by using Eq.8. A wide or narrow band input can be approximately achieved by adjusting the parameter ζ_g of Eq.7. One example is indicated in Fig.1 and the corresponding values of modified coefficient are shown in Fig.2. On the other hand, general shape of the $\rho_{0,ij}$ which always exists in the form of the product by modified coefficient is shown in Fig.3. It is understood from this figure that the influence of the correlation between modal responses is limited to the neighborhood of (ω_i/ω_j)=1.0 where the closely spaced degree between natural frequencies is very large. Then, the modified coefficient becomes important in the evaluation when (ω_i/ω_j) exists in the neighborhood of 1.0. Putting this respect on the mind and seeing Fig.2, it is understood that Eq.6 which Kiureghian has assumed from intuitive consideration is certainly correct to the wide-band random excitation, because of the modified coefficient being about one. The modified coefficient, however, can occasionally take the value of about two due to the narrow-band input. Thus, it turns out that the ordinary CQC method has the higher possibility to give the evaluation of non-safety side for the case where a structure has closely spaced natural frequencies and is excited by the narrow-band input like a secondary system.



(a) Wide Band Input (ζ_g=1.0) (b) Narrow Band Input (ζ_g=0.15)

Fig.I Input Waves for ζ_g=1.0 and 0.15



(a) Ratio to Wide Band Input (b) Ratio to Narrow Band Input

Fig.2 Ratios to Wide and Narrow Band Inputs

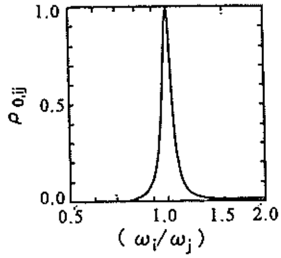


Fig.3 Typical Example of $\rho_{0,ij}$

From the above-mentioned results, it can be said that the response spectrum method with high accuracy can be realized instead of the ordinary CQC method, if Eq.5 is calculated by using the modified coefficient obtained from Eq.8. The calculation with Eq.8, however, is complex and lacks practical use as a seismic analysis tool at usual design level. Then, the practicable seismic analysis technique is proposed in the next chapter.

3. DEVELOPMENT OF PRACTICABLE RESPONSE SPECTRUM METHOD CONSIDERING MODIFIED COEFFICIENT

3.1 Setting of Modified Coefficient

Modified coefficient R_{ij} is composed of ω_g, ζ_g, ω_i, ω_j, h_i, h_j, T, β_i, β_j, ϕ_i, and ϕ_j as shown in Eq.8. Therefore, in order to evaluate the contribution of each parameter to R_{ij}, 70 cases for R_{ij} are obtained to the combination of various parameters. Based on such parametric studies, a simple method to presume the modified coefficient is newly derived as follows:

For wide-band input ;	R_{ij}=const.=1.0	(10)
For narrow-band input ;		
R_{ij}=1.0	for $0.98 \leqq (\omega_i/\omega_j) \leqq 1.02$	(11)
R_{ij}=2.0	for $(\omega_i/\omega_j) < 0.98$ or $1.02 < (\omega_i/\omega_j)$	(12)

In order to set the above expression, the following informations are considered.

(1) Disagreement of R_{ij} derived from Eqs.10 to 12 with one obtained by numerical

calculation increases in the region where the closely spaced degree between natural frequencies becomes small (that is, the region where the value of (ω_i/ω_j) exists apart from 1.0) . However, because $\rho_{0,ij}$ rapidly becomes small in this region, the disagreement of R_{ij} does not affect the accuracy for the estimation of response .

(2) Eq.10 is the expression to follow the region where the ordinary CQC method is effective.

(3) Some combination cases with parameters show that the estimation of R_{ij} with the wider range of (ω_i/ω_j) than the range defined on Eq.11 is good agreement with the numerical calculation. However, for more safety side evaluation to a lot of designs with various parameters, it is desirable to reduce the possibility of adopting Eq.11 and to enlarge the possibility that R_{ij}=2 is adopted oppositely. Thus, the range of (ω_i/ω_j) in Eq.11 becomes small. And, (1 ± 0.02) of the upper and lower limit values of (ω_i/ω_j) are set as the region having the average value of R_{ij} =1.2 .

When the modified coefficient is obtained by Eqs.10 to 12, it is necessary to give the judgment to the input, wide-band or narrow-band process. However, it is difficult in general and not realistic to judge the frequency characteristics of the input at design level. Then, the method by which the judgment can be easily given at the design stage is presented in next paragraph.

3.2 Judgment of Frequency Characteristics of Input

It is assumed that power spectral density (P.S.D.) of the input acceleration can be approximately expressed by Eq.7. In this case, it is most important problem how to presume ζ_g from the given response spectrum (or, floor response spectrum) because ζ_g becomes an index by which the input is distinguished between wide-band and narrow-band process.

The response spectrum has shape in Fig.4 in general. Also, S(T,h) is usually given as the maximum values of absolute response acceleration of single degree-of-freedom system. $S(T_o,h)$ is almost equal to the maximum value of the input acceleration. Because of the shape of response spectrum being affected by P.S.D. of the input acceleration, T_g in Fig.4 may be correlated with ω_g in Eq.7 by the following relationship:

$$(2\pi/T_g) \cong \omega_g \tag{13}$$

On the other hand, this response spectrum, sometimes at various designs, is not given as the shape of Fig.4, but might be arranged in the form of the response factor μ which is a ratio of the maximum value of the absolute response acceleration and one of the input acceleration. Therefore, response factor μ_g at T_g in Fig.4 is represented as follows:

$$\mu_g = \frac{S(T_g,h)}{\text{maximum value of input acceleration}} \cong \frac{S(T_g,h)}{S(T_0,h)} \tag{14}$$

It is also more possible to show μ as the following expression from the random vibration theory (5) :

$$\mu = \frac{\text{standard deviation of absolute response acceleration } (\sigma_a)}{\text{standard deviation of input acceleration } (\sigma_E)} \tag{15}$$

The response factor μ estimated at T_g is represented as $\mu_g=(\sigma_{ag}/\sigma_E)$, and also σ_{ag} is related to σ_{dg} (standard deviation of the relative response displacement) as follows:

$$\sigma_{ag} = \omega_g^{\ 2}\sigma_{dg} \tag{16}$$

Thus, the following expression is obtained from Eqs.15 and 16:

$$\mu_g = \omega_g^{\ 2}\sigma_{dg} / \sigma_E \tag{17}$$

where

$$\sigma_E^{\ 2} = \int_0^\infty G_F(\omega)d\omega = \frac{\pi G_0 \omega_g \sqrt{1+4\zeta_g^{\ 2}}}{4\zeta_g} \tag{18}$$

$$\sigma_{dg} = \int_0^\infty G_S(\omega)d\omega = \left(\frac{\pi G_0}{4h\omega_g^{\ 3}}\right)\frac{1+4\zeta_g(\zeta_g+h)}{4\zeta_g(\zeta_g+h)} \tag{19}$$

($G_S(\omega)$; P.S.D. of relative displacement of single degree-of-freedom system with natural period T_g)

Eventually, the following relationship is given from Eqs.17 to 19:

$$\mu_g^{\ 2} = \frac{1+4\zeta_g(\zeta_g+h)}{4h\sqrt{1+4\zeta_g^{\ 2}}(\zeta_g+h)} \tag{20}$$

Eq.20 is graphed as shown in Fig.5, and if this figure is used, ζ_g is easily obtained.

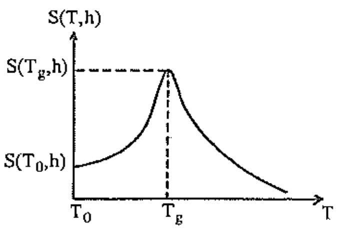


Fig.4 Typical Example of Response Spectrum

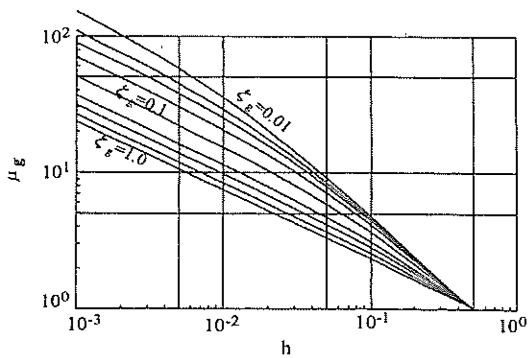


Fig.5 μ_g—h—ζ_g Curve

Next, importance is to set the value of ζ_g which becomes the boundary of wide-band and narrow-band. ζ_g=1.0 is set by the under mentioned reason in this research though Kiureghian proposes ζ_g=0.6 (1) .

(1) Setting of the boundary value of ζ_g is required only to obtain the modified coefficient, and it is not the main theme of this research to define or evaluate the boundary of wide-band and narrow-band processes, technologically. Adopting ζ_g with a big value as the boundary one means that many inputs are judged as the narrow-band process. As a result, the possibility that the modified coefficient of 2.0 is adopted becomes higher, and the

presumption at safety side is expected.

(2) Estimation with ξ_g=1.0 as the boundary value is good agreement with simulation results compared with ξ_g=0.6 .

(3) There are no theoretical grounds in ξ_g=0.6.

4. PROPOSAL OF NEW RESPONSE SPECTRUM METHOD (REVISED CQC METHOD)

The outline of the proposed response spectrum method has been already described in chapters 2 and 3. Because the proposed technique is an improvement of the ordinary CQC method, this new method is named RCQC (Revised Complete Quadratic Combination) method. Fig.6 shows the flow chart of seismic design by RCQC method.

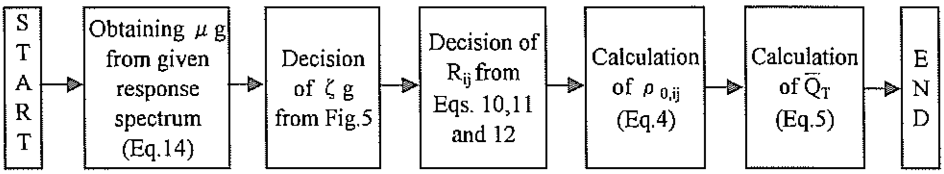


Fig.6 Flow Chart of RCQC Method

The characteristics of RCQC method are as follows.

(1) RCQC method corresponds to the ordinary SRSS and CQC methods without contradiction.

$R_{ij}=1$ → ordinary CQC method

$(R_{ij}\rho_{0,ij})=\delta^{ij}$ → ordinary SRSS method (δ^{ij} is Kronecker'delta)

(2) All areas of wide-band and narrow-band processes to the input are covered owing to the introduction of index ξ_g, and, in addition, RCQC method covers the closely spaced eigenvalue problem as well as the ordinary CQC method. Therefore, if a designer calculates according to Fig.6, the seismic design can be done considering neither the frequency characteristics of the input nor the correlation between modal responses by the closely spaced natural frequencies.

(3) In spite of the same calculation time by RCQC as one by ordinary SRSS or CQC, the accuracy of presumption with RCQC is better than one with SRSS or CQC.

5. VERIFICATION OF RCQC METHOD BY SIMULATION

Two degree-of-freedom system with 1% modal damping is adopted as a model for the verification, and two kinds of examinations are executed as follows.

(1) When the structural system is different, with the same input

The response maximum values to three kinds of structural systems due to a narrow-band random process are compared by using different methods such as SRSS, CQC, RCQC and numerical simulation. The result is shown in Table 1.

(2) When the input is different, with the same structural system

The response maximum values due to three kinds of random waves in Fig.7 are compared with the model which has the structural characteristics of case 1 in Table 1. The result is shown in Table 2.

As a result of the above-mentioned verification, RCQC method is confirmed as the useful response spectrum method.

Table 1 Comparison for Different Structures

Case	freq.(Hz) f_1, f_2	R_{12}	$\rho_{0,12}$	SRSS	CQC	RCQC	Simulation
1	4.43 4.57	2	0.286	714	827	927	1038
2	1.97 2.03	2	0.308	153	175	195	208
3	1.90 2.10	2	0.039	150	153	156	148

Table 2 Comparison for Different Waves

Case	input wave	R_{12}	$\rho_{0,12}$	SRSS	CQC	RCQC	Simulation
4	A	1	0.286	298	336	336	335
5	B	2	0.286	97	109	121	122
6	C	1	0.286	369	418	418	414

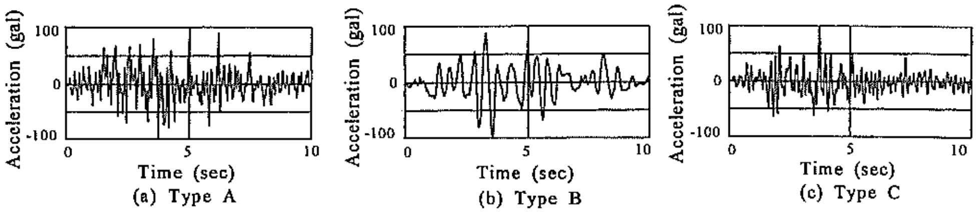


Fig.7 Input Waves for Comparison

6. SUMMARY

The revised CQC method (RCQC method) is developed in order to accurately analyze the secondary system which has the closely spaced natural frequencies. In addition, the validity of RCQC method is confirmed by comparing with numerical simulations.

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